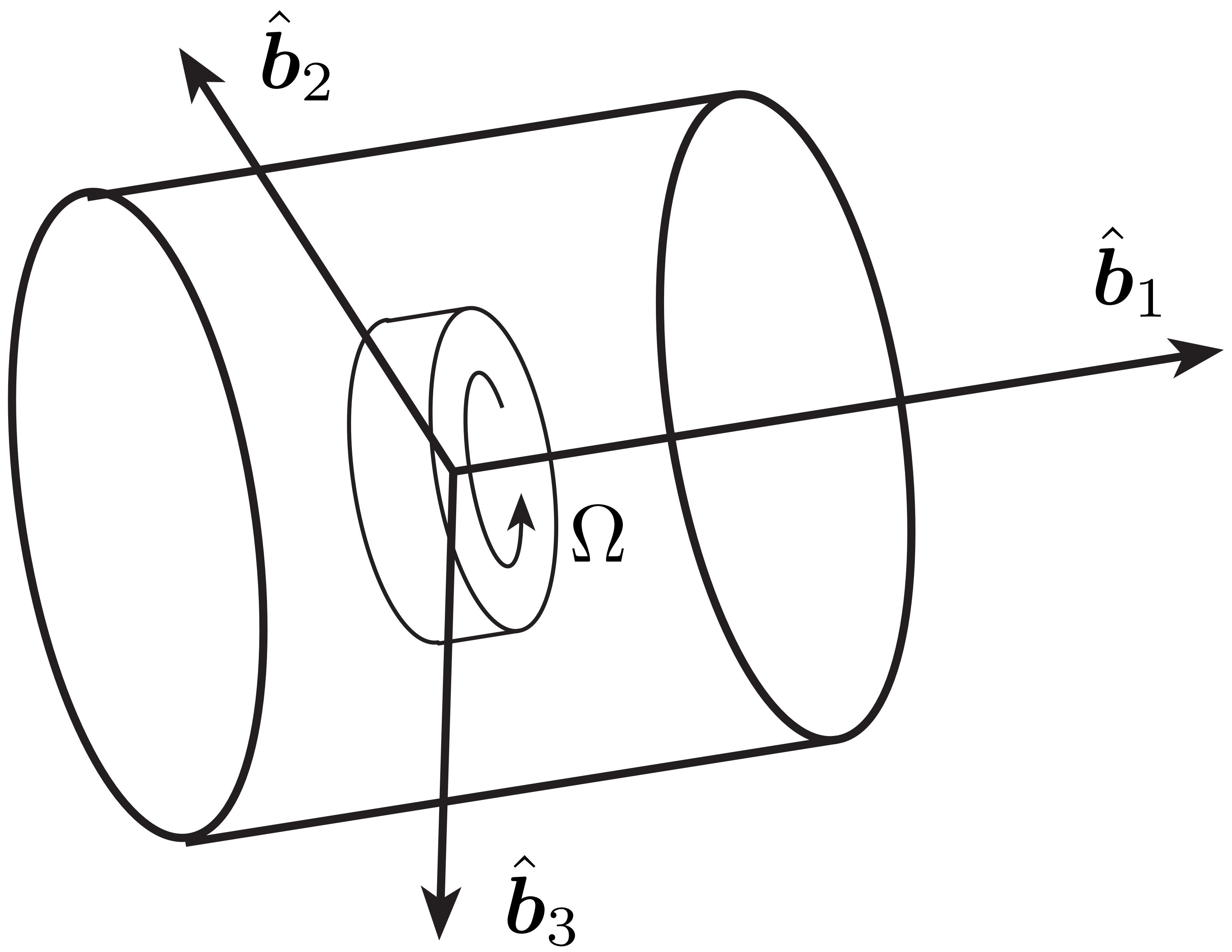
Dual Spin Spacecraft

Elegant attitude stabilization method...

University of Colorado
Boulder

Equations of Motion

- Assume a rigid spacecraft has an internal fly wheel, whose *constant* spin axis is aligned with the first body axis.



Note: The spacecraft inertia magnitude about each body axis is still free to be chosen.

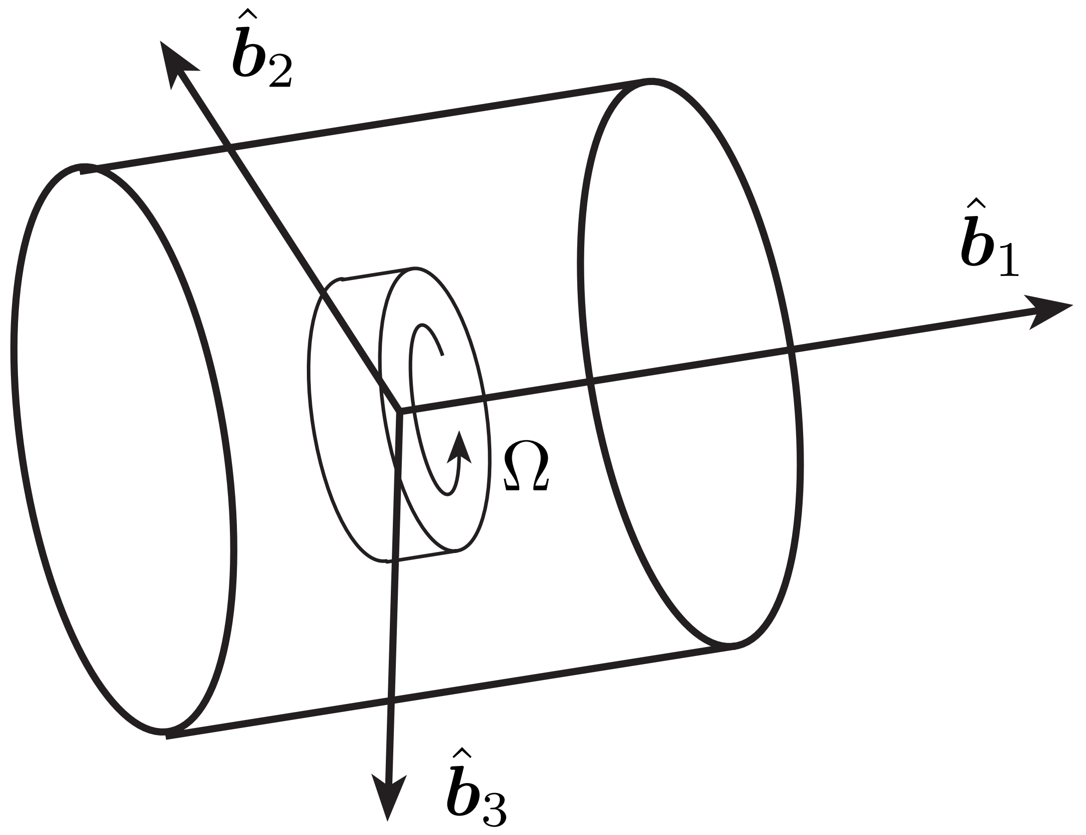
Total inertia matrix:

$$[I] = [I_s] + [I_W] = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}$$

Total ang. momentum:

$$\boldsymbol{H} = [I]\boldsymbol{\omega} + \underbrace{I_W \Omega}_{h} \hat{\boldsymbol{b}}_1$$

$$\boldsymbol{\omega} = \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}$$



Differentiate to use Euler's equation:

$$\dot{\boldsymbol{H}} = \frac{{}^{\mathcal{B}}\mathrm{d}}{\mathrm{d}t}(\boldsymbol{H}) + \boldsymbol{\omega} \times \boldsymbol{H}$$
$$= [I]\dot{\boldsymbol{\omega}} + \dot{h}\hat{\boldsymbol{b}}_1 + [\tilde{\boldsymbol{\omega}}][I]\boldsymbol{\omega} + \boldsymbol{\omega} \times (h\hat{\boldsymbol{b}}_1) = \boldsymbol{L}$$

Differential equations of motion with no external torque:

$$\boxed{[I]\dot{\boldsymbol{\omega}} = -[\tilde{\boldsymbol{\omega}}][I]\boldsymbol{\omega} - \dot{h}\hat{\boldsymbol{b}}_1 - h\omega_3\hat{\boldsymbol{b}}_2 + h\omega_2\hat{\boldsymbol{b}}_3}$$

with $\dot{h} = I_W\dot{\Omega}$

- Using Euler's equation, we find the spacecraft equations of motion with a constant speed fly wheel to be:

$$\begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \begin{pmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{pmatrix} = - \begin{pmatrix} (I_3 - I_2)\omega_2\omega_3 \\ (I_1 - I_3)\omega_1\omega_3 \\ (I_2 - I_1)\omega_1\omega_2 \end{pmatrix} + I_W \begin{pmatrix} -\dot{\Omega} \\ -\Omega\omega_3 \\ \Omega\omega_2 \end{pmatrix}$$

- This vector equation can also be written as three scalar equations:

$$\dot{\omega}_1 = \frac{I_2 - I_3}{I_3}\omega_2\omega_3 - \frac{I_W}{I_1}\dot{\Omega}$$
$$\dot{\omega}_2 = \frac{I_3 - I_1}{I_2}\omega_1\omega_3 - \frac{I_W}{I_2}\omega_3\Omega$$
$$\dot{\omega}_3 = \frac{I_1 - I_2}{I_3}\omega_1\omega_2 + \frac{I_W}{I_3}\omega_2\Omega$$

Linear Stability Analysis

- To determine the stability of this dual-spin spacecraft with constant wheel rate, we assume that the angular rate vector is an equilibrium rotation rate $\boldsymbol{\omega}_e$.
- Next, we study small variations in angular rates about this equilibrium position.
- For the equilibrium motion, note that

$$\boldsymbol{\omega} = \boldsymbol{\omega}_e + \delta\boldsymbol{\omega}$$

$$\dot{\omega}_{e_1} = \frac{I_2 - I_3}{I_1}\omega_{e_2}\omega_{e_3} = 0$$
$$\dot{\omega}_{e_2} = \frac{I_3 - I_1}{I_2}\omega_{e_1}\omega_{e_3} - \frac{I_{W_s}}{I_2}\omega_{e_3}\Omega = 0$$
$$\dot{\omega}_{e_3} = \frac{I_1 - I_2}{I_3}\omega_{e_1}\omega_{e_2} + \frac{I_{W_s}}{I_3}\omega_{e_2}\Omega = 0$$

- Substituting $\boldsymbol{\omega} = \boldsymbol{\omega}_e + \delta\boldsymbol{\omega}$ into the rigid body equations of motion yields:

$$(\dot{\omega}_{e_1} + \delta\dot{\omega}_1) = \frac{I_2 - I_3}{I_1}(\omega_{e_2} + \delta\omega_2)(\omega_{e_3} + \delta\omega_3)$$

$$(\dot{\omega}_{e_2} + \delta\dot{\omega}_2) = \frac{I_3 - I_1}{I_2}(\omega_{e_1} + \delta\omega_1)(\omega_{e_3} + \delta\omega_3) - \frac{I_{W_s}}{I_2}(\omega_{e_3} + \delta\omega_3)\Omega$$

$$(\dot{\omega}_{e_3} + \delta\dot{\omega}_3) = \frac{I_1 - I_2}{I_3}(\omega_{e_1} + \delta\omega_1)(\omega_{e_2} + \delta\omega_2) + \frac{I_{W_s}}{I_3}(\omega_{e_2} + \delta\omega_2)\Omega$$

- Next, assume that the space craft is spinning nominally about its first body axis

$$\boldsymbol{\omega}_e = \omega_{e_1}\hat{\boldsymbol{b}}_1 = \begin{pmatrix} \omega_{e_1} \\ 0 \\ 0 \end{pmatrix}$$

$$\omega_{e_2} = \omega_{e_3} = 0$$

- Dropping the higher order terms, and assuming that the equilibrium spin condition of interest is $\boldsymbol{\omega}_e = \omega_{e_1}\hat{\boldsymbol{b}}_1$, we find the following departure motion differential equations of motion.
-

$$\begin{aligned}\delta\dot{\omega}_1 &= 0\\ \delta\dot{\omega}_2 &= \frac{I_3 - I_1}{I_2}\omega_{e_1}\delta\omega_3 - \frac{I_{W_s}}{I_2}\delta\omega_3\Omega\\ \delta\dot{\omega}_3 &= \frac{I_1 - I_2}{I_3}\omega_{e_1}\delta\omega_2 + \frac{I_{W_s}}{I_3}\delta\omega_2\Omega\end{aligned}$$

- Note that $\delta\omega_1$ is constant and does not appear in the other two equations (decoupled from them).

$$\begin{aligned}\delta\dot{\omega}_2 &= \left(\frac{I_3 - I_1}{I_2}\omega_{e_1} - \frac{I_{W_s}}{I_2}\Omega\right)\delta\omega_3\\ \delta\dot{\omega}_3 &= \left(\frac{I_1 - I_2}{I_3}\omega_{e_1} + \frac{I_{W_s}}{I_3}\Omega\right)\delta\omega_2\end{aligned}$$

- Next, we take the derivative of $\delta\dot{\omega}_2$:

$$\delta\ddot{\omega}_2 = \left(\frac{I_3 - I_1}{I_2}\omega_{e_1} - \frac{I_{W_s}}{I_2}\Omega\right)\delta\dot{\omega}_3$$

- Substituting in the $\delta\dot{\omega}_3$ result from the previous page, we find the following decoupled body rate departure dynamics about the 2nd body axis:

$$\delta\ddot{\omega}_2 + \underbrace{\left(\frac{I_1 - I_3}{I_2}\omega_{e_1} + \frac{I_{W_s}}{I_2}\Omega\right)\left(\frac{I_1 - I_2}{I_3}\omega_{e_1} + \frac{I_{W_s}}{I_3}\Omega\right)}_{k}\delta\omega_2 = 0$$

Compare this to: $\delta\ddot{\omega}_2 + k\delta\omega_2 = 0$

Stability requires that $k > 0$

- The parameter k can be written as:

$$k = \frac{\omega_{e_1}^2}{I_2 I_3}\left(I_1 - I_3 + I_{W_s}\hat{\Omega}\right)\left(I_1 - I_2 + I_{W_s}\hat{\Omega}\right)$$

where $\hat{\Omega} = \dfrac{\Omega}{\omega_{e_1}}$

Zero RW Spin Rate

- First, let's verify the classical rigid body spin stability analysis if the RW spin rate is zero.
- For stability, we require *k*>0:

$$k = \frac{\omega_{e_1}^2}{I_2 I_3}\left(I_1 - I_3\right)\left(I_1 - I_2\right) > 0$$

True if:

$I_1 > I_3 \qquad I_1 > I_2 \Rightarrow$ Max. Inertia Case

$I_1 < I_3 \qquad I_1 < I_2 \Rightarrow$ Min. Inertia Case

Non-Zero RW Spin

- Next, let's look at the stability requirement if the RW spin rate is nonzero:

$$k = \frac{\omega_{e_1}^2}{I_2 I_3}\left(I_1 - I_3 + I_{W_s}\hat{\Omega}\right)\left(I_1 - I_2 + I_{W_s}\hat{\Omega}\right) > 0$$

True if:

$I_1 > I_3 - I_{W_s}\hat{\Omega}$	$I_1 > I_2 - I_{W_s}\hat{\Omega}$
$I_1 < I_3 - I_{W_s}\hat{\Omega}$	$I_1 < I_2 - I_{W_s}\hat{\Omega}$

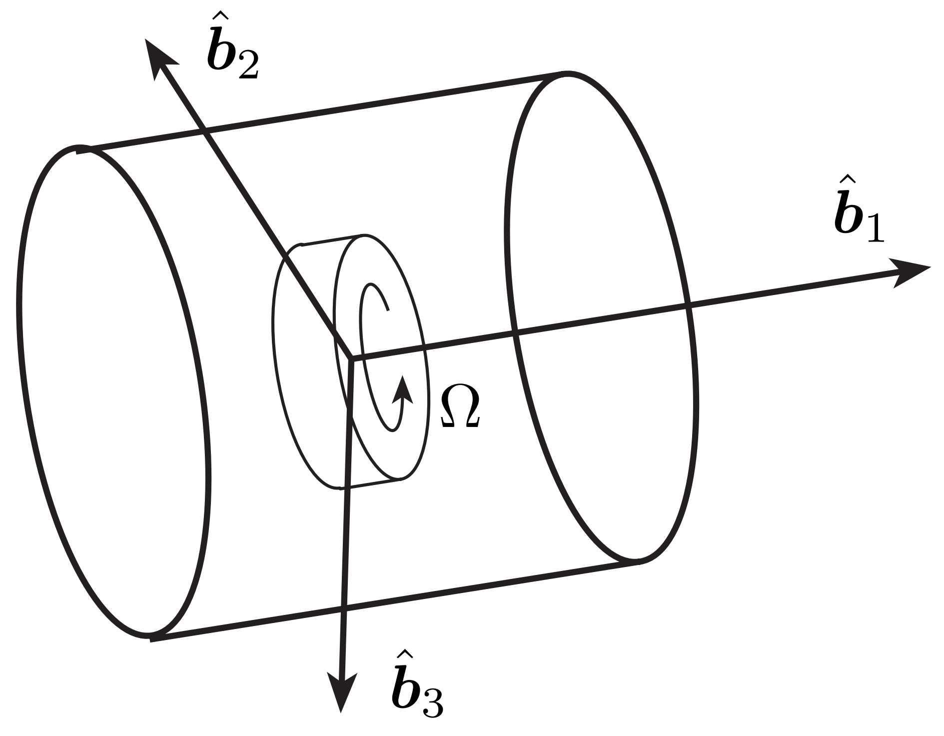
Note: The spacecraft spin can be made stable, regardless if I_1 is a major, intermediate or minor inertia!

Note: Careless use of the RW mode can also cause the spacecraft spin too become unstable.

Example

- Consider a spacecraft to have the following inertias:

$$
\begin{aligned}
I_1 &= 350\ \text{kgm}^2 \\
I_2 &= 300\ \text{kgm}^2 \\
I_3 &= 400\ \text{kgm}^2 \\
I_{W_s} &= 10\ \text{kgm}^2
\end{aligned}
$$



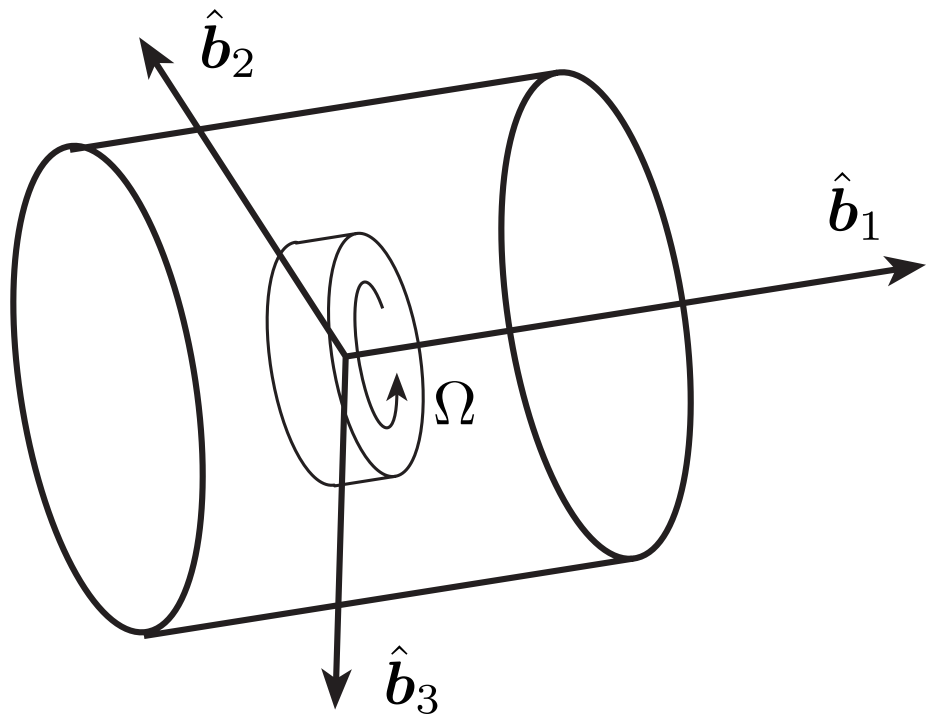
- Without the fly-wheel, note that spinning abut the first body axis would be unstable.
- The spacecraft spin about $\hat{\boldsymbol{b}}_1$ is 60 RPM.
- How fast does the wheel have to spin to make this spacecraft a stable dual-spin system?

- The stability conditions are:

Set 1: $I_1 > I_3 - I_{W_s}\hat{\Omega}$ $\quad$ $I_1 > I_2 - I_{W_s}\hat{\Omega}$

Set 2: $I_1 < I_3 - I_{W_s}\hat{\Omega}$ $\quad$ $I_1 < I_2 - I_{W_s}\hat{\Omega}$

Since $I_1 > I_2$, the second condition of set 1 is satisfied if $\hat{\Omega} > -5$

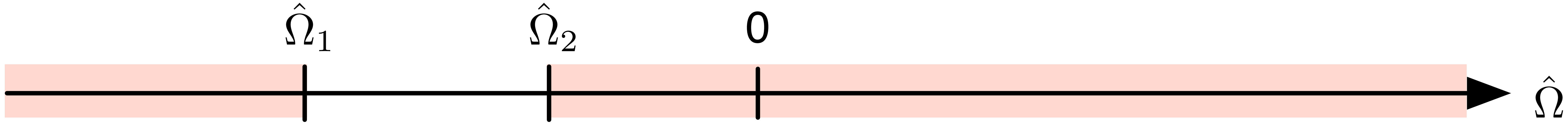


The first condition of set 1 then requires that:

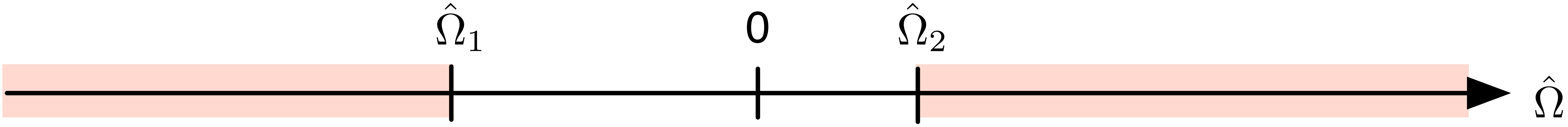
$$
\begin{aligned}
I_1 &> I_3 - I_{W_s}\hat{\Omega} \\
I_{W_s}\hat{\Omega} &> I_3 - I_1 \\
\hat{\Omega} &> \frac{I_3 - I_1}{I_{W_s}} \\
\hat{\Omega} &> 5
\end{aligned}
$$

$$
\begin{aligned}
\Omega &= \hat{\Omega}\omega_{e_1} \\
&= 300 \text{ RPM}
\end{aligned}
$$

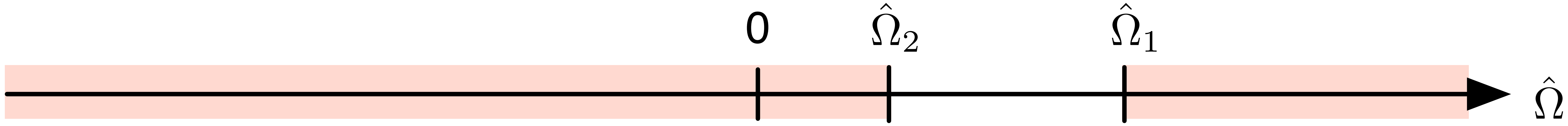
Rotor spinning about major axis:



Rotor spinning about intermediate axis:

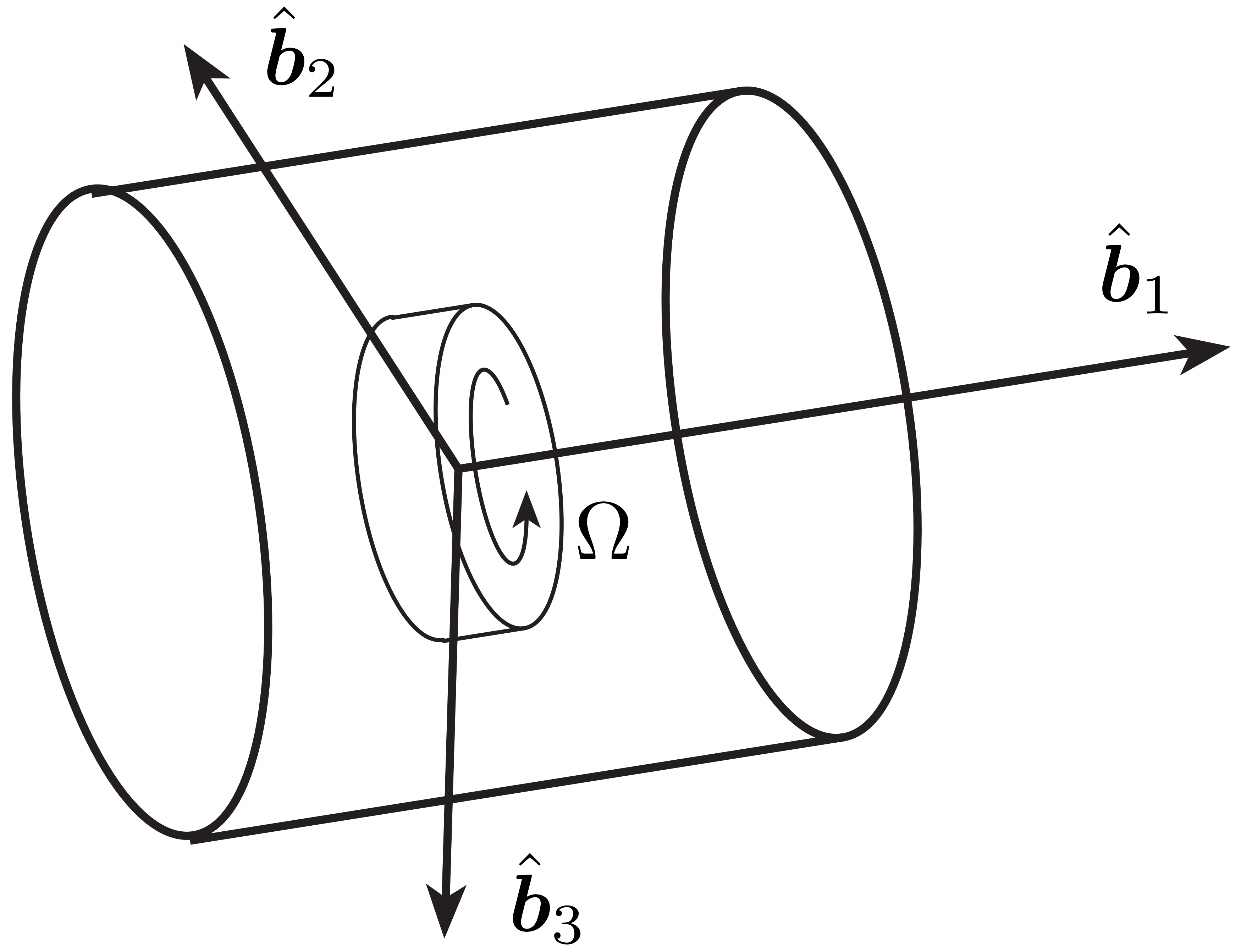


Rotor spinning about minor axis:



Spin-Up Study

- Next we investigate a classical spin-up maneuver with a dual-spin spacecraft. Assume the wheel is initial at rest relative to the spacecraft.
- The spacecraft is assumed to have a pure spin about a principal axis which his not aligned with the reaction wheel spin axis.
- Then the RW is spun up until it has the same amount of angular momentum as the spacecraft had initially.
- What will happen to the spacecraft attitude during this spin-up maneuver?

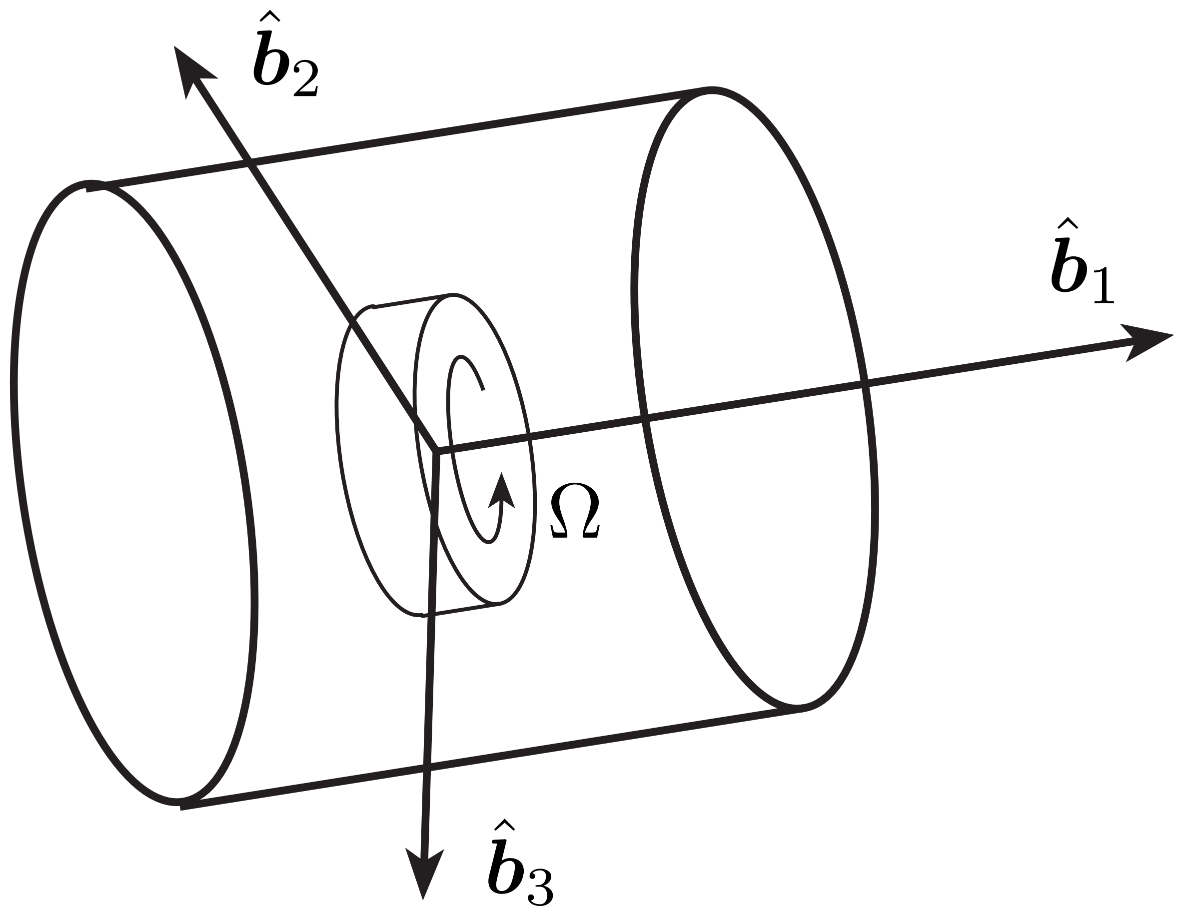


- Because no external torques are present, the angular momentum magnitude is constant and given by:

$$H = |\boldsymbol{H}(t_0)|$$

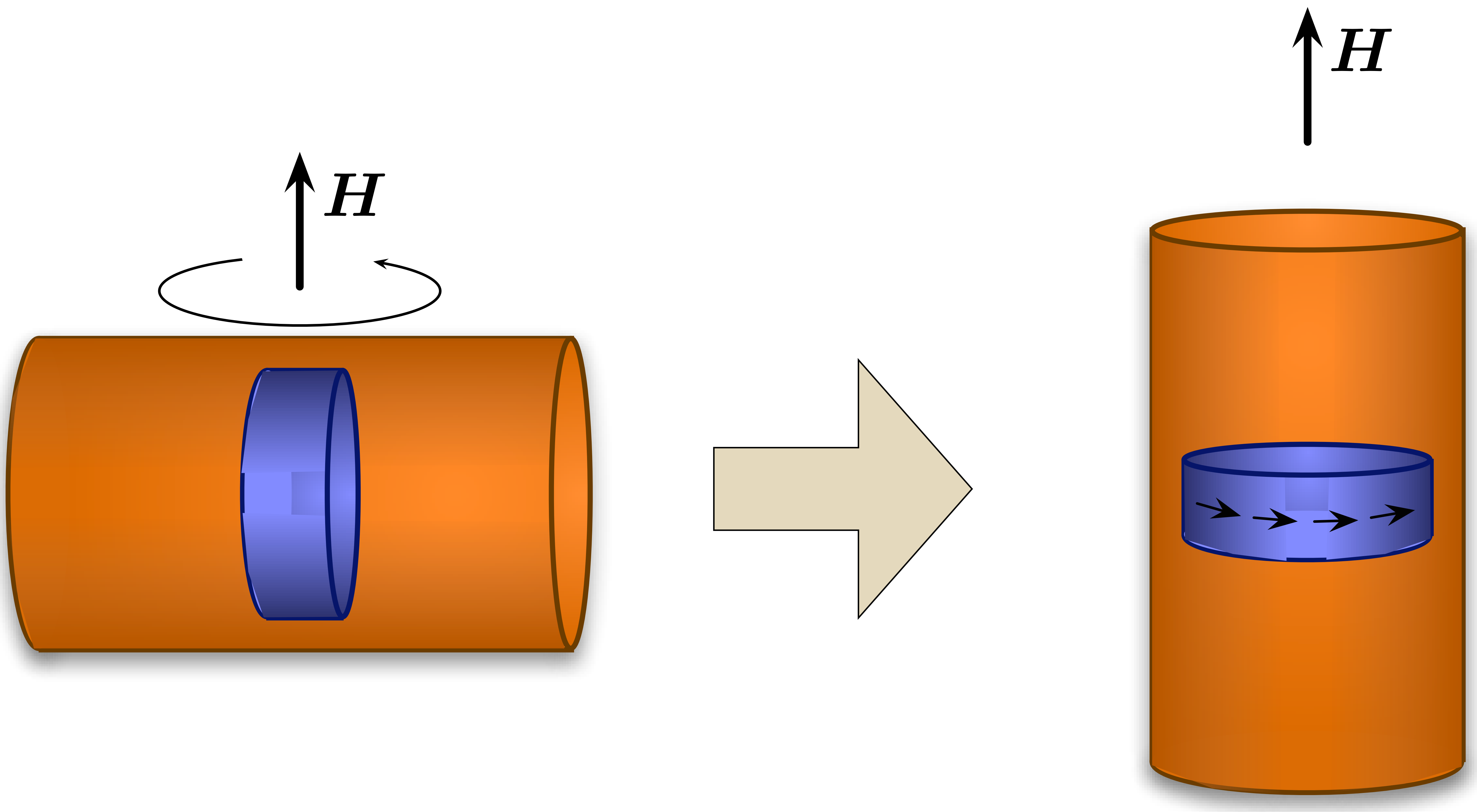
- The wheel angular momentum is increased at a constant rate through:

$$\dot{h} = I_W \dot{\Omega} = C = \text{constant}$$
$$h(t) = Ct$$



- The total maneuver time is

$$T_{\max} = \frac{H}{C} = \frac{H}{I_W \dot{\Omega}}$$



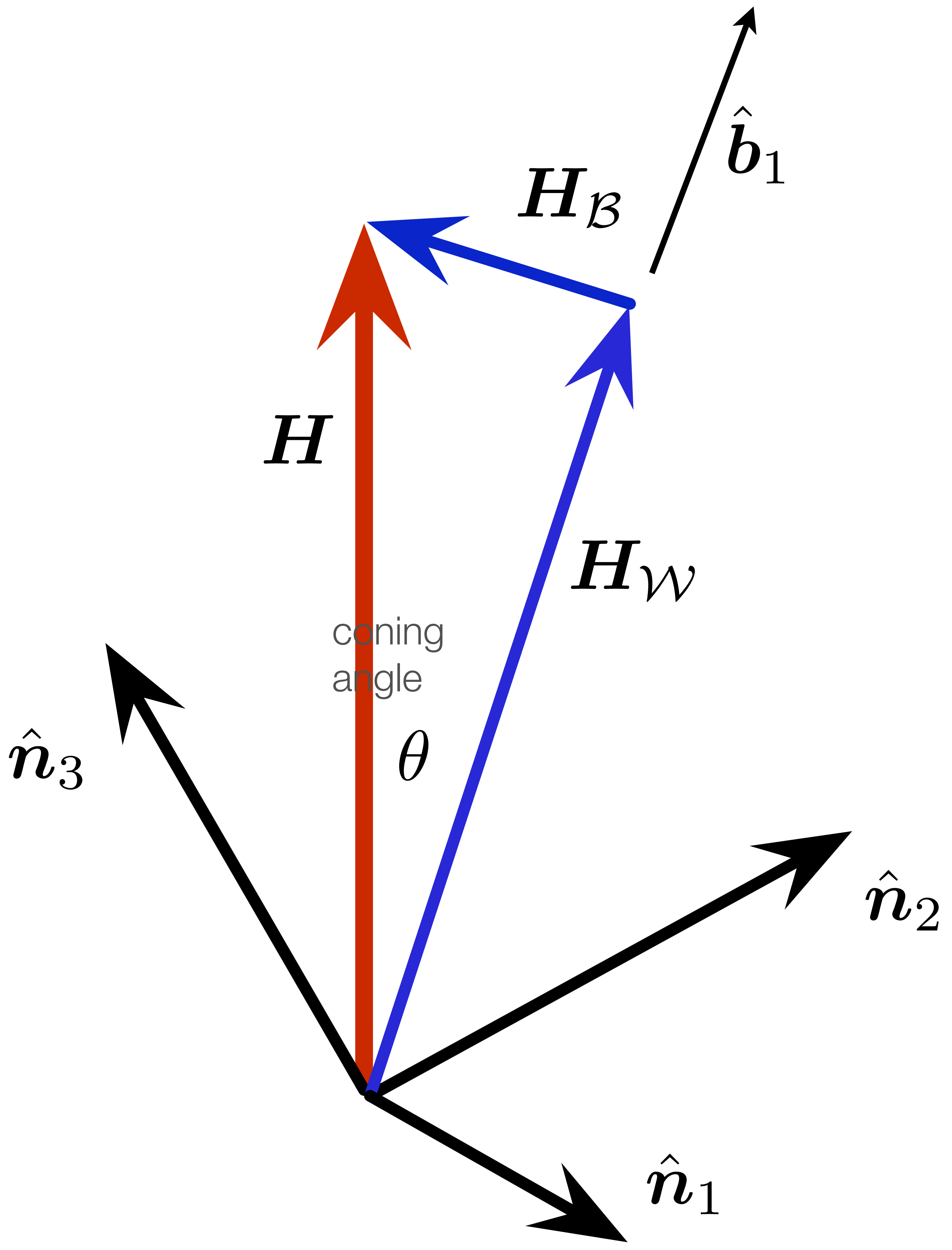
Question: As the RW assumes the same amount of angular momentum as the initially spinning spacecraft possessed, won't the angular momentum conservation cause the craft to realign RW spin axis along the momentum vector with the spacecraft at rest?

The answer is, not necessarily...

We are only controlling a single-degree of freedom, which is influencing the three-dimension motion of the spacecraft.

If the wheel angular momentum has the same magnitude as the initial system angular momentum $\boldsymbol{H}$, this does not mean that the wheel angular momentum $\boldsymbol{H}_W$ is aligned with the $\boldsymbol{H}$ vector.

Let's study this through an numerical example...



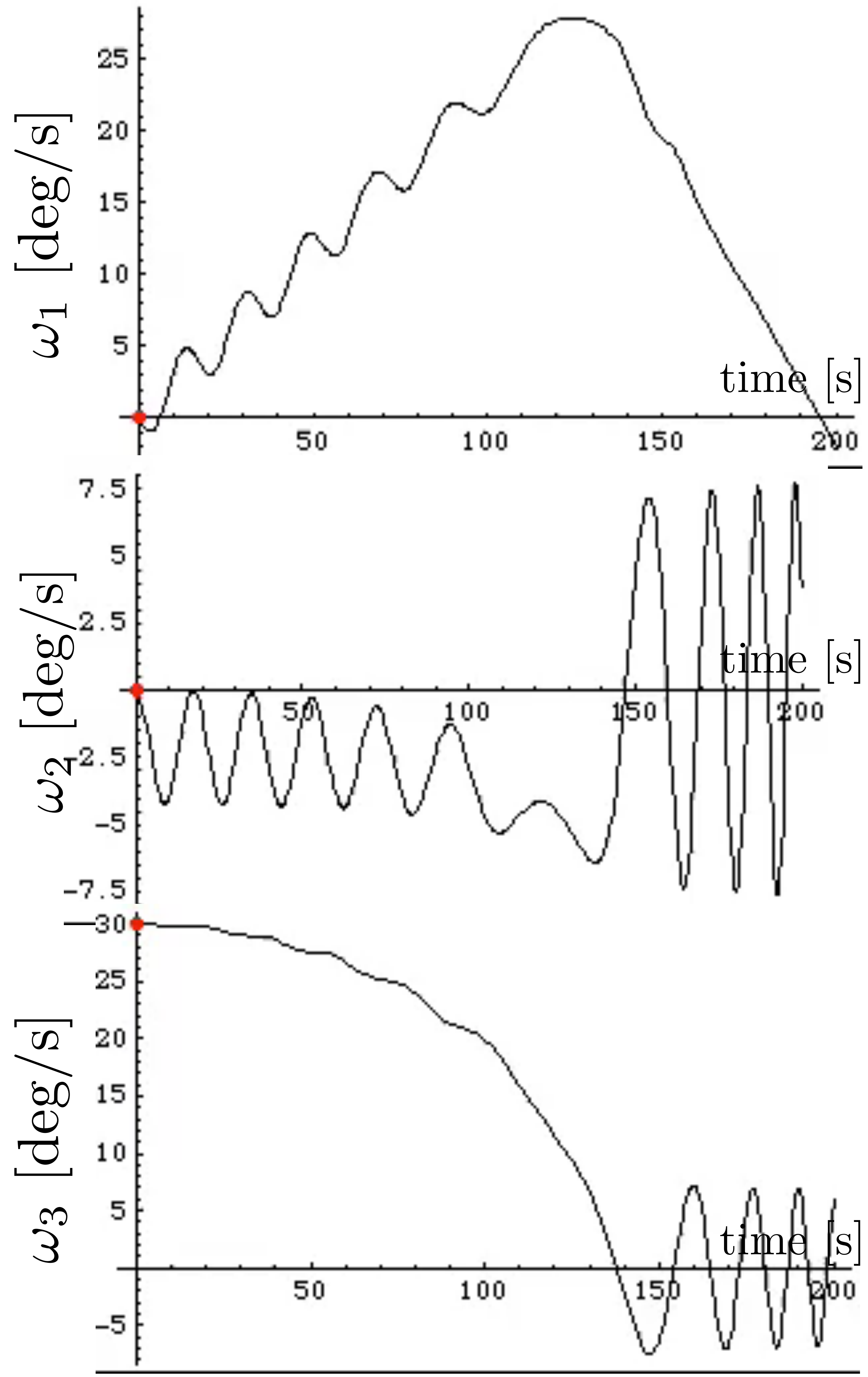
Example: Numerical Simulations of Spin up Maneuvers

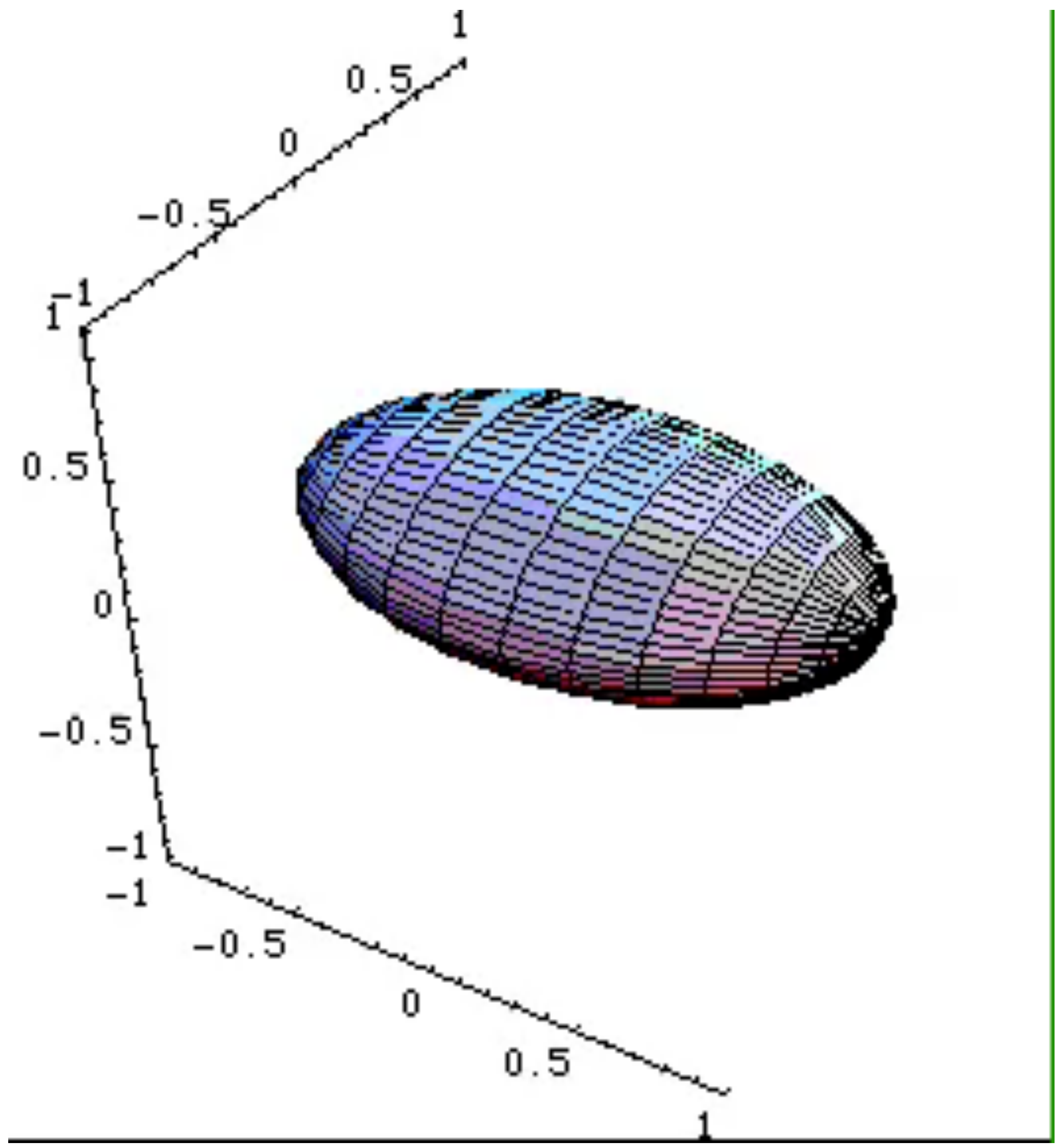
Spacecraft is initially in a flat spin about the major inertia principal axis.

The RW is spun up at different rates and the final coning angle θ is investigated.

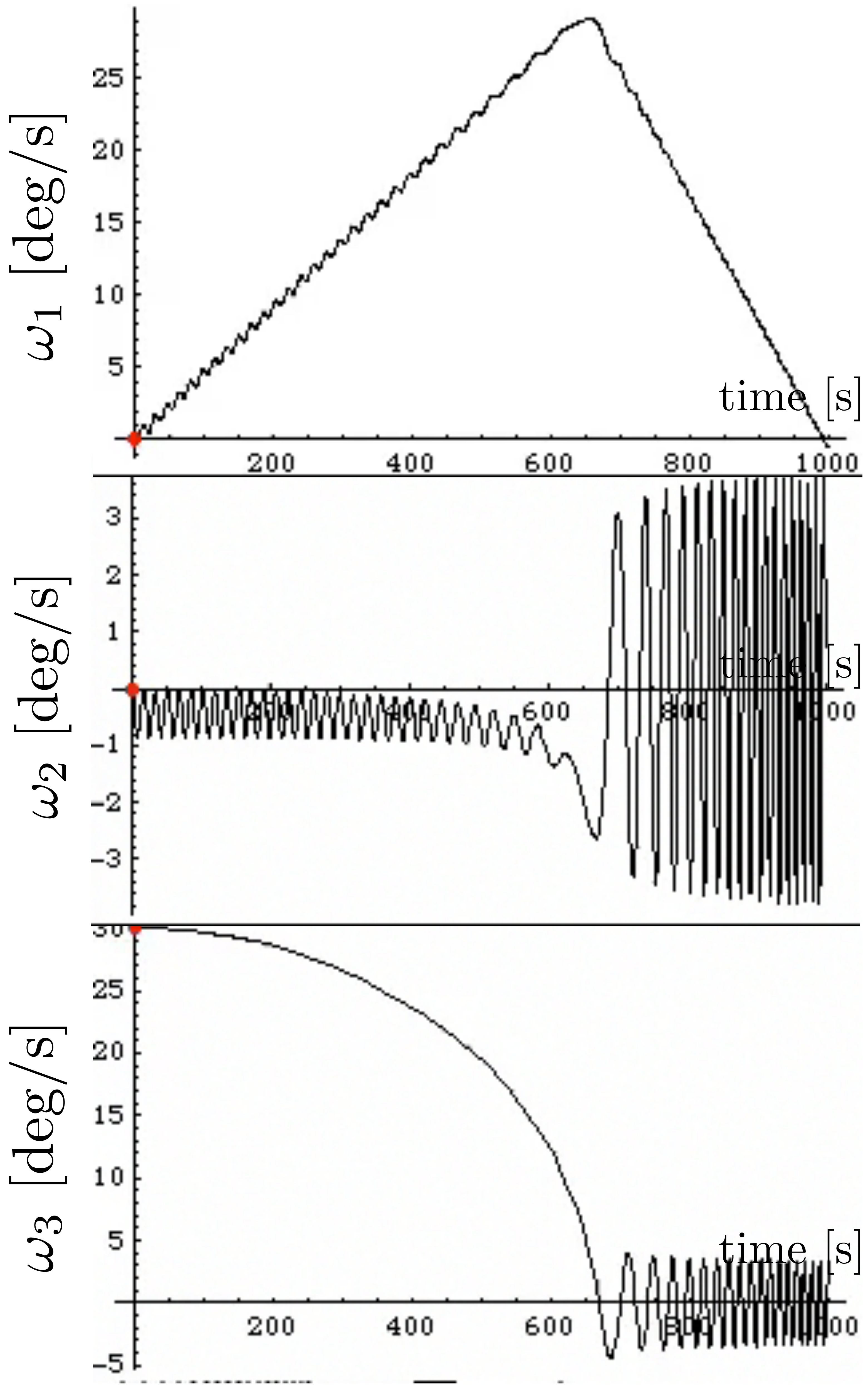
Simulation Parameters

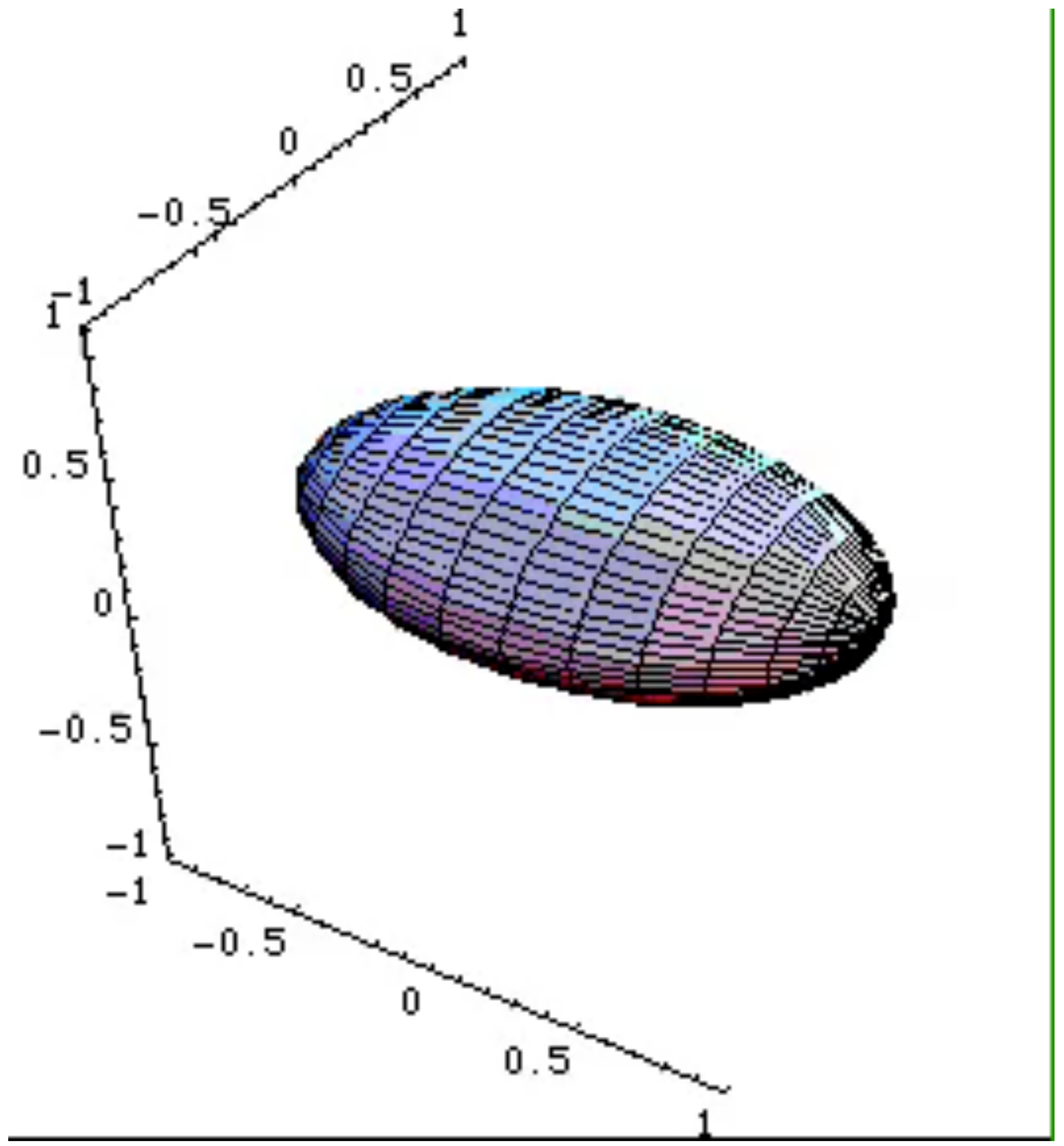
I_1	9.47 kgm^2
I_2	21.90 kgm^2
I_3	27.57 kgm^2
$\omega_3(t_0)$	30 °/s
$\omega_1(t_0)$, $\omega_2(t_0)$	0 °/s
$\Omega(t_0)$	0 °/s
I_W	1.89 kgm^2



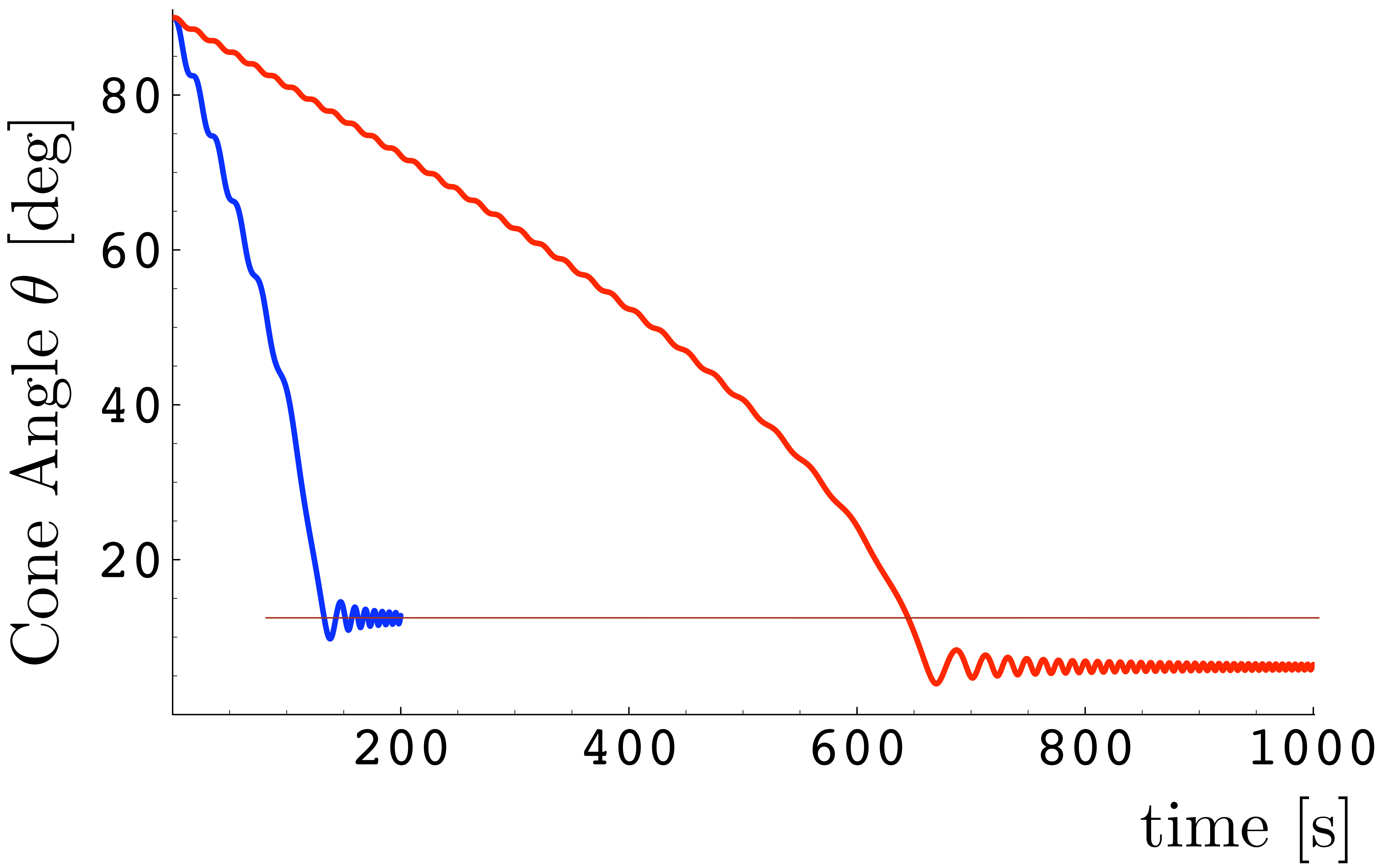


Maneuver Time: 200 seconds





Maneuver Time: 1000 seconds


Cone Angle θ [deg]
80
60
40
20
200
400
600
800
1000
time [s]

- To study the spin-up dynamics, we can use the momentum sphere –energy ellipsoid method.*

Spacecraft Kinetic Energy:

$$E^* = \frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2)$$

The control objective is to drive this positive definite measure of the spacecraft motion to zero!

Momentum Sphere:

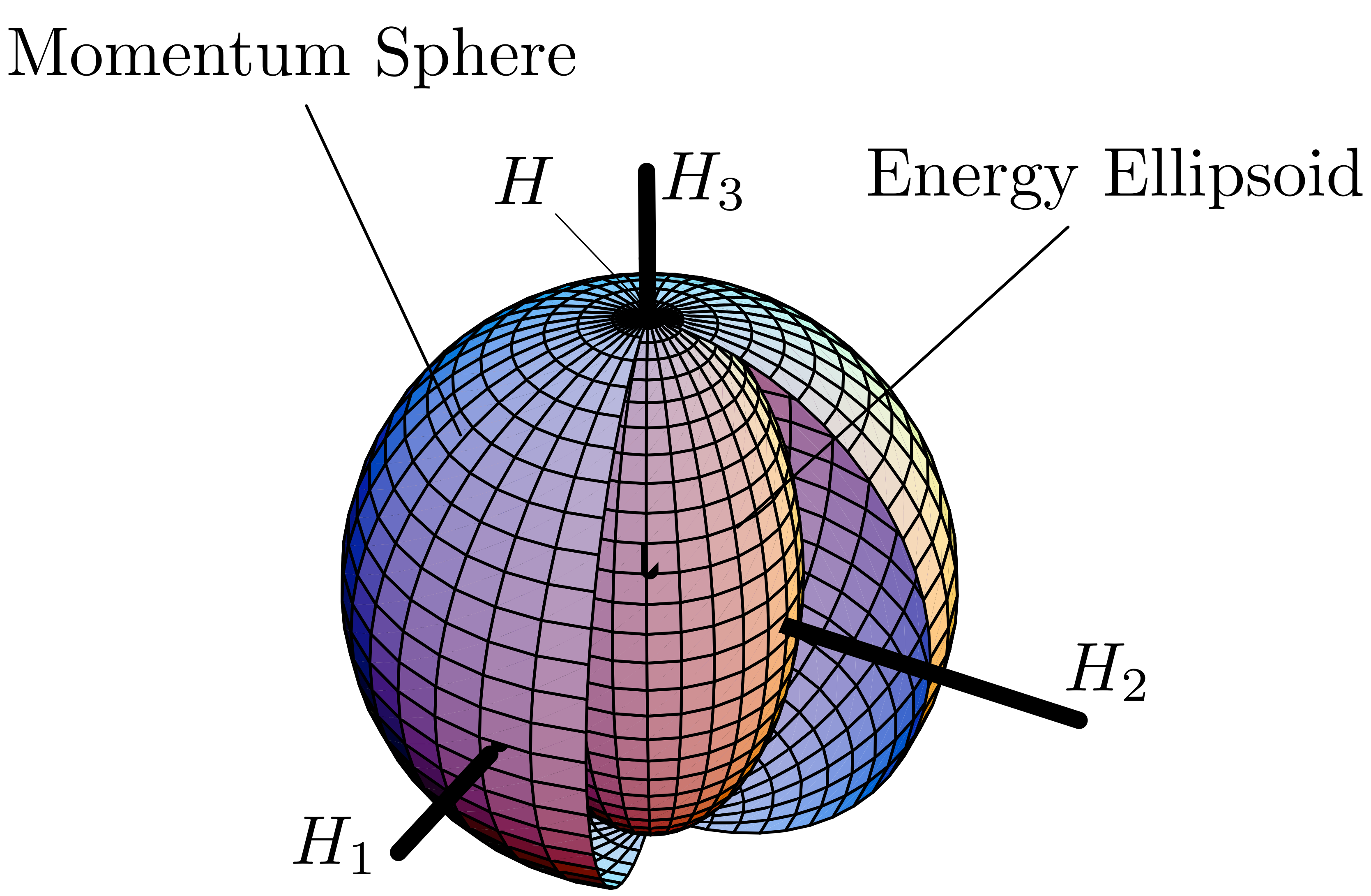
$$H^2 = H_1^2 + H_2^2 + H_3^2$$

Energy Ellipsoid:

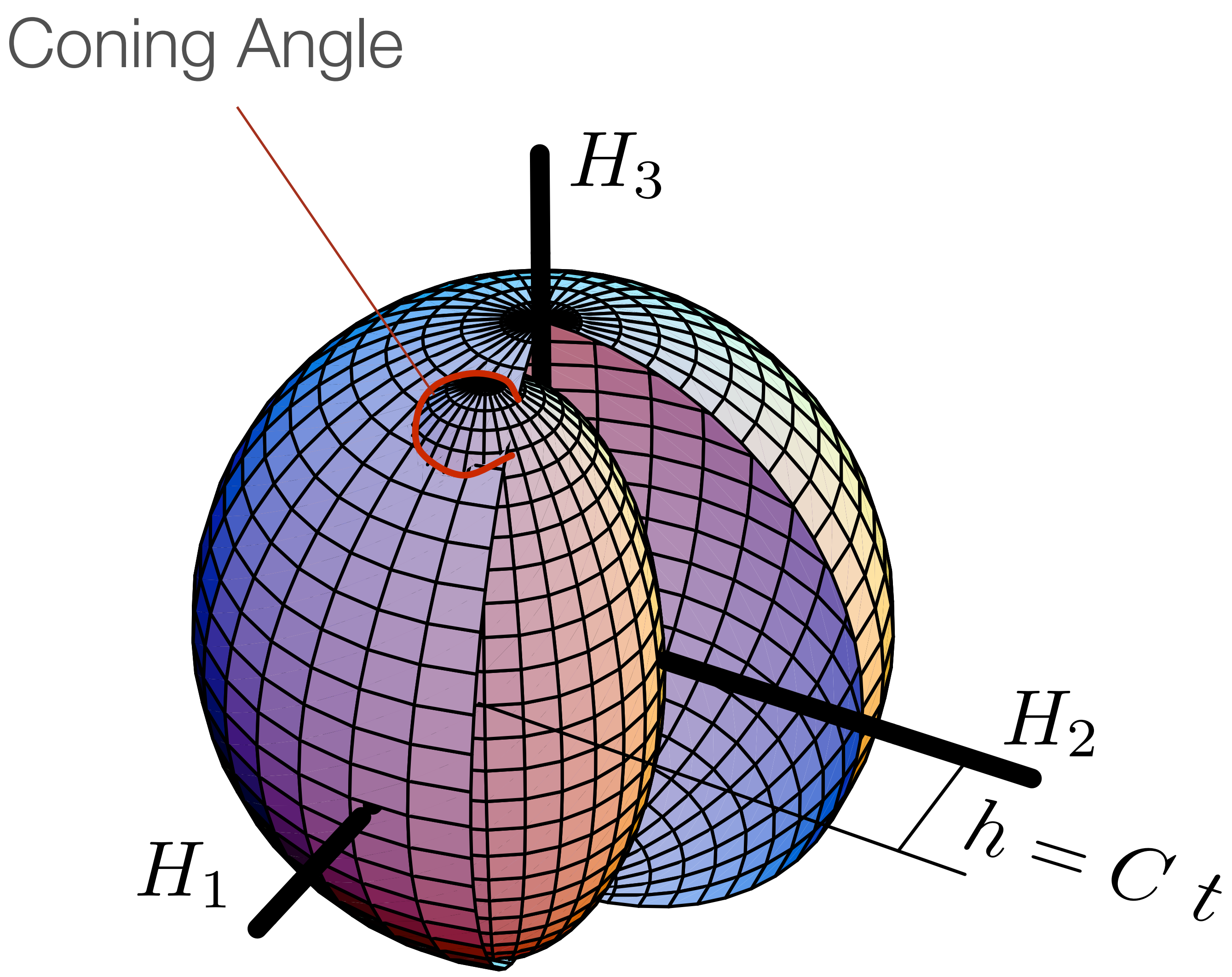
$$1 = \frac{(H_1 - h)^2}{2I_1E^*} + \frac{H_2^2}{2I_21E^*} + \frac{H_3^2}{2I_3E^*}$$

Note how the energy ellipsoid size will vary as the spacecraft kinetic energy is reduced, and how the ellipsoid center will shift along the first body axis.

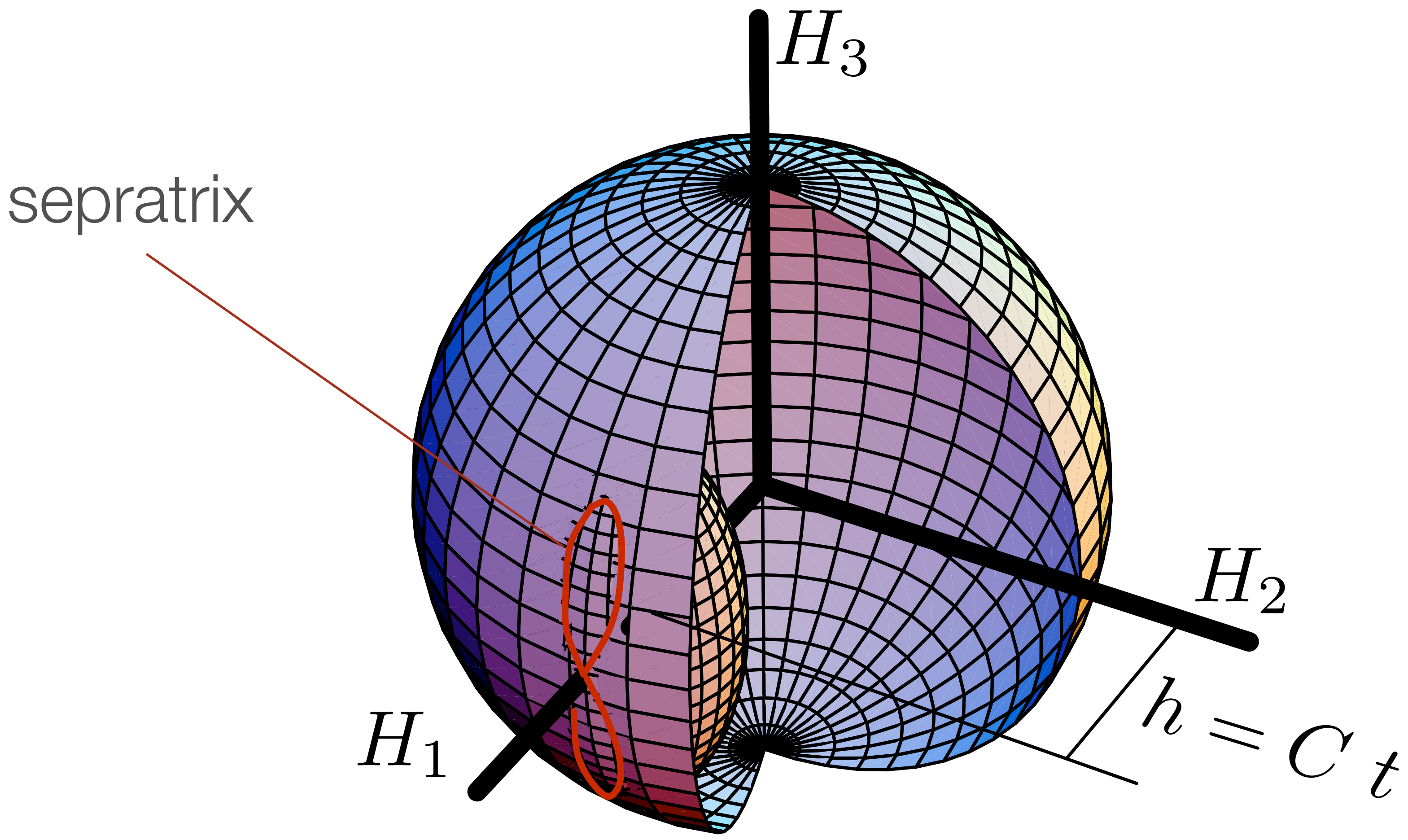
*Barba, P., and Auburn, J., "Satellite Attitude Acquisition by Momentum Transfer," Paper #AAS-75-053, Presented at the AAS/AIAA Astrodynamics Conference, Nasau, Bahamas, July 1975.



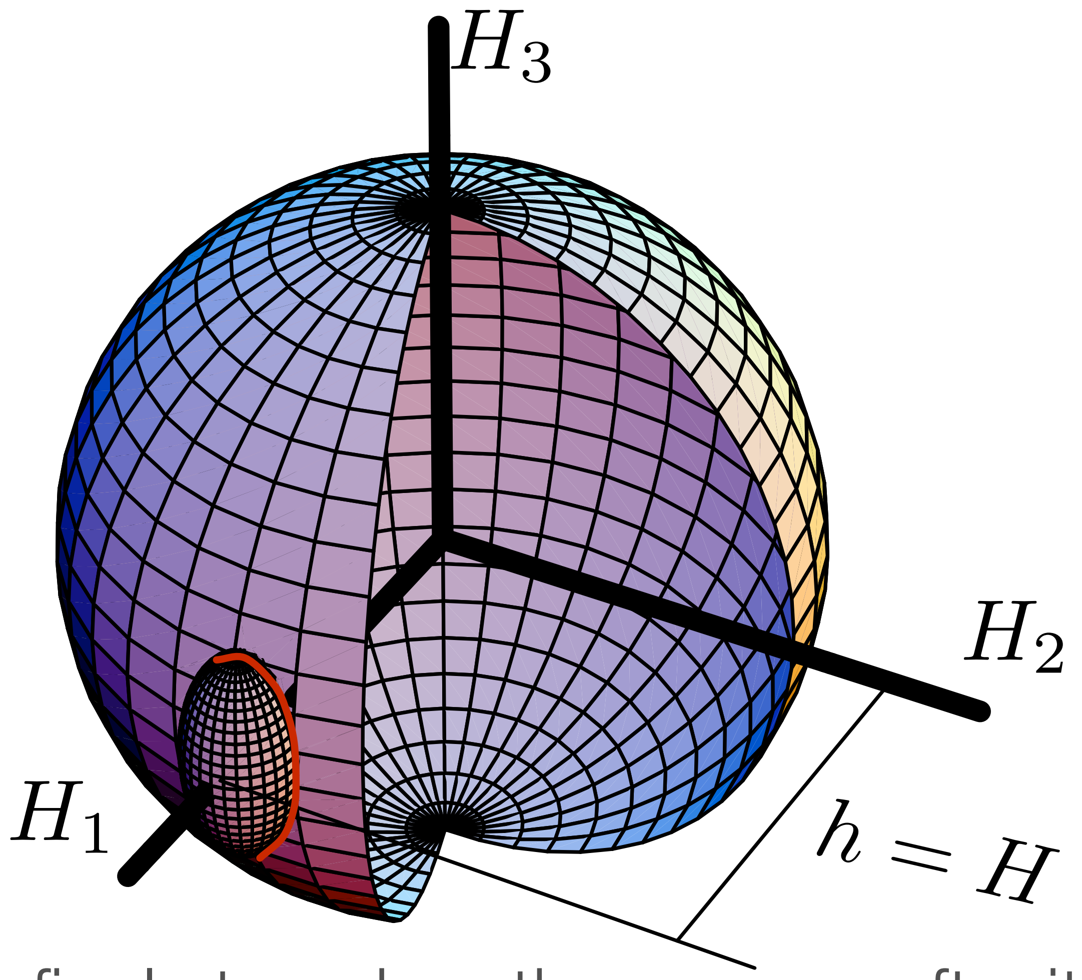
Spacecraft with pure spin about 3rd axis, RW at rest



The RW is spun up and reduces the spacecraft kinetic energy.



A critical energy condition is reached with the sepratrix.



The final stage has the spacecraft with a non-zero energy state, and some coning.