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(2.)

$$\det(A_\alpha - \lambda I) = \begin{vmatrix} \alpha-\lambda & 1 & 0 \\ 1 & \alpha-\lambda & 1 \\ 0 & 1 & \alpha-\lambda \end{vmatrix} = (\alpha-\lambda)^3 - 2(\alpha-\lambda) = 0$$

$$\begin{matrix} \alpha-\lambda & 1 & 0 \\ 1 & \alpha-\lambda & 1 \end{matrix}$$

$$(\alpha-\lambda)[(\alpha-\lambda)^2-2]=0$$

$\lambda_1 = \alpha$

$\lambda_2 = \alpha - \sqrt{2}$

$\lambda_3 = \alpha + \sqrt{2}$

$\text{spec}(A_\alpha) = \{\alpha-\sqrt{2}, \alpha, \alpha+\sqrt{2}\}$

$$\begin{pmatrix} \alpha & 1 & 0 \\ 1 & \alpha & 1 \\ 0 & 1 & \alpha \end{pmatrix} \qquad \alpha + 2\sqrt{2} = x$$

$\cancel{\alpha = \sqrt{2}}$

$\cancel{\ker(A_{\sqrt{2}} - \lambda I)}$

$$\cancel{\left(\begin{array}{ccc|c} \sqrt{2}-(\alpha-\sqrt{2}) & 1 & 0 & 0 \\ 1 & \sqrt{2}-(\alpha-\sqrt{2}) & 1 & 0 \\ 0 & 1 & \sqrt{2}-(\alpha-\sqrt{2}) & 0 \end{array}\right)} \sim \cancel{\left(\begin{array}{ccc|c} \alpha+2\sqrt{2} & 1 & 0 & 0 \\ 1 & \alpha+2\sqrt{2} & 1 & 0 \\ 0 & 1 & \alpha+2\sqrt{2} & 0 \end{array}\right)} \sim$$

$$\sim \cancel{\left(\begin{array}{ccc|c} x & 1 & 0 & 0 \\ 1 & x & 1 & 0 \\ 0 & 1 & x & 0 \end{array}\right)} \sim \cancel{\left(\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 0 & 1-x^2 & -x & 0 \\ 0 & 1 & x & 0 \end{array}\right)} \sim$$

$\dim(\mathbb{R}^3) = 3$

ak je ~~sa~~ $\text{spec}(A_\alpha)$ trojprvková, má A_α $\dim(\mathbb{R}^3) = 3$ rôzne vlastné hodnoty, tak je A_α diagonalizovateľná pre $\forall \alpha \in \mathbb{R}$, lebo ~~nikdy~~ $\alpha - \sqrt{2} \neq \alpha \neq \alpha + \sqrt{2}$ nikdy.

$\to$ A_α je diagonalizovateľná pre $\alpha \in \mathbb{R}$

$\to$ A_α má vlastnú hodnotu rovnú $\sqrt{2}$ ~~ak~~ pre $\alpha = 2\sqrt{2}$ alebo $\alpha = 0$