① $\mathcal{U} = \mathcal{L}o\{(1,1,1),(3,1,-1)\}$

$\mathbb{R}^3$ so skalárnym súčinom — Karolína Vallová

$\mathcal{U}$ je podpriestor $\mathbb{R}^3$

báza $\mathbb{R}^3$ $((0,0,1),(0,1,0),(1,0,0))$

$$\mathcal{U} - \mathcal{L}o\{\overset{y_1}{(1,1,1)}, \overset{y_2}{(3,1,-1)}\}$$

báza $Z = (\vec{x}_1, \vec{x}_2)$

ortogonálna báza

$$\vec{x}_1 = \vec{y}_1 = (1,1,1)$$

$$\vec{x}_2 = y_2 - proj_{\vec{x}_1}(\vec{y}_2) = y_2 - \frac{\langle \vec{y}_2, \vec{x}_1\rangle}{\langle \vec{x}_1, \vec{x}_1\rangle}\vec{x}_1 = \vec{y}_2 - \frac{3}{3}\vec{x}_1 = \vec{y}_2 - \vec{y}_1 \boxed{=}$$

$$\langle \vec{x}_1, \vec{x}_1\rangle = 3$$

$$\langle \vec{y}_2, \vec{x}_1\rangle = 3 + 1 - 1 = 3$$

$$\boxed{=} (2,0,-2) = x_2$$

$$\langle x_2, x_2\rangle = 4 + 4 = 8$$

$$Z = (\overset{x_1}{(1,1,1)}, \overset{x_2}{(2,0,-2)})$$

$$X = \left(\frac{x_1}{\|x_1\|}, \frac{x_2}{\|x_2\|}\right) = \left(\frac{x_1}{\sqrt{\langle x_1, x_1\rangle}}, \frac{x_2}{\sqrt{\langle x_2, x_2\rangle}}\right) = \left(\frac{x_1}{\sqrt{3}}, \frac{x_2}{\sqrt{8}}\right) = \left(\frac{1}{\sqrt{3}}(1,1,1), \frac{1}{\sqrt{8}}(2,0,-2)\right)$$

ortonormálna báza X

$$\mathcal{U}^{\perp} = \{\vec{w} \in \mathbb{R}^3 : \vec{w} \perp \mathcal{U}\}$$

$$\langle \vec{w}_1, \vec{y}_1\rangle = 0 \qquad \vec{w}_1 = (-1,1,1)$$

$$\langle \vec{w}_2, \vec{y}_2\rangle = 0 \qquad \vec{w}_2 = (-1,1,3)$$

$1 + 1 + 1$

$-1 + 1 + 3$

$$\vec{x}_1' = \vec{w}_1$$

$$X' = \left(\frac{\vec{x}_1'}{\sqrt{3}}, \frac{\vec{x}_2'}{\sqrt{8}}\right) = \left(\frac{1}{\sqrt{3}}(-1,1,1), \frac{1}{\sqrt{8}}(2,0,2)\right)$$

$$\vec{x}_2' = \vec{w}_2 - proj_{\vec{x}_1}(\vec{w}_2) = \vec{w}_2 - \frac{\langle \vec{w}_2, x_1'\rangle}{\langle \vec{x}_1, \vec{x}_1\rangle}\vec{x}_1 = \vec{w}_2 - \frac{3}{3}x_1 = \vec{w}_2 - \vec{w}_1 = (2,0,2) = \vec{x}_2'$$

$$Z' = ((-1,1,1),(2,0,2)) \qquad X' = \left(\frac{\vec{x}_1'}{\sqrt{3}}, \frac{\vec{x}_2'}{\sqrt{8}}\right)$$

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