RW Control Devices

- **First let us develop a feedback control law for a spacecraft with N reaction wheels with general orientation.**

  EOM:

  G_s = [g1 g2 . . .gN]

  $$[I_{RW}]\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega} \times [I_{RW}]\boldsymbol{\omega} - \boldsymbol{\omega} \times [G_s]\boldsymbol{h_s} - [G_s]\boldsymbol{u_s} + \boldsymbol{L}$$

  With

  I_RW fixed as seen by the body

  $$\boldsymbol{h_s} = \begin{pmatrix} \vdots \\ J_{s_i}(\omega_{s_i} + \Omega_i \\ \vdots \end{pmatrix}$$

  h: angular momentum of RW

  J: inertial of RW

  Inertial Matrix:

  $$[I_{RW}] = [I_s] + \sum_{i=1}^{N} (J_{t_i}\hat{\boldsymbol{g}}_{t_i}\hat{\boldsymbol{g}}_{t_i}^T + J_{g_i}\hat{\boldsymbol{g}}_{g_i}\hat{\boldsymbol{g}}_{g_i}^T)$$

  The RW motor control torque vector is:

  $$\boldsymbol{u_s} = \begin{pmatrix} \vdots \\ J_{s_i}(\dot{\Omega} + \hat{\boldsymbol{g}}_{s_i}^T\dot{\boldsymbol{\omega}} \\ \vdots \end{pmatrix}$$

- **Spacecraft Tracking Errors:**

  $\boldsymbol{\sigma}$: MRP vector of body frame relative to reference frame

  $\delta\boldsymbol{\omega} = \boldsymbol{\omega} - \boldsymbol{\omega_r}$: Body angular velocity tracking error vector

- *Lyapunov Function:*

  $$V(\boldsymbol{\sigma}, \delta\boldsymbol{\omega}) = \frac{1}{2}\delta\boldsymbol{\omega}^{\boldsymbol{T}}[I_{RW}]\delta\boldsymbol{\omega} + 2K ln(1 + \boldsymbol{\sigma}^{\boldsymbol{T}}\boldsymbol{\sigma})$$

  Components taken in the B frame

- *Let's set the Lyapunov Rate to:*

$$\dot{V} = -\delta\boldsymbol{\omega}[P]\delta\boldsymbol{\omega} \leq 0$$

$$[I_{RW}]\frac{{}^{B}d}{dt}(\delta\boldsymbol{\omega}) + K\boldsymbol{\sigma} + [P]\delta\boldsymbol{\omega} = 0$$

Close-loop dynamics

$$[G_s]\boldsymbol{u_s} = \underbrace{\boxed{K\boldsymbol{\sigma} + [P]\delta\boldsymbol{\omega} - [\tilde{\boldsymbol{\omega}}]([I_{RW}]\boldsymbol{\omega} + [G_s]\boldsymbol{h_s}) - [I_{RW}](\dot{\boldsymbol{\omega}}_{\boldsymbol{r}} - \boldsymbol{\omega} \times \boldsymbol{\omega_r}) + \boldsymbol{L}}}_{\boldsymbol{L_r}}$$

- *Control condition:*

$$[G_s]\boldsymbol{u_s} = \boldsymbol{L_r}$$

Case 1: 3 RWs aligned with principal axes of spacecraft.

$$\boldsymbol{u_s} = \boldsymbol{L_r}$$

Case 2: *N* RWs aligned generally.

$$\boldsymbol{u_s} = [G_s]^T([G_s][G_s]^T)^{-1}\boldsymbol{L_r}$$

Minimum-norm inverse

Energy rate:

$$\dot{T} = \boldsymbol{\omega}^{\boldsymbol{T}}\boldsymbol{L} + \sum_{i=1}^{N} \Omega_i u_{s_i}$$

Work/energy principle