QComp Homework 1

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September 16, 2020

1 Problem 3

For an arbitrary 1-qbit gate W, 2-qbit matrix for cW_{01} is obtained using

$$cW_{01} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes I + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes W.$$

Also, 2-qbit matrix for W_0 (which only applies W to the first qbit) is obtained using

$$W_0 = W \otimes I.$$

1.1 1

$$HXH = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} = Z$$

1.2 2

$$cZ_{01} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = cZ_{10}$$

1.3 3

$$H_0H_1 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

$$H_0H_1cX_{01}H_0H_1 = \frac{1}{4}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\end{pmatrix}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{pmatrix} =$$

$$\frac{1}{4}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & -1 & -1 & 1\\ 1 & 1 & -1 & -1\end{pmatrix}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{pmatrix} =$$

$$\frac{1}{4}\begin{pmatrix}4 & 0 & 0 & 0\\ 0 & 4 & 0 & 0\\ 0 & 0 & 0 & 4\\ 0 & 0 & 4 & 0\end{pmatrix} = cX_{10}$$

1.4 4

$$c(e^{i\alpha})_{01} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & e^{i\alpha} & 0\\ 0 & 0 & 0 & e^{i\alpha}\end{pmatrix} = U_1(\alpha)_{(0)}$$

2 Problem 4

Separable states should allow representation in the form

$$(a\,|0\rangle + b\,|1\rangle) \otimes (c\,|0\rangle + d\,|1\rangle) = ac\,|00\rangle + ad\,|01\rangle + bc\,|10\rangle + bd\,|11\rangle\,.$$

2.1 1

If the state is separable, then

$$ac = \frac{2}{3}, \quad ad = \frac{1}{3}, \quad bc = 0, \quad bd = -\frac{2}{3}.$$

It is clear that none of a, b, c, d are 0, so $\frac{c}{d} = \frac{ac}{ad} = 2$. Thus $bc = bd\frac{c}{d} = 2bd = -\frac{4}{3}$. Contradiction, so the state is entangled.

2.2 2

$$\frac{1}{2}(|00\rangle - i\,|01\rangle + i\,|10\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle - i\,|i\rangle)$$

Separable state.

2.3 3

If the state is separable, then (omitting the normalization coefficient)

$$ac = 1, \quad ad = -1, \quad bc = 1, \quad bd = 1.$$

It is clear that none of a, b, c, d are 0, so $\frac{c}{d} = \frac{ac}{ad} = -1$. Thus $bc = bd\frac{c}{d} = -bd = -1$. Contradiction, so the state is entangled.

3 Problem 6

3.1 a

After the measurement of m qbits, the state is $|x\rangle_m |\Phi(x)\rangle_{n-m}$ with probability $|a_x|^2$. When another k qbits are measured, since the state is already equal to a computational basis vector, it does not change, and the possible final states are $|x\rangle_m |\Phi(x)\rangle_{n-m}$ with probability $|a_x|^2$.

3.2 b

In general, after the measurement of $m+k$ qbits the state could be $\sum_z b_z |x\rangle_m |y\rangle_k |z\rangle_{n-m-k}$. The probability amplitudes b_z are given by

$$b_z = \langle x|_m \langle y|_k \langle z|_{n-m-k} \sum_{x'} a_{x'} |x'\rangle_m |\Phi(x')\rangle_{n-m} = \sum_{x'} a_{x'} \langle x|_m \langle y|_k \langle z|_{n-m-k} |x'\rangle_m |\Phi(x')\rangle_{n-m} .$$

It is clear that the only possibly non-zero summand is when $x' = x$, so

$$b_z = a_x \langle x|_m \langle y|_k \langle z|_{n-m-k} |x\rangle_m |\Phi(x)\rangle_{n-m} = \begin{cases} a_x, & \text{if } |y\rangle_k |z\rangle_{n-m-k} = |\Phi(x)\rangle_{n-m} ; \\ 0 & \text{otherwise.} \end{cases}$$

Thus, the possible final states are $|x\rangle_m |\Phi(x)\rangle_{n-m}$ with probability $|a_x|^2$, the same result as in (a).