Задача 2.

а) $\begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix} = U \Sigma V^+$ $\qquad \Sigma = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$

$$\boxed{\begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{U} \underbrace{\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}}_{\Sigma} \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}_{V^+}}$$

б) $A = \begin{pmatrix} 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$A^T A = \begin{pmatrix} 0 & 0 \\ 0 & 4 \end{pmatrix}$ $\quad \lambda_{1,2} = 4, 0 \quad \sigma_{1,2} = 2, 0$

в) $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$A^T A = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$ $\quad \lambda = 4, 0 \quad \Sigma = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$

$a_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad a_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$A^T A = \underbrace{\begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}}_{V} \begin{pmatrix} 0 & 0 \\ 0 & 4 \end{pmatrix} \underbrace{\begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}}_{V^T}$$

$$\boxed{A = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}}$$