Language Models for Law and Social Science

4. Supervised Learning with Text

Response Essay RE01, First Draft Due Friday, March 24th

- ▶ Info linked in syllabus, at "Response Essays with Applications Readings"
- ▶ Critically read and review a social science paper using NP.
- ▶ 300 words is the minimum for a passing grade (2+) but 500+ words would be expected for high grade (4-5).
- ▶ Anonymize your submission – do not include your name anywhere in the document. Submit as TXT or PDF to EduFlow.

What can I write about?

- Info linked in syllabus, at "Response Essays with Applications Readings"
- Can read/respond to an off-syllabus paper if it applies tools from one of the first four lectures.
  - dictionary methods, n-grams, document similarity, topic models, machine learning.
  - No papers allowed from the in-class presentation list.
  - Please confirm off-syllabus readings with me by email.
- Note:
  - if a paper has an appendix, you are responsible for reading it!

What to think about

See response essays page.

- Apply what we have been doing in lecture and in the in-class presentations.
- Focus on identifying problems with the paper, rather than summarizing it.
- Example essays from previous years are available from info page.
- We will continue to practice criticizing the required reading next week.

Example: Unsupervised Discovery of Gendered Language (Hoyle et al 2019)

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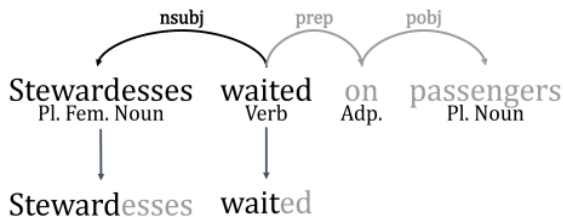▶ This paper builds on the "gender bias" NLP papers by adding in syntactic information:

Figure 2: An example sentence with its labeled dependency parse (top) and lemmatized words (bottom).

▶ Corpus: dependency parse of 3.5 million books from Goldberg and Orwant (2013).

  ▶ 37 million noun-adjective pairs
  ▶ 41-million subject-verb pairs
  ▶ 14 million verb-object pairs

▶ Extract pairs that have high PMI with the gender of the noun.

Gendered Adjectives

$\tau_{\text{MASC-POS}}$		$\tau_{\text{MASC-NEG}}$		$\tau_{\text{MASC-NEU}}$		$\tau_{\text{FEM-POS}}$		$\tau_{\text{FEM-NEG}}$		$\tau_{\text{FEM-NEU}}$	
Adj.	Value	Adj.	Value	Adj.	Value	Adj.	Value	Adj.	Value	Adj.	Value
faithful	2.3	unjust	2.4	german	1.9	pretty	3.3	horrible	1.8	virgin	2.8
responsible	2.2	dumb	2.3	teutonic	0.8	fair	3.3	destructive	0.8	alleged	2.0
adventurous	1.9	violent	1.8	financial	2.6	beautiful	3.4	notorious	2.6	maiden	2.8
grand	2.6	weak	2.0	feudal	2.2	lovely	3.4	dreary	0.8	russian	1.9
worthy	2.2	evil	1.9	later	1.6	charming	3.1	ugly	3.2	fair	2.6
brave	2.1	stupid	1.6	austrian	1.2	sweet	2.7	weird	3.0	widowed	2.4
good	2.3	petty	2.4	feudatory	1.8	grand	2.6	harried	2.4	grand	2.1
normal	1.9	brutal	2.4	maternal	1.6	stately	3.8	diabetic	1.2	byzantine	2.6
ambitious	1.6	wicked	2.1	bavarian	1.5	attractive	3.3	discontented	0.5	fashionable	2.5
gallant	2.8	rebellious	2.1	negro	1.5	chaste	3.3	infected	2.8	aged	1.8
mighty	2.4	bad	1.9	paternal	1.4	virtuous	2.7	unmarried	2.8	topless	3.9
loyal	2.1	worthless	1.6	frankish	1.8	fertile	3.2	unequal	2.4	withered	2.9
valiant	2.8	hostile	1.9	welsh	1.7	delightful	2.9	widowed	2.4	colonial	2.8
courteous	2.6	careless	1.6	ecclesiastical	1.6	gentle	2.6	unhappy	2.4	diabetic	0.7
powerful	2.3	unsung	2.4	rural	1.4	privileged	1.4	horrid	2.2	burlesque	2.9
rational	2.1	abusive	1.5	persian	1.4	romantic	3.1	pitiful	0.8	blonde	2.9
supreme	1.9	financial	3.6	belted	1.4	enchanted	3.0	frightful	0.5	parisian	2.7
meritorious	1.5	feudal	2.5	swiss	1.3	kindly	3.2	artificial	3.2	clad	2.5
serene	1.4	false	2.3	finnish	1.1	elegant	2.8	sullen	3.1	female	2.3
godlike	2.3	feeble	1.9	national	2.2	dear	2.2	hysterical	2.8	oriental	2.2
noble	2.3	impotent	1.7	priestly	1.8	devoted	2.0	awful	2.6	ancient	1.7
rightful	1.9	dishonest	1.6	merovingian	1.6	beauteous	3.9	haughty	2.6	feminist	2.9
eager	1.9	ungrateful	1.5	capetian	1.4	sprightly	3.2	terrible	2.4	matronly	2.6
financial	3.3	unfaithful	2.6	prussian	1.4	beloved	2.5	damned	2.4	pretty	2.5
chivalrous	2.6	incompetent	1.7	racial	0.9	pleasant	1.8	topless	3.5	asiatic	2.0

Gendered Verbs (as subject)

| $\tau_{\text{MASC-POS}}$		$\tau_{\text{MASC-NEG}}$		$\tau_{\text{MASC-NEU}}$		$\tau_{\text{FEM-POS}}$		$\tau_{\text{FEM-NEG}}$		$\tau_{\text{FEM-NEU}}$	
Verb	Value	Verb	Value	Verb	Value	Verb	Value	Verb	Value	Verb	Value
succeed	1.6	fight	1.2	extend	0.7	celebrate	2.4	persecute	2.1	faint	0.7
protect	1.4	fail	1.0	found	0.8	fascinate	0.8	faint	1.0	be	1.1
favor	1.3	fear	1.0	strike	1.3	facilitate	0.7	fly	1.0	go	0.4
flourish	1.3	murder	1.5	own	1.1	marry	1.8	weep	2.3	find	0.1
prosper	1.7	shock	1.6	collect	1.1	smile	1.8	harm	2.2	fly	0.4
support	1.5	blind	1.6	set	0.8	fan	0.8	wear	2.0	fall	0.1
promise	1.5	forbid	1.5	wag	1.0	kiss	1.8	mourn	1.7	wear	0.9
welcome	1.5	kill	1.3	present	0.9	champion	2.2	gasp	1.1	leave	0.7
favour	1.2	protest	1.3	pretend	1.1	adore	2.0	fatigue	0.7	fell	0.1
clear	1.9	cheat	1.3	prostrate	1.1	dance	1.7	scold	1.8	vanish	1.3
reward	1.8	fake	0.8	want	0.9	laugh	1.6	scream	2.1	come	0.7
appeal	1.6	deprive	1.5	create	0.9	have	1.4	confess	1.7	fertilize	0.6
encourage	1.5	threaten	1.3	pay	1.1	play	1.0	get	0.5	flush	0.5
allow	1.5	frustrate	0.9	prompt	1.0	give	0.8	gossip	2.0	spin	1.6
respect	1.5	fright	0.9	brazen	1.0	like	1.8	worry	1.8	dress	1.4
comfort	1.4	temper	1.4	tarry	0.7	giggle	1.4	be	1.3	fill	0.2
treat	1.3	horrify	1.4	front	0.5	extol	0.6	fail	0.4	fee	0.2
brave	1.7	neglect	1.4	flush	0.3	compassionate	1.9	fight	0.4	extend	0.1
rescue	1.5	argue	1.3	reach	0.9	live	1.4	fake	0.3	sniff	1.6
win	1.5	denounce	1.3	escape	0.8	free	0.9	overrun	2.4	celebrate	1.1
warm	1.5	concern	1.2	gi	0.7	felicitate	0.6	hurt	1.8	clap	1.1
praise	1.4	expel	1.7	rush	0.6	mature	2.2	complain	1.7	appear	0.9
fit	1.4	dispute	1.5	duplicate	0.5	exalt	1.7	lament	1.5	gi	0.8
wish	1.4	obscure	1.4	incarnate	0.5	surpass	1.7	fertilize	0.5	have	0.5
grant	1.3	damn	1.4	freeze	0.5	meet	1.1	feign	0.5	front	0.5

Gendered Verbs (as object)

| $\tau_{\text{MASC-POS}}$		$\tau_{\text{MASC-NEG}}$		$\tau_{\text{MASC-NEU}}$		$\tau_{\text{FEM-POS}}$		$\tau_{\text{FEM-NEG}}$		$\tau_{\text{FEM-NEU}}$	
Verb	Value	Verb	Value	Verb	Value	Verb	Value	Verb	Value	Verb	Value
praise	1.7	fight	1.8	set	1.5	marry	2.3	forbid	1.3	have	1.0
thank	1.7	expel	1.8	pay	1.2	assure	3.4	shame	2.5	expose	0.8
succeed	1.7	fear	1.6	escape	0.4	escort	1.2	escort	1.3	escort	1.4
exalt	1.2	defeat	2.4	use	2.1	exclaim	1.0	exploit	0.9	pour	2.1
reward	1.8	fail	1.3	expel	0.9	play	2.7	drag	2.1	marry	1.3
commend	1.7	bribe	1.8	summon	1.7	pour	2.6	suffer	2.2	take	1.1
fit	1.4	kill	1.6	speak	1.3	create	2.0	shock	2.1	assure	1.6
glorify	2.0	deny	1.5	shop	2.6	have	1.8	fright	2.4	fertilize	1.6
honor	1.6	murder	1.7	excommunicate	1.3	fertilize	1.8	steal	2.0	ask	1.0
welcome	1.9	depose	2.3	direct	1.1	eye	0.9	insult	1.8	exclaim	0.6
gentle	1.8	summon	2.0	await	0.9	woo	3.3	fertilize	1.6	strut	2.3
inspire	1.7	order	1.9	equal	0.4	strut	3.1	violate	2.4	burn	1.7
enrich	1.7	denounce	1.7	appoint	1.7	kiss	2.6	tease	2.3	rear	1.5
uphold	1.5	deprive	1.6	animate	1.1	protect	2.1	terrify	2.1	feature	0.9
appease	1.5	mock	1.6	follow	0.7	win	2.0	persecute	2.1	visit	1.3
join	1.4	destroy	1.5	depose	1.8	excel	1.6	cry	1.8	saw	1.3
congratulate	1.3	deceive	1.7	want	1.1	treat	2.3	expose	1.3	exchange	0.8
extol	1.1	bore	1.6	reach	0.9	like	2.2	burn	2.6	shame	1.6
respect	1.7	bully	1.5	found	0.8	entertain	2.0	scare	2.0	fade	1.2
brave	1.7	enrage	1.4	exempt	0.4	espouse	1.4	frighten	1.8	signal	1.2
greet	1.6	shop	2.7	tip	1.8	feature	1.2	distract	2.3	see	1.2
restore	1.5	elect	2.2	elect	1.7	meet	2.2	weep	2.3	present	1.0
clear	1.5	compel	2.1	unmake	1.5	wish	1.9	scream	2.3	leave	0.8
excite	1.2	offend	1.5	fight	1.2	fondle	1.9	drown	2.1	espouse	1.3
flatter	0.9	scold	1.4	prevent	1.1	saw	1.8	rape	2.0	want	1.1

Machine Learning Essentials

Ensemble Learning with XGBoost
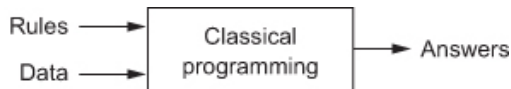
Deep Learning Essentials
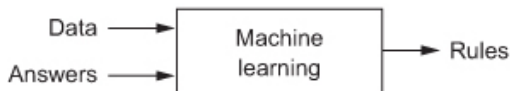
What is machine learning?
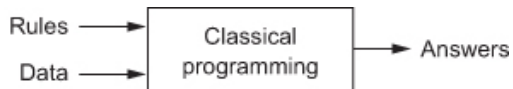
- ▶ In classical computer programming, humans input the rules and the data, and the computer provides answers.
- ▶ In (supervised) machine learning, humans input the data and the answers, and the computer learns the rules.
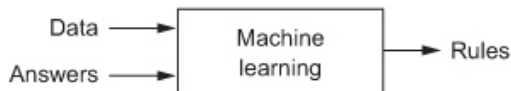
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▶ In (supervised) machine learning, humans input the data and the answers, and the computer learns the rules.

```
>>> from sklearn.datasets import load_iris
>>> from sklearn.linear_model import LogisticRegression
>>> X, y = load_iris(return_X_y=True)
>>> clf = LogisticRegression(random_state=0).fit(X, y)
>>> clf.predict(X[:2, :])
array([0, 0])
>>> clf.predict_proba(X[:2, :])
array([[9.8...e-01, 1.8...e-02, 1.4...e-08],
       [9.7...e-01, 2.8...e-02, ...e-08]])
>>> clf.score(X, y)
0.97...
```

What do ML Algorithms do? Fit a function to data points
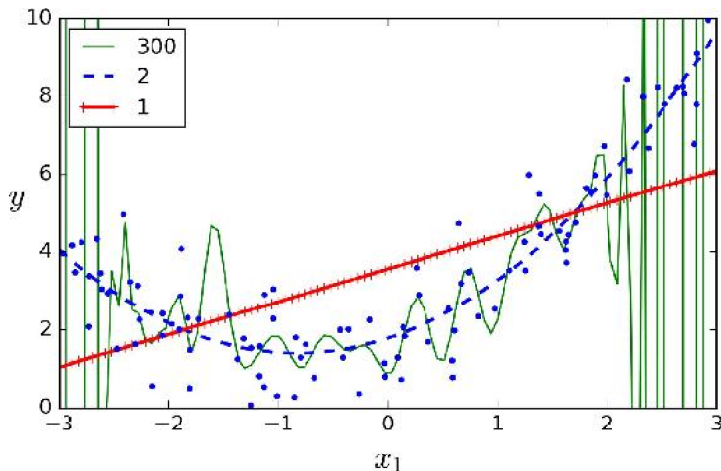
Figure 4-14. High-degree Polynomial Regression

What do ML Algorithms do? Minimize a cost function

▶ A typical cost function (or loss function) for regression problems is Mean Squared Error (MSE):

$$\text{MSE}(\theta) = \frac{1}{n_D} \sum_{i=1}^{n_D} (h(x_i; \theta) - y_i)^2$$

  ▶ n_D, the number of rows/observations
  ▶ x, the matrix of predictors, with row x_i
  ▶ y, the vector of outcomes, with item y_i
  ▶ $h(x_i; \theta) = \hat{y}$ the model prediction (hypothesis)

The **data** (x, y) are taken as given, and the ML algorithm searches for **parameters** θ to minimize the cost function.

e.g., Linear Regression is Machine Learning

▶ Ordinary Least Squares Regression (OLS) assumes the functional form
$f(x;\theta) = x_i'\theta$ and minimizes the mean squared error (MSE)

$$\min_{\hat{\theta}} \frac{1}{n_D} \sum_{i=1}^{n_D} (x_i'\hat{\theta} - y_i)^2$$

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$$\min_{\hat{\theta}} \frac{1}{n_D} \sum_{i=1}^{n_D} (x_i'\hat{\theta} - y_i)^2$$

▶ This minimand has a closed form solution

$$\hat{\theta} = (\mathbf{x}'\mathbf{x})^{-1}\mathbf{x}'\mathbf{y}$$

  ▶ most machine learning models do **not** have a closed form solution $\rightarrow$ use numerical optimization instead (gradient descent).

$$\text{MSE}(\theta) = \frac{1}{n_D} \sum_{i=1}^{n_D} (h(\theta; \boldsymbol{x}_i) - y_i)^2$$

▶ The partial derivative for feature j is

$$\frac{\partial \text{MSE}}{\partial \theta_j} = \frac{2}{n_D} \sum_{i=1}^{n_D} (\underbrace{h(\theta; \boldsymbol{x}_i) - y_i}_{\text{error for this obs}}) \underbrace{\frac{\partial h(\theta; \boldsymbol{x}_i)}{\partial \theta_j}}_{\text{how } \theta_j \text{ shifts } h(\cdot)}$$

  ▶ $\rightarrow$ estimates how changing θ_j would reduce the error across the whole dataset.

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▶ The **gradient** ∇ gives the vector of these partial derivatives for all features:

$$\nabla_\theta \text{MSE} = \begin{bmatrix} \frac{\partial \text{MSE}}{\partial \theta_1} \\ \frac{\partial \text{MSE}}{\partial \theta_2} \\ \vdots \\ \frac{\partial \text{MSE}}{\partial \theta_{n_x}} \end{bmatrix}$$

▶ **Gradient descent** nudges θ against the gradient (the direction that reduces MSE):

$$\theta_{t+1} = \theta_t - \eta \nabla_\theta \text{MSE}$$

▶ $\eta = $ learning rate

▶ If the cost function is convex, gradient descent is guaranteed to find the global minimum.

Data Prep for Machine Learning

Data Prep for Machine Learning

▶ Data Pre-Processing: See Geron Chapter 2 for pandas and sklearn syntax:
  ▶ imputing missing values.
  ▶ feature scaling (often helpful/necessary for ML models to work well)
    ▶ if predictors are sparse (e.g. bag-of-words), use `StandardScaler(with_mean=False)`.
  ▶ encoding categorical variables.
  ▶ **Best practice: reproducible data pipeline**.

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  - ▶ encoding categorical variables.
  - ▶ **Best practice: reproducible data pipeline**.

- ▶ Train/Test Split:
  - ▶ ML models can achieve arbitrarily high accuracy in-sample, so performance should be evaluated out-of-sample.
  - ▶ standard approach: randomly sample 80% training dataset to learn parameters, form predictions in 20% testing dataset for evaluating performance.

```python
from sklearn.model_selection import train_test_split
X_train, X_test, y_train, y_test = train_test_split( X, y, test_size=0.2, random_state=4)
```

Machine Learning with Text Data

- We have a corpus (or dataset) D of $n_D \geq 1$ documents d_i (or data points).
- Each document i has an associated outcome or label $\mathbf{y}_i$ with dimensions $n_y \geq 1$
- Some documents are labeled and some are unlabeled $\rightarrow$
  - we would like to learn a function $\hat{\mathbf{y}}(d_i)$ based on the labeled data ...
  - ... to machine-classify the unlabeled data.

Text Features as ML Features

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Text Features as ML Features

- Each document is a sequence of symbols d_i, while (standard) ML algorithms work on numbers.
- The solution: all the methods from Weeks 1, 2, 3 for extracting informative numerical information from documents:
    - style features
    - counts over dictionary patterns
    - tokens
    - n-grams
    - principal components
    - topic shares
    - etc.
- Represent documents as a matrix $\boldsymbol{x}$ with $n_x \geq 1$ features.

Supervised Feature Selection

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- ▶ e.g., for classification (discrete labels), use chi2 (chi-squared metric)
  - ▶ very fast, works on sparse matrices
  - ▶ features must be non-negative (with neg predictors, use f_classif)
- ▶ For regression tasks (continuous outcome), use f_regression.
- ▶ **Remember: only in the training set**

Three Types of (Standard) Machine Learning Problems

Determined by the data type of the outcome variable (or label):

▶ **Regression**: a one-dimensional, continuous, real-valued outcome.
  ▶ e.g., number of days of prison assigned
▶ **Binary classification**: two choices, normalized to zero and one.
  ▶ e.g., guilty or innocent
▶ **Multinomial Classification**: Three or more discrete, un-ordered outcomes.
  ▶ e.g., predict what judge is assigned to a case: Alito, Breyer, or Cardozo

Regression models $\leftrightarrow$ Continuous outcome

▶ If the outcome is continuous (e.g., Y = tax revenues collected, or criminal sentence imposed in months of prison):
  ▶ Need a regression model.
▶ Problems with OLS:
  ▶ tends to over-fit training data.
  ▶ cannot handle multicollinearity.
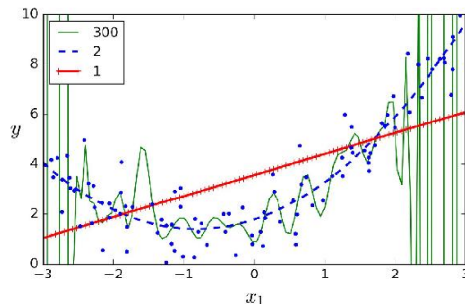
Figure 4-14. High-degree Polynomial Regression

▶ **Regularization**: model training methods designed to reduce/prevent over-fitting.

Regularized Loss Function

$$\hat{\boldsymbol{\theta}} = \arg\min_{\theta} \frac{1}{n_D} \sum_{i=1}^{n_D} L(h(\boldsymbol{x}_i; \theta), \boldsymbol{y}_i) + \lambda R(\theta)$$

- $R(\theta)$ is a "regularization function" or "regularizer", designed to reduce over-fitting.
- λ is a hyperparameter where higher values increase regularization.

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In particular:

- "**Lasso**" (or L1) penalty:

$$R_1 = \|\theta\|_1 = \sum_{j=1}^{n_x} |\theta_j|$$

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- "**Ridge**" (or L2) penalty:

$$R_2 = \|\theta\|_2^2 = \sum_{j=1}^{n_x} (\theta_j)^2$$

  - shrinks coefficients toward zero and helps select between collinear predictors.

- **Elastic Net**: $R_{\text{enet}} = \lambda_1 R_1 + \lambda_2 R_2$

Evaluating Regression Models

```python
1   from sklearn.metrics import mean_squared_error, r2_score
2
3   # model evaluation for training set
4   y_train_predict = house_model.predict(X_train)
5   rmse = (np.sqrt(mean_squared_error(y_train, y_train_predict)))
6   r2 = r2_score(y_train, y_train_predict)
7
8   print('RMSE for training is ', rmse)
9   print('R2 score for training is ', r2)
10
11  y_test_predict = house_model.predict(X_test)
12  rmse = (np.sqrt(mean_squared_error(y_test, y_test_predict)))
13  r2 = r2_score(y_test, y_test_predict)
14
15  print('RMSE for testing is ', rmse)
16  print('R2 score for testing is ',r2)
```

Binary Outcome $\leftrightarrow$ Binary Classification

- Binary classifiers try to match a boolean outcome $y \in \{0, 1\}$.
    - The standard approach is to apply a transformation (e.g. sigmoid/logit) to normalize $\hat{y} \in [0, 1]$.
    - Prediction rule is 0 for $\hat{y} < .5$ and 1 otherwise.

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  ▶ Prediction rule is 0 for $\hat{y} < .5$ and 1 otherwise.
▶ The binary cross-entropy (or log loss) is:

$$L(\theta) = \underbrace{-\frac{1}{n_D}}_{\text{negative}} \sum_{i=1}^{n_D} [\underbrace{y_i}_{y_i=1} \underbrace{\log(\hat{y}_i)}_{\log \text{ prob} y_i=1} + \underbrace{(1-y_i)}_{y_i=0}\underbrace{\log(1-\hat{y}_i)}_{\log \text{ prob} y_i=0}]$$

▶ In **logistic regression** we use a sigmoid transformation:
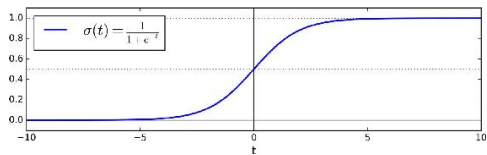
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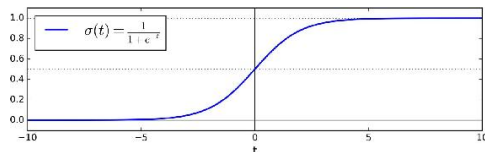
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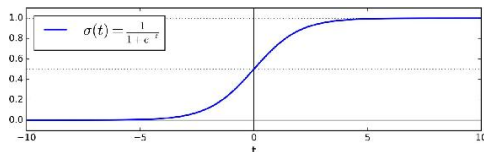
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  ▶ does not have a closed form solution, but it is convex (guaranteeing that gradient descent will find the global minimum).

▶ Like linear regression, logistic regression can be regularized with L1 and/or L2 penalties.

A **Confusion Matrix** is a nice way to visualize classifier performance:

		Predicted Class	
		Negative	Positive
True Class	Negative	**# True Negatives**	# False Positives
	Positive	# False Negatives	**# True Positives**

▶ Cell values give counts in the **test set**.

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AUC-ROC = Area Under the Receiver Operating Characteristic Curve

▶ provides an aggregate measure of performance across all possible classification thresholds.

▶ Interpretation: randomly sample one positive and one negative example. AUC = probability that the model correctly guesses which is which.

Evaluating Classification Models: Calibration Curves

Calibration plot
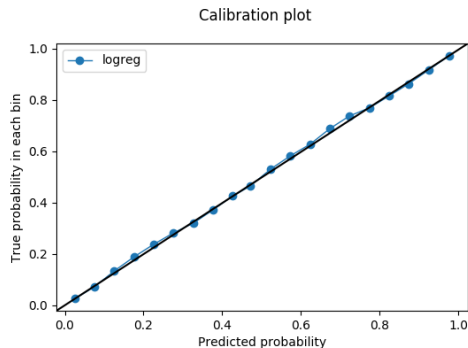
- ▶ Plotting the binned fraction in a category (Y axis) against the predicted probability in a category (X axis):
- ▶ Provides evidence of whether the classifer is replicating the conditional distribution of the outcome.

```
from seaborn import regplot
regplot(y_test, y_pred, x_bins=20)
```

Application: Predicting Political Party from Text

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In years that classifier is more accurate, speech is more polarized:
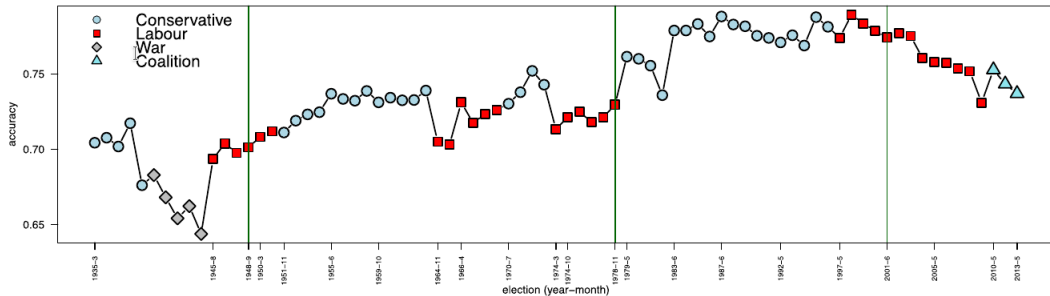
Multi-Class Classification

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▶ The outcome is $y_i \in \{1, ..., k, ..., n_y\}$ output classes, which can also be represented as a one-hot vector

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▶ We want to learn a vector function

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}, \theta)$$

taking text features $\boldsymbol{x}$ as inputs and outputing a vector of probabilities across outcome classes:

$$\hat{\boldsymbol{y}} = \{\hat{y}^1, ..., \hat{y}^{n_y}\}, \sum_{k=1}^{n_y} \hat{y}^k = 1, \hat{y}^k \geq 0 \ \forall k$$

▶ for prediction step, can select the highest-probability class:

$$\tilde{y} = \arg \max_k \hat{y}_{[k]}$$

Categorical Cross Entropy

▶ The standard loss function in multinomial classification is **categorical cross entropy**:

$$L(\theta) = -\sum_{k=1}^{n_y} \boldsymbol{y}^k \log(\hat{y}^k(\boldsymbol{x}, \theta))$$

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▶ Since there is just one true class ($y = 1$ for one class k^*, and zero for others), simplifies to

$$L(\theta) = -\log(\hat{y}^{k^*}(\boldsymbol{x}, \theta))$$

   ▶ Rewards putting higher probability on the true class, ignores distribution of probabilities on other classes.

Multinomial Logistic Regression

Multinomial logistic regression computes probabilities for each class k using the softmax transformation

$$\hat{y}_k(\boldsymbol{x}_i) = \Pr(y_i = k) = \frac{\exp(\theta'_k \boldsymbol{x}_i)}{\sum_{l=1}^{n_y} \exp(\theta'_l \boldsymbol{x}_i)}$$

▶ softmax is the multiclass generalization of sigmoid → can then interpret $\hat{y}$ as probabilities.

▶ n_x features and n_y output classes → there is a $n_y \times n_x$ parameter matrix Θ, where the parameters for each class θ_k are stored as rows.

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The **L2-penalized logistic regression** has loss function

$$\mathcal{L}(\theta) = -\frac{1}{n_D} \sum_{i=1}^{n_D} \log \frac{\exp(\theta_{k^*}' \boldsymbol{x}_i)}{\sum_{l=1}^{n_y} \exp(\theta_l' \boldsymbol{x}_i)} + \lambda \sum_{j=1}^{n_x} \sum_{k=1}^{n_y} (\theta_{[j,k]})^2$$

▶ λ = strength of L2 penalty (could also add lasso penalty)
  ▶ as before, predictors should be scaled to the same variance.

		Predicted Class		
		Class A	Class B	Class C
True Class	Class A	Correct A	A, classed as B	A, classed as C
	Class B	B, classed as A	Correct B	B, classed as C
	Class C	C, classed as A	C, classed as B	Correct C

More generally, with **multi-class confusion matrix** M with items M_{ij} (row i, column j):

$$\text{Precision for } k = \frac{\text{True Positives for } k}{\text{True Positives for } k + \text{False Positives for } k} = \frac{M_{kk}}{\sum_l M_{lk}}$$

$$\text{Recall for } k = \frac{\text{True Positives for } k}{\text{True Positives for } k + \text{False Negatives for } k} = \frac{M_{kk}}{\sum_l M_{kl}}$$

$$F_1(k) = 2 \times \frac{\text{precision}(k) \times \text{recall}(k)}{\text{precision}(k) + \text{recall}(k)}$$

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Can average these metrics across classes to get aggregate metrics.

▶ e.g., balanced accuracy = unweighted average of recalls across classes.

▶ can weight classes by their frequency in dataset

Overview of Text Analysis Methods (Osnabruegge et al 2021

	Dictionaries (Custom)	Dictionaries (Generic)	Topic Modeling	Within-Domain Supervised Learning	Cross-Domain Supervised Learning
Design/Annotation Costs	High	Low	Low	High	Moderate

Specificity	High	Moderate	Low	High	Moderate
Interpretability	High	High	Moderate	High	High

Validatability	Low	Low	Low	High	High

Review: NLP "Bias" is statistical bias

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Review: NLP "Bias" is statistical bias

- ▶ Supervised learning algorithms trained on annotated datasets learn both the annotated label and other language information that is correlated with the label (confounders).
- ▶ e.g., sentiment scores that are trained on annotated datasets also learn from the correlated non-sentiment information:


```
text_to_sentiment("Let's go get Italian food")
2.0429166109
text_to_sentiment("Let's go get Chinese food")
1.4094033658
text_to_sentiment("Let's go get Mexican food")
0.3880198556

text_to_sentiment("My name is Emily")
2.2286179365
text_to_sentiment("My name is Heather")
1.3976291151
text_to_sentiment("My name is Yvette")
0.9846380213
text_to_sentiment("My name is Shaniqua")
-0.4704813178
```

True Sentiment in Language → **Sentiment Score**

Confounding language [e.g. race, ethnicity]
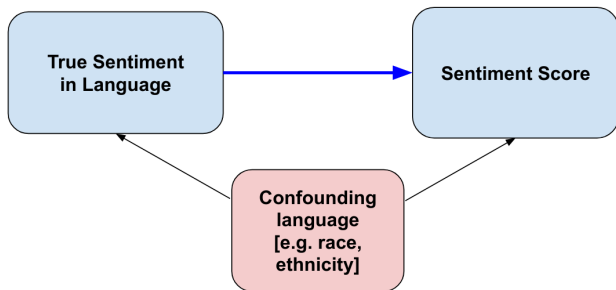
Review: Classifiers Learn Confounders
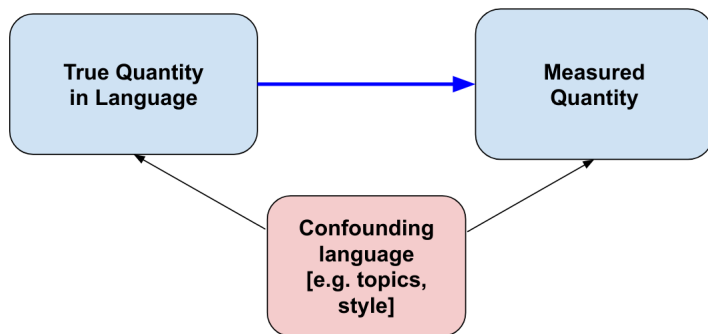
- supervised models (classifiers, regressors) learn features that are correlated with the label being annotated.
  - when taken to new data, they predicted label might reflect the presence of the confounders, rather than the presence of the true label.
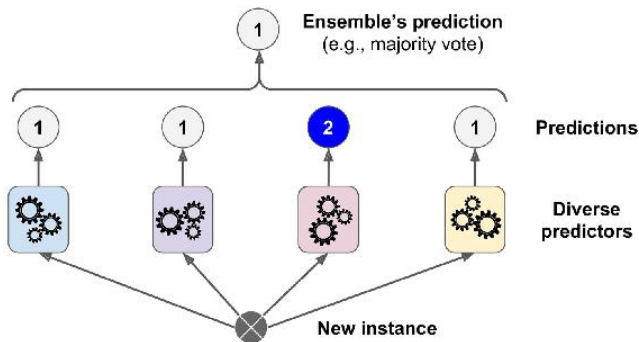
Machine Learning Essentials

Ensemble Learning with XGBoost

Deep Learning Essentials

XGBoost: Overview
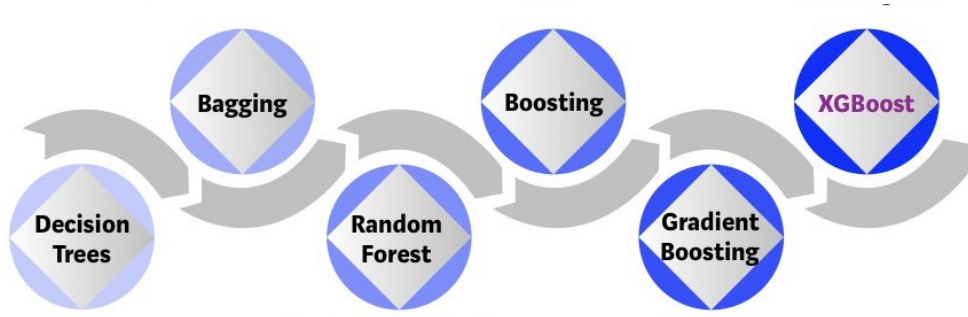
Voting Classifiers

- ▶ voting classifiers (ensembles of different models that vote on the prediction) generally out-perform the best classifier in the ensemble.
  - ▶ more diverse algorithms will make different types of errors, and improve your ensemble's robustness.

Random Forests

Random Forests are optimized ensembles of bootstrapped decision trees:

```
from sklearn.ensemble import RandomForestClassifier
rfc = RandomForestClassifier()
rfc.fit(X,y)
```

1. Each voting tree gets its own sample of data.
2. At each tree split, a random sample of features is drawn, only those features are considered for splitting.
3. For each tree, error rate is computed using data outside its bootstrap sample.

XGBoost

- ▶ Feurer et al (2018) find that XGBoost beats a sophisticated AutoML procedure with grid search over 15 classifiers and 18 data preprocessors.
- ▶ A good starting point for any machine learning task.

- ▶ easy to use
- ▶ actively developed
- ▶ efficient / parallelizable
- ▶ provides model explanations
- ▶ takes sparse matrices as input

```python
from xgboost import XGBClassifier
model = XGBClassifier()

model.fit(X_train, y_train,
          early_stopping_rounds=10,
          eval_metric="logloss",
          eval_set=[(X_eval, y_eval)]
          )

y_pred = model.predict(X_test)
accuracy = accuracy_score(y_test, y_pred)
```

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    - solve machine learning problems, just like logistic regression or gradient boosted machines
    - use torch, rather than sklearn.

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- **why not use neural nets?**
  - usually worse than standard ML on standard problems, and harder to implement.
  - Computational constraints: they require specialized processors, can be expensive to train

A "Neuron"
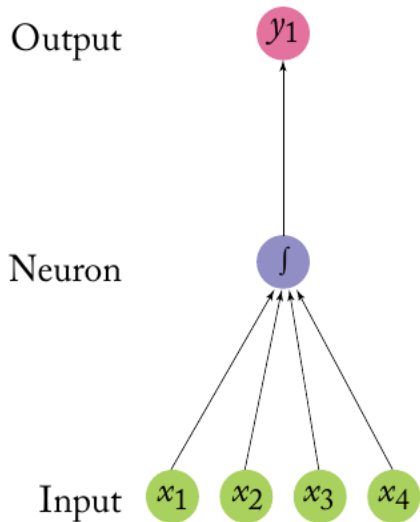
Output y_1

Neuron $\int$

Input x_1 x_2 x_3 x_4

- applies dot product to vector of numerical inputs:
  - multiplies each input by a learned weight (parameter or coefficient)
  - sums these products
- applies a non-linear "activation function" to the sum
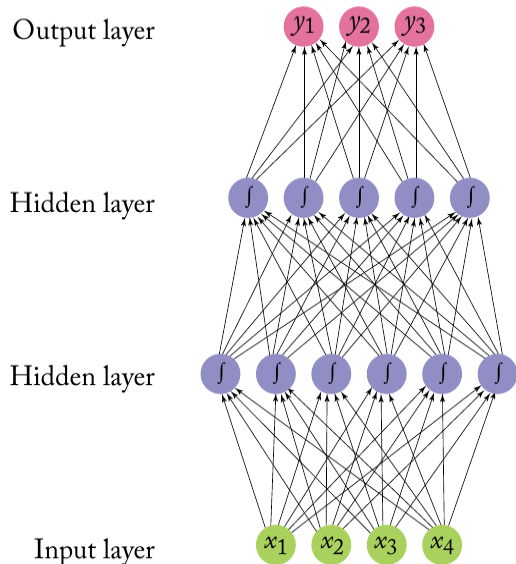 - (e.g., the $\int$ shape indicates a sigmoid transformation)
- passes the output.

"Neuron" = Logistic Regression

$$\hat{y} = \text{sigmoid}(\boldsymbol{x} \cdot \theta) = \frac{1}{1 + \exp(-\boldsymbol{x} \cdot \theta)}$$
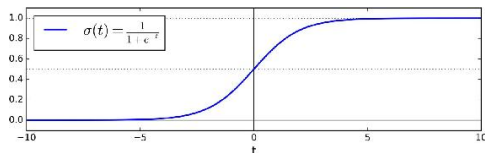
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- ▶ applies a non-linear "activation function" (sigmoid) to the sum
- ▶ passes the output.

Multi-Layer Perceptron (MLP)

- ▶ A multilayer perceptron (also called a feed-forward network or sequential model) stacks neurons horizontally and vertically.

- ▶ alternatively, think of it as a stacked ensemble of logistic regression models.

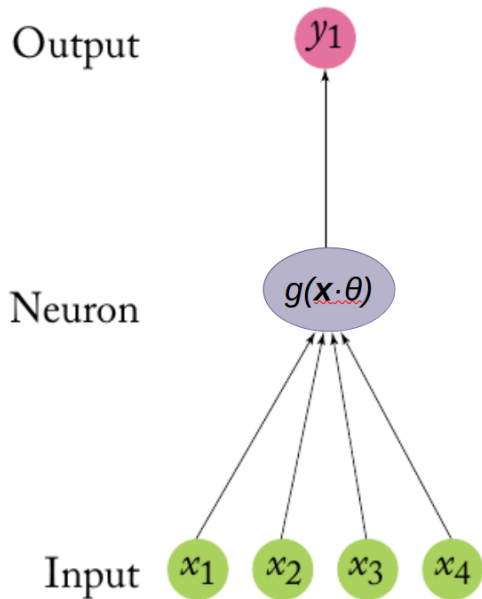- ▶ this vertical stacking is the "deep" in "deep learning"!

Activation functions $g(\boldsymbol{x} \cdot \theta)$

Output — y_1

Neuron — $g(\boldsymbol{x} \cdot \theta)$

Input — x_1 x_2 x_3 x_4
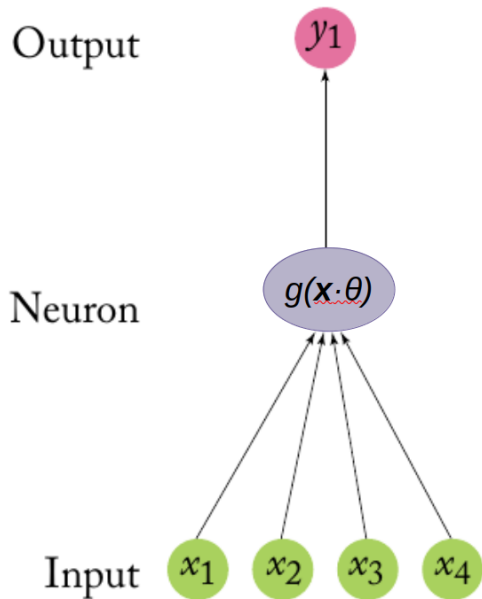
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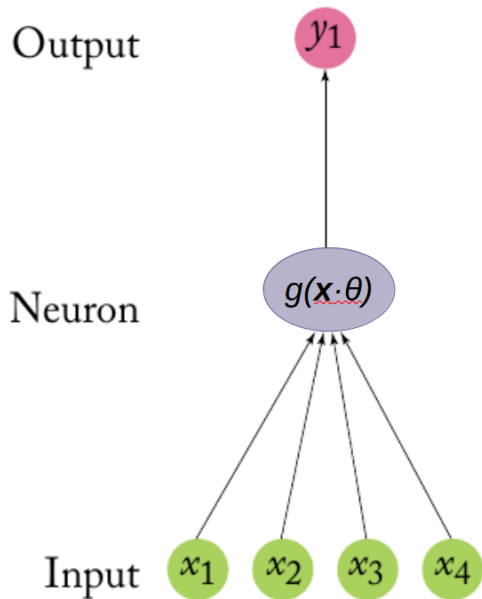
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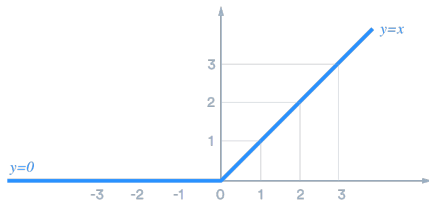
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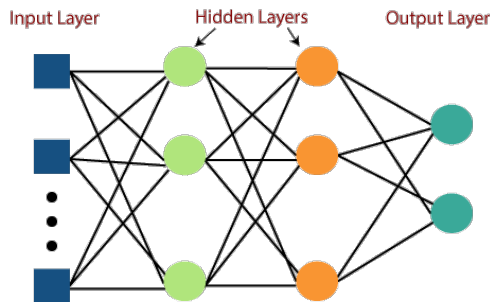
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ReLU (rectified linear unit) function:

$$g(\mathbf{x} \cdot \theta) = \text{ReLU}(\mathbf{x} \cdot \theta) = \max\{0, \mathbf{x} \cdot \theta\}$$

Equation Notation: Multi-Layer Perceptron

▶ An multi-layer perceptron (MLP) with two hidden layers is

$$\boldsymbol{y} = \boldsymbol{g}_2(\boldsymbol{g}_1(\boldsymbol{x} \cdot \boldsymbol{\omega}_1) \cdot \boldsymbol{\omega}_2) \cdot \boldsymbol{\omega}_y$$

$$\boldsymbol{y} \in \{0,1\}^{n_y}, \boldsymbol{x} \in \mathbb{R}^{n_x}, \boldsymbol{\omega}_1 \in \mathbb{R}^{n_x \times n_1}, \boldsymbol{\omega}_2 \in \mathbb{R}^{n_1 \times n_2}, \boldsymbol{\omega}_y \in \mathbb{R}^{n_2 \times n_y}$$

   ▶ n_1, n_2 = dimensionality in first and second hidden layer.
   ▶ $\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \boldsymbol{\omega}_y$ = set of learnable weights for the first hidden, second hidden, and output layer
   ▶ $\boldsymbol{g}_1(\cdot), \boldsymbol{g}_2(\cdot)$ = element-wise non-linear functions (e.g. ReLU) for first and second layer.

Google Developers Guide for Text Classification:

▶ with relatively few, longer documents, use an MLP:

  ▶ $x =$ **tf-idf-weighted bigrams** as a baseline specification for text classification tasks.

    ▶ select 20,000 features using supervised feature selection in training set.

  ▶ $f(\cdot) =$ MLP with two hidden layers.

▶ A simple MLP is one of the models tried by the U.K. parliament paper (Peterson and Spirling 2018).
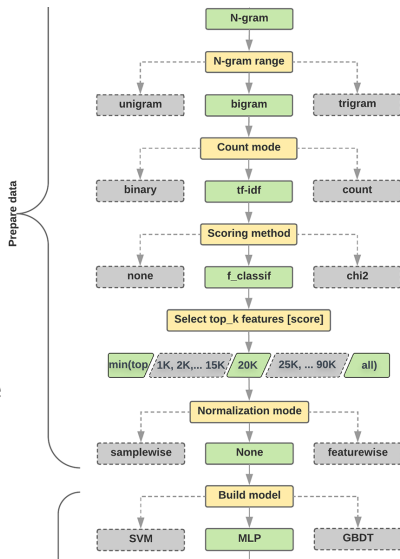
Dropout

An elegant regularization technique:

- at every training step, every neuron has some probability (typically $p = 0.5$) of being temporarily dropped out, so that it will be ignored at this step.
- at test time, neurons dont get dropped anymore but coefficients are down-weighted by p.

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- ▶ at test time, neurons dont get dropped anymore but coefficients are down-weighted by p.

Why it works:

- ▶ Neurons cannot co-adapt with neighbors; they must be independently useful.
- ▶ Layers cannot rely excessively on just a few inputs.
- ▶ Approximates an ensemble of N models (where N is the number of neurons).

Early Stopping

- A second elegant regularization technique, used in both xgboost and in neural nets:
  - gradually train the model gradients while checking fit in a held-out validation set.
  - when model starts overfitting, stop training.
- Requires user to specify two additional parameters:
  - validation set: a third sample of the data, separate from training set and test set
  - early stopping rounds: stop training if we have done this many epochs without improving validation set performance.

True/False Quiz

1. Good performance with stochastic gradient descent requires that the loss function is convex, so that is why it is used for training neural nets rather than regular (non-stochastic) gradient descent.
2. Feature scalers and feature selectors should be fitted in the training set and applied in the test set.
3. Mean squared error (MSE) is preferred to R-squared in evaluating/selecting regression models.
4. An advantage of xgboost over elastic net or neural nets is that it is less likely to learn from confounding text characteristics, and therefore generalize better out of domain.
5. Rectified linear unit (ReLU) should be used as the activation function in MLPs.
6. Neural nets tend to out-perform xgboost on text classification tasks.
7. Early stopping means splitting into three sets (train, validation, test), and training the model until performance starts decreasing in the validation set.
8. Number of hidden layers, and number of neurons per layer, are hyperparameters that can be learned by cross-validation in the training set.

Simple machine learning with long documents

1. Take TF-IDF bigrams as inputs X.
   - if too high-dimensional, use supervised feature selection and/or filter on parts of speech
2. Select a machine learning model for predicting outcome y:
   - For regression (y is one-dimensional and continuous), elastic net or gradient boosted regressor.
   - For classification (y is discrete), L2-penalized logistic regression or gradient boosted classifier.
3. Use cross-validation grid search in training set to select model hyperparameters.
   - For classification, use cross entropy; for regression, use mean squared error.
4. Evaluate model in held-out test set:
   - For classification, use balanced accuracy, confusion matrix, and calibration plot.
   - For regression, use R squared and binscatter plot.
5. Interpret the model predictions:
   - for gradient boosting, use feature importance ranking.
   - for linear models, examine coefficients
   - look at highest and lowest ranked documents for $\hat{y}$
6. Answer the research question!

Video Presentation: "Media Slant is Contagious"