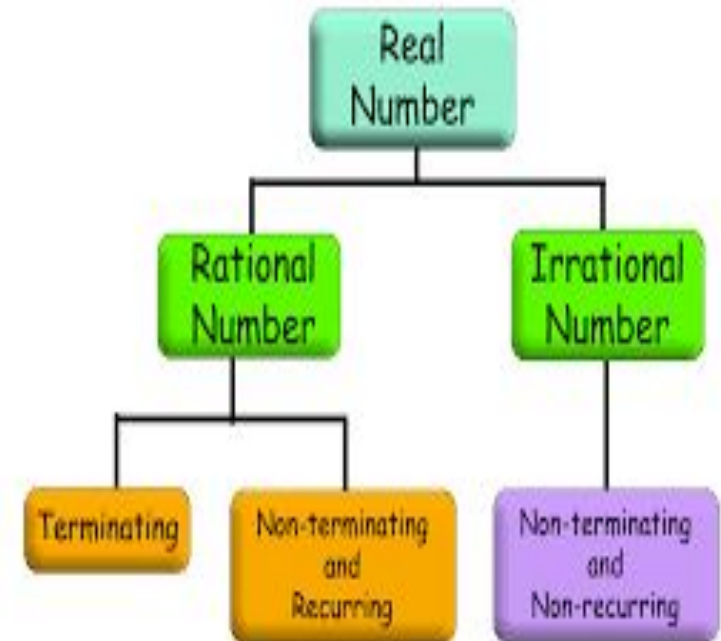
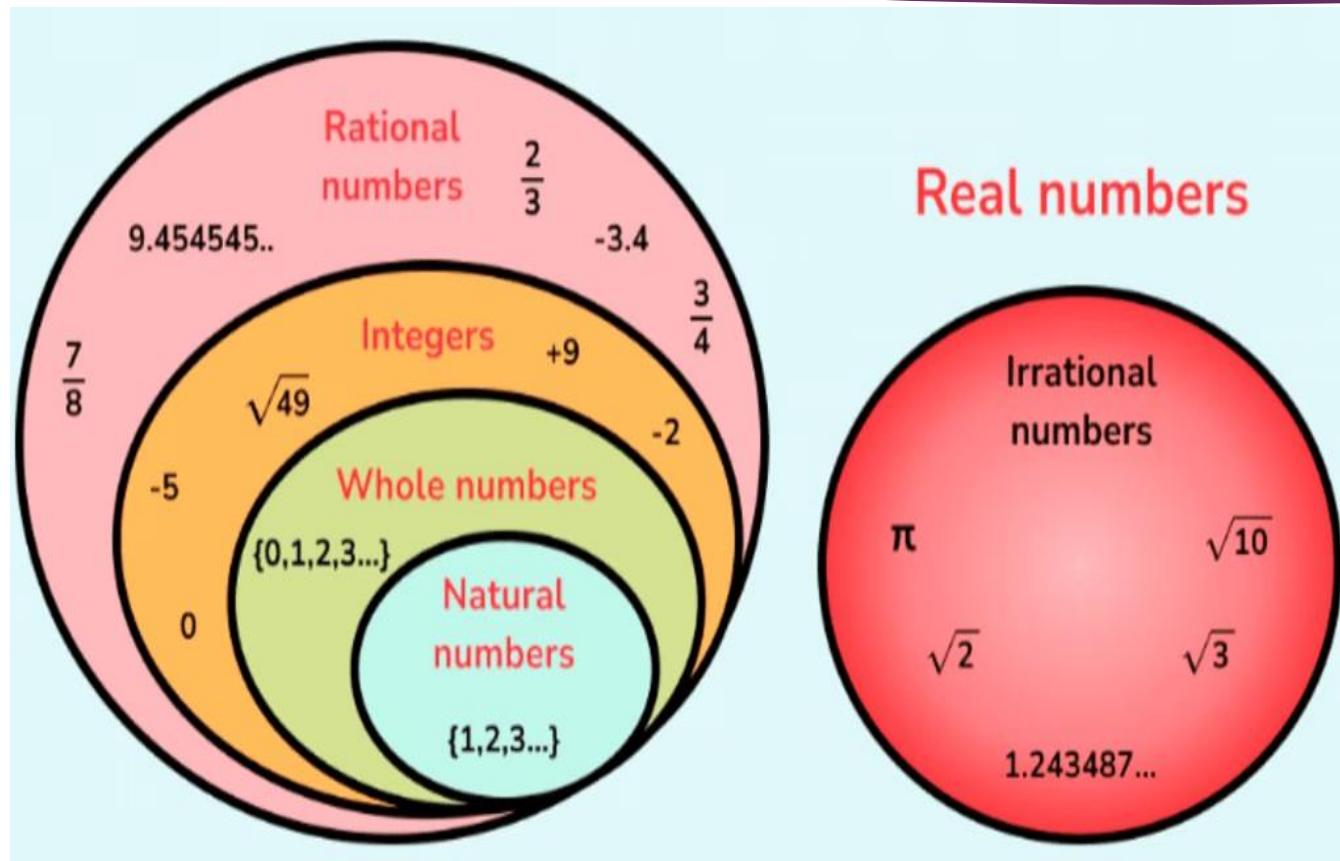


# Number System

# Topics:

- ▶ **Classification of numbers**
- ▶ **Conversions**
- ▶ **Divisibility rules**
- ▶ **HCF-LCM**
- ▶ **Factors and Factorial**
- ▶ **Remainder theorem , Fermet theorem , Wilson theorem.**
- ▶ **Number of zeroes in factorial.**
- ▶ **Unit digit**

# Classification of numbers



# Questions on Conversion--

Q1) Convert  $0.93939393$  into  $p/q$  form. (terminating recurring)

Q2) Convert  $0.1787878$  into  $p/q$  form. (terminating recurring)

Q3) Convert  $5.343434\dots$  into  $p/q$  form. (non-terminating recurring)

Q4) Convert  $2.35222222\dots$  into the  $p/q$  form. (non terminating recurring)

# Some facts :

- ▶ **Prime numbers:** Prime numbers are numbers greater than 1 that only have two factors, 1 and the number itself.
  - Prime numbers are natural numbers greater than 1.
  - 2 is the smallest prime number.
  - 2 is the only even prime number. All the prime numbers except 2 are odd.
  - Any two prime numbers are always coprime.
  - Every prime number can be written in the form of  $6n + 1$  or  $6n - 1$  say 79.
  - Between 1-50 there are 15 prime numbers , and between 50-100 there are 10.
  - **Composite numbers:** A composite number is a natural number or a positive integer which has more than two factors.
  - $O+O \rightarrow E$     $E+E \rightarrow E$     $E+O \rightarrow O$ .

# Divisibility Rules

**Divisibility by 2:** A number is divisible by 2 if its units place is either 0 or multiple of 2.

**Divisibility by 3:** A number is divisible by 3 if the sum of digits is a multiple of 3.

**Divisibility by 4:** A number is divisible by 4, if the number formed by its last 2 digits is divisible by 4.

**Divisibility by 5:** A number is divisible by 5 if its units place is 0 or 5.

**Divisibility by 6:** A number is divisible by 6 if it is divisible by 2 and 3 both.

**Divisibility by 7:** We need to double the last digit of the number and then subtract it from the remaining number. If the result is divisible by 7, then the original number will also be divisible by 7.

**Divisibility by 8:** A number is divisible by 8 if the numbers formed by the last 3 digits is divisible by 8.

**Divisibility by 9:** A number is divisible by 9 if the sum of its digits is divisible by 9.

**Divisibility by 10:** A number is divisible by 10 if it has zero (0) in its units place.

**Divisibility by 11:** A number is divisible by 11 if the sum of the digits in the odd places and the sum of the digits in the even places difference is a multiple of 11 or zero.

**Divisibility by 12:** A number is divisible by 12, if it is divisible by co-prime 12 i.e., 3 and 4.

**Divisibility by 15:** A number is divisible by 15, if it is divisible by co-prime 15 i.e., 3 and 5.

**Divisibility by 18:** A number is divisible by 18, if it is divisible by co-prime 18 i.e., 2 and 9.

**Divisibility by 45:** A number is divisible by 45, if it is divisible by co-prime 45 i.e., 5 and 9.



### Divisibility Rule for 13

- ▶ **Add** 4 times the last digit to the remaining truncated number. Repeat the step as necessary. If the result is divisible by 13, the original number is also divisible by 13. For example: **3146**

### Divisibility Rule for 17

- ▶ **Subtract** 5 times the last digit from remaining truncated number. Repeat the step as necessary. If the result is divisible by 17, the original number is also divisible by 17 . For example: **2278** .
- ▶ **Any number of the form abab is always divisible by 101.**
- ▶ **Any number of the form abcabc is always divisible by 1001.**
- ▶ **Any number of the form ababab is always divisible by 10101.**

Q1) What least value should be assigned to \* so that the number  $63576*2$  is divisible by 8?

- A.2                  B. 1                  C.4                  D. 3.

# Questions on divisibility

**Q1) 5742a62b is divisible by 9 and b is an even number. Find the sum of all values of a ?**

- (a) 45                      (b) 21.                      (c) 78                      (d) 43

**Q2) If 256X561 is divisible by 11, then what can be the value of 'X'?**

- A) 3.                      B) 0                      C) 6                      D) 8

**Q3) If a number 968A96B is to be divisible by 72, the respective values of A and B can be?**

- (a) 7 and 8                      B) 7 and 0.                      C) 5 and 8                      D) 0 and 8

**Q4) How many pairs of a and b are possible in the number 763a1b2 is divisible by 9 ?**

- (a) 12                      B) 11.                      C) 8                      D) 6



# HCF-LCM

**Q1) Three bells toll at intervals of 9, 12 and 15 minutes respectively. All three begin to toll at 8 a.m. At what time will they first toll together again?**

- A. 11 a.m .      B. 8:30 a.m      C. 10 a.m      D. 10:30 a.m**

**Q2) Five bells begin to toll together at intervals of 9 s, 6 s, 4 s, 10 s and 8 s, respectively. How many times will they toll together in the span of one hour (excluding the toll at the start)?**

- A. 5      B. 8      C. 10.      D. None of these**

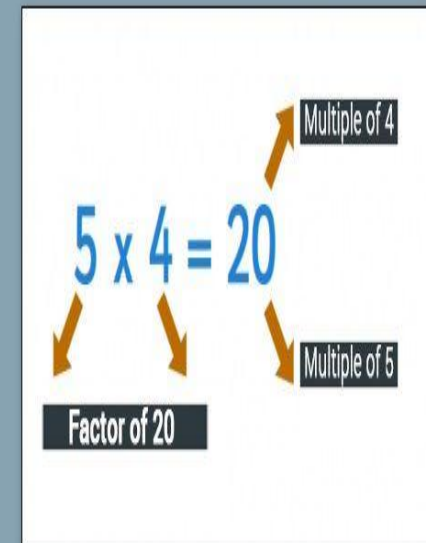
**Q3) A person has to completely put each of the three liquids i.e. 403 liters of petrol, 465 litres of diesel and 496 liters of oil in bottles of equal size without mixing any of the three types of liquids such that each bottle is completely filled. What is the least possible number of bottles required?**

- A. 44 .      B. 34      C. 31      D. None of these**

# Factors

- ▶ Number of odd factors of  $N$  = product of only odd numbers power increased by 1.
- ▶ • Number of even factors of  $N$  = Total factors – odd factors
- ▶ • Number of prime factors of  $N$  = addition of powers =  $a+b+c$ .
- ▶ • Product of factors of  $N$  =  $N^{\text{No. of factors}/2}$
- ▶ • Sum of factors of  $N$  =  $(p_0+p_1+\dots+p_A. (q_0+q_1+\dots+q_B. (r_0+r_1+\dots+r_C.$

## Factors And Multiples



# Factors:

- ▶ **Example:120.**
- ▶ Find total ,even and number of odd factors.
- ▶ Find sum of even and odd factors.
- ▶ Find product of factors.
- ▶ Find total prime factors.
- ▶ Find distinct prime factors.

# Questions on Factors .

**Q1) Find the following for the number 84?**

- I. Number of odd factors.**  
**II. Number of even factors.**
- A. 4,8.      B. 5,5      C. 8,12      D. 7,9**

**Q2) How many factors of 1200 are odd integers?**

- A. 6 .      B. 8      C.12      D. 22**

**Q3)Find the sum of factors of 18?**

- A. 6      B. 13      C. 39.      D. 35**

**Q4)Find the multiplication of factors of 18?**

- A. 6232      B.  $18^{39/2}$       C.  $12^{54}$       D. 12999**

# Number of zeroes in Factorial

Q1) Find the number of trailing zeroes in the expansion of  $23!$ ?

- A. 5                      B. 4                      C. 20                      D. 21

Q2) Find the number of zeros in  $2*3*4*5*.....*125$ ?

- A. 30                      B. 35                      C. 38                      D. 31 .

Q3) Find the highest power of 30 in  $40!$ ?

- A. 12                      B. 10                      C. 8                      D. 9 .

Q4) Find the highest power of 24 in  $150!$ ?

- A. 48.                      B. 72                      C. 58                      D. 45

Find the trailing zeros in  $102!$

$102/5 + 102/25 = 20+4=24$  (Here  $100/125$  is not possible, so divide by 5's powers till it is less or equal to number) So,  $102!$  will have 24 zeros.

# Remainders:

## Question:

$233+123+445+990+4444+1112$  is divided by 2 , find the remainder ?

## Fermat's theorem-

If  $p$  is a prime and  $a$  is an integer co-prime to  $p$ , then  $[a^{(p-1)}]/p$  gives remainder 1.

Example:- Find the remainder when  $72^{40}$  divide by 41?

## Wilson's Theorem-

This theorem state that for a prime number  $p$  ,  $(p-1)!$  Divide by  $p$  , then the remainder is  $p-1$ .

Example: - Find the remainder when  $16!$  is divided by 17.

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder.}$$

Theorem1:  $a^n + b^n$  is divisible by  $a + b$  when  $n$  is ODD.

Theorem 2:  $a^n - b^n$  is divisible by  $a + b$  when  $n$  is EVEN.

Theorem 3:  $a^n - b^n$  is ALWAYS divisible by  $a - b$ .

# Remainders:

## To Remember:

Theorem 1:  $a^n + b^n$  is divisible by  $a + b$  when  $n$  is **ODD**.

Theorem 2:  $a^n - b^n$  is divisible by  $a + b$  when  $n$  is **EVEN**.

Theorem 3:  $a^n - b^n$  is **ALWAYS** divisible by  $a - b$ .



# Questions on Remainders.

**Q1) A number when divided by 54 leaves a remainder of 31. Find the remainder when the same number is divided by 27?**

**A. 4.            B. 23                      C. 15                      D. A. or B.**

**Q2) Find the remainder when  $2^{93}$  is divided by 7?**

**A. 1.                      B. 2                      C. 4                      D. 6**

**Q3) Find the remainder  $7^{13}/13$  ?**

**A. 0                      B. 1                      C. 4                      D. None of these.**

**Q4) The remainder, when  $(15^{23} + 23^{23})$  is divided by 19, is?**

**A. 4                      B. 15                      C. 0.                      D. 18**

**Q5) What is the remainder when  $4^{96}$  is divided by 6?**

**A. 0                      B. 2                      C. 3                      D. 4.**

# unit digit.

All numbers have cyclicity of 4

2	3	4	5	6	7	8	9
$2^1=$ 2	$3^1=$ 3	$4^1=$ 4	$5^1=$ 5	$6^1=$ 6	$7^1=$ 7	$8^1=$ 8	$9^1=$ 9
$2^2=$ 4	$3^2=$ 9	<b><math>4^2=</math> 6</b>	$5^2=$ 5	$6^2=$ 6	$7^2=$ 9	$8^2=$ 4	<b><math>9^2=</math> 1</b>
$2^3=$ 8	$3^3=$ 7	$4^3=$ 4	$5^3=$ 5	$6^3=$ 6	$7^3=$ 3	$8^3=$ 2	$9^3=$ 9
$2^4=$ 6	$3^4=$ 1	$4^4=$ 6	$5^4=$ 5	$6^4=$ 6	$7^4=$ 1	$8^4=$ 6	$9^4=$ 1
<b><math>2^5=</math> 2</b>	<b><math>3^5=</math> 3</b>	$4^5=$ 4	$5^5=$ 5	$6^5=$ 6	<b><math>7^5=</math> 7</b>	<b><math>8^5=</math> 8</b>	$9^5=$ 9
$2^6=$ 4	$3^6=$ 9	$4^6=$ 6	$5^6=$ 5	$6^6=$ 6	$7^6=$ 9	$8^6=$ 4	$9^6=$ 1
$2^7=$ 8	$3^7=$ 7	$4^7=$ 4	$5^7=$ 5	$6^7=$ 6	$7^7=$ 3	$8^7=$ 8	$9^7=$ 9

# Questions on Unit Digit:

**Q1) What is the unit digit in  $2^9$ ?**

- A. 1                      B. 3                      C. 2.                      D. 4**

**Q2) What is the unit's digit of the number  $(6^{256} - 4^{256})$ ?**

- A.0.                      B. 1                      C. 4                      D. 7**

**Q3) Find the unit digit of  $23^{251}$  ?**

- A. 7.                      B. 2                      C. 3                      D. 1**

**Q4) The unit digit of  $((137)^{13})^{47}$  is?**

- A. 1                      B. 3                      C. 5                      D. 7.**

**Q5) The unit digit of  $35^{87} + 93^{46}$  is?**

- A. 2                      B. 4 .                      C. 6                      D. 8**



**THANKYOU**