Nonlinear Spacecraft Control

ASEN 5010

Prof. H. Schaub
hanspeter.schaub@colorado.edu

University of Colorado
Boulder

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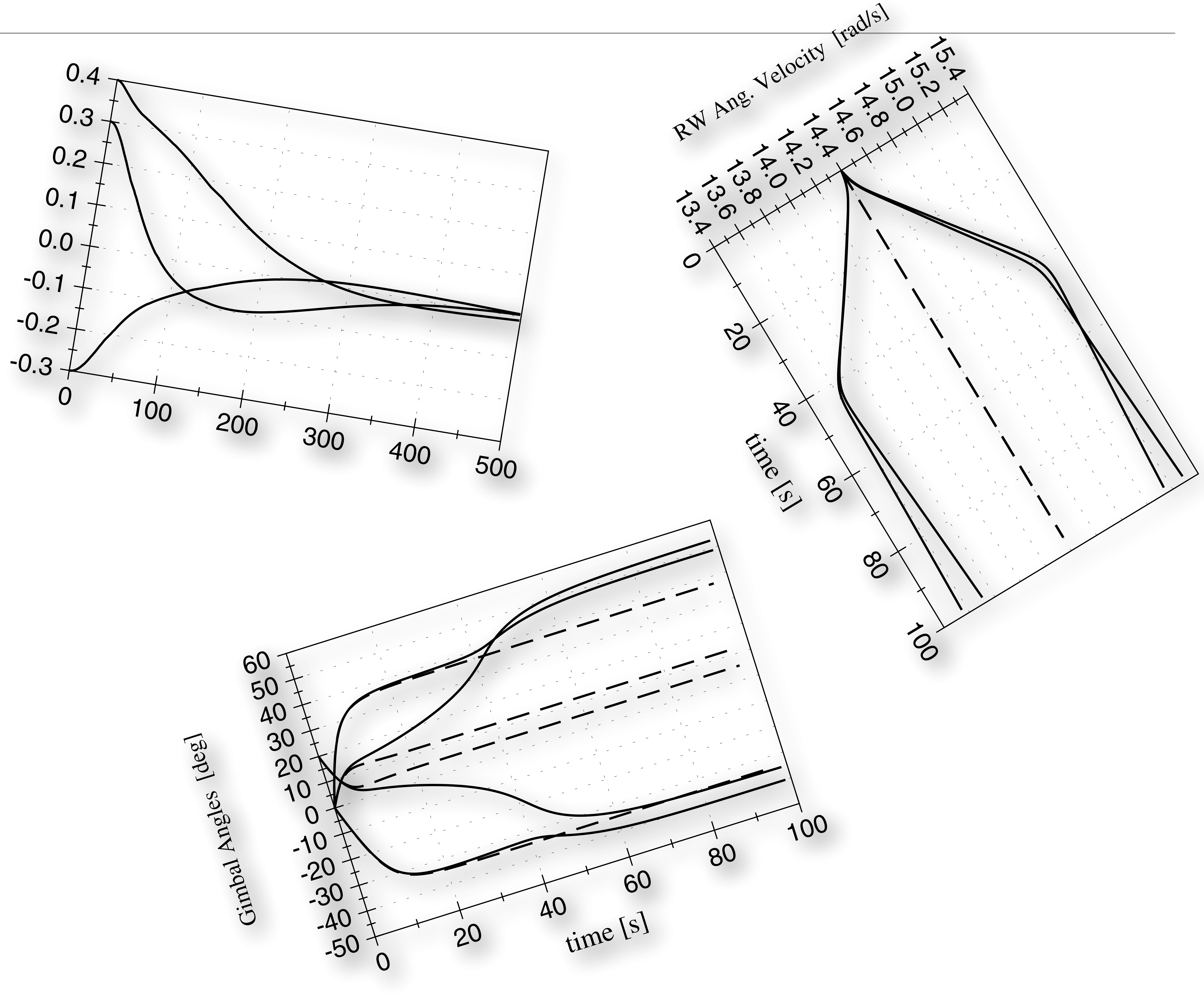
- Lyapunov Optimal Feedback
- Linear Closed-Loop Dynamics



Stability Definitions

Why isn’t stable just stable?

Definitions

State Vector: $\boldsymbol{x} = (x_1 \cdots \ x_N)^T$

EOM:

$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, t)$ —— Non-Autonomous System

$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$ —— Autonomous System

Control Vector: $\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{x})$

Closed-Loop System: $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t)$

Equilibrium State: A state vector point $\boldsymbol{x}_e$ is said to be an equilibrium state (or equilibrium point) of a dynamical system described by $\dot{\boldsymbol{x}}=\boldsymbol{f}(\boldsymbol{x},t)$ at time t_0 if

$$\boldsymbol{f}(\boldsymbol{x}_e, t) = 0 \qquad \forall\ t > t_0$$

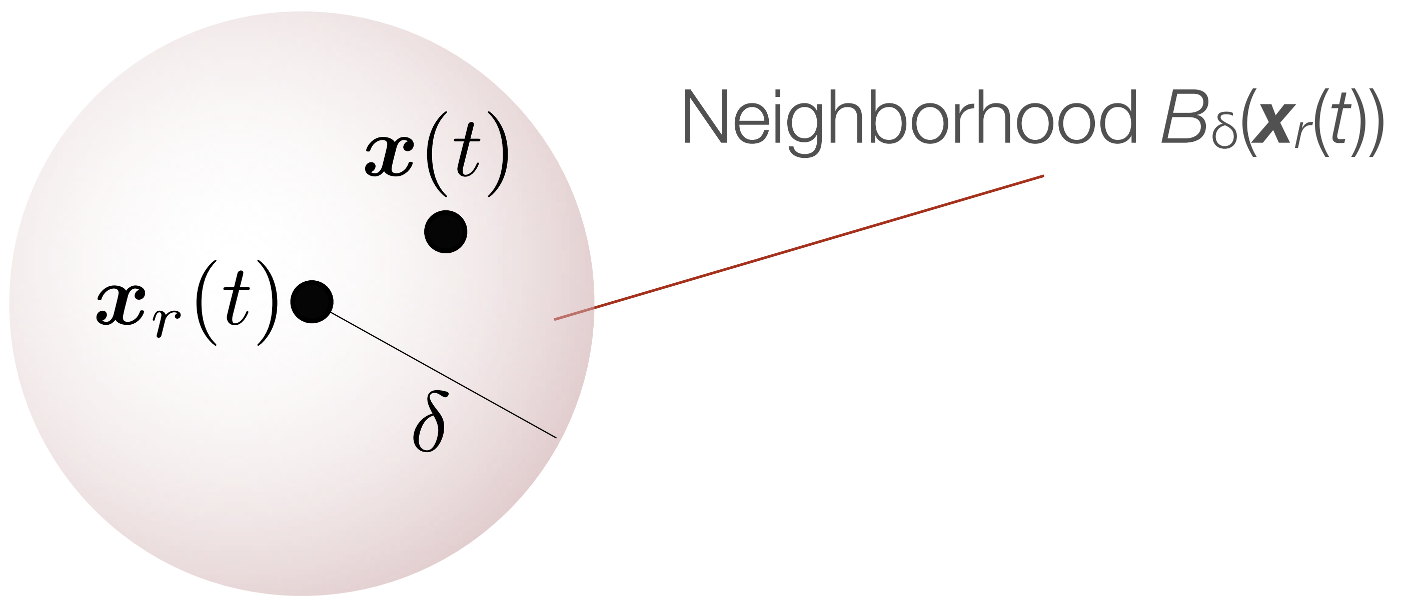
$$\dot{\boldsymbol{x}}_e = 0 \qquad \boldsymbol{x}_e = \text{constant}$$

Neighborhood: Given δ>0, a state vector $\boldsymbol{x}(t)$ is said to be in the neighborhood $B_\delta(\boldsymbol{x}_r(t))$ of the state $\boldsymbol{x}_r(t)$ if

$$||\boldsymbol{x}(t) - \boldsymbol{x}_r(t)|| \; < \; \delta$$

then

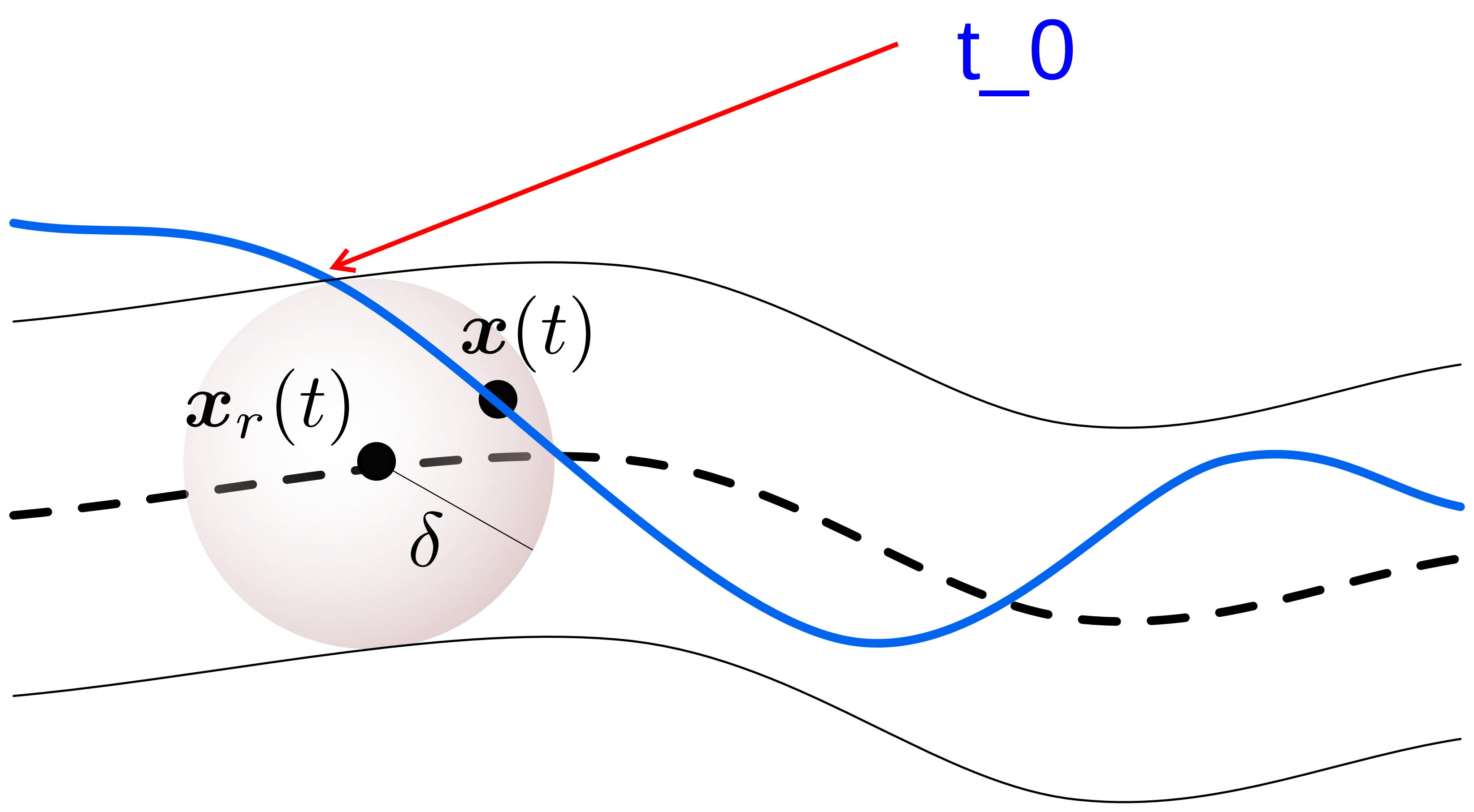
$$\boldsymbol{x}(t) \in B_\delta(\boldsymbol{x}_r(t))$$



Doesn't depend on initial condition

Lagrange Stability: The motion $\boldsymbol{x}(t)$ is said to be Lagrange stable (or bounded) relative to $\boldsymbol{x}_r(t)$ if there exists a δ>0 such that

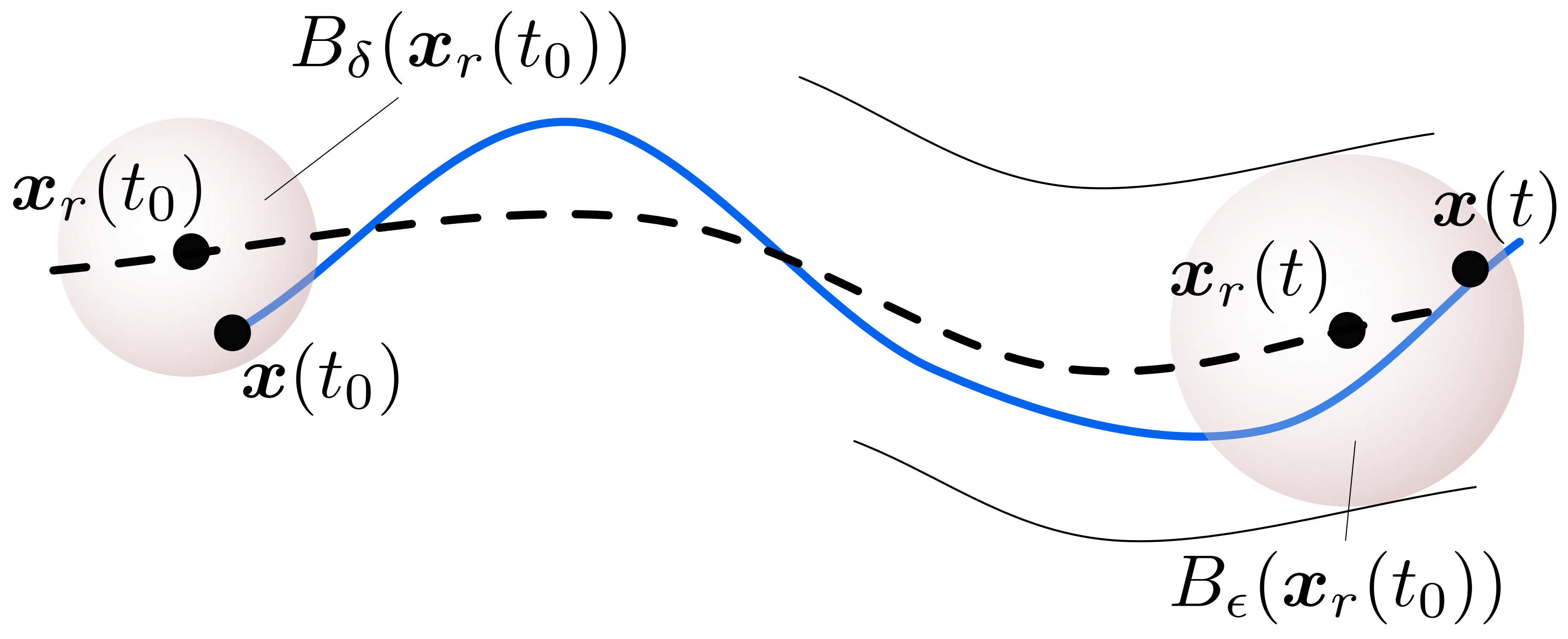
$$\boldsymbol{x}(t) \in B_\delta(\boldsymbol{x}_r(t)) \qquad \forall \; t > t_0$$



Depend on initial condition

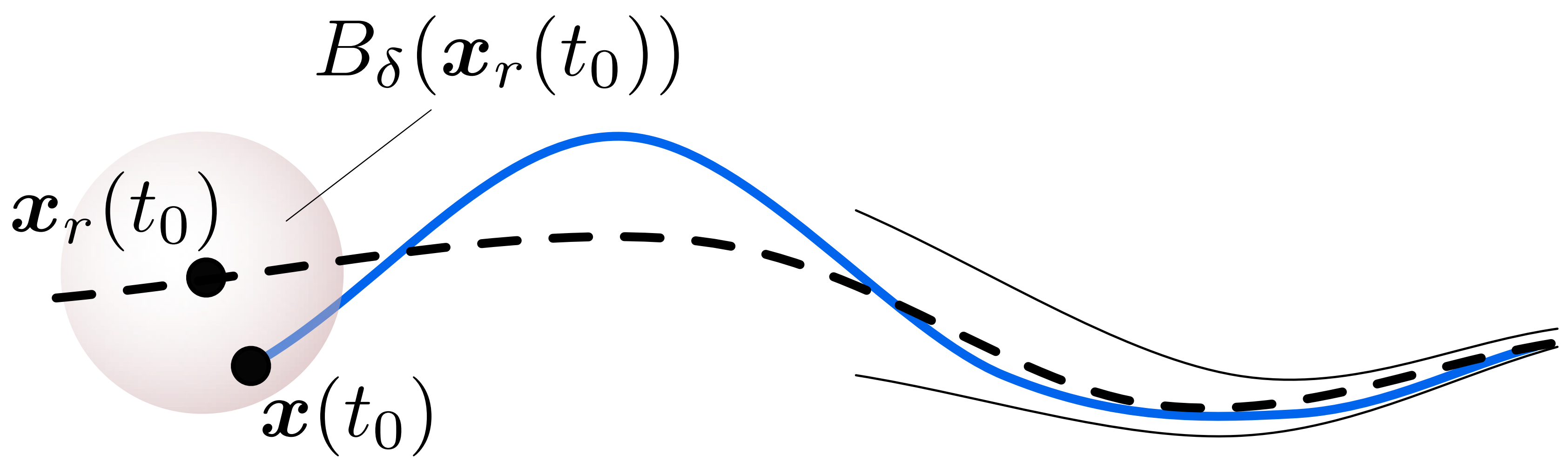
Lyapunov Stability: The motion $\boldsymbol{x}(t)$ is said to be Lyapunov stable (or stable) relative to $\boldsymbol{x}_r(t)$ if for each ε>0 there exists a δ(ε)>0 such that

$$\boldsymbol{x}(t_0) \in B_\delta(\boldsymbol{x}_r(t_0)) \Longrightarrow \boldsymbol{x}(t) \in B_\epsilon(\boldsymbol{x}_r(t)) \quad \forall \; t > t_0$$

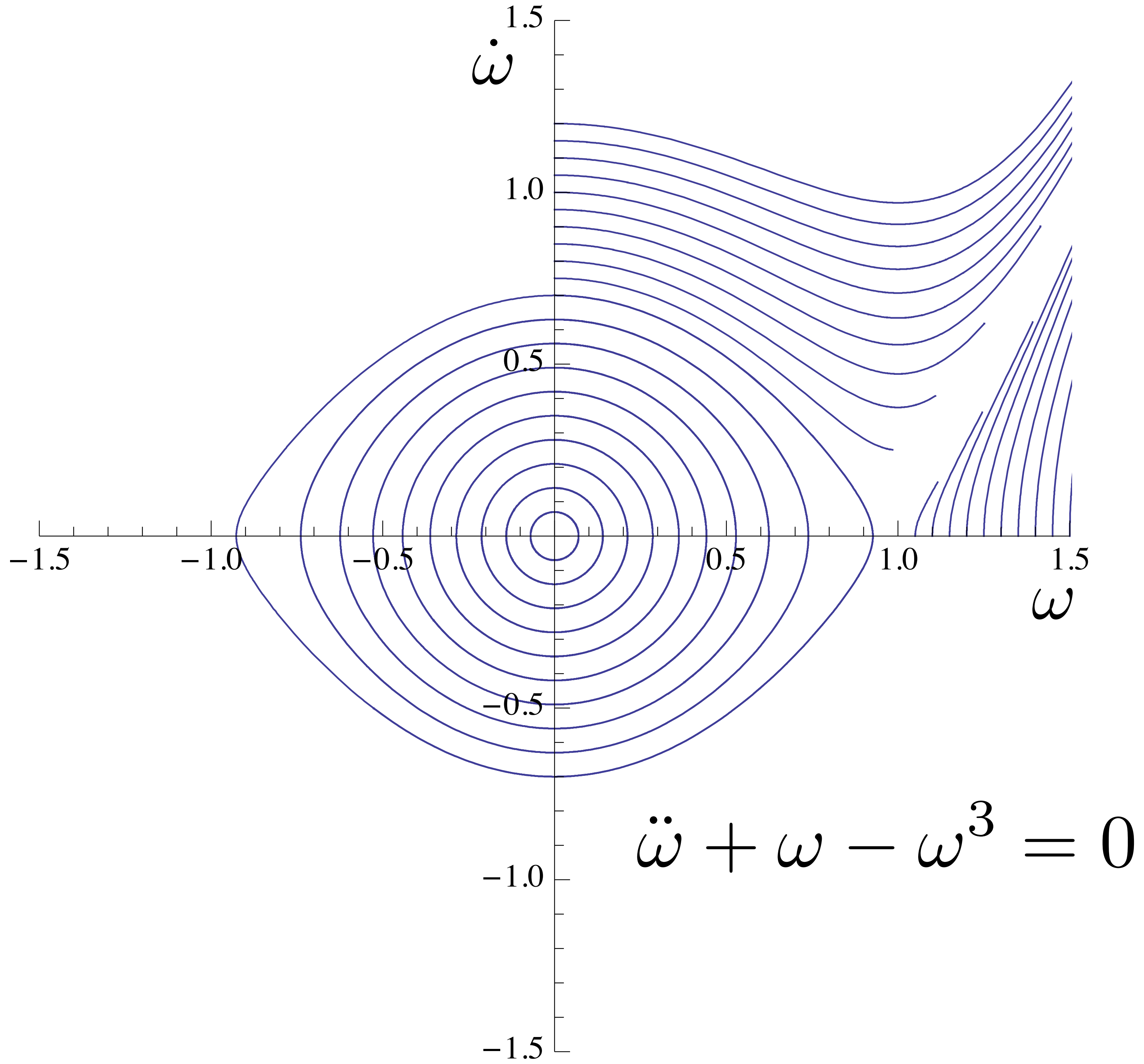


Asymptotic Stability: The motion $\boldsymbol{x}(t)$ is asymptotically stable relative to $\boldsymbol{x}_r(t)$ if $\boldsymbol{x}(t)$ is Lyapunov stable and there exists a $\delta>0$ such that

$$\boldsymbol{x}(t_0) \in B_\delta(\boldsymbol{x}_r(t_0)) \Longrightarrow \lim_{t\to\infty} \boldsymbol{x}(t) = \boldsymbol{x}_r(t)$$



Global Stability: The motion $\boldsymbol{x}(t)$ is globally stable relative to $\boldsymbol{x}_r(t)$ if $\boldsymbol{x}(t)$ is stable for any initial state vector $\boldsymbol{x}(t_0)$.



$$\ddot{\omega} + \omega - \omega^3 = 0$$

(Show Mathematica Example)

Linearization of Dynamical System

Reference motion given by:

Feedforward control

$$\dot{\boldsymbol{x}}_r = \boldsymbol{f}(\boldsymbol{x}_r, \boldsymbol{u}_r)$$

Nonlinear EOM:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$$

Feedback control:

$$\delta \boldsymbol{u} = \boldsymbol{u} - \boldsymbol{u}_r$$

Departure motion:

$$\delta \boldsymbol{x} = \boldsymbol{x} - \boldsymbol{x}_r$$

Performing a Taylor Series expansion of $\boldsymbol{x}$ about ($\boldsymbol{x}_r$,$\boldsymbol{u}_r$) we obtain

$$\delta \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}_r, \boldsymbol{u}_r) + \frac{\partial \boldsymbol{f}(\boldsymbol{x}_r, \boldsymbol{u}_r)}{\partial \boldsymbol{x}} \delta \boldsymbol{x} + \frac{\partial \boldsymbol{f}(\boldsymbol{x}_r, \boldsymbol{u}_r)}{\partial \boldsymbol{u}} \delta \boldsymbol{u} + H.O.T - \boldsymbol{f}(\boldsymbol{x}_r, \boldsymbol{u}_r)$$

$$\delta \dot{\boldsymbol{x}} \simeq \frac{\partial \boldsymbol{f}(\boldsymbol{x}_r, \boldsymbol{u}_r)}{\partial \boldsymbol{x}} \delta \boldsymbol{x} + \frac{\partial \boldsymbol{f}(\boldsymbol{x}_r, \boldsymbol{u}_r)}{\partial \boldsymbol{u}} \delta \boldsymbol{u}$$

Let us define:

$$[A] = \frac{\partial \boldsymbol{f}(\boldsymbol{x}_r, \boldsymbol{u}_r)}{\partial \boldsymbol{x}}$$

$$[B] = \frac{\partial \boldsymbol{f}(\boldsymbol{x}_r, \boldsymbol{u}_r)}{\partial \boldsymbol{u}}$$

The linearized system is then written in standard form as

$$\boxed{\delta \dot{\boldsymbol{x}} \simeq [A] \delta \boldsymbol{x} + [B] \delta \boldsymbol{u}}$$

If the nominal reference motion is an equilibrium state $\boldsymbol{x}_e$, then the linearized EOM simplify to:

$$\dot{\boldsymbol{x}} \simeq [A] \boldsymbol{x} + [B] \boldsymbol{u}$$