*The orientation of the body frame with respect to the reference frame is given by satellite's attitude.*

*This orientation is represented by a proper orthogonal matrix called as rotation matrix or attitude matrix.*

**COURSE I: Kinematics: Describing the Motion of Spacecraft**

**1. Performed Attitude Method:**

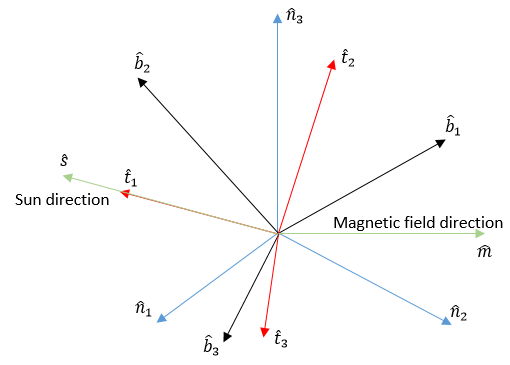
+ A minimum of three coordinates is required to describe the relative angular displacement between two reference frames.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Parameter-  ization | Dimen-sion | Attitude Matrix | Kinematic Equations | Singularities | Constraints |
| DCM | 9 |  |  | None |  |
| Euler Angles | 3 |  |  | Symmetric set:  Asymmetrix set: | None |
| Quaternions | 4 |  |  | None |  |
| CRP | 3 |  |  |  | None |
| MRP | 3 |  |  |  | None |

**2. Attitude Determination:**

+ It needs a minimum of two observation vectors to determine the three dimensional orientation.

* *TRIAD Method:*



+ Input: 2 direction vectors (Sun, Earth, Magnetic field direction, Stars, Moon, …).

+ Output: DCM from Inertial frame to Body fixed frame.

+ Advantage: Easy to operate and calculate.

+ Disadvantage: Exist case that is 2 vectors parallel each other.

* *Wahba’s Problem:*

+ Evaluate the measurements by loss function:

* *Devenport’s q-Method:*

+ Input: 2 direction vectors (Sun, Earth, Magnetic field direction, Stars, Moon, …).

+ Output: A quaternions is eigenvector corresponding the largest eigenvalue of matrix.

+ Advantage: Minimize the loss function J.

+ Disadvantage: Hard to find eigenvalues and eigenvector of matrix.

* *QUEST Method:*

+ Input: 2 direction vectors (Sun, Earth, Magnetic field direction, Stars, Moon, …).

+ Output: A CRP vector corresponding the optimal eigenvalue of matrix.

+ Advantage:

* It uses a classic Newton-Raphson to find optimal eigenvalue. This allows us to avoid the numerically intensive eigenvalue problem.
* It introduces CRP vector that is easier to calculate a matrix.

+ Disadvantage: optimal eigenvalue is a approximate value, therefore accuracy of measurements is lower than q-Method.

**COURSE II: Kinetics: Studying Spacecraft Motion**

**1. Rigid Body Dynamics:**

**+** Total Energy:

+ Energy Rate:

+ Angular Momentum:

+ Equations of Motion:

* Euler’s Equation:
* Euler’s rotational equations of motion:

**2. Momentum/Energy Surface:**

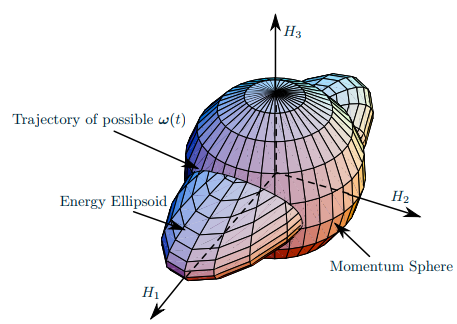
+ No external torque os acting on the body Energy and Momentum are conserved.

+ Momentum:

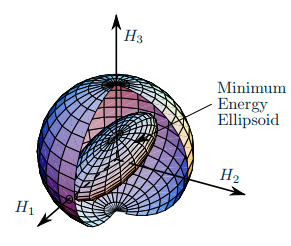
+ Energy:

The admissible angular velocities will be on the intersection of these two ellipsoids.

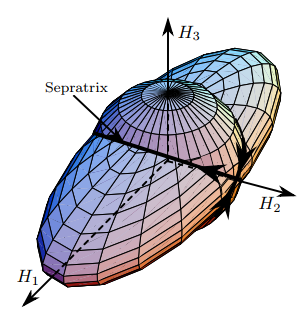
+ Assume:



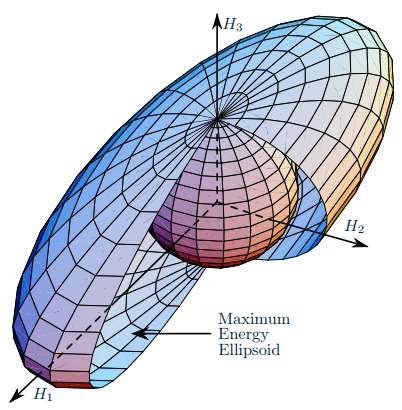
* Minimum energy case:



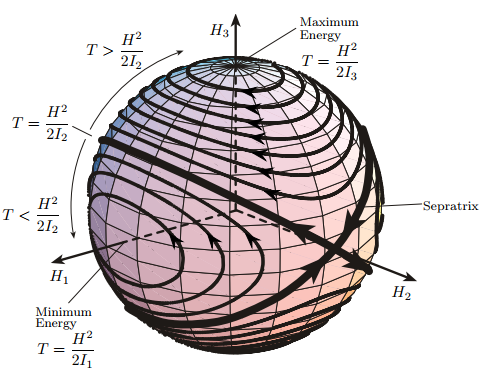
* Intermadiate energy case:



* Maximum energy case:



* Family of energy ellipsoid and momentum sphere intersections.

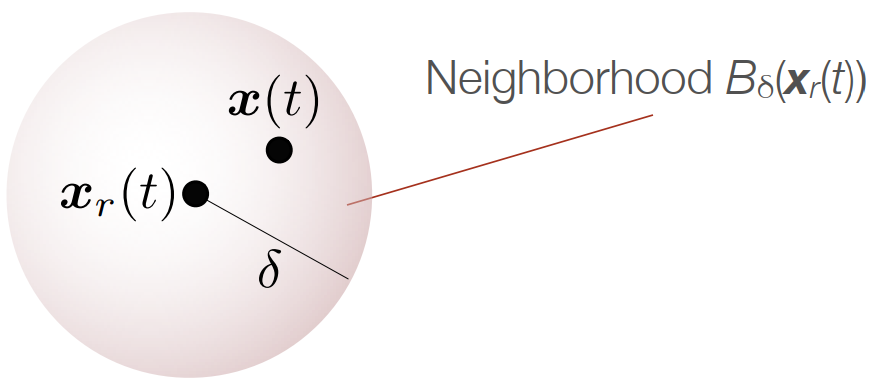


**COURSE III: Control of Nonlinear Spacecraft Attitude Motion**

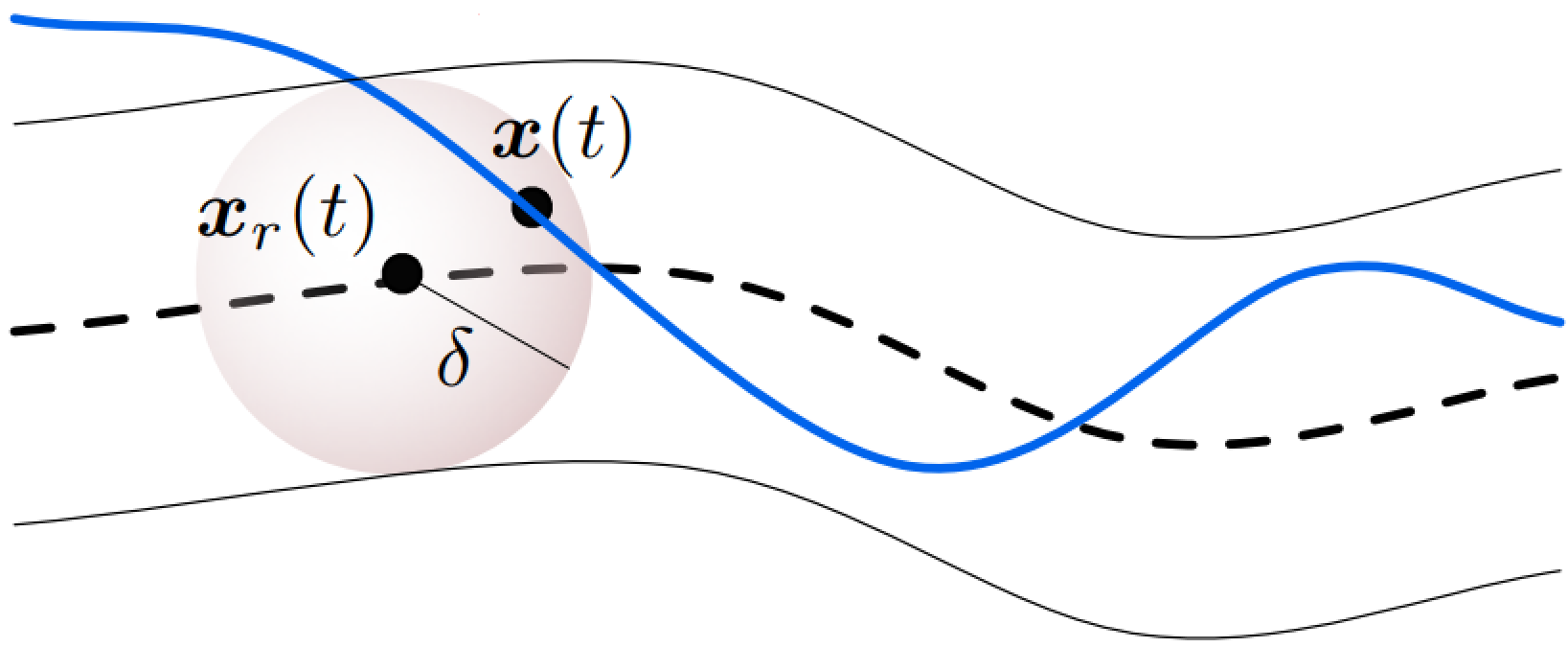
**1. Stability Definitions:**

* *Neighborhood*: Give , a state vector is said to be in the neighborhood of the state if:

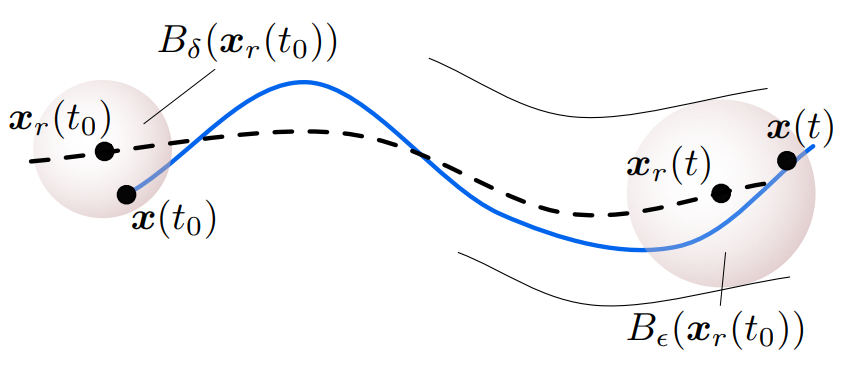
then



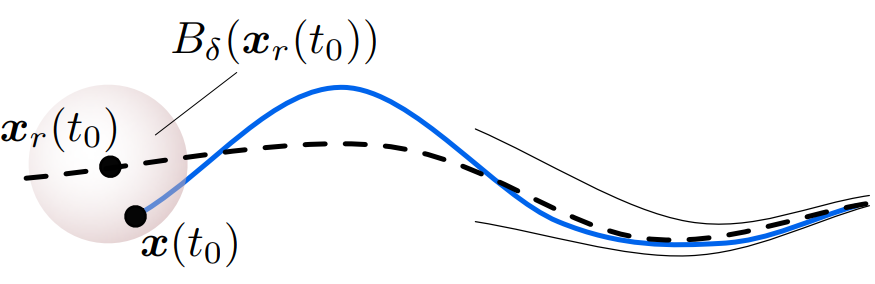
* *Lagrange Stability*: The motion is said to be Lagrange stable (or bound) relative to if there exists a such that:



* *Lyapunov Stability*: The motion is said to be Lyapunov stable (or bound) relative to if for each there exists a such that:



* *Asymptotic Stability*: The motion is said to be asymptotic stable relative to if is Lyapunov stable and there exists a such that:

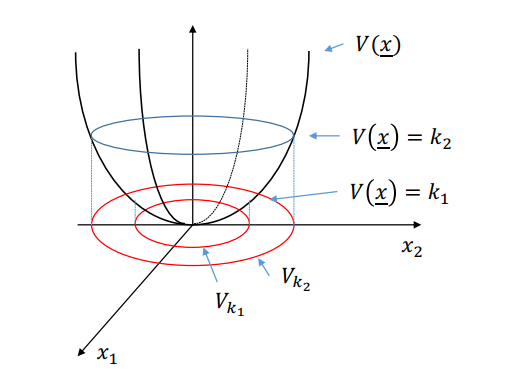


* *Global Stability*: The motion is said to be asymptotic stable relative to if is stable for any initial state vector.

**2. Lyapunov Function:**

+ The scalar function is a Lyapunov function for the dynamical system if it is continuos and there exists a such that for any :

1. is a positive define function about .
2. V(**x**) has continuous partial derivatives.
3. is negative semi-difinite.



**3. Lyapunov Function for Difference Goal:**

+ Equation of Motions:

* Rigid Body Detumbling:

+ State Vector: + Goal:

Lyapunov Function:

+ Reference: + Goal:

Lyapunov Function:

* Tracking:

+ State Vector: + Goal:

Lyapunov Function: