MOOC Spacecraft Dynamics Capstone Project

Attitude Dynamics and Control of a Nano-Satellite Orbiting Mars

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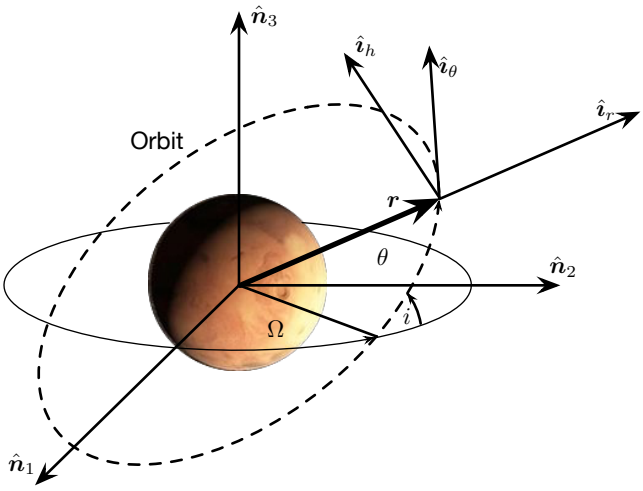

Figure 1: Illustration of the Inertial frame $\mathcal{N} : \{\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}_3\}$ and Hill frame $\mathcal{H} = \{\hat{\boldsymbol{\imath}}_r, \hat{\boldsymbol{\imath}}_\theta, \hat{\boldsymbol{\imath}}_h\}$ base vectors

1 Project Overview

This project considers a small satellite orbiting Mars at a low altitude gathering science data. However, this small satellite needs to transfer this data to a larger mother satellite at a higher altitude. Further, to keep the batteries from draining all the way, periodically the satellite must point its solar panels at the sun to recharge. Thus, three mission goals must be considered by the satellite: 1) point sensor platform straight down at Mars, 2) point communication platform at the mother satellite, and 3) point the solar arrays at the sun. In all scenarios the small spacecraft and mother craft are on simple circular orbits whose motion is completely known.

The high-level goal of this project is to design a thruster-based attitude control to achieve these attitude control scenarios. In order to satisfy the mission requirements, the spacecraft's body-frame $\mathcal{B}$ must be driven towards various reference body-frames $\mathcal{R}$ that corresponds to the desired attitude. The reference attitude is computed from the knowledge of the spacecraft position and velocity about Mars, as well as the knowledge of the mother satellite motion for the communication scenario and the knowledge of the sun heading for the power generation scenario. Once this reference is derived, you will implement the torque control law $\boldsymbol{u}$ that drives the current attitude MRP $\boldsymbol{\sigma}_{B/N}$ and the angular velocity $\boldsymbol{\omega}_{B/N}$ towards their reference values ($\boldsymbol{\sigma}_{R/N}$, $\boldsymbol{\omega}_{R/N}$). The scope of this project encompasses reference frame generation, attitude characterization and feedback control. By the end of this project, you will have gained valuable practical experience in the aforementioned areas thanks to analytic derivations and software implementation of the different project milestones.

2 Mission Description

A nano-satellite in on a circular low Mars orbit (LMO) mars to observe the non-sunlit Mars surface. A second satellite, the mother spacecraft, is on a circular geosynchronous mars orbit (GMO). The nano-satellite is to either point a sensor at the surface when in science mode, point the solar panels in the sun direction when in power mode, and point the communication dish at the mothercraft when in communication mode.

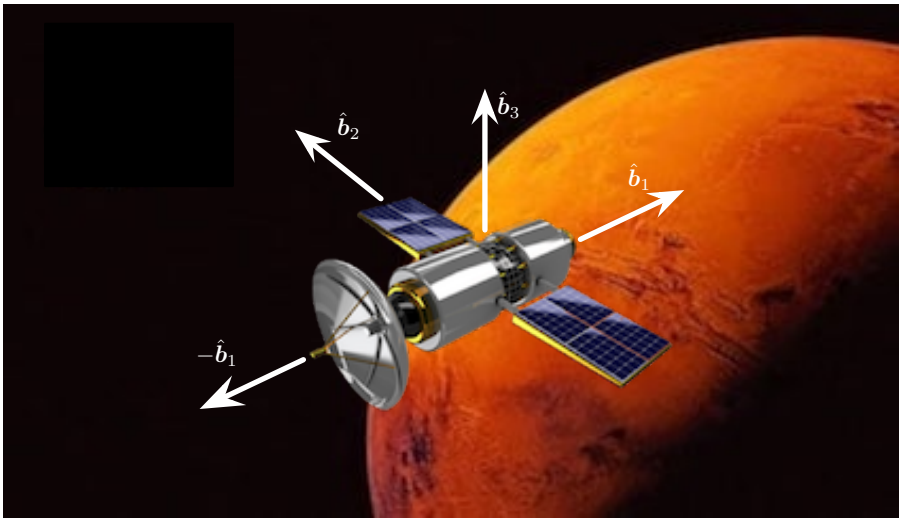

Figure 2: Illustration of the spacecraft component directions.

For simplicity, it is assumed that both the nano and the mothercraft are in simple two-body circular orbits. The nano-satellite's orbit is characterized by an altitude of $h = 400$ km (i.e., the orbit radius is $r_{\text{LMO}} = R_{\mathord{\male}} + h$ where $R_{\mathord{\male}} = 3396.19$ km), and the (3-1-3) Euler angle set right ascension Ω, inclination angle i and the true latitude angle θ as illustrated in Figure 1. Let the Mars gravity constant be given by $\mu = 42828.3$ km^3/s^2. The constant orbit rate is then given by $\dot{\theta}_{\text{LMO}} = \sqrt{\mu/r^3} = 0.000884797$ rad/sec.

Mars has a rotational period of 1 day and 37 minutes. Thus the geosynchronous mothercraft must have the same orbit period. This leads to a constant orbit radius of r_{GMO} = 20424.2 km. As the mothercraft orbit is in the equatorial plane, it has a zero inclination angle. The GMO orbit rate is $\dot{\theta}_{\text{GMO}} = 0.0000709003$ rad/s. Because both the LMO and GMO orbits are circular, their angular rates $\dot{\theta} = \sqrt{\mu/r^3}$ are constant.

These two circular orbits can easily be described in the so-called Hill frame (denoted $\mathcal{H}$). Figure 1 shows the orientation of the Hill frame with respect to the inertial frame (denoted $\mathcal{N}$). In the Hill frame, $\hat{\boldsymbol{i}}_r$ points to the spacecraft, $\hat{\boldsymbol{i}}_h$ is the direction of the angular momentum vector, and $\hat{\boldsymbol{i}}_\theta = \hat{\boldsymbol{i}}_h \times \hat{\boldsymbol{i}}_r$. It is important to note that the Hill frame is given through the (3-1-3) Euler angle set (Ω, i, θ).

In order to make this a realistic simulation, the LMO spacecraft is equipped with an antenna to communicate with the mothercraft (aligned with $-\hat{\boldsymbol{b}}_1$), a sensor to observe to observe Mars (aligned with $+\hat{\boldsymbol{b}}_1$), and solar panels for power (panel normal is along $+\hat{\boldsymbol{b}}_3$). Each of these devices are located on a different face of the spacecraft. Figure 2 shows the layout of the LMO spacecraft. In the following sections, you will derive the expressions of the attitude reference that the satellite must track, as well as the associated attitude error. Once these quantities are know and related to the spacecraft's current attitude and position, you will eventually implement a control law that drives the spacecraft attitude $\mathcal{B}$ towards its reference $\mathcal{R}$ for the two mission phases.

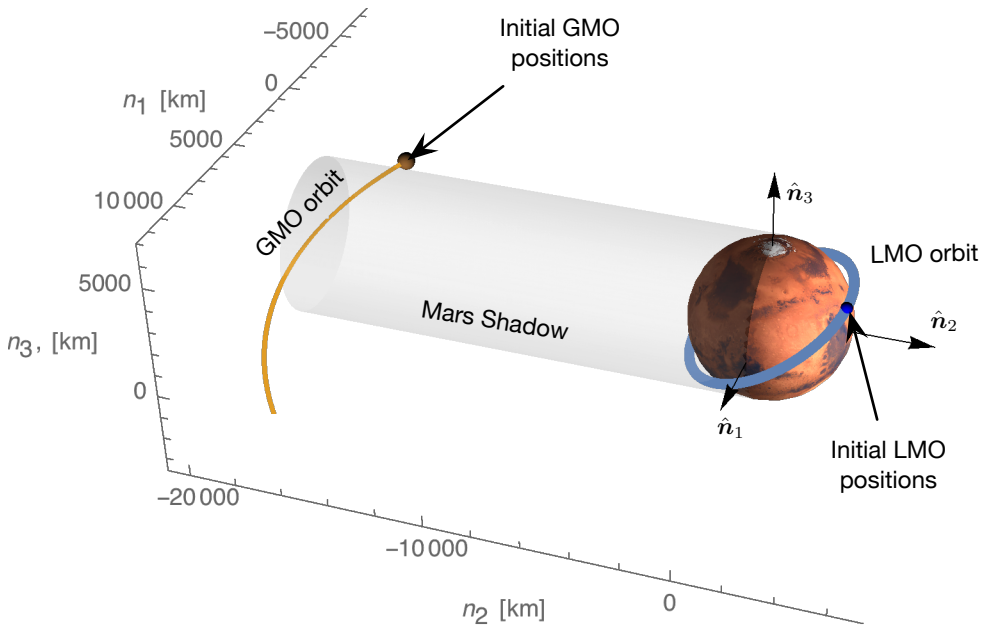

Figure 3: Mission Orbit Scenario Illustration

3 Mission Scenario Definitions

3.1 Spacecraft Attitude States

The spacecraft's initial attitude is given through the following Modified Rodrigues Parameter (MRP) set[1]:

$$\boldsymbol{\sigma}_{B/N}(t_0) = \begin{bmatrix} 0.3 \\ -0.4 \\ 0.5 \end{bmatrix} \tag{1}$$

The initial body angular velocity is given by:

$${}^{\mathcal{B}}\boldsymbol{\omega}_{B/N}(t_0) = {}^{\mathcal{B}}\begin{bmatrix} 1.00 \\ 1.75 \\ -2.20 \end{bmatrix} \text{deg/s} \tag{2}$$

The rigid LMO spacecraft inertia tensor is given by:

$${}^{\mathcal{B}}[I] = {}^{\mathcal{B}}\begin{bmatrix} 10 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7.5 \end{bmatrix} \text{ kg m}^2 \tag{3}$$

The spacecraft is assumed to be able to create any three-dimensional control torque vector $\boldsymbol{u}$ using a set of thrusters.

3.2 Mission Pointing Scenarios

This section defines the various attitude pointing modes for the spacecraft. For simplification, it can be assumed that the sun is an infinite distance away and always in the $\hat{n}_2$ direction as illustrated in Figure 3.

During the mission, the control law must detumble the satellite and either point its antenna ($-\hat{\boldsymbol{b}}_1$) towards the GMO mother satellite, its sensor ($\hat{\boldsymbol{b}}_1$) at Mars (nadir direction or $-\boldsymbol{r}$ direction) or its solar panels ($\hat{\boldsymbol{b}}_3$) at the sun. Due to the high power demands of the telescope, the spacecraft needs to point its solar panels at the sun whenever the spacecraft is on the sunlit side of Mars (i.e. positive satellite $\hat{\boldsymbol{n}}_2$ position coordinate). In other words, when pointing at the Sun, the spacecraft must point its solar panels axis $\hat{\boldsymbol{b}}_3$ in the $\hat{\boldsymbol{n}}_2$ direction. To complete this 3D frame definition, assume that $\hat{\boldsymbol{r}}_1$ must point in the $-\hat{\boldsymbol{n}}_1$ direction.

When the spacecraft on the shaded side of Mars (i.e., over the hemisphere with $-\hat{\boldsymbol{n}}_2$ position coordinates) it must either enter communication or science mode. In science mode the science platform axis $\hat{\boldsymbol{b}}_1$ must point at the center of Mars, thus in the nadir direction. To complete the reference frame, assumed that $\hat{\boldsymbol{r}}_2$ must line up with the orbit along-track axis $\hat{\boldsymbol{\imath}}_\theta$. The communication mode requires that the LMO and GMO satellite position vectors have an angular difference of less than 35 degrees. In this mode the nano-satellite communication platform axis $-\hat{\boldsymbol{b}}_1$ must point towards the GMO mothercraft. Table 1 outlines a summary of these pointing requirement.

Table 1: Spacecraft Pointing Scenario Summary

Orbital Situation	**Primary Pointing Scenario Goals**
SC on sunlit Mars side	Point Solar Panels axis $\hat{\boldsymbol{b}}_3$ at Sun
SC not on sunlit Mars side & GMO Visible	Point Antenna axis $-\hat{\boldsymbol{b}}_1$ at GMO
SC not on sunlit Mars side & GMO not Visible	Point sensor axis $\hat{\boldsymbol{b}}_1$ along Mars nadir direction

3.3 Mission Orbit Overview

The initial orbit frame angles of both satellites are are listed in Table 2. The corresponding initial orbit positions are visualized in Figure 3.

Table 2: Initial Orbit Frame Orientation Angles

Spacecraft	Ω	i	$\theta(t_0)$
LMO	20°	30°	60°
GMO	0°	0°	250°

3.4 Attitude Control Law Overview

In this project we employ the simple proportional-derivative (PD) attitude control law:

$$^{\mathcal{B}}\boldsymbol{u} = -K\boldsymbol{\sigma}_{B/R} - P\,{}^{\mathcal{B}}\boldsymbol{\omega}_{B/R} \tag{4}$$

It is used to drive the body frame $\mathcal{B}$ towards its reference $\mathcal{R}$.

4 Description of Project Tasks

4.1 Task 1: Orbit Simulation (5 points)

Assume the general orbit frame $\mathcal{O} : \{\hat{\boldsymbol{\imath}}_r, \hat{\boldsymbol{\imath}}_\theta, \hat{\boldsymbol{\imath}}_h\}$ as illustrated in Figure 1. This allows the position vector on a circular orbit to be written as $\boldsymbol{r} = r\hat{\boldsymbol{\imath}}_r$ where r is here a constant radius. Your tasks are:

- Derive the inertial spacecraft velocity vector $\dot{\boldsymbol{r}}$. Note that for a circular orbit $\dot{\theta}$ is constant.
- Write a function whose inputs are the radius r and the (3-1-3) Euler angles (Ω, i, θ) and the outputs are the inertial position vector ${}^{\mathcal{N}}\boldsymbol{r}$ and velocity ${}^{\mathcal{N}}\dot{\boldsymbol{r}}$ of the associated circular orbit.
- Confirm the operation of this program by entering the inertial position and velocity vector results for $\Big(r_{\text{LMO}}, \Omega_{\text{LMO}}, i_{\text{LMO}}, \theta_{\text{LMO}}(450\text{s})\Big)$ and $\Big(r_{\text{GMO}}, \Omega_{\text{GMO}}, i_{\text{GMO}}, \theta_{\text{GMO}}(1150\text{s})\Big)$

4.2 Task 2: Orbit Frame Orientation (5 points)

Let Hill frame $\mathcal{H} = \{\hat{\boldsymbol{i}}_r, \hat{\boldsymbol{i}}_\theta, \hat{\boldsymbol{i}}_h\}$ be the orbit frame of the LMO satellite. These base vectors are generally defined as

$$\hat{\boldsymbol{i}}_r = \frac{\boldsymbol{r}_{\text{LM}}}{|\boldsymbol{r}_{\text{LM}}|}, \qquad \hat{\boldsymbol{i}}_\theta = \hat{\boldsymbol{i}}_h \times \hat{\boldsymbol{i}}_r, \qquad \hat{\boldsymbol{i}}_h = \frac{\boldsymbol{r}_{\text{LM}} \times \dot{\boldsymbol{r}}_{\text{LMO}}}{|\boldsymbol{r}_{\text{LM}} \times \dot{\boldsymbol{r}}_{\text{LMO}}|}$$

Your tasks are:

- Determine an analytic expressions for [$\mathcal{HN}$]
- Write a function whose input is simply time t, and the output is the DCM $[HN(t)]$
- Validate the operation of this function by computing $[HN(t = 300\text{s})]$

4.3 Task 3: Sun-Pointing Reference Frame Orientation (10 points)

To point the spacecraft solar panels axis $\hat{\boldsymbol{b}}_3$ at the sun, the reference frame $\mathcal{R}_s$ must be chosen such that $\hat{\boldsymbol{r}}_3$ axis points in the sun direction ($\hat{\boldsymbol{n}}_2$ in this scenario). Further, assume the first axis $\hat{\boldsymbol{r}}_1$ points in the $-\hat{\boldsymbol{n}}_1$ direction. Your tasks are:

- Determine an analytic expressions for the sun pointing reference frame $\mathcal{R}_s$ by defining the DCM $[R_sN]$
- Write a function that returns $[R_sN]$.
- Validate the evaluation of $[R_sN]$ by providing the numerical values for $t = 0$s.
- What is the angular velocity ${}^{\mathcal{N}}\boldsymbol{\omega}_{R_s/N}$?

4.4 Task 4: Nadir-Pointing Reference Frame Orientation (10 points)

To point the spacecraft sensor platform axis $\hat{\boldsymbol{b}}_1$ towards the center of Mars or nadir direction, the reference frame $\mathcal{R}_n$ must be chosen such that $\hat{\boldsymbol{r}}_1$ axis points towards the planet. Further, assume the second axis $\hat{\boldsymbol{r}}_2$ points in the velocity direction $\hat{\boldsymbol{i}}_\theta$. Your tasks are:

- Determine an analytic expressions for the nadir pointing reference frame $\mathcal{R}_n$ by defining the DCM $[R_nN]$
- Write a function that returns $[R_nN]$ as a function of time.
- Write a function that determines the angular velocity vector ${}^{\mathcal{N}}\boldsymbol{\omega}_{R_n/N}$.
- Validate the evaluation of $[R_nN]$ by providing the numerical values for $t = 330$s.
- What is the angular velocity ${}^{\mathcal{N}}\boldsymbol{\omega}_{R_n/N}(330\text{s})$?

4.5 Task 5: GMO-Pointing Reference Frame Orientation (10 points)

To point the nano-satellite communication platform axis $-\hat{\boldsymbol{b}}_1$ towards the GMO mother spacecraft, the communication reference frame $\mathcal{R}_c$ must be chosen such that $-\hat{\boldsymbol{r}}_1$ axis points towards the GMO satellite location. Assume $\Delta\boldsymbol{r} = \boldsymbol{r}_{\text{GMO}} - \boldsymbol{r}_{\text{LMO}}$. Further, to fully define a three-dimensional reference frame, assume the second axis is defined as

$$\hat{\boldsymbol{r}}_2 = \frac{\Delta\boldsymbol{r} \times \hat{\boldsymbol{n}}_3}{|\Delta\boldsymbol{r} \times \hat{\boldsymbol{n}}_3|}$$

while the third is then defined as $\hat{\boldsymbol{r}}_3 = \hat{\boldsymbol{r}}_1 \times \hat{\boldsymbol{r}}_2$. Your tasks are:

- Determine an analytic expressions for the communication mode reference frame $\mathcal{R}_c$ by defining the DCM $[R_cN]$
- Write a function that returns $[R_cN]$ as a function of time.
- Write a function that determines the angular velocity vector ${}^{\mathcal{N}}\boldsymbol{\omega}_{R_c/N}$. Note that an analytical expression for this body rate vector is very challenging. It is ok to use numerical differences in these frames to compute $\mathrm{d}([R_cN])/\mathrm{d}t$, and then relate this to $\boldsymbol{\omega}_{R_c/N}$. As the orbit trajectories are given as a function of time, it is straight forward to include a time-based numerical difference method.
- Validate the evaluation of $[R_cN]$ by providing the numerical values for $t = 330$s.
- Validate the angular velocity ${}^{\mathcal{N}}\boldsymbol{\omega}_{R_c/N}(330\text{s})$.

4.6 Task 6: Attitude Error Evaluation (10 points)

Any attitude feedback control algorithm requires the ability to compute the attitude and angular velocity tracking errors of the current body frame $\mathcal{B}$ relative to the reference frame R. Your tasks are:

- Write a function that has the inputs of time, the current body attitude states $\boldsymbol{\sigma}_{B/N}$ and ${}^{\mathcal{B}}\boldsymbol{\omega}_{B/N}$, as well as the current references frame frame orientation $[RN]$ and rates ${}^{\mathcal{N}}\boldsymbol{\omega}_{R/N}$, and returns the associated tracking errors $\boldsymbol{\sigma}_{B/R}$ and ${}^{\mathcal{B}}\boldsymbol{\omega}_{B/R}$.
- Validate this function by computing $\boldsymbol{\sigma}_{B/R}$ and ${}^{\mathcal{B}}\boldsymbol{\omega}_{B/R}$ at the initial time t_0 for the sun-pointing, nadir pointing and GMO reference orientations above. Just compute the LMO tracking errors at t_0, with the initial conditions given in section 3.1, for the three reference frames at the initial time.

4.7 Task 7: Numerical Attitude Simulator (10 points)

In order to integrate the attitude you need a numerical integrator. The LMO and GMO orbit satellite positions are given already as a function of time from your earlier tasks. Let the propagated attitude state $\boldsymbol{X}$ be

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{\sigma}_{B/N} \\ {}^{\mathcal{B}}\boldsymbol{\omega}_{B/N} \end{bmatrix} \tag{5}$$

Assume that the spacecraft is rigid and its dynamics obey

$$[I]\dot{\boldsymbol{\omega}}_{B/N} = -[\tilde{\boldsymbol{\omega}}_{B/N}][I]\boldsymbol{\omega}_{B/N} + \boldsymbol{u} \tag{6}$$

where $\boldsymbol{u}$ is your external control torque vector.

- Write an RK4 integrator using your programing language of choice (i.e., do not use a built-in integrator such as Matlab's ode45). Make sure your time step is chosen small enough such that numerical integrator errors are not visible in your simulation result plots. With RK4 it is ok to use a fixed integration time step of 1 second. Further, hold the control $\boldsymbol{u}$ vector piece-wise constant over the RK4 integration to the next time step. You can updated the control $\boldsymbol{u}$ at every time step in advance fo the RK4 integration step.
- Demonstrate that your integrator works properly by integrating $\boldsymbol{X}$ forward for 500 seconds with $\boldsymbol{u} = 0$. Provide $\boldsymbol{H} = [I]\boldsymbol{\omega}_{B/N}$ at 500s and express the $\boldsymbol{H}$ vector in the $\mathcal{B}$ frame.
- Provide the rotational kinetic energy $T = \frac{1}{2}\boldsymbol{\omega}_{B/N}^T[I]\boldsymbol{\omega}_{B/N}$ at 500 seconds.
- Provide the MRP attitude $\boldsymbol{\sigma}_{B/N}(500s)$
- Provide angular momentum vector ${}^{\mathcal{N}}\boldsymbol{H}(500s)$ in inertial frame components.
- If you apply a fixed control torque ${}^{\mathcal{B}}\boldsymbol{u} = (0.01, -0.01, 0.02)$ Nm, provide the attitude $\boldsymbol{\sigma}_{B/N}(t = 100s)$.

4.8 Task 8: Sun Pointing Control (10 points)

First the individual control pointing modes are developed and tested on their own. Next they are combined into a full mission scenario simulation in section 4.11. In the current task, use the initial spacecraft attitude and orbit conditions in section 3.1 and Table 2 and assume the spacecraft is to engage directly into a sun-pointing mode starting at t_0. The attitude control law is the simple PD control shown in Eq. (4). Your tasks are:

- Numerically implement this control law in your earlier simulation and ensure that sun-pointing is achieved with the desired closed loop performance.
- Use linearized closed loop dynamics of a regular problem, such as this sun pointing control, to determine the K and P feedback gains such that the slowest decay response time (i.e. time for tracking errors to be $1/e$ the original size) is 120 seconds. This means all decay time constants should be 120 seconds or less. Further, the closed loop response for all $\boldsymbol{\sigma}_{B/N}$ components should be either critically damped or under-damped. Thus at least must be critically damped with $\xi = 1$, while the other modes will have $\xi \leq 1$.
- Validate the response by providing the $\boldsymbol{\sigma}_{B/N}$ states at t = 15s, 100s, 200s and 400s. Note that you must always provide the MRP corresponding to the short rotation.

4.9 Task 9: Nadir Pointing Control (10 points)

Next the nadir pointing attitude mode is created and tested. Use the same gains K and P as developed for sun-pointing, and the PD control in Eq. (4). Your tasks are:

- Numerically implement the nadir pointing control mode and for now just assume that at t_0 nadir pointing is required, even though the satellite is in the sunlight at this time.
- Validate the response by providing the $\boldsymbol{\sigma}_{B/N}$ states at t = 15s, 100s, 200s and 400s. Note that you must always provide the MRP corresponding to the short rotation.

4.10 Task 10: GMO Pointing Control (10 points)

Next the GMO pointing attitude mode is created and test. Use the same gains K and P as developed for sun-pointing, and the PD control in Eq. (4). Your tasks are:

- Numerically implement the GMO pointing control mode and for now just assume that at t_0 nadir pointing is required, even though the satellite is in the sunlight at this time.
- Validate the response by providing the $\boldsymbol{\sigma}_{B/N}$ states at t = 15s, 100s, 200s and 400s. Note that you must always provide the MRP corresponding to the short rotation.

4.11 Task 11: Mission Scenario Simulation (10 points)

Finally you are ready to simulate the full on mission scenario. Use the initial spacecraft attitude and orbit conditions in section 3.1 and Table 2, then propagate 6500 seconds to demonstrate how the attitude pointing performance of the nano-satellite as it enters different control modes. Your tasks are:

- Numerically implement a login into your above simulation such that the satellite automatically computer the proper reference frame stages as it switches between sun-pointing, nadir-pointing and communication modes.
- Validate the response by providing the $\boldsymbol{\sigma}_{B/N}$ states at t = 300s, 2100s, 3400s, 4400s and 5600s. Note that you must always provide the MRP corresponding to the short rotation.

5 Concluding Remarks

This project illustrates how to evaluate and implement a range of attitude control modes. Besides the simple PD control uses here, other attitude tracking control solutions could readily be substituted. When studying the response of such simulations, it is very helpful to create a three-dimensional visualization where the simulation states are used to illustrate how a spacecraft model is pointing in inertial space. This provides the three-dimensional context that the above simulations are indeed achieving the desired attitude pointing behavior.

References

[1] Schaub, H. and Junkins, J. L., *Analytical Mechanics of Space Systems*, AIAA Education Series, Reston, VA, 4th ed., 2018, doi:10.2514/4.105210.