

---

### Harmonious numbers

---

[Amicable numbers](#) are pairs of numbers such that the sum of the proper divisors of each one is equal to the other. The smallest such pair of numbers is 220 and 284.

Your great aunt Maude has long believed that it is a fundamental error to consider 1 to be a proper divisor of any number<sup>1</sup>, so she calls a pair of numbers *harmonious* if the sum of the proper divisors in her sense of either one is equal to the other. She has tasked you with providing a catalogue of all pairs of harmonious numbers where the smaller one is less than 2,000,000.

---

### Task

Write a program that computes all pairs of *harmonious* numbers and outputs them, one pair to a line, separated by a space, to `stdout`. The smaller number of each pair should be listed first, and there should be no duplicates.

The pairs must be written in increasing order of the smaller number. That is, looking down the output, the first column increases.

(2 points, Individual)

---

### Notes

There is a link to Amicable numbers. Be careful: you are to find *harmonious* numbers, which are similar but different.

This would be easy if you had an upper bound on the *larger* number, but you have an upper bound on the *lower* number.

Remember that 2,000,000 is *not* to be considered.

---

<sup>1</sup>She claims, quite reasonably, that if  $a$  is a proper divisor of  $n$  with  $ab = n$  then  $b$  should also be a proper divisor. So, since  $n$  is not considered a proper divisor of  $n$ , neither should 1 be.

Do not use floating point numbers unless you are prepared to provide a formal proof that your program is correct.

---

### **Testing**

Make sure you can find the Maude-proper divisors of a number.

You can find the divisors of  $n$  in  $O(\sqrt{n})$  time.