Chapter

Introduction to Satellite Attitude Control

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Abstract

This chapter will introduce the space environment satellites must operate in, the motion they make in orbit, and their orientation while in orbit. The forces acting on the spacecraft will be considered, along with the implications of conservation of energy. The fundamentals of orbital mechanics will be presented, so common orbits can be visualized and discussed in terms of the six classical orbital elements. Perturbations impacting the orbit are covered for a better understanding of how orbits change over time. The inertial frame of reference will be defined and then transformed into body coordinates of the satellite using the direction cosine matrix and quaternions to describe the attitude of the spacecraft. A variety of modern attitude control techniques will be developed in the following chapters.

Keywords: satellite, space environment, gravitational force, conservation of momentum, orbital mechanics, classical orbital elements, orbital perturbations, frame of reference, LEO, MEO, GEO, HEO, direction cosine matrix, quaternion

1. Introduction

Controlling satellites begins with understanding the space environment they operate in and what forces are acting on them. Along with the solution to the two-body problem, the motion of satellites can be visualized quickly with a basic knowledge of the six classical orbital elements (COEs). Several common orbits are described in terms of their COEs. Lastly, the orientation of the satellite is described relative to an inertial frame of reference using the direction cosine matrix and quaternions.

2. Background

2.1 Environment

The space environment generally refers to the conditions existing above the earth's atmosphere. Since the atmosphere gradually dissipates as altitude increases, there is no fixed line of demarcation to define the edge of space. One convention defines the edge of the atmosphere where space begins at 100 km above the earth's surface, which will serve the purposes of this book. Once out of the atmosphere, the satellite will operate in a vacuum. There are several consequences to a vacuum environment, including outgassing, cold welding, and no heat transfer through convection. Without the atmosphere's protection, spacecraft are also susceptible micro-meteors. Similarly, the earth's magnetosphere protects the spacecraft from electromagnetic radiation and charged particles. There are a lot of advantages to staying in the atmosphere, but satellites need to leave the relative safety of the atmosphere to reach the altitudes their missions demand.

But of all the effects in the space environment, the greatest is arguably gravity.

For reasons we will touch on shortly, the force due to gravity causes the spacecraft to move through space in a very specific and predictable way. All objects have mass, the amount of "stuff" that gravitationally attracts other objects. And significantly for this book, how that mass is arranged impacts how an object resists changes in motion. Weight is the force of gravitational attraction of two objects, and we are most familiar with this force when we step on a scale. As seen in Eq. (1), weight is dependent on the distance between the two masses.

$$F_{gravity} = a_{gravity} m_{satellite} = G\frac{m_{earth}}{R^2} m_{satellite} = \mu_{earth}\frac{m_{sat}}{R^2} \tag{1}$$

Gravity is then seen as the interaction of two masses at a given distance from each other. The earth's mass can be treated as a constant point mass, and the satellite's mass when at the surface is 6378 km above the center of the earth. A quick calculation of this situation shows the acceleration due to gravity on the earth's surface is the familiar 9.8 m/s^2.

$$F_{gravity@surface} = 3.986x10^{14}\frac{m_{sat}}{(6378000)^2} = 9.8 m_{sat} \tag{2}$$

There are other forces acting on the spacecraft, and for more accurate results they need to be considered.

$$\Sigma F_{external} = F_{gravity} + F_{drag} + F_{thrust} + F_{3rd\ body} + F_{other} = m_{sat} a \tag{3}$$

Drag is the force the satellite feels as it passes through the atmosphere, similar to the force a hand feels when stuck out a window of a moving car. Even though the atmosphere dwindles off to nothing around 600 km above the earth's surface, its effects are still felt by satellites in low earth orbit (LEO). LEO is described in detail in Section 4.1. Thrust can be generated from rockets on the spacecraft for various purposes, but for now we will leave thrusters off. Other celestial bodies like the moon and Jupiter can also impart a gravitational force on the spacecraft. However, to understand the basic principles of satellite motion in space, the earth's force due to gravity is the most significant and sufficient. Adding a unit vector to Eq. (1) to show gravity pulling the satellite in the opposite direction, towards the center of the earth, yields:

$$\Sigma F_{external} = F_{gravity} = -\mu_{earth}\frac{m_{sat}}{R^2}\hat{R} = m_{sat}\vec{a} = m_{sat}\vec{\ddot{R}} \tag{4}$$

This results in the traditional two-body problem differential equation:

$$\vec{\ddot{R}} + \mu\frac{m_{sat}}{R^2}\hat{R} = 0 \tag{5}$$

With the solution

$$R = \frac{a(1-e^2)}{1+ecos\upsilon} \tag{6}$$

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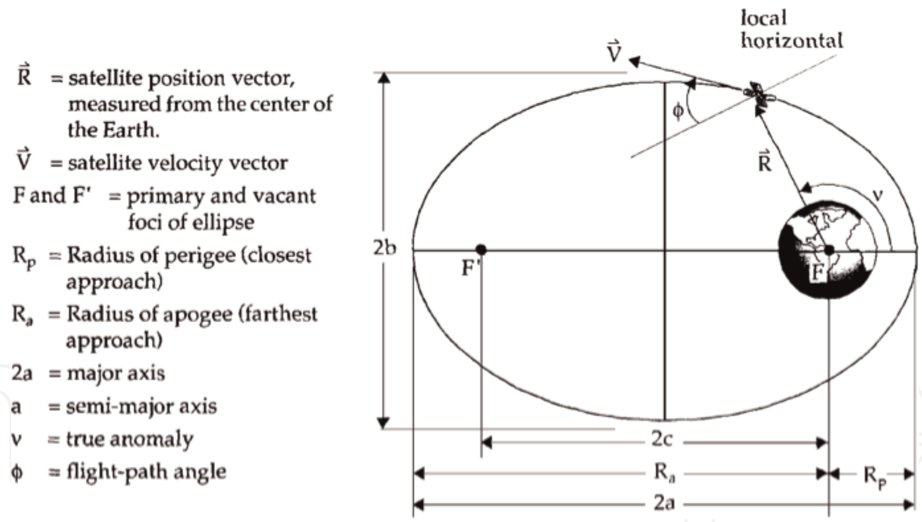

Figure 1.
Elliptical orbit [1].

where the variables are labeled in **Figure 1**. Please take time to familiarize yourself with the naming conventions used here. This solution gives the distance of the satellite from the center of the earth at all points in its orbit.

2.2 Conservation of energy

Intuitively, the more mass a satellite has, the more energy is required to achieve a particular orbit. When a satellite is launched, the chemical energy of the rockets is converted to kinetic energy to move the satellite. The higher the satellite goes, the more potential energy it has. At some point during launch, the rocket must turn over on its side to generate enough horizontal speed for the satellite to stay in orbit—otherwise it would just fall back down to the ground like a ball.

As a thought exercise, imagine a goalie throwing a soccer ball parallel to the ground. The harder the ball is thrown, the further it goes before it hits the ground. Keeping the initial trajectory parallel to the ground, throwing the ball with enough energy will cause the ball to go past the horizon of the earth. But gravity will still pull it back towards earth, or will it? With enough kinetic energy, the ball will move beyond the pull of the earth, beyond even the pull of the sun. The speed required to do so is called the escape velocity. Since the satellite is intended to orbit the earth, the energy imparted onto the satellite needs to be limited to what the earth's gravity can keep in orbit.

Through the law of conservation of energy, we can calculate how much chemical energy is required to put the satellite into the desired orbit. As you have undoubtedly seen, satellites require very, very large rockets to achieve orbit around the earth.

3. Classical orbital elements

To better understand the motion of the satellite in space, this section will elaborate on the location of the satellite in its orbital path. There are six orbital elements, collectively referred to as the classical orbital elements (COEs).

3.1 Two-dimensional elements

3.1.1 Orbit shape

From Kepler's first law, the shape of the orbit, or its path, is one of the four conic sections: circle, ellipse, parabola, or hyperbola [2]. This shape is defined by the orbit's eccentricity, e, which is the ratio of the difference and sum of the perigee and apogee distances.

$$e = \frac{R_{apogee} - R_{perigee}}{R_{apogee} + R_{perigee}} \tag{7}$$

For satellites orbiting the earth, only the circular and elliptical orbits are of interest. From the above formula, the circle is seen as a special case of the ellipse where the distances to perigee and apogee are equal. Keep in mind the radius at perigee must be greater than 6378 km (earth's radius) + 160 km (altitude above significant atmospheric effects). Since the satellite stays in elliptical orbit as long as $e < 1$, apogee could be much, much greater than the radius at perigee. However, at some point, gravitational forces from the sun and moon will come into play and invalidate the radius solution from the two-body problem presented earlier. An eccentricity of 0.7 is considered highly elliptical, and beyond that is seldom used.

3.1.2 Orbit size

The size of the orbit represents how much energy is in the orbit, and is calculated as the semi-major axis of the orbit. The larger the semi-major axis, the more energy is present. In fact, the specific mechanical energy of the orbit,, is defined as

$$\varepsilon = -G\frac{m_{earth}}{2a} = -\frac{\mu_{earth}}{2a} \tag{8}$$

Through calculations not shown here, the velocity in the satellite's direction of motion can be computed using:

$$V = \sqrt{2\left(\frac{\mu}{R} + \varepsilon\right)} \tag{9}$$

where R is the distance to the center of the earth. To give a sense of the speeds involved, a satellite in LEO will be traveling about 7.5 km/s, or 4000 mph. It is obvious from this equation that a circular orbit has a constant velocity. Less obvious, but naturally following, is the velocity of the satellite in an elliptical orbit is always changing. As the spacecraft passes perigee at maximum speed, it is slowing down as its kinetic energy is transferred into potential energy. When the satellite reaches apogee, it has reached its maximum potential energy. From there it gains speed as it falls towards perigee again, in a never-ending transfer of energy.

From Kepler's third law, the amount of time required for the satellite to make one orbit is known as the period of the orbit and is also defined by the size of the orbit [2].

$$P = 2\pi\sqrt{\frac{a^3}{\mu}} \tag{10}$$

3.1.3 True anomaly

The eccentricity and semi-major axis have defined the shape and size of the orbit, but where is the satellite along that path in space? The satellite's position on the path is measured counter-clockwise from perigee and is called the true anomaly, ν.

Much can be known about a satellite's path through space knowing size, shape, and true anomaly. To get the complete picture, we need to discuss the remaining three classical orbital elements. But before we do so, we must make the jump from the two-dimensional discussion so far into three-dimensional space. The orbital path described in this section can be rotated in three-dimensional space, and those rotations make up the remaining three classical orbital elements.

3.2 Frame of reference

To describe a location in space, we define a frame of reference that is nonrotating relative to the stars. The geocentric-equatorial coordinate system, with the origin at the earth's center and orthogonal vectors **I**, **J**, and **K** is one such example (**Figure 2**).

The fundamental plane is the **I**, **J** plane intersecting the equator, and **K** points to the north pole. To orient the fundamental plane, the principal direction, **I,** is defined to point towards the sun at vernal equinox, when the earth passes above the celestial equator of the sun on the first day of spring (**Figure 3**).

We can then describe the location of a satellite using a position vector, **R**, and a velocity vector, **V**, in the geocentric-equatorial coordinate system. Of note, each vector has three components, so together there are six components—the same number as the number of classical orbital elements. With six numbers, the satellite's path can be uniquely determined.

Since it is difficult to visualize the motion of a spacecraft in orbit using **R** and **V** vectors ([4234,2342,3] km and [7.5, 1, 1] km/s anyone?), another frame of reference

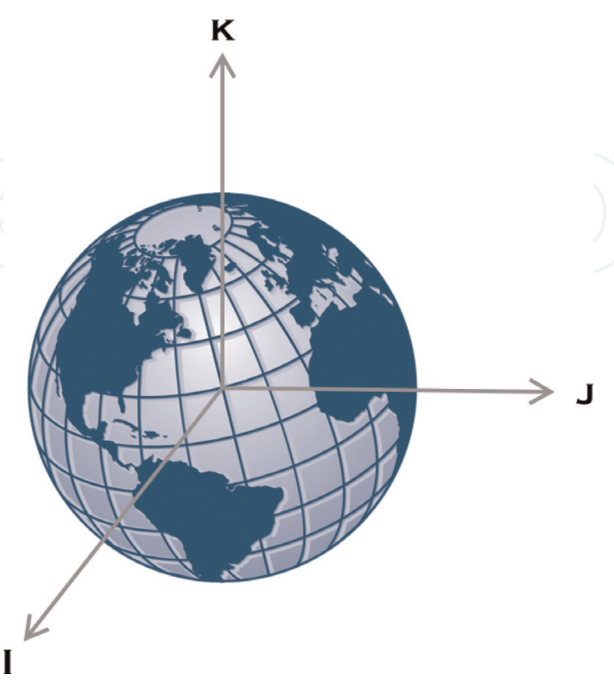

Figure 2.
Geocentric-equatorial coordinate system [3].

is used to describe locations of spacecraft in orbit around the earth. The Perifocal coordinate system uses the orthogonal unit vectors **P**, **Q**, and **W** to describe position. The satellite's orbital plane discussed in the previous section makes up the fundamental plane, with **P** pointing to periapsis and **Q** rotated 90° in the direction of satellite motion. **W** is then perpendicular to the orbital plane.

3.3 Three-dimensional elements

3.3.1 Inclination

Starting with an equatorial orbit, the orbital plane can be tilted up. The angle it is tilted up from the equator is referred to as the inclination angle, i. Since the center of the earth (the source of gravitational pull) must always be in the orbital plane, the point in the orbit where the satellite passes the equator on its way up is referred to as the ascending node, and the point where the satellite passes the equator on the way down is unsurprisingly referred to as the descending node. Drawing a line through these two points on the equator is what defines the line of nodes (**Figure 4**).

Inclining the orbital plane can be visualized as pivoting the plane about the line of nodes. Tilting the orbit 90° creates a polar orbit. A prograde orbit has an inclination between 0 and 90°. A retrograde orbit has an inclination between 90 and 180°. Tilting the orbital plane an extra 180° ($180 < i < 360$) results in the same plane in three-dimensional space.

3.3.2 Right ascension of the ascending node

The inclined orbit can be swiveled about the north pole by rotating the line of nodes counter-clockwise away from the direction of vernal equinox. The rotation of

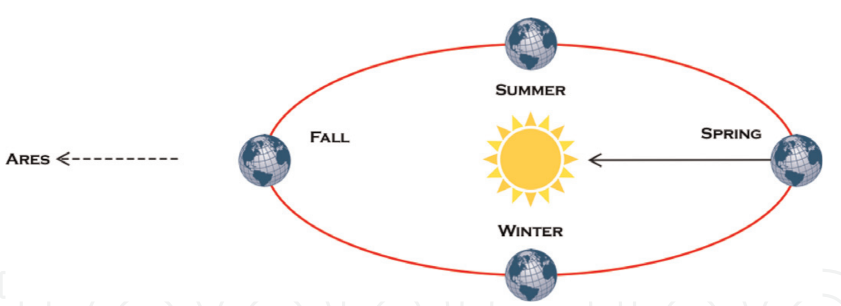

Figure 3.
Vernal equinox.

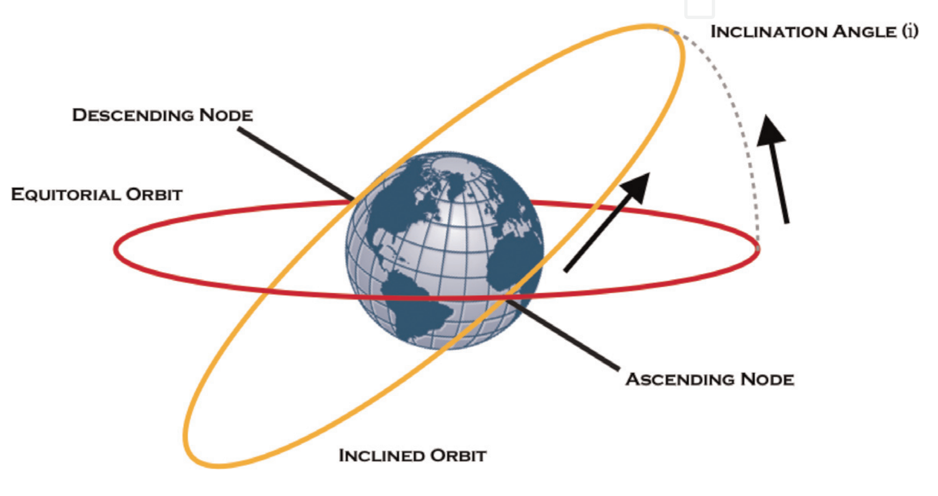

Figure 4.
Inclined orbit.

the right ascension of the ascending node (RAAN), Ω, can be any number between 0 and 360°. In the special case of an equatorial orbit, there is no ascending or descending node, therefore there is no line of nodes and omega is not defined (**Figure 5**).

3.3.3 Argument of perigee

The final rotation is not a rotation of the orbital plane, but the orientation of the orbit within the orbital plane. Rotating the orbit 90° counter-clockwise inside the orbital plane would put perigee at the ascending node. Rotating the orbit 270° would put perigee at the descending node. Perigee can be rotated 360°, so $0 < \omega < 360$. For the special case of a circular orbit, any rotation would result in the same orbit, so perigee is defined to be in the vernal equinox direction (**Figure 6**).

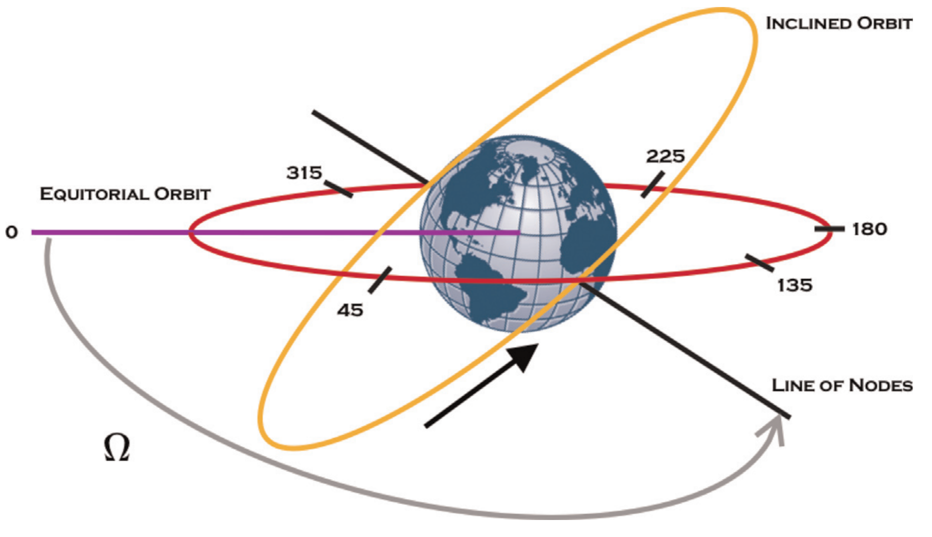

Figure 5.
Swiveled orbit.

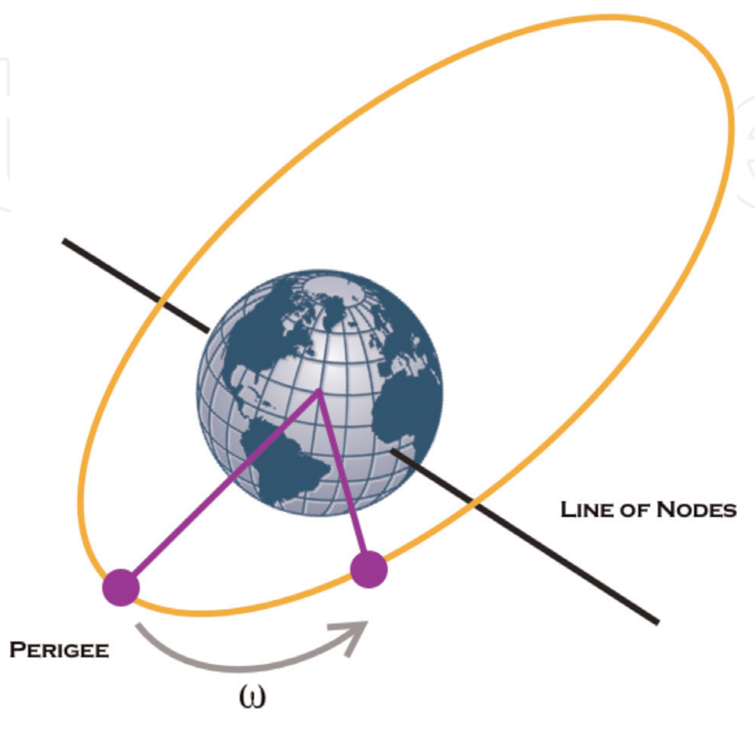

Figure 6.
Oriented orbit.

In summary, the six classical orbital elements are:

a, semi-major axis
e, eccentricity
i, inclination
Ω, right ascension of the ascending node (RAAN)
ω, argument of perigee
ν, true anomaly

The spacecraft's location in orbit can be visualized easily with these six values. Recall the **R** and **V** vectors also make up six components (each are three dimensional vectors), but do not paint the same intuitive mental picture as using the COEs.

3.4 Orbital perturbations

There are several disturbances that act on the satellite and can cause the orbital plane to shift over time. Most are related to factors we excluded to simply the math to a two-body problem.

For example, atmospheric drag is a force that takes energy out of the orbit with a perigee less than 700 km. Each pass at perigee happens with less speed, which reduces apogee. The effect is to circularize the orbit, and then spiral into the earth. The lower the perigee, the greater the effects of atmospheric drag.

Another example is the oblate earth. The earth is not a perfect sphere, which means its mass is not evenly distributed radially in all directions. Instead the mass of the earth is "squashed" like a pumpkin and produces what is referred to as the J2 effect. Since the mass of the earth is constant and gravity is conservative, the size and shape of the orbit is not impacted. Due to the symmetry above and below the equator, the inclination also does not change. However, the pull on the satellite from this "bulge" of the earth's mass will cause the line of nodes to rotate. Also due to symmetry, a polar orbit will not experience a J2 effect. Of note, the J2 can be used to create an orbital plane that always points to the sun. Satellites in LEO with i = 98°will rotate at the same 1°/day as the sun, creating a sun synchronous orbit.

Other disturbances include longitudinal drift, perigee rotation, 3rd body effects (gravitational pull from the sun, moon, jupiter, etc.), and solar radiation pressure. Of course, impacts with micro-meteoroids or other space debris can also impact the satellite's motion.

While many of the perturbations discussed in this section can be modeled with modern software, the simplified two-body problem is generally sufficient for a basic understanding of the satellite's location in space.

4. Useful orbits

Each of the orbits described below have been used successfully for space missions. They each have their own unique advantages and disadvantages, so selecting one for a mission hinges on which advantages are critical and which disadvantages can be lived with.

4.1 Low earth orbit (LEO)

LEOs can be circular or elliptical orbits with an altitude between 160 and 6000 km. Their relative closeness to the surface of the earth make them useful for several reasons. Since LEOs have the smallest size, they require the least amount of energy (smaller rocket) to get the satellite into orbit. Additionally, sensors collect more energy (such as light for photographs) the closer they are to the emitting source. Energy dissipates as the square of the distance, and these R2 losses only get worse the higher the satellite goes.

However, there are compromises to staying in LEO. As mentioned earlier, the atmosphere will impart drag on the satellites below about 700 km, robbing the orbit of energy and reducing the semi-major axis. This is a vicious circle, pun intended, and ultimately results in the satellite burning up during the transfer of kinetic energy into thermal energy.

Being that close to the earth's surface also means the satellite's sensors cannot "see" as much as if it were higher. The area on the earth that has line of sight to the spacecraft is called the footprint, and is larger the higher the satellite is. Consider what can be seen looking down from a tower compared to a helicopter, compared to an airplane. Staying too low comes with a limited view. Using multiple satellites in slightly offset orbits at the same altitude can overcome this limitation, but at significant expense.

4.2 Medium earth orbit (MEO)

At MEO, with an altitude of around 20,000 km, the orbits tend to be circular, though they do not need to be. This altitude equates to a period of 12 hours, which makes revisit times of the satellite consistent and predictable over a given area of the earth. Sending satellites higher also gives them a larger footprint across the surface of the earth, which means fewer satellites are necessary to cover the entirety of the earth's surface. Instead of 60+ satellites at LEO, 24 satellites can provide continuous coverage, as is done with GPS satellites.

Being above the atmosphere does have its drawbacks, as charged particles collected in regions called the Van Allen belts can adversely impact the electronics onboard the satellite. And even at this high altitude, the satellite is still not high enough to see the entire disk of the earth.

4.3 Geosynchronous earth orbit (GEO)

As the name implies, satellites at GEO complete one orbit in the same amount of time it takes the earth to make one rotation on its axis. Eq. (10) can be used to determine the semi-major axis to be 35,780 km. Like LEOs and MEOs, GEOs can also be circular or elliptical with any inclination. The high altitude means one satellite can see the entire disk of the earth, so only four satellites are required to provide continuous coverage over the entire surface of the earth.

The primary disadvantage at GEO are the R^2 losses of signals as they cross the vast distance to the surface. Of course, getting to GEO is no mean feat, and very large rockets are required to carry the energy to get there.

Geostationary orbits are special case of GEO, and deserve special mention here. They are defined to have a semi-major axis of 35,780 km, eccentricity of 0, an inclination near 0. These circular orbits allow the satellite to always be over the same point of the equator. This in turn means the ground station in communication with the satellite is always in view, which is especially useful for TV and radio satellites.

4.4 Highly elliptical orbit

The eccentricity of an elliptical orbit can be anywhere from 0 to less than 1. As the name suggests, the eccentricity of HEOs is far from 0. In fact, an eccentricity around 0.7 is common. One consequence is a very low perigee, which provides all the benefits found at LEO. And the orbit has a very high apogee, giving all the benefits of GEO. While it appears to be the best of both worlds, it also shares the disadvantages of each.

A special HEO is the Molniya orbit. This orbit has an inclination of 63 or 117° to keep the argument of perigee constant. It also has a 500 km perigee and a 40,000 km apogee to produce a semi-major axis corresponding to a 12-hour period.

5. Attitude

With the spacecraft's location described, the next step in establishing its pose is to define the orientation of the spacecraft. In later chapters we will be able to discuss equipment used for changing attitude and the control algorithms used to drive the satellite to the desired attitude.

5.1 Orientation

The spacecraft has its own three-dimensional orientation, known as body coordinates. The three orthogonal, right-handed unit vectors, **u**, **v**, and **w** are typically selected in some meaningful way, such as along the edges of the satellite body. This body coordinate system then needs to be compared to some inertial reference system so the changes can be measured.

5.1.1 Direction cosine matrix

Comparing the body coordinate system to the geocentric-equatorial coordinate system shows that each axis of one system can be represented as a vector sum of the three components of the other system.

In other words, **u**, **v**, and **w** (x, y, and z in **Figure** 7) each project some amount onto the **I** unit vector (e_1 of **Figure** 7) of the geocentric-equatorial coordinate system. The same is true for the **J** and **K** unit vectors. In matrix form,

$$A = \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & \mathrm{w}_2 & w_3 \end{bmatrix} \tag{11}$$

Thus, the direction cosine matrix specifies the orientation of the spacecraft relative to the inertial reference frame, and can be used to map a vector in one coordinate system to another. For example, to transform a vector in geocentric-equatorial coordinates to a vector in the body coordinate system through matrix multiplication:

$$\boldsymbol{a_b} = A\boldsymbol{a_{ge}} = \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{bmatrix} \begin{bmatrix} a_I \\ a_J \\ a_K \end{bmatrix} = \begin{bmatrix} a_u \\ a_v \\ a_w \end{bmatrix} \tag{12}$$

This can be seen as a single a rotation about some axis, which is called the Eigen axis. The eigenvector, **e**, is the unit vector in the direction the rotation is about. However, it is difficult to visualize this singular axis, so instead three rotations are made about the three principal orthogonal axes by an, Φ, to arrive at the same result as rotation about the single eigenvector.

$$\mathbf{a}_b = A_2 A_1 A_3 \mathbf{a}_{\mathbf{ge}} = \begin{bmatrix} a_u \\ a_v \\ a_w \end{bmatrix} \tag{13}$$

where the direction cosine matrix, A, in Eq. (11) is the product of:

$$\mathrm{A}_3 = \begin{bmatrix} \cos\Phi & \sin\Phi & 0 \\ -\sin\Phi & \cos\Phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

$$\mathrm{A}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Phi & \sin\Phi \\ 0 & -\sin\Phi & \cos\Phi \end{bmatrix} \tag{15}$$

$$\mathrm{A}_2 = \begin{bmatrix} \cos\Phi & 0 & -\sin\Phi \\ 0 & 1 & 0 \\ \sin\Phi & 0 & \cos\Phi \end{bmatrix} \tag{16}$$

Recall that matrix multiplication is not commutative, so the order of rotations definitely matters. The order is right to left, so Eq. (12) is a 3-1-2 rotation. Because transformations of an orthogonal matrix preserve the length of vectors, the transformation performed by the direction cosine matrix is seen to be a rotation of the original vector.

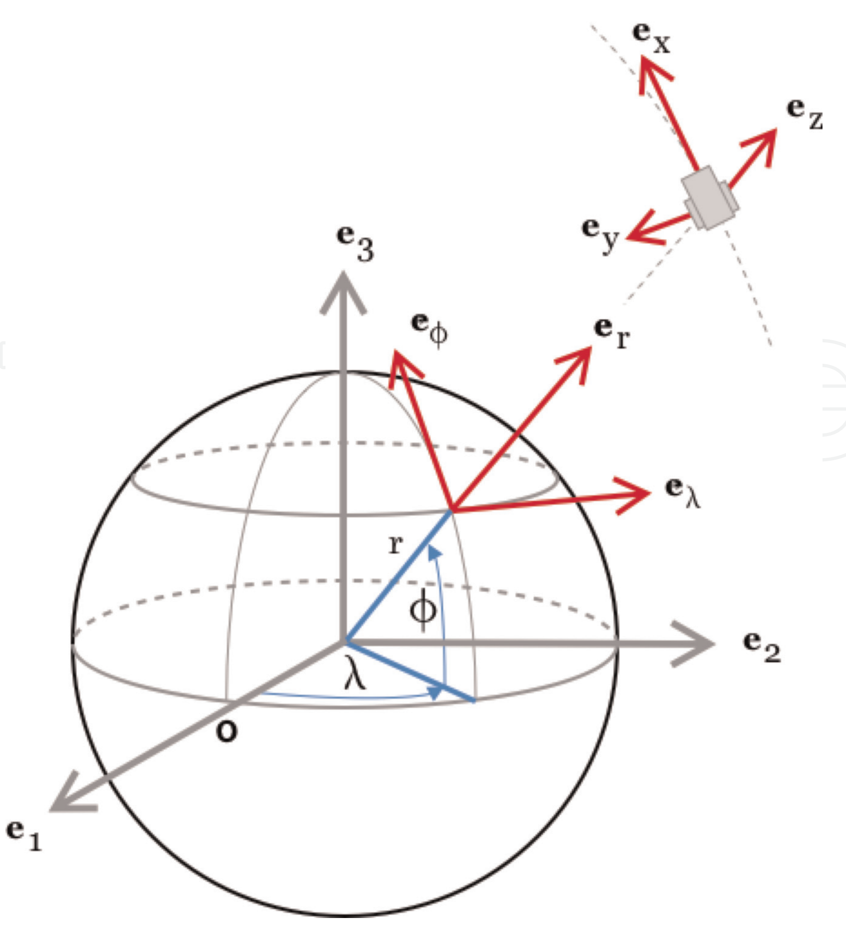

Figure 7.
Different coordinate systems. [4].

5.1.2 Quaternions

Parameterizing the direction cosine matrix with quaternions will help in future calculations, so it is presented here. Using the previously described terms for Eigen axis and angle of rotation, **e** and Φ, quaternions are defined as:

$$\boldsymbol{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} e_1 \sin\dfrac{\Phi}{2} \\ e_2 \sin\dfrac{\Phi}{2} \\ e_3 \sin\dfrac{\Phi}{2} \\ \cos\dfrac{\Phi}{2} \end{bmatrix} \tag{17}$$

with the constraint equation,

$$q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1 \tag{18}$$

In terms of quaternions, the direction cosine matrix, A, can be written as:

$$A = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix} \tag{19}$$

5.1.3 Euler angles

As seen in Eq. (18), three independent parameters are needed to describe the spacecraft's orientation. Another way to represent the spacecraft with only three parameters is through the use of Euler Angles. Instead of a single angle of rotation about the eigen axis, three rotations are made about the principle axes. The rotations can be made in any order and with any axis (i.e., A_{313} or A_{312}), but of course resulting in a different direction cosine matrix.

As with aircraft, these rotations are often referred to in terms pitch, roll, and yaw. By way of an example, let us consider a yaw, roll, pitch sequence. The first rotation around a principle axis is through an angle ϕ, then a second rotation around another principle axis by angle θ, a final rotation around the last principle axis by angle ψ. One example of this would be to rotate about the **K** axis, then the **I** axis and then the **J** axis. This is referred to as a 3-1-2 sequence and the direction cosine matrix can be written as:

$$\mathrm{A}_{312}(\phi, \theta, \psi) = \begin{bmatrix} \cos\psi\ \cos\phi - \sin\theta\ \sin\psi\ \sin\phi & \cos\psi\ \sin\phi + \sin\theta\ \sin\psi\ \cos\phi & -\cos\theta\ \sin\psi \\ -\cos\theta\ \sin\phi & \cos\theta\ \cos\phi & \sin\theta \\ \sin\psi\ \cos\phi + \sin\theta\ \cos\psi\ \sin\phi & \sin\psi\ \sin\phi - \sin\theta\ \cos\psi\ \cos\phi & \cos\theta\ \cos\psi \end{bmatrix} \tag{20}$$

Given the direction cosine matrix, the rotation angles for the 3-1-2 sequence can be calculated to be:

$$\begin{bmatrix} \theta \\ \phi \\ \psi \end{bmatrix} = \begin{bmatrix} arcsinA_{23} \\ -arctan\left(\frac{A_{21}}{A_{22}}\right) \\ -arctan\left(\frac{A_{13}}{A_{33}}\right) \end{bmatrix} \tag{21}$$

5.1.4 Small angle approximation

Sometimes the spacecraft will only rotate by a small amount. This is especially true as a move transient settles into the steady state attitude. Recall that for a small angle, θ,

$$cos\theta \cong 1 \tag{22}$$

$$sin\theta \cong \theta \tag{23}$$

$$sin\theta sin\theta = 0 \tag{24}$$

Using the small angle approximation reduces Eq. (20) to:

$$A_{312}(\phi, \theta, \psi) = \begin{bmatrix} 1 & \phi & -\psi \\ -\phi & 1 & \theta \\ \psi & -\theta & 1 \end{bmatrix} \tag{25}$$

For small angles, quaternions can be expressed in terms of the Euler angles by:

$$\boldsymbol{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \cong \begin{bmatrix} \frac{1}{2}\theta \\ \frac{1}{2}\psi \\ \frac{1}{2}\phi \\ 1 \end{bmatrix} \tag{26}$$

As with all algebraic calculations, care must be taken not to divide by zero. This is especially true when using trigonometric functions, but is also a concern when a quaternion is close to zero. Numerical techniques for handling these situations are beyond the scope of this introduction, but must be considered nonetheless.

5.2 Determination equipment

A variety of sensor are available to provide the information needed to determine the spacecraft's attitude. The constant position of the stars makes them especially useful for navigation. Star sensors take images of the star fields above them and compare the images to those in a catalog.

The magnetic field around the earth is not as constant, but has been modeled well enough to provide useful information for attitude determination. Magnetometers measure the strength of the electric field in a given axis. Using three orthogonal magnetometers can provide a good attitude estimate from the models.

The sun and moon locations at a given point in time are a known quantity and so sun sensors and moon sensors that detect them can be part of the solution. Similarly, horizon detectors can find the edge of the earth and use that information for reference.

Gyroscopes measure the rate of change and are especially useful in tracking changes to the satellite's attitude.

There are many methods to calculate the attitude of a spacecraft, including geometric, algebraic, covariance and q-method. However, the method used depends greatly on the spacecraft and its mission, so a discussion of determination methods is not presented here.

6. Control

Now that the attitude of the spacecraft can be defined and measured, the motion of the spacecraft can be controlled using control laws using any number of active and passive methods to achieve a set of desired input angles.

$$\begin{bmatrix} \theta_d \\ \phi_d \\ \psi_d \end{bmatrix} \tag{27}$$

We begin with a review of the kinematics involved [5].

6.1 Kinematics

The satellite is in a continuous state of free fall, and forces acting on the satellite will cause it to rotate about its center of mass. The resistance to that rotation is described by the moments of inertia and is represented by the inertia matrix

$$\mathrm{J} = \begin{bmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{zx} & J_{zy} & J_{zz} \end{bmatrix} \tag{28}$$

The torque of the rotational motion in the inertial frame is calculated by

$$\mathrm{T} = \mathrm{J}\dot{\omega} \tag{29}$$

and from [5] the desired torques on the satellite in body coordinates becomes

$$\mathrm{T}_d = \mathrm{J}\dot{\omega}_d + \omega_d \times \mathrm{J}\omega_d \tag{30}$$

The required torques can calculated using the sinusoidal trajectory

$$\theta = \frac{1}{2}\left(\mathrm{A} + \mathrm{A}\sin\left(\omega_f \mathrm{t} + \varphi\right)\right) \tag{31}$$

The angular velocity of the body, ω_B, calculated from the torque

$$\mathrm{T} = \dot{\mathrm{H}}_i = \mathrm{J}\dot{\omega}_i + \omega_i \times \mathrm{J}\omega_i \tag{32}$$

The Euler angles are then found by integrating $\dot{\omega}_i$ to get

$$\omega^{NB} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{33}$$

The satellite's pose in body coordinates relative to the orbital frame can then be found by taking the difference of the body and orbital frames relative to the inertial frame.

$$\omega^{OB} = \omega^{NB} - \omega^{NO} \tag{34}$$

6.2 Control

The motion of the satellite described by the kinetics of Section 6.1 must be controlled. There are two types of control systems, open loop and closed loop [6]. The open loop system has inputs that do not rely on the output. Examples would include systems where inputs are based on a clock, where the same input is put into the system at a given time. A closed loop system, on the other hand, closes the loop by sending the output information through a feedback mechanism to modify the input.

For satellite control, we are primarily concerned with comparing our observed state with some desired state using a closed loop control law. If the system is controllable, an unconstrained control vector, i.e., torque input, can take the system from an initial state to any other state in a finite interval of time [6].

One way to produce the required torque input is through the use of control moment gyros (CMGs). CMGs are momentum exchange devices, essentially rotating spinning discs which impart a rotation about the axis to which they are pointed. The larger the disc and the faster the spinning, the more torque is produced. A CMG inherently has a singular direction where no torque can be created [7]. To maintain controllability using critical components, a skewed pyramid orientation with advanced technique aim to avoid the impact of these singularities [8].

7. Conclusion

This chapter introduced the space environment and provided an overview of the forces acting on a satellite in orbit. The two-body problem was presented to show how the motion of satellites can be calculated and visualized in terms of the six COEs. The advantages and disadvantages of the most common orbits were discussed. Then the satellite's frame of reference was transformed to an inertial frame of reference with the direction cosine matrix or quaternions. Finally, basic spacecraft kinematics and dynamics were introduced along with the concept of controllability.

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