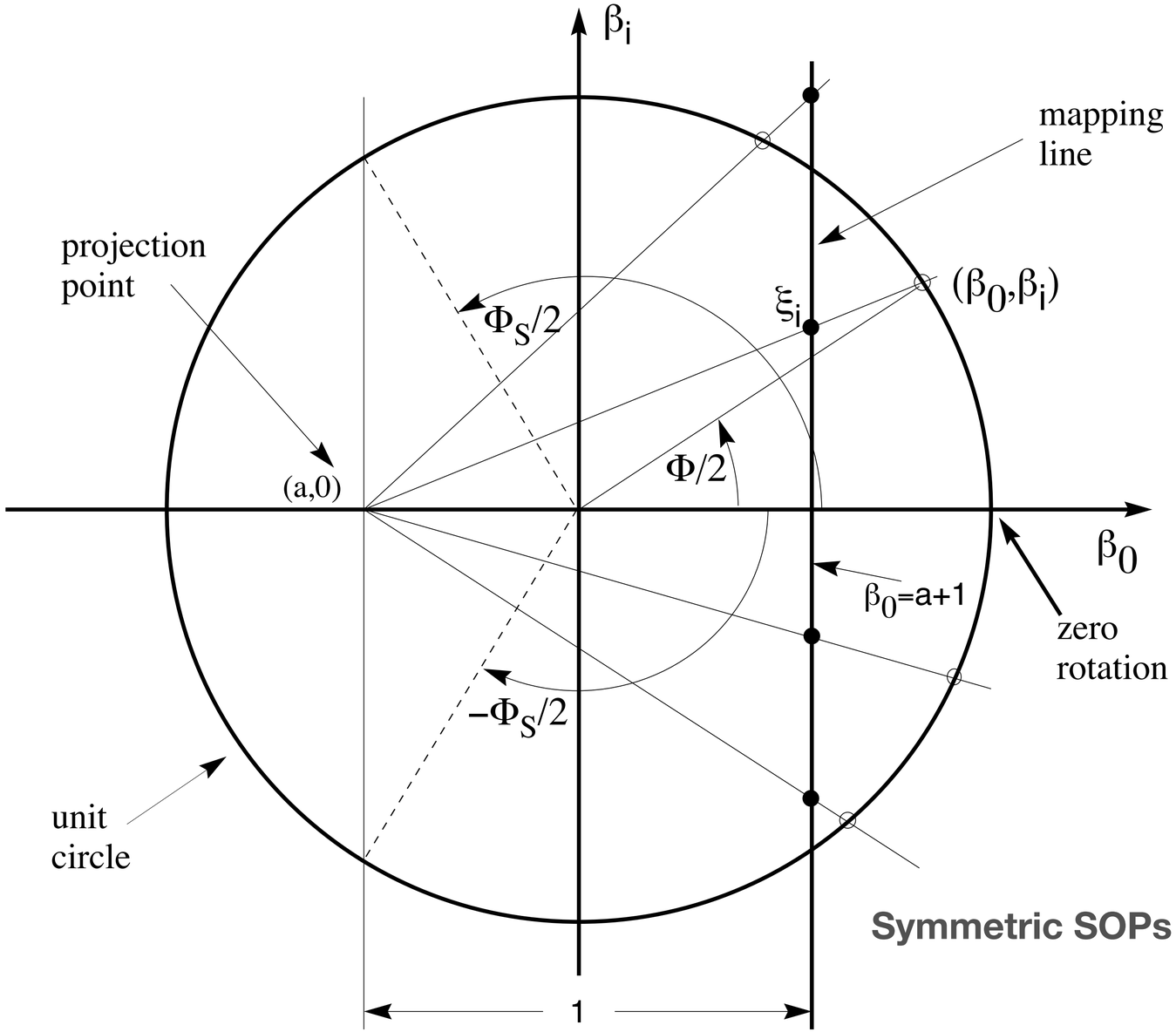
Stereographic Orientation Parameters (SOPs)

Elegant family of attitude coordinates...

Quick facts...

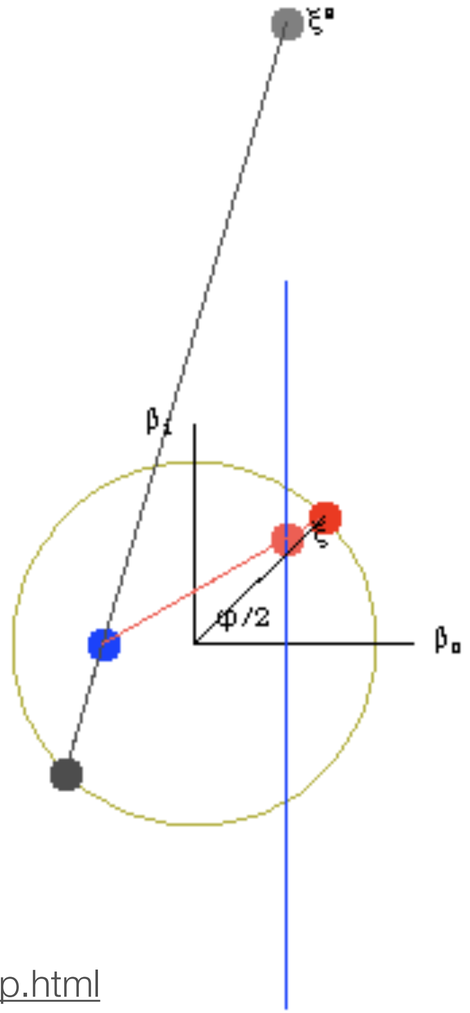
- The Stereographic Orientation Parameters are a class of attitude parameters that generalize the previously discussed classical and modified Rodrigues parameters.
- The are two types of SOPs:
  - Symmetric Set: Goes singular if a $\pm\Phi$ principal rotation is performed.
  - Asymmetric Set: Goes singular at either Φ_1 or Φ_2, and this rotation must be about a particular axis.
- References:

- H. Schaub and J.L. Junkins. ``Stereographic Orientation Parameters for Attitude Dynamics: A Generalization of the Rodrigues Parameters.'' *Journal of the Astronautical Sciences*, Vol. 44, No. 1, Jan.–Mar. 1996, pp. 1–19.
- C. M. Southward, J. Ellis and H. Schaub, “Spacecraft Attitude Control Using Symmetric Stereographic Orientation Parameters,” *Journal of Astronautical Sciences*, Vol. 55, No. 3, July–September, 2007, pp. 389–405.

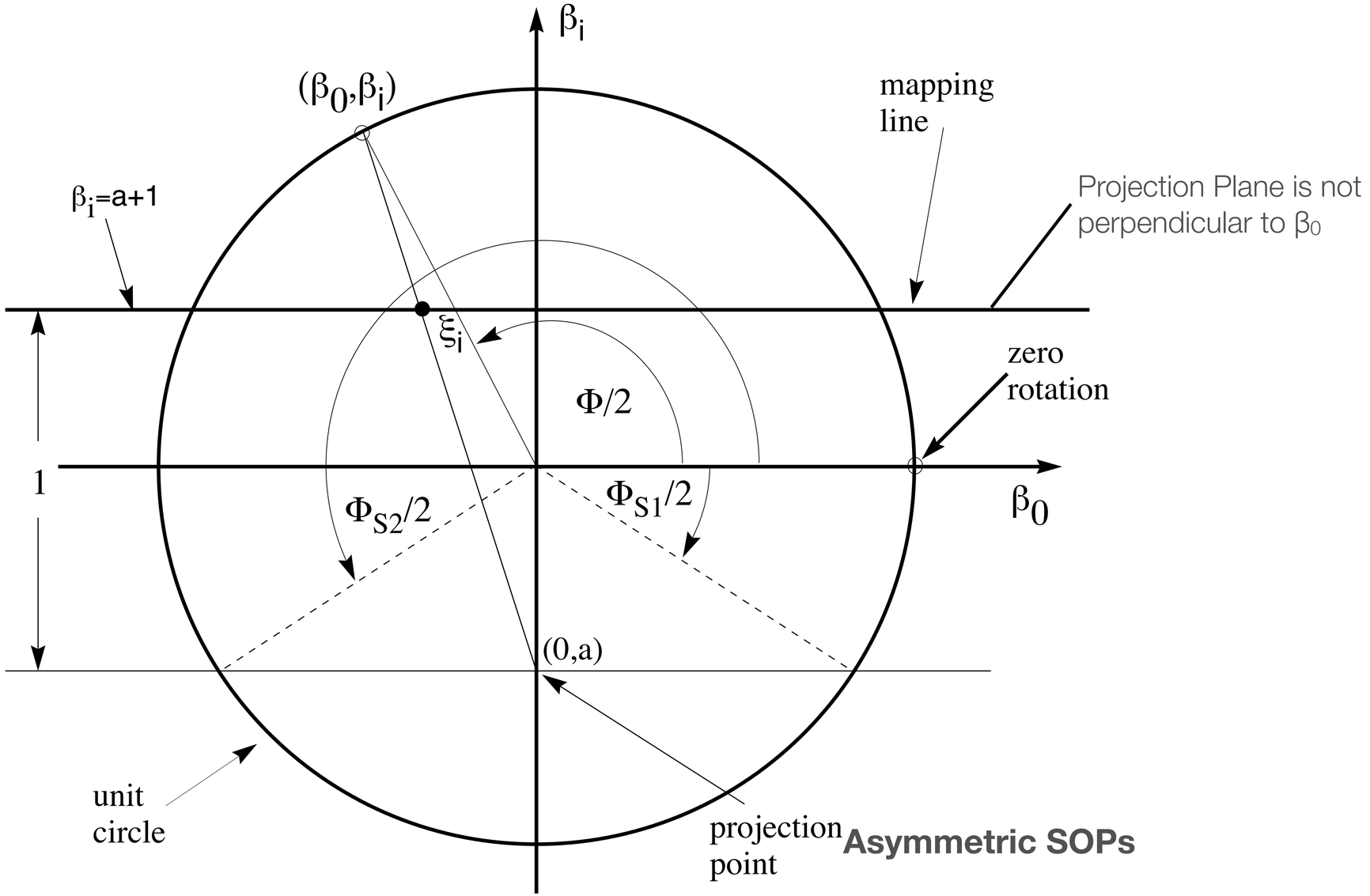


Symmetric SOPs

SSOP's

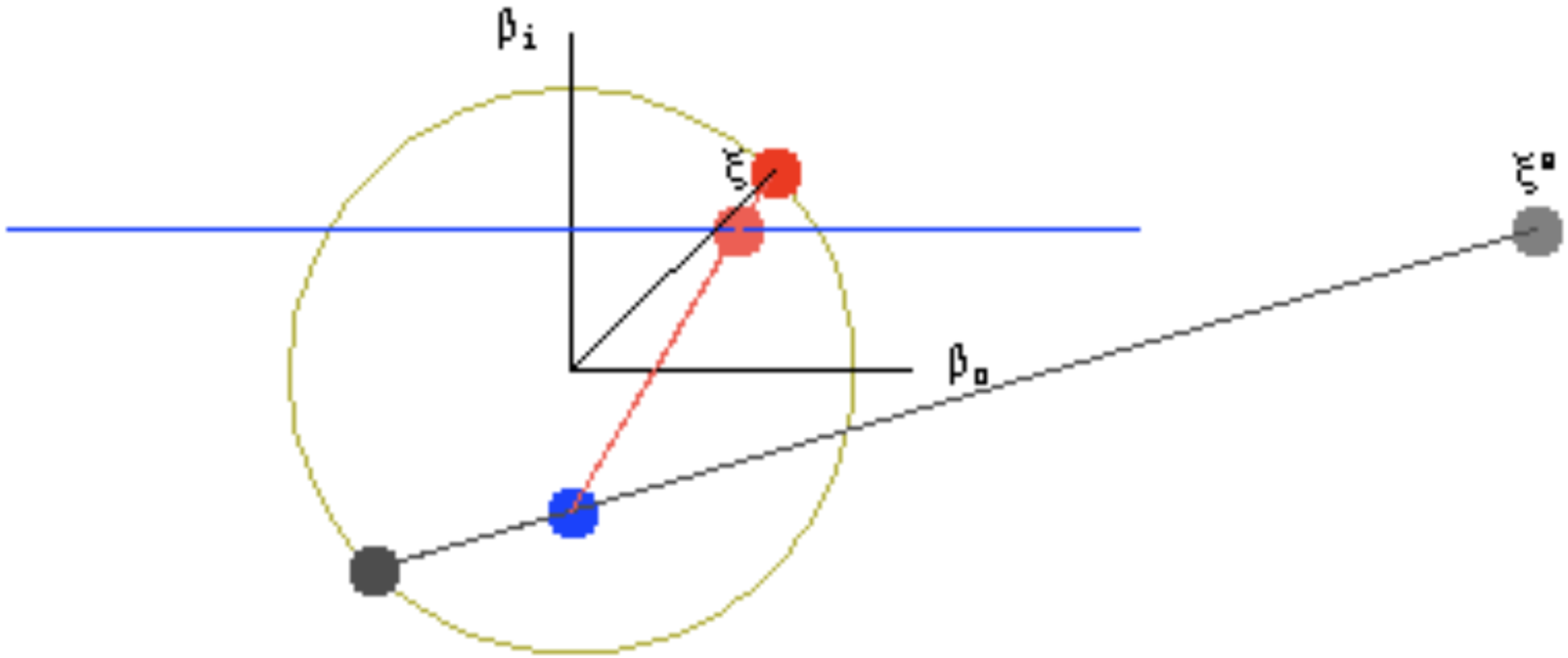


http://hanspeterschaub.info/ssop.html



Asymmetric SOPs

SOP's

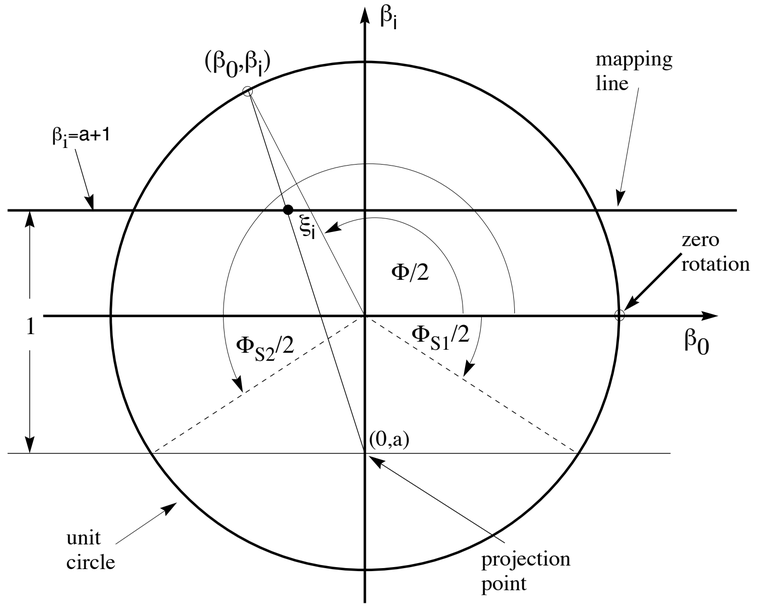


http://hanspeterschaub.info/assop.html

Example: asymmetric SOP

Projection plane: $\beta_1 = 0$

Projection point: $\beta_1 = -1$



Mapping from EP:

$$\eta_1 = \frac{\beta_0}{1+\beta_1} \quad \eta_2 = \frac{\beta_2}{1+\beta_1} \quad \eta_3 = \frac{\beta_3}{1+\beta_1}$$

Mapping to EP:

$$\beta_0 = \frac{2\eta_1}{1+\eta^2} \quad \beta_1 = \frac{1-\eta^2}{1+\eta^2} \quad \beta_2 = \frac{2\eta_2}{1+\eta^2} \quad \beta_3 = \frac{2\eta_3}{1+\eta^2} \qquad \eta^2 = \boldsymbol{\eta}^T\boldsymbol{\eta}$$

Singular behavior:

$$\beta_1 \to -1 \begin{cases} \Phi_1 = -180^o \\ \Phi_2 = +540^o \end{cases}$$

Shadow set:

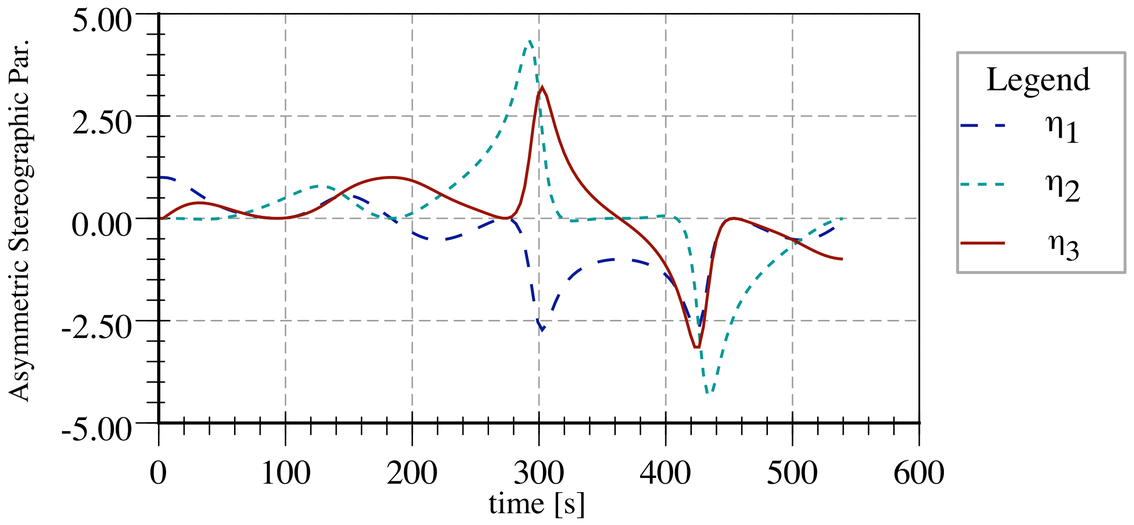
$$\boldsymbol{\eta}^S = -\frac{\boldsymbol{\eta}}{\boldsymbol{\eta}^T\boldsymbol{\eta}}$$

Prescribed 3-1-3 Euler Angle time histories:

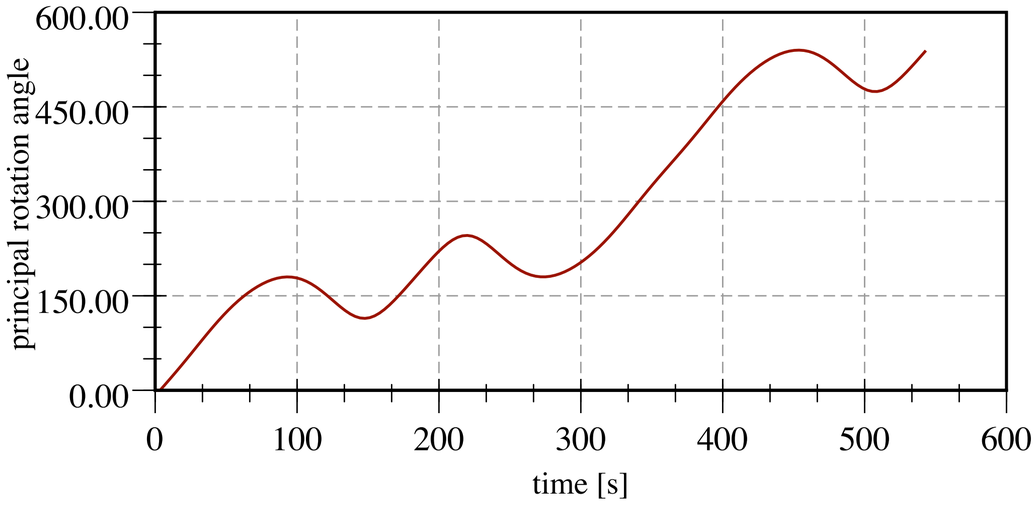
$$\theta_1(t) = t$$

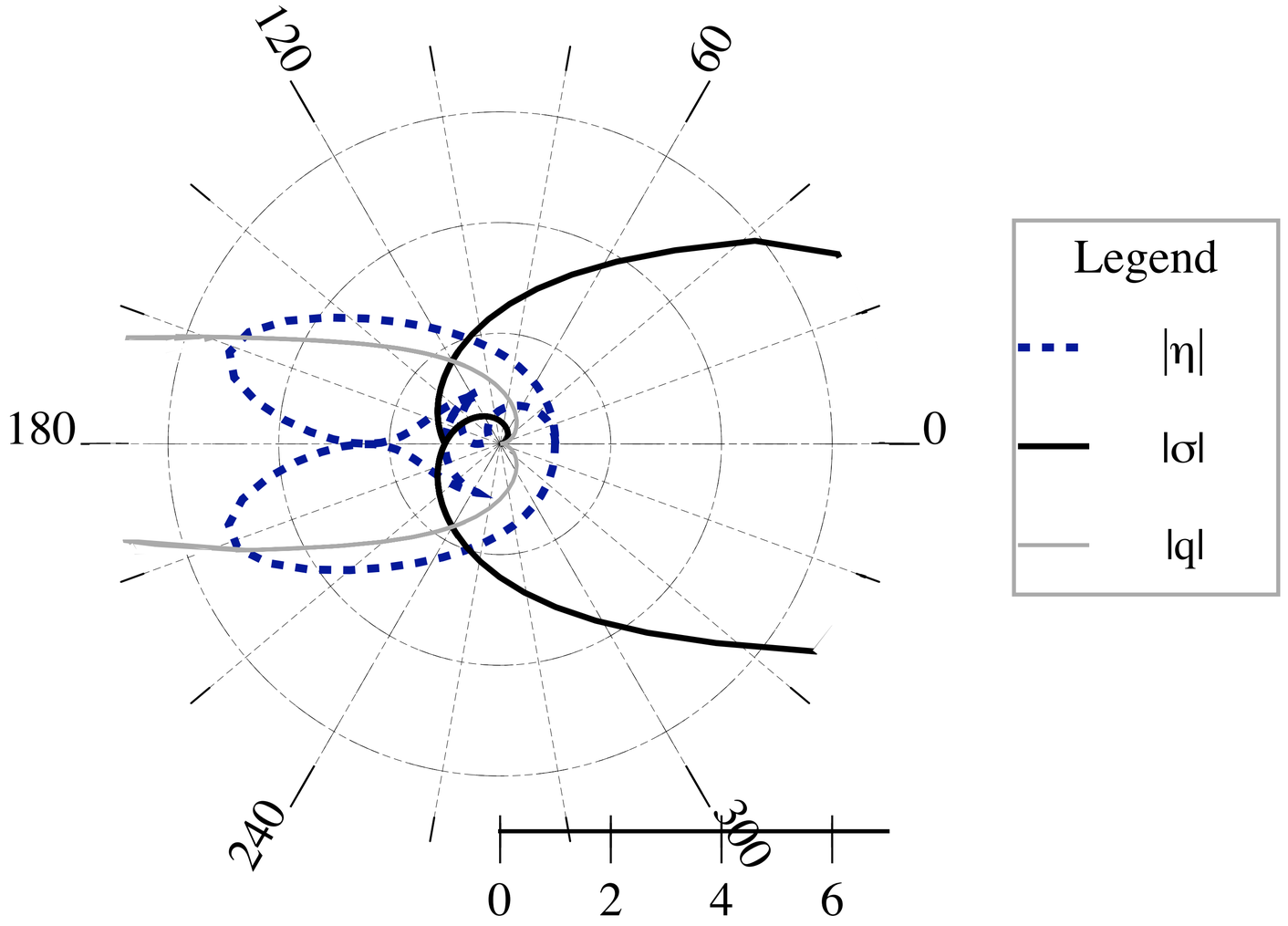
$$\theta_2(t) = (1 - \cos 2t)\frac{\pi}{2}$$

$$\theta_3(t) = (\sin 2t)\frac{\pi}{4}$$

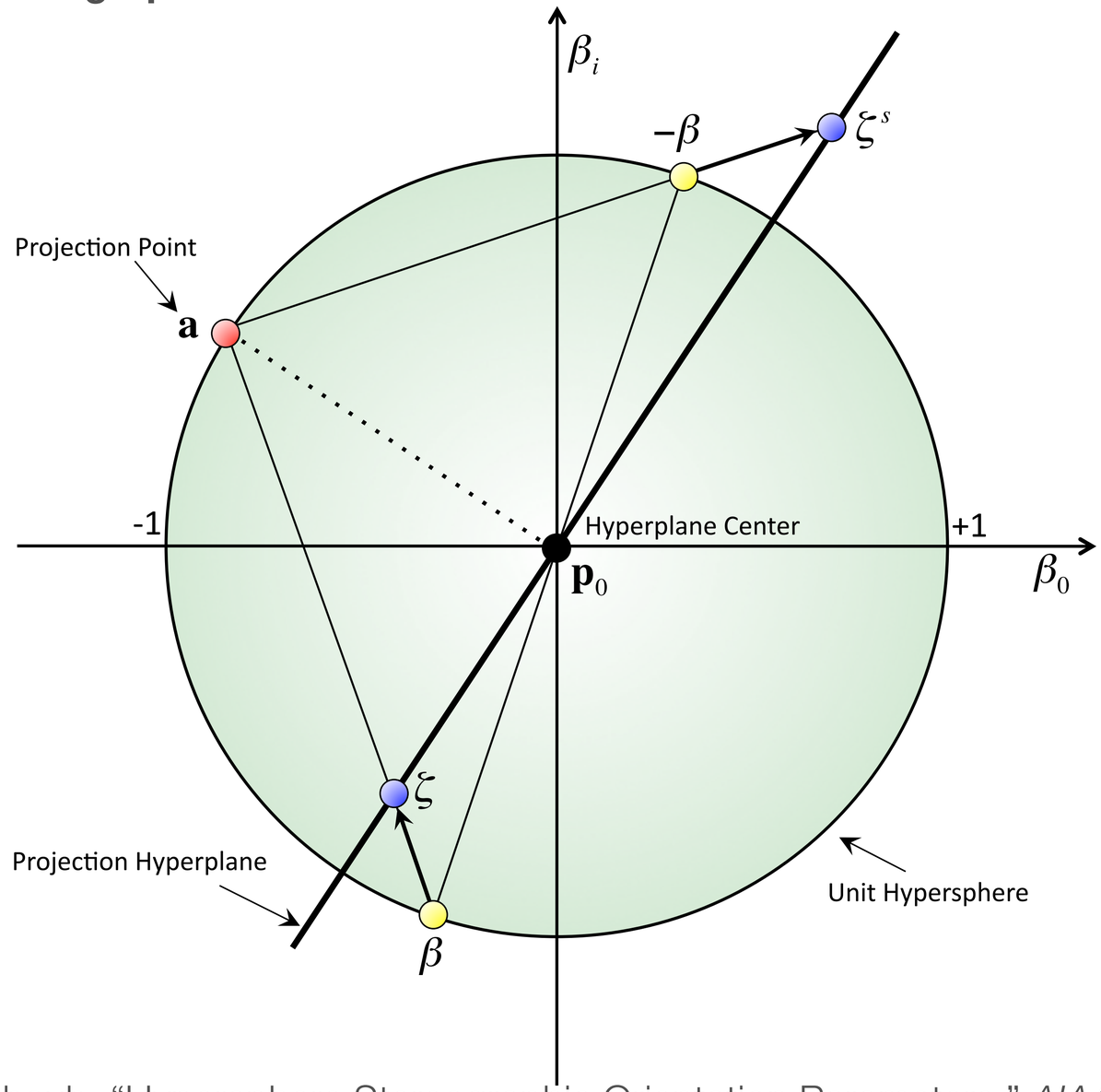


The body is essentially doing a tumble about the 1st body axis, while doing sinusoidal wobbles about the other axes.




120
60
180
0
240
300
0
2
4
6
Legend
|η|
|σ|
|q|

Hyper-Surface Stereographic Orientation Parameters



Mullen and H. Schaub, "Hypersphere Stereographic Orientation Parameters," *AIAA Journal of Guidance, Control and Dynamics*, Vol. 33, No. 1, Jan.–Feb., 2010, pp. 249–254. doi:10.2514/1.46783