QComp Homework 1

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1 Problem 3

For an arbitrary 1-qbit gate W, 2-qbit matrix for cW_{01} is obtained using

$$cW_{01} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes I + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes W.$$

Also, 2-qbit matrix for W_0 (which only applies W to the first qbit) is obtained using

$$W_0 = W \otimes I.$$

1.1 1

$$HXH = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} = Z$$

1.2 2

$$cZ_{01} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = cZ_{10}$$

1.3 3

$$H_0H_1 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

$$\begin{aligned} H_0H_1cX_{01}H_0H_1 &= \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} = \\ &\frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} = \\ &\frac{1}{4}\begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 4 & 0 \end{pmatrix} = cX_{10} \end{aligned}$$

1.4 4

$$c(e^{i\alpha})_{01} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{i\alpha} & 0 \\ 0 & 0 & 0 & e^{i\alpha} \end{pmatrix} = U_1(\alpha)_{(0)}$$

2 Problem 4

Separable states should allow representation in the form

$$(a|0\rangle + b|1\rangle) \otimes (c|0\rangle + d|1\rangle) = ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle .$$

2.1 1

If the state is separable, then

$$ac = \frac{2}{3}, \quad ad = \frac{1}{3}, \quad bc = 0, \quad bd = -\frac{2}{3}.$$

It is clear that none of a, b, c, d are 0, so $\frac{c}{d} = \frac{ac}{ad} = 2$. Thus $bc = bd\frac{c}{d} = 2bd = -\frac{4}{3}$. Contradiction, so the state is entangled.

2.2 2

$$\frac{1}{2}(|00\rangle - i|01\rangle + i|10\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle - i|i\rangle)$$

Separable state.

2.3 3

If the state is separable, then (omitting the normalization coefficient)

$$ac = 1, \quad ad = -1, \quad bc = 1, \quad bd = 1.$$

It is clear that none of a, b, c, d are 0, so $\frac{c}{d} = \frac{ac}{ad} = -1$. Thus $bc = bd\frac{c}{d} = -bd = -1$. Contradiction, so the state is entangled.

3 Problem 6

Let $l = n - m - k$. We can represent the states $|\Phi_x\rangle$ as

$$|\Phi_x\rangle_{n-m} = \sum_y b_{x,y} |y\rangle_k |\Omega_{x,y}\rangle_l .$$

3.1 a

After the measurement of m qbits, the state is $|x\rangle_m |\Phi_x\rangle_{n-m} = \sum_y b_{x,y} |x\rangle_m |y\rangle_k |\Omega_{x,y}\rangle_l$ with probability $|a_x|^2$. When another k qbits are measured, any vector y, for which $b_{x,y} \neq 0$, can be result of measurement, and the state collapses to $|x\rangle_m |y\rangle_k |\Omega_{x,y}\rangle$ with probability $|b_{x,y}|^2$. So the possible final states are $|x\rangle_m |y\rangle_k |\Omega_{x,y}\rangle_l$ with probability $|a_x|^2 |b_{x,y}|^2$.

3.2 b

In general, after the measurement of $m + k$ qbits, the state could be one of $\sum_z c_z |x\rangle_m |y\rangle_k |z\rangle_l$ for some x and y. The probability amplitudes c_z are given by

$$c_z = \langle x|_m \langle y|_k \langle z|_l |\Psi\rangle_n = \langle x|_m \langle y|_k \langle z|_l \sum_{x'} a_{x'} |x'\rangle_m |\Phi_{x'}\rangle_{n-m} =$$

$$\langle x|_m \langle y|_k \langle z|_l \sum_{x'} a_{x'} \sum_{y'} b_{x',y'} |x'\rangle_m |y'\rangle_k |\Omega_{x',y'}\rangle_l =$$

$$\sum_{x'} \sum_{y'} a_{x'} b_{x',y'} \langle x|_m \langle y|_k \langle z|_l |x'\rangle_m |y'\rangle_k |\Omega_{x',y'}\rangle_l .$$

It is clear that the only possibly non-zero summand is when $x' = x$, and $y = y'$, so $c_z = a_x b_{x,y} \langle z|_l |\Omega_{x,y}\rangle_l$. Now,

$$\sum_z c_z |x\rangle_m |y\rangle_k |z\rangle_l = a_x b_{x,y} \sum_z \langle z|_l |\Omega_{x,y}\rangle_l |z\rangle_l = a_x b_{x,y} |x\rangle_m |y\rangle_k |\Omega_{x,y}\rangle_l$$

Thus, the possible final states are $|x\rangle_m |y\rangle_k |\Omega_{x,y}\rangle_l$ with probability $|a_x|^2 |b_{x,y}|^2$, the same result as in (a).