Introduction

- Fuzzy logic:
  - A way to represent variation or imprecision in logic
  - A way to make use of natural language in logic
  - Approximate reasoning
- Humans say things like "If it is sunny and warm today, I will drive fast"
- Linguistic variables:
  - Temp: {freezing, cool, warm, hot}
  - Cloud Cover: {overcast, partly cloudy, sunny}
  - Speed: {slow, fast}

Crisp (Traditional) Variables

- Crisp variables represent precise quantities:
  - x = 3.1415296
  - A $\in \{0,1\}$
- A proposition is either True or False
  - $\mathbf{A} \wedge \mathbf{B} \Rightarrow \mathbf{C}$
- King(Richard) $\wedge$ Greedy(Richard) $\Rightarrow$ Evil(Richard)
- Richard is either greedy or he isn't:
  - Greedy(Richard) $\in \{0,1\}$

Fuzzy Sets

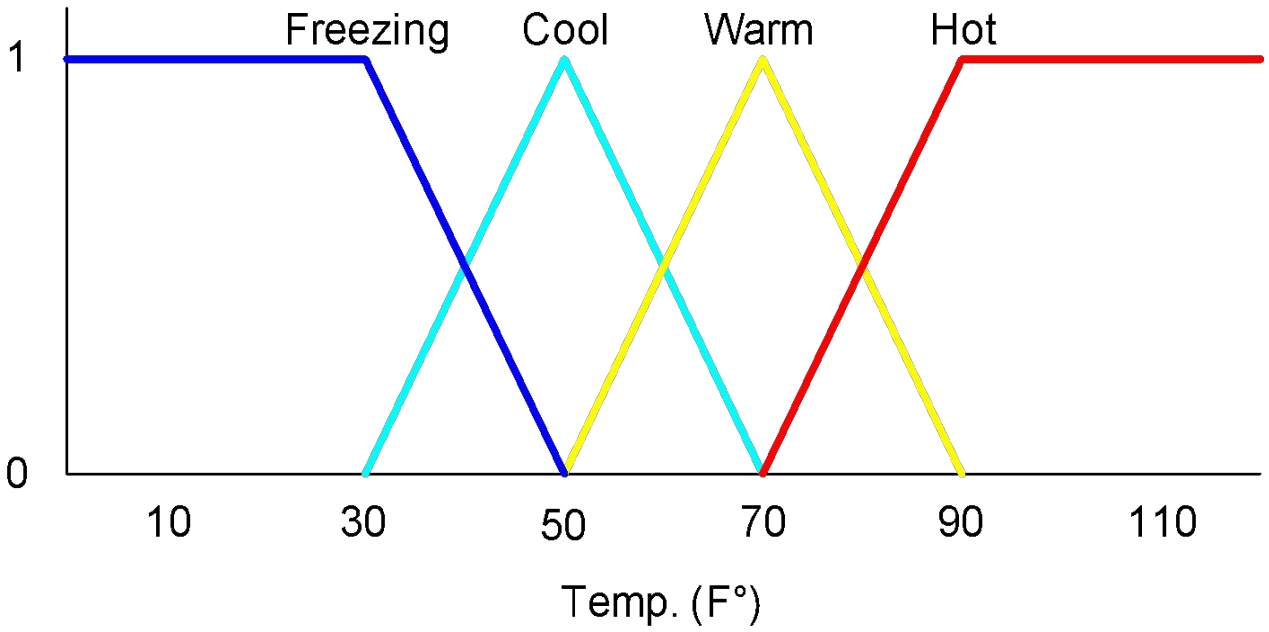
- What if Richard is only somewhat greedy?
- Fuzzy Sets can represent the degree to which a quality is possessed.
- Fuzzy Sets (Simple Fuzzy Variables) have values in the range of [0,1]
- Greedy(Richard) = 0.7
- Question: How evil is Richard?

Fuzzy Linguistic Variables

- Fuzzy Linguistic Variables are used to represent qualities spanning a particular spectrum
- Temp: {**Freezing**, **Cool**, **Warm**, **Hot**}
- Membership Function
- Question: What is the temperature?
- Answer: It is warm.
- Question: How warm is it?

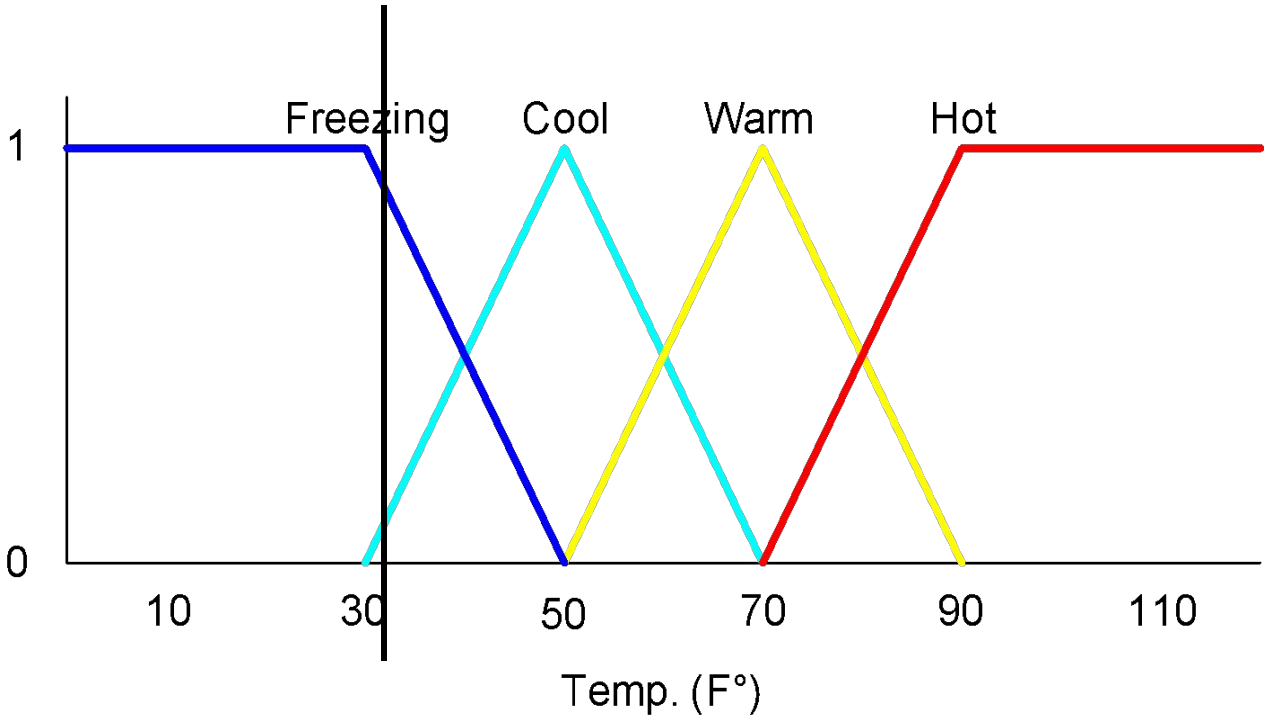
Membership Functions

- Temp: {Freezing, Cool, Warm, Hot}
- Degree of Truth or "Membership"



Membership Functions

- How cool is 36 F° ?



Membership Functions

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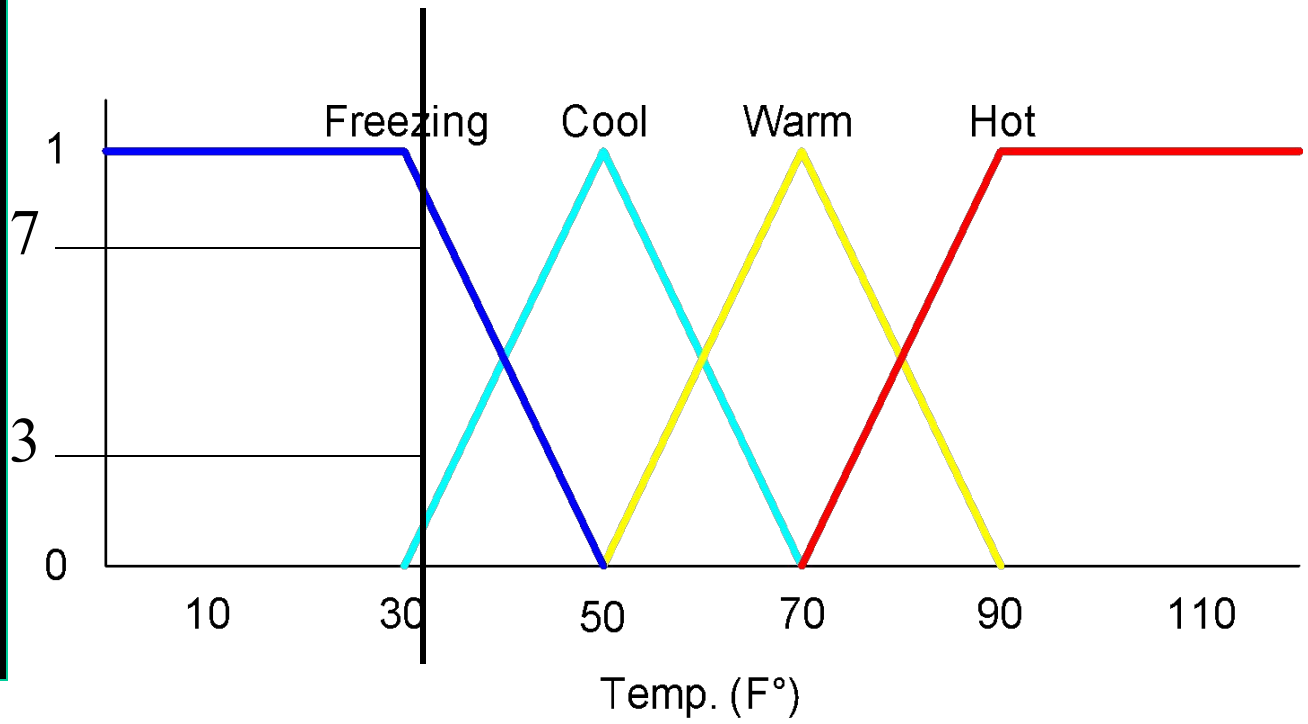
- Crisp Variables
- Fuzzy Sets
- Linguistic Variables
- **Membership Functions**
- Fuzzy Logic
  - Fuzzy OR
  - Fuzzy AND
  - Example
- Fuzzy Control
  - Variables
  - Rules
  - Fuzzification
  - Defuzzification
- Summary

- How cool is 36 F° ?
- It is 30% Cool and 70% Freezing

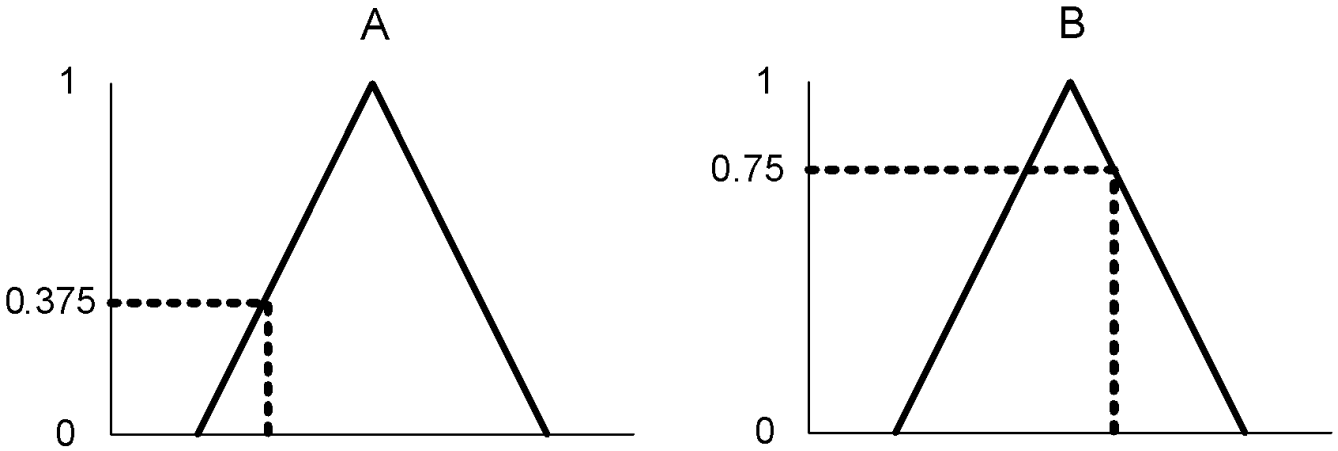


Fuzzy Logic

- How do we use fuzzy membership functions in predicate logic?
- Fuzzy logic Connectives:
  - Fuzzy Conjunction, $\wedge$
  - Fuzzy Disjunction, $\vee$
- Operate on degrees of membership in fuzzy sets

Fuzzy Disjunction

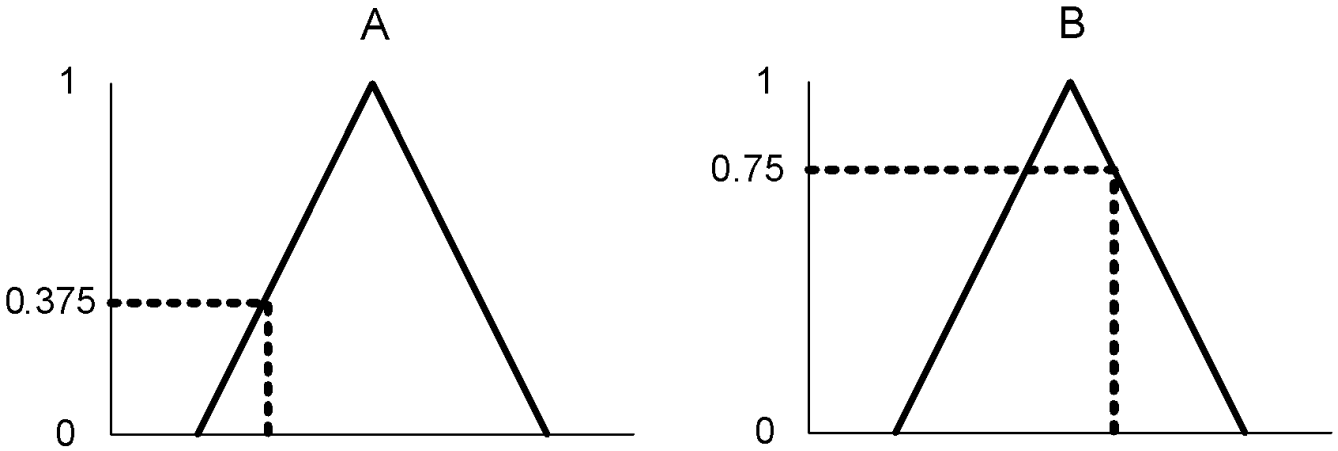
- $A \vee B \triangleq \max(A, B)$
- $A \vee B = C$ "Quality C is the disjunction of Quality A and B"



- $(A \vee B = C) \Rightarrow (C = 0.75)$

Fuzzy Conjunction

- $A \wedge B \triangleq \min(A, B)$
- $A \wedge B = C$ "Quality C is the conjunction of Quality A and B"



- $(A \wedge B = C) \Rightarrow (C = 0.375)$

Example: Fuzzy Conjunction

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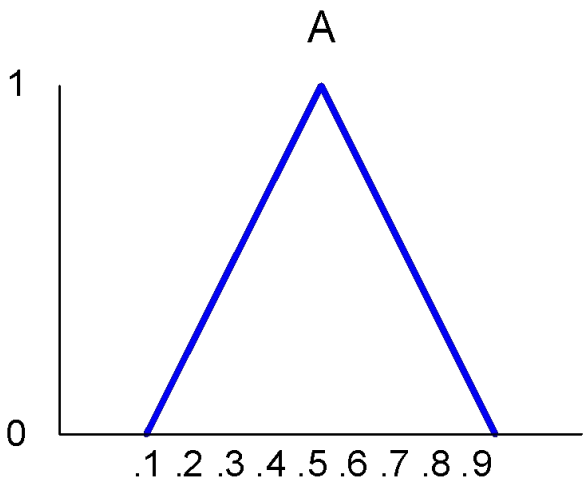
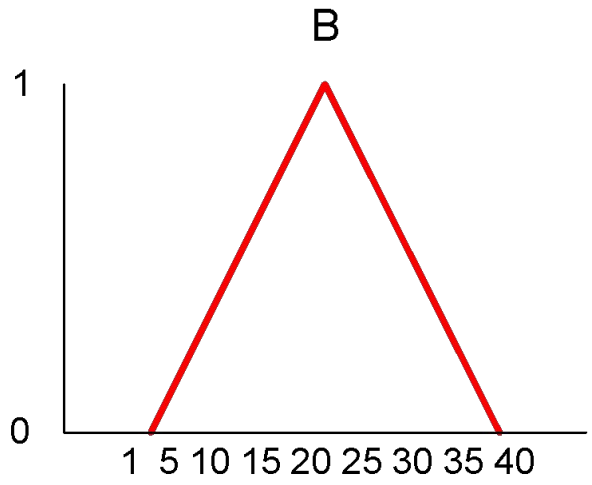
Calculate A ∧ B given that A is .4 and B is 20





Example: Fuzzy Conjunction

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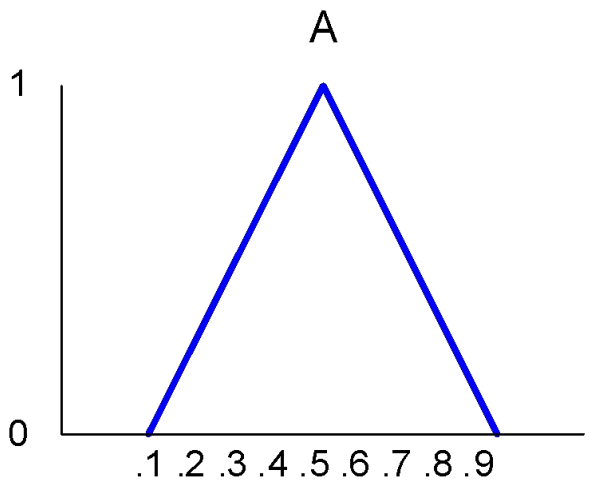

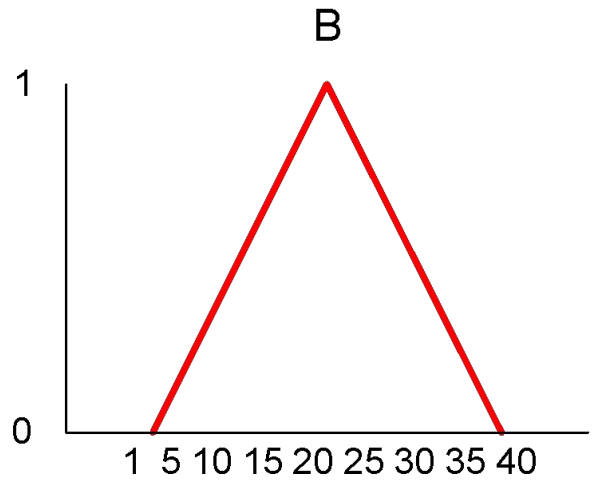

- Determine degrees of membership:

Example: Fuzzy Conjunction

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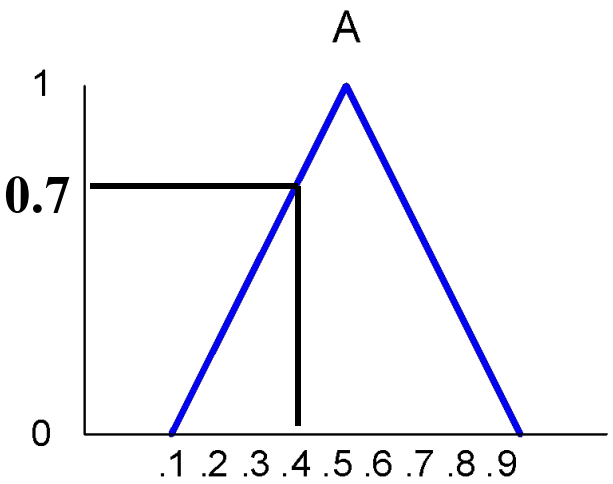

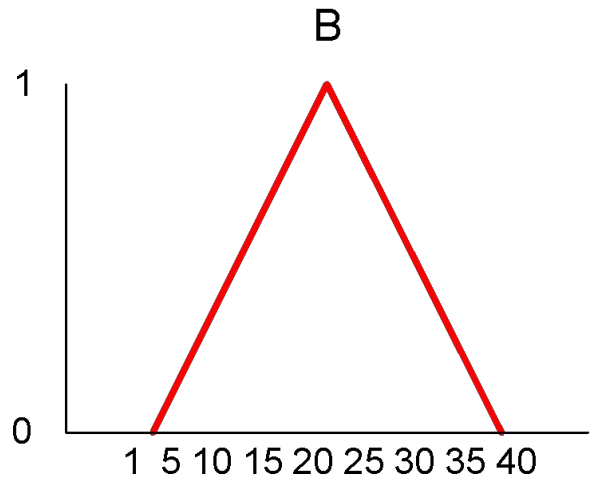

- Determine degrees of membership:
  - A = 0.7

Example: Fuzzy Conjunction

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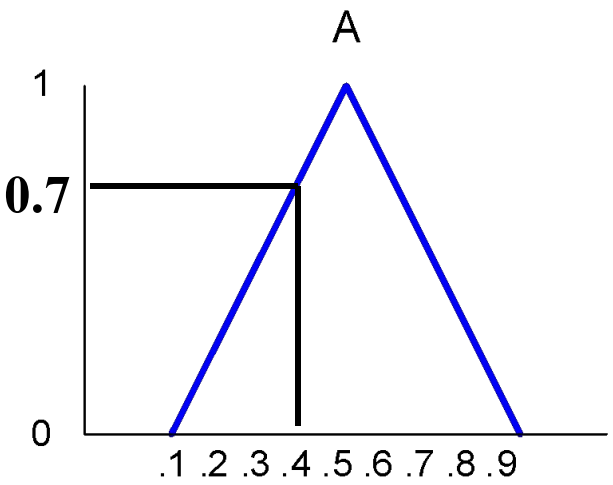
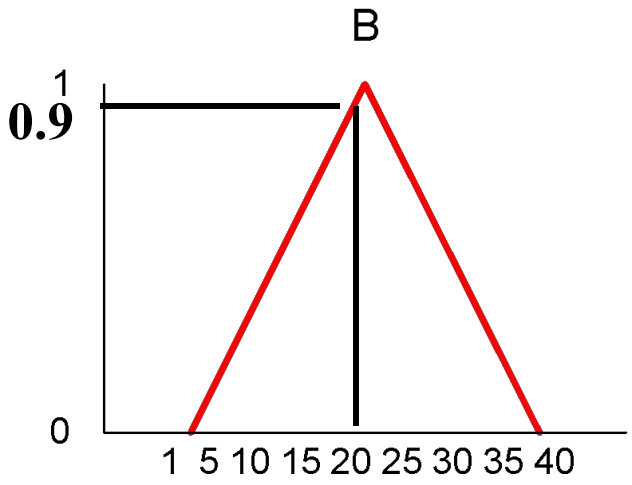
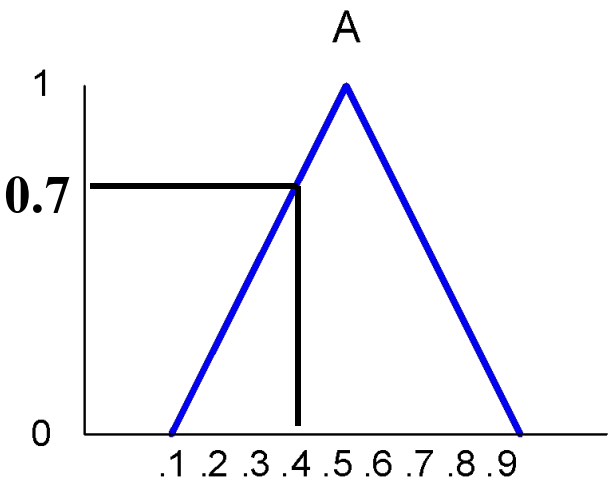


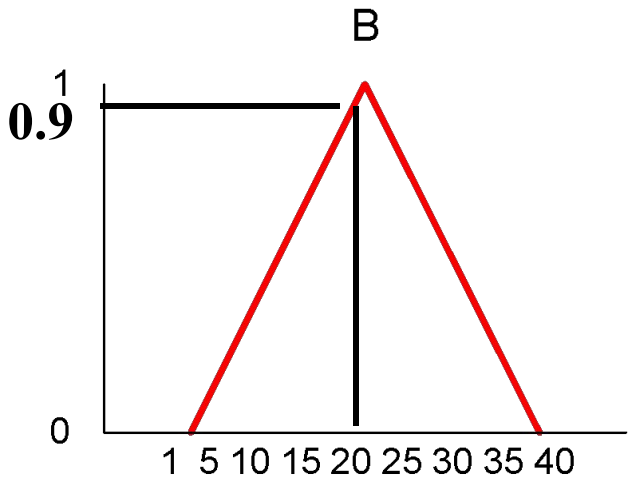


- Determine degrees of membership:
  - A = 0.7 B = 0.9

Example: Fuzzy Conjunction

Calculate A ∧ B given that A is .4 and B is 20





- Determine degrees of membership:
  - A = 0.7 B = 0.9
- Apply Fuzzy AND
  - A ∧ B = min(A, B) = 0.7

Fuzzy Control

- Fuzzy Control combines the use of fuzzy linguistic variables with fuzzy logic
- Example: Speed Control
- How fast am I going to drive today?
- It depends on the weather.
- Disjunction of Conjunctions

Inputs: Temperature

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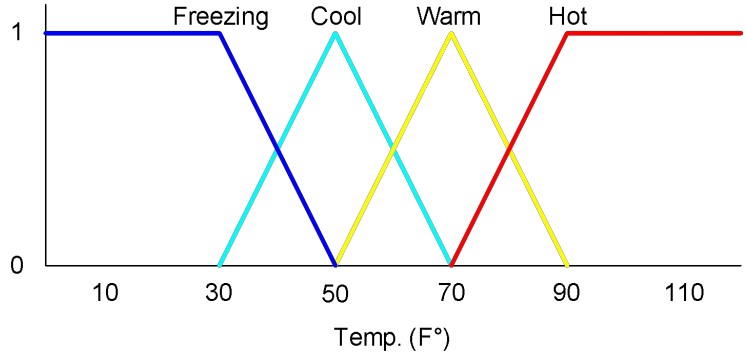
- Temp: {Freezing, Cool, Warm, Hot}



Inputs: Temperature, Cloud Cover

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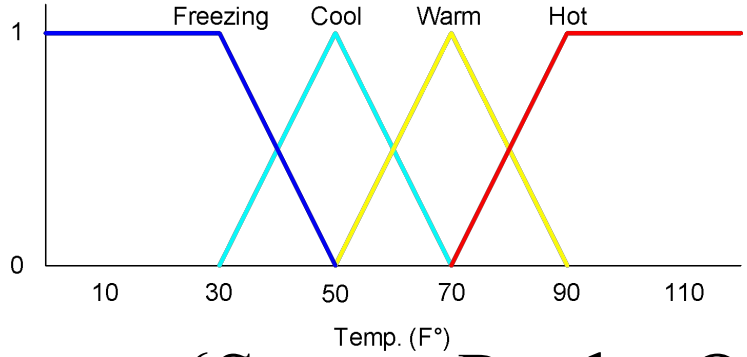
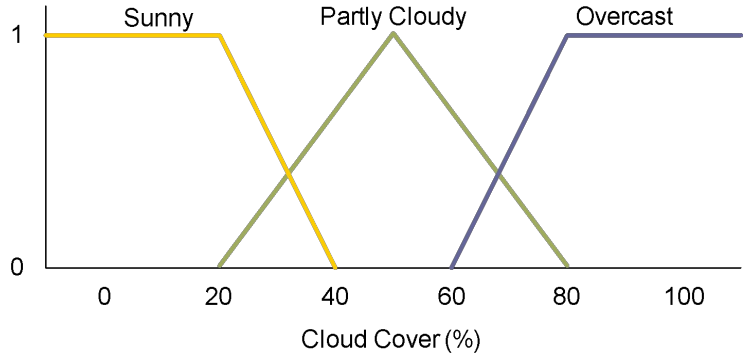
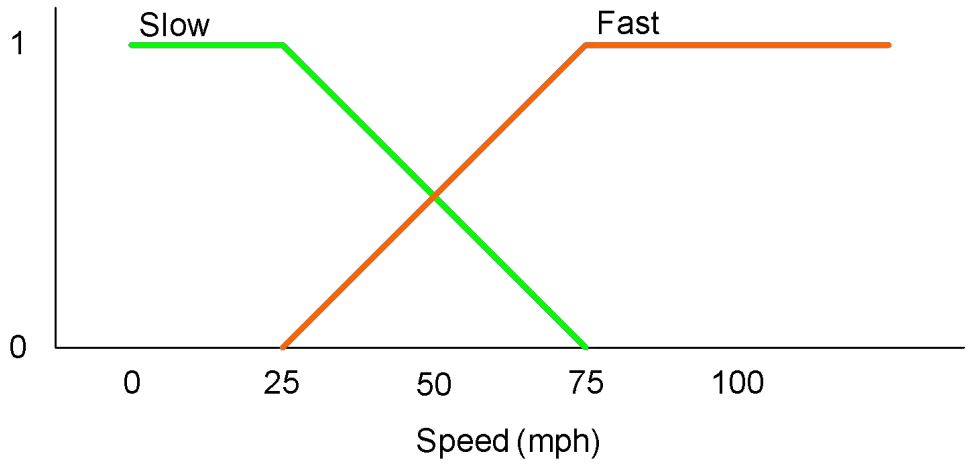


- Cover: {Sunny, Partly, Overcast}



Output: Speed

- Speed: {Slow, Fast}



Rules

- If it's Sunny and Warm, drive Fast

  $\text{Sunny(Cover)} \wedge \text{Warm(Temp)} \Rightarrow \text{Fast(Speed)}$

- If it's Cloudy and Cool, drive Slow

  $\text{Cloudy(Cover)} \wedge \text{Cool(Temp)} \Rightarrow \text{Slow(Speed)}$

- Driving Speed is the combination of output of these rules...

Example Speed Calculation

- How fast will I go if it is
  - 65 F°
  - 25 % Cloud Cover ?

Fuzzification: Calculate Input Membership Levels

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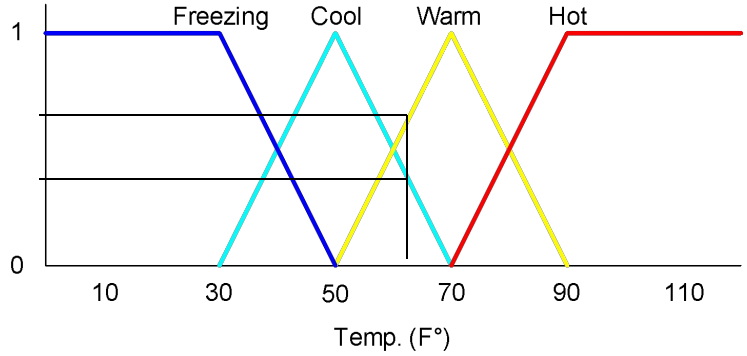
- 65 F° ⇒ Cool = 0.4, Warm= 0.7



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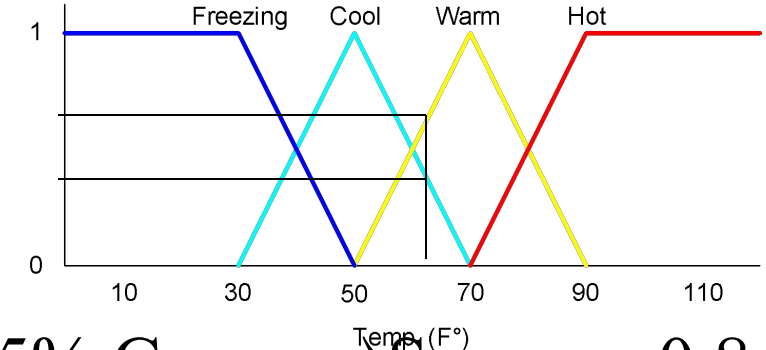
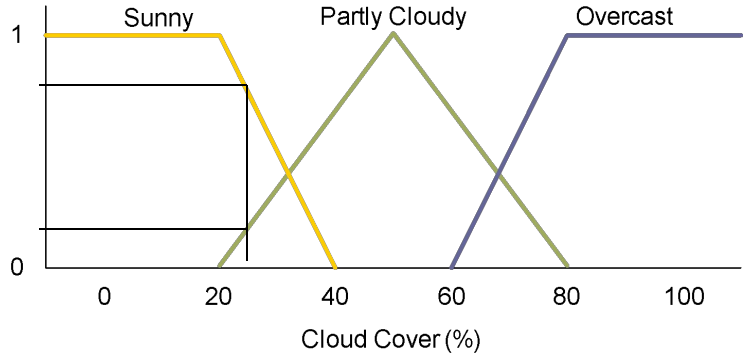


- 25% Cover ⇒Sunny = 0.8, Cloudy = 0.2



...Calculating...

- If it's Sunny and Warm, drive Fast

$$\text{Sunny(Cover)} \wedge \text{Warm(Temp)} \Rightarrow \text{Fast(Speed)}$$

$$0.8 \wedge 0.7 = 0.7$$

$$\Rightarrow \textbf{Fast = 0.7}$$

- If it's Cloudy and Cool, drive Slow

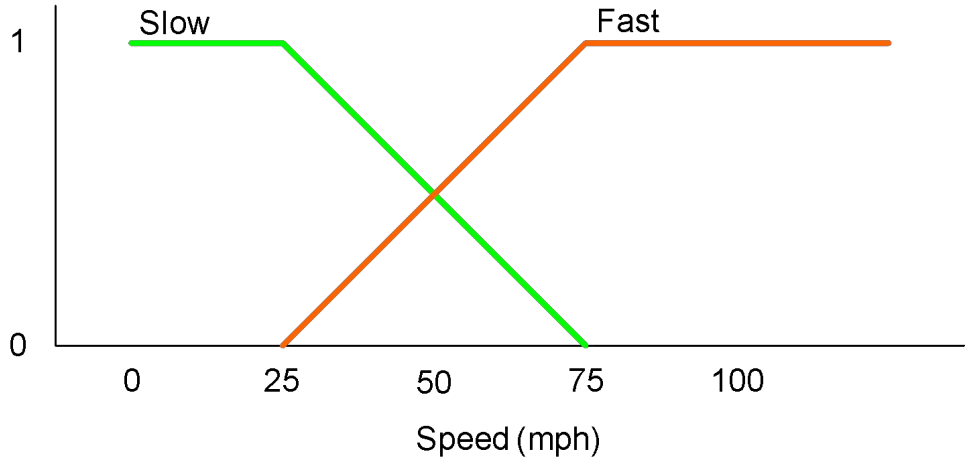
$$\text{Cloudy(Cover)} \wedge \text{Cool(Temp)} \Rightarrow \text{Slow(Speed)}$$

$$0.2 \wedge 0.4 = 0.2$$

$$\Rightarrow \textbf{Slow = 0.2}$$

Defuzzification: Constructing the Output

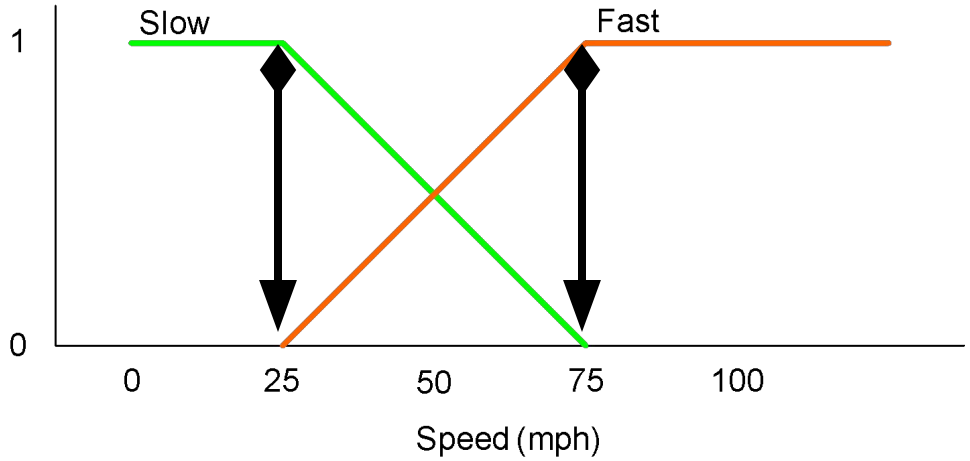
- Speed is 20% Slow and 70% Fast



- Find centroids: Location where membership is 100%

Defuzzification: Constructing the Output

- Speed is 20% Slow and 70% Fast



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Defuzzification: Constructing the Output

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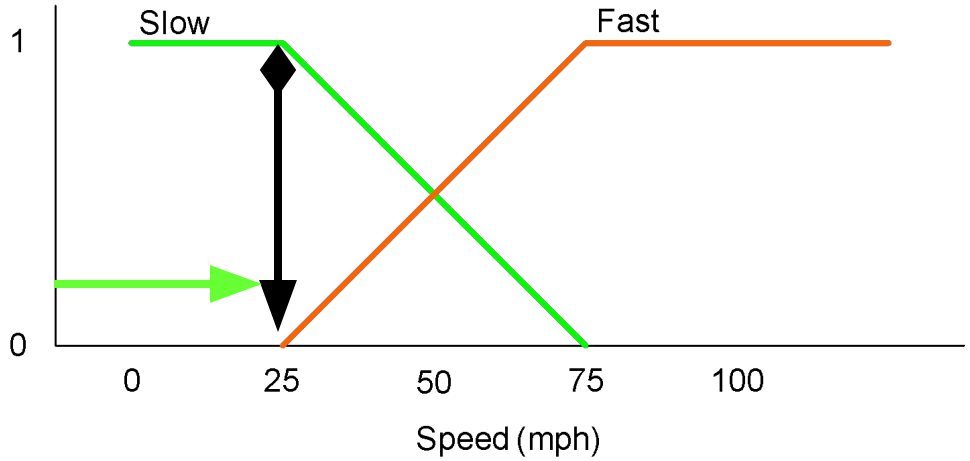
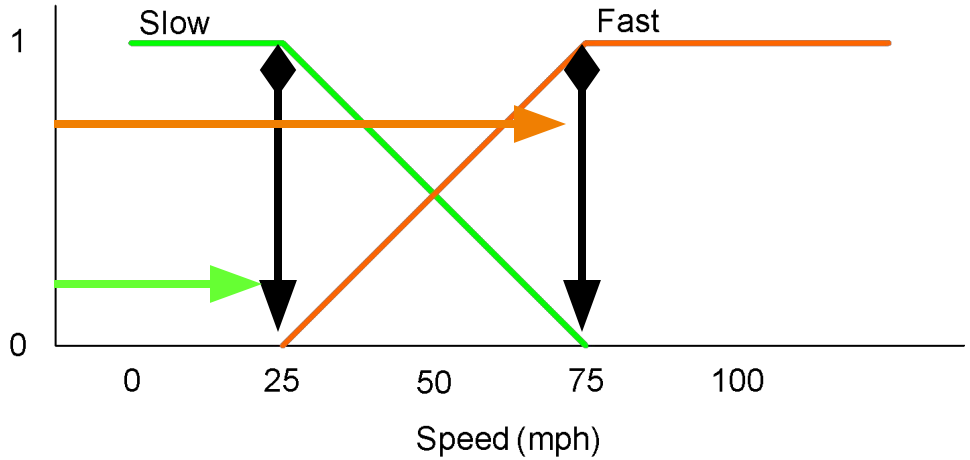


- Speed = weighted mean

  = (2*25+...

Defuzzification: Constructing the Output

- Speed is 20% Slow and 70% Fast



- Speed = weighted mean

  = (2*25+7*75)/(9)

  = 63.8 mph

Notes: Follow-up Points

- Fuzzy Logic Control allows for the smooth interpolation between variable centroids with relatively few rules
- This does not work with crisp (traditional Boolean) logic
- Provides a natural way to model some types of human expertise in a computer program

Notes: Drawbacks to Fuzzy logic

- Requires tuning of membership functions
- Fuzzy Logic control may not scale well to large or complex problems
- Deals with imprecision, and vagueness, but not uncertainty

Summary

- Fuzzy Logic provides way to calculate with imprecision and vagueness
- Fuzzy Logic can be used to represent some kinds of human expertise
- Fuzzy Membership Sets
- Fuzzy Linguistic Variables
- Fuzzy AND and OR
- Fuzzy Control