Electron Diffraction in Graphene

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Abstract
Electrons diffract quickly from a polycrystalline graphene layer, creating interference rings on a fluorescent screen due to random orientation of bonds. In this experiment, the graphene interplanar spacing is determined by measuring diffraction ring diameters and applying an accelerating voltage. The de Broglie equation is used to calculate wavelength based on anode voltage. The results are close to theoretical values and within 10% error margin. The errors have been discussed and addressed.

Keywords
Electron diffraction–Bragg'law–Crystal planes–Graphene

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Contents

Objectives

- Measure the diameter of the two smallest diffraction rings under varying anode voltages.
- Calculate the wavelength of electrons based on these anode voltages, utilizing the de Broglie equation.
- To find the interplanar spacing of graphene.

1. Introduction

Interference is the phenomenon when two waves interact with each other to superpose at a point based on their relative phases, intensities and path differences. In this experiment we use this property of electrons after getting scattered from graphene planes to give interference rings. Interference in electron diffraction occurs when high-speed electrons interact with a material, such as a polycrystalline graphene layer, while diffracting through its lattice planes. The de Broglie wavelength λ of electrons is determined by their momentum gained from acceleration voltage[2].
Mathematically:

$$\lambda = \frac{h}{p} \tag{1}$$

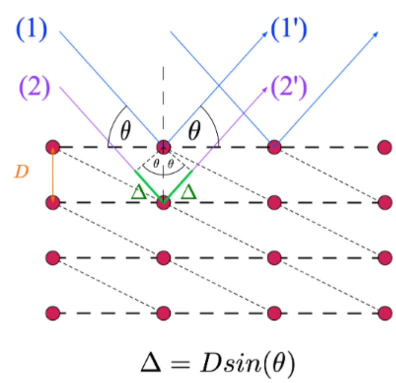

Figure 1. Reflection of electron waves from atomic planes. Ray 1 and 2 have no path length difference, 2Δ is the path length difference between ray 1 from the top plane and ray 3 from the one below. The short-dashed lines show another possible set of atomic planes.[1]

where h is the plancks constant and p is the momenta. The momenta is given by the energy - acceleration potential relation as:

$$\frac{1}{2}mv^2 = \frac{p^2}{2m} = e.U \tag{2}$$

where e is electronic charge, U is the acceleting potential, m is the mass of electron and v is the velocity of the electron. Thus, using eq(1), we can get:

$$\lambda = \frac{h}{\sqrt{2meU}}$$

Bragg's law states that when a ray is incident onto a crystal surface, its angle of incidence, θ, will reflect with the same angle of scattering, θ. The rays are assumed to reflect from crystal planes. From this assumptions, we can derive the

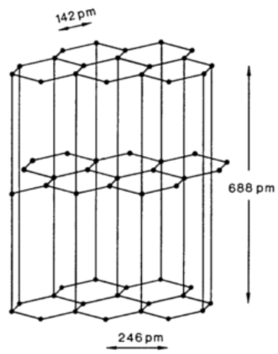

Figure 2. Crystal lattice of graphene[3].

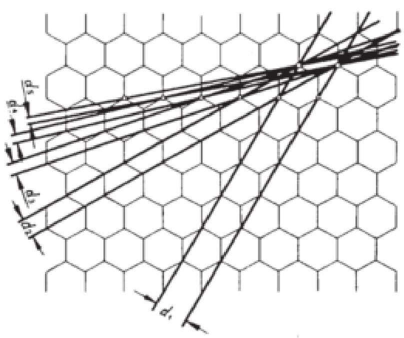

Figure 3. Interplanar spacing in graphene : D_5 = 213 pm, D_4 = 123 pm, D_3 = 80.5 pm D_2 = 59.1 pm, D_1 = 46.5 pm.

Bragg's equation that gives the condition for constructive interference. The Bragg's equation is:

$$2dsin(\theta) = n\lambda \tag{3}$$

where d is the inter atomic spacing and n is a positive integer. In the experiment, we use sample of polycrystalline graphene in which inter planar bonds are broken, leading to random orientation of planes. Thus, when a light ray is reflected from the planes, it forms rings due to constructive interference [4]. The Bragg's angle θ can be found using these rings. The deviation angle (α), and θ are related with the following relation:

$$\alpha = 2\theta \tag{4}$$

The eq(4) can be derived from observation in fig(5). Using some elementary maths, we get:

$$sin(2\alpha) = \frac{r}{R} \tag{5}$$

Now, if α is small, we get:

$$sin(2\alpha) \sim 2sin(\alpha) \tag{6}$$

Simnilarly, using for θ:

$$sin(\alpha) = sin(2\theta) \sim 2sin(\theta) \tag{7}$$

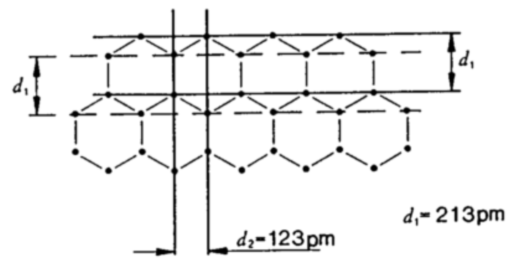

Figure 4. Graphite planes for first two interference rings.

From eq(7), eq(3) and eq(5), we get:

$$r = \frac{2R}{d}.n.\lambda \tag{8}$$

The inner two interference rings occur due to reflection from teh lattice planes d1 and d2 for n = 1, as shown in fig(4)

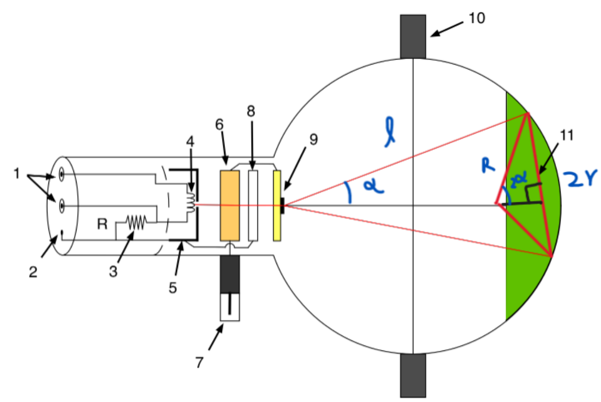

Figure 5. Overview of the electron diffraction tube.(1)4-mm socket for filament heating supply, (2) 2-mm socket for cathode connection, (3) internal resistor, (4) filament. (5) cathode, (6) anode, (7) 4-mm plug for anode connection (HV), (8) focusing electrode, (9) polycrystalline graphite grating, (10) Boss, (11) fluorescent screen.[1]

2. Experimentation

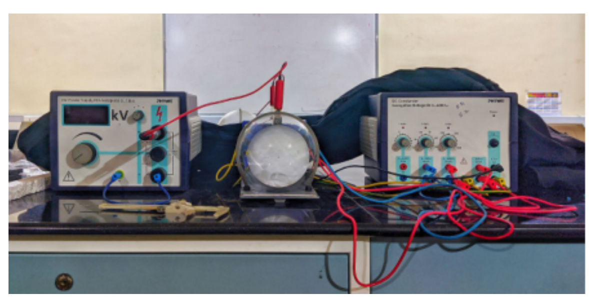

Figure 6. Experimental setup

2.1 Apparatus required

- Electron diffraction tube and mounting

- High-value resistor
- High voltage supply unit
- Connecting cord
- Vernier caliper
- Power supply

The experiment relies on an electron gun to generate a focused electron beam by heating a cathode using thermionic emission. A control grid fine-tunes the beam, while a strong electric field accelerates electrons to 10 kV between the cathode and anode. These high-energy electrons interact with a thin graphene foil, resulting in diffraction and dispersion. When diffracted electrons hit a fluorescent screen, they produce light and form a visible diffraction pattern and ring patterns are visible on the screen. To prevent beam attenuation caused by collisions with air molecules, the entire setup is enclosed in a vacuum tube.

3. Observations and calculations

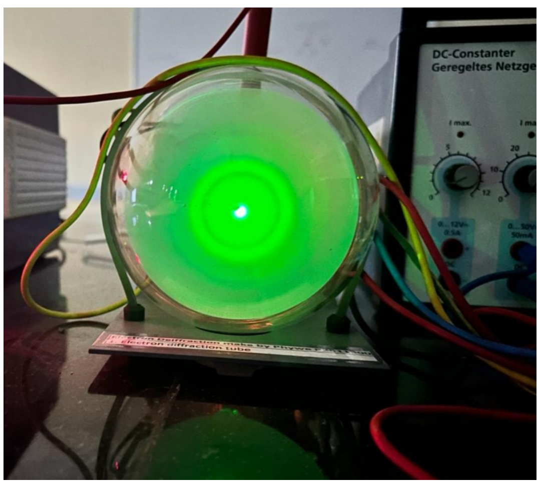

Figure 7. Rings observation

Rings were observed on the fluorescent screen when increasing the voltage. The radius of the rings reduces when the accelerating potential is increased. The applied accelerating current values were kept constant (maximum) throughout the experiment since we could not observe diffraction patterns at lower current values because the intensity of the formed diffraction rings was very low. The rings were observed as shown in fig(7). Using relation for λ with U, we get the following table 1: Using the eq(8), we can get the values of the d1, d2 thorugh the data taken, as shown in table 2. Averaging the d1, d2 values, we get the spacings are 108.386 pm and 57.516 pm. This corresponds roughly to the d2 and d4 planes of the graphene lattice. From the graph between radius and λ, we get:

$$m_1 = (1.426 \pm 0.062) \cdot 10^9$$

Table 1. Observation table

U (kV)	Radius 1 (cm)	Radius 2 (cm)	λ (pm)
4.7	2.192	4.062	17.884
5.3	2.095	3.812	16.841
5.7	1.936	3.614	16.240
6.3	1.907	3.484	15.447
6.9	1.822	3.314	14.760
7.3	1.726	3.255	14.350
8.4	1.573	3.092	13.377
9.2	1.488	2.870	12.783
10.2	1.373	2.740	12.140

Table 2. Interplanar spacing

Radius 1 (cm)	Radius 2 (cm)	λ (pm)	d1(pm)	d2(pm)
2.192	4.062	17.884	106.067	57.237
2.095	3.812	16.841	104.508	57.435
1.936	3.614	16.240	109.051	58.418
1.907	3.484	15.447	105.305	57.64
1.822	3.314	14.760	105.317	57.902
1.726	3.255	14.350	108.086	57.313
1.573	3.092	13.377	110.561	56.246
1.488	2.870	12.783	111.68	57.902
1.373	2.740	12.140	114.948	57.599

$$m_2 = (2.237 \pm 0.066) \cdot 10^9$$

From, thhis and using the expression for d, we get the table 2

The errors in the least counts can be taken considered only and other errors cannot be measured. The least cound for the applied voltage is 0.1 kV, vernier calipers had a lc of 0.01cm, leading to 0.01cm uncertainty in r. Thus, the error in d is given by:

$$\frac{\delta d}{d_{mean}} = \sqrt{(\frac{\delta r}{r})^2 + (\frac{\delta \lambda}{\lambda})^2} \tag{9}$$

Hence, the results after error corection is:

$$d_1 = 108.386pm \pm 5.453pm, d_2 = 57.516pm \pm 2.880pm$$

Thus, the interplanar spacings were found using the electron diffraction and the expreiment was concluded.

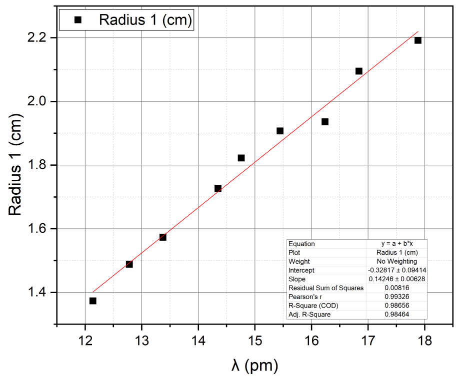

Figure 8. radius 1 vs λ

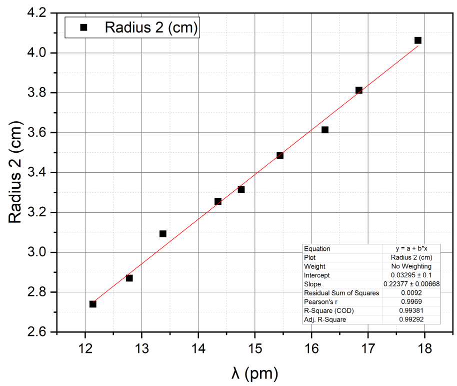

Figure 9. radius 2 vs λ

4. Results

- Rings were successfully observed with electron diffraction.
- The graph between radius and λ was a staight line.
- The fringes corresponded to interplanar spacing of d_1 = 108.386 pm± 5.453 pm and d_2 = 57.516 pm ± 2.880 pm. These values are nearest to D_2 and D_4 planes of graphene lattice respectively.

5. Discussion and conclusion

The results support de Broglie's hypothesis by demonstrating an electron diffraction pattern on a fluorescent screen. Errors in calculated electron wavelengths at different anode voltages are based on the least count, which includes instrumental and random errors.
anual readings were taken in the dark with vernier callipers, making it difficult to correctly identify the radius because the rings were spread out and blurry. This could have caused measurement errors. This highlights the significance of precise measurement techniques. Future experiments may benefit from improved tools or automated systems.

The measurement of diffraction ring radii helped determine graphene interplanar spacing, which aligned with the D2 and D4 planes. However, first planes showed a consistent instrumental error of around 10 pm, indicating that some erroraneous readings may be taken, although d4 plane has exact overlap with the D4 plane. The calculated interplanar spacings are close to the D2 and D4 planes, indicating success with the experiment's methodology. Overall, the experiment successfully supported de Broglie's hypothesis.

References

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