In [1]:

```
from sklearn.datasets import load_iris
import numpy as np
import matplotlib.pyplot as plt
plt.rcParams['figure.figsize'] = (12, 8)
```

In [2]:

```
X, y = load_iris(return_X_y=True)
```

In [3]:

```
c1 = np.array([X[i] for i in range(len(y)) if y[i] == 0])
c2 = np.array([X[i] for i in range(len(y)) if y[i] == 1])
c3 = np.array([X[i] for i in range(len(y)) if y[i] == 2])
```

In [4]:

```
def cov(data):
    X = data - data.mean(axis=0, keepdims=True)
    return 1.0/len(data)*np.dot(X.transpose(), X)
```

In [5]:

```
a1, cov1 = c1.mean(axis=0), cov(c1)
a2, cov2 = c2.mean(axis=0), cov(c2)
a3, cov3 = c3.mean(axis=0), cov(c3)
```

Матожидания:

In [6]:

```
print a1
print a2
print a3
```

```
[ 5.006  3.418  1.464  0.244]
[ 5.936  2.77   4.26   1.326]
[ 6.588  2.974  5.552  2.026]
```

Матрицы ковариаций:

In [7]:

```
print cov1, '\n\n'
print cov2, '\n\n'
print cov3
```

```
[[ 0.121764  0.098292  0.015816  0.010336]
 [ 0.098292  0.142276  0.011448  0.011208]
 [ 0.015816  0.011448  0.029504  0.005584]
 [ 0.010336  0.011208  0.005584  0.011264]]


[[ 0.261104  0.08348   0.17924   0.054664]
 [ 0.08348   0.0965    0.081     0.04038 ]
 [ 0.17924   0.081     0.2164    0.07164 ]
 [ 0.054664  0.04038   0.07164   0.038324]]


[[ 0.396256  0.091888  0.297224  0.048112]
 [ 0.091888  0.101924  0.069952  0.046676]
 [ 0.297224  0.069952  0.298496  0.047848]
 [ 0.048112  0.046676  0.047848  0.073924]]
```

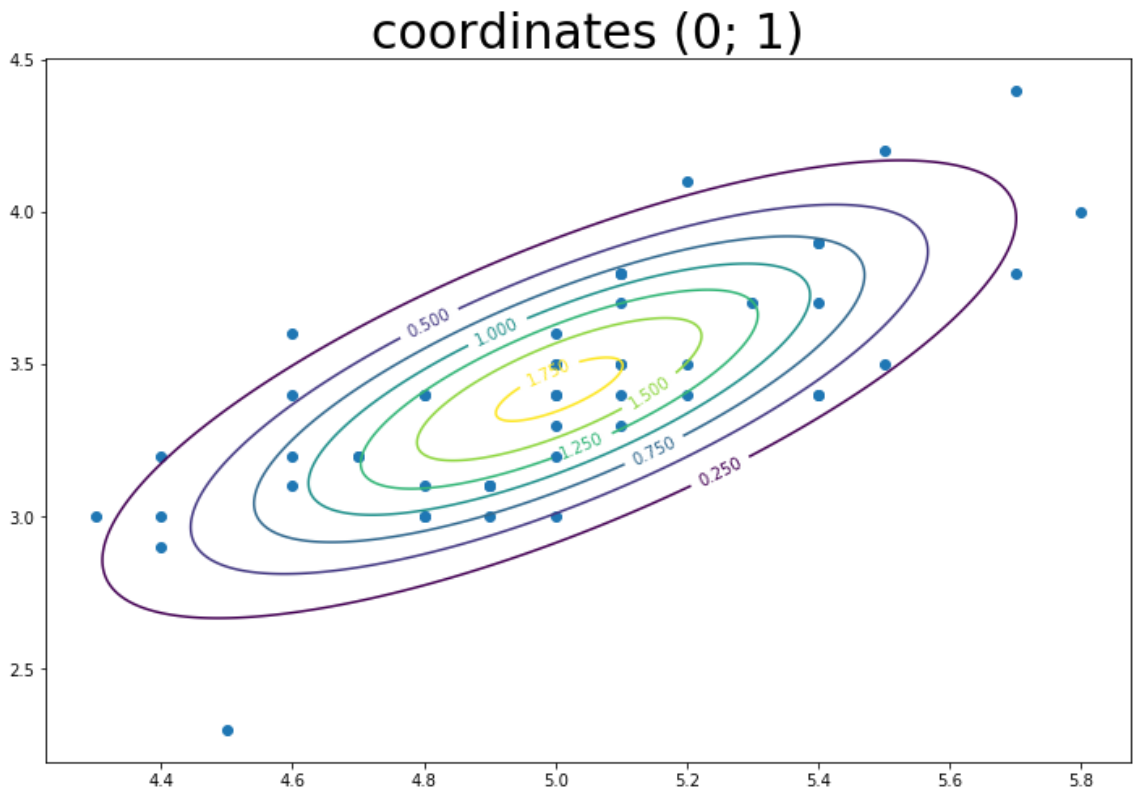
In [8]:

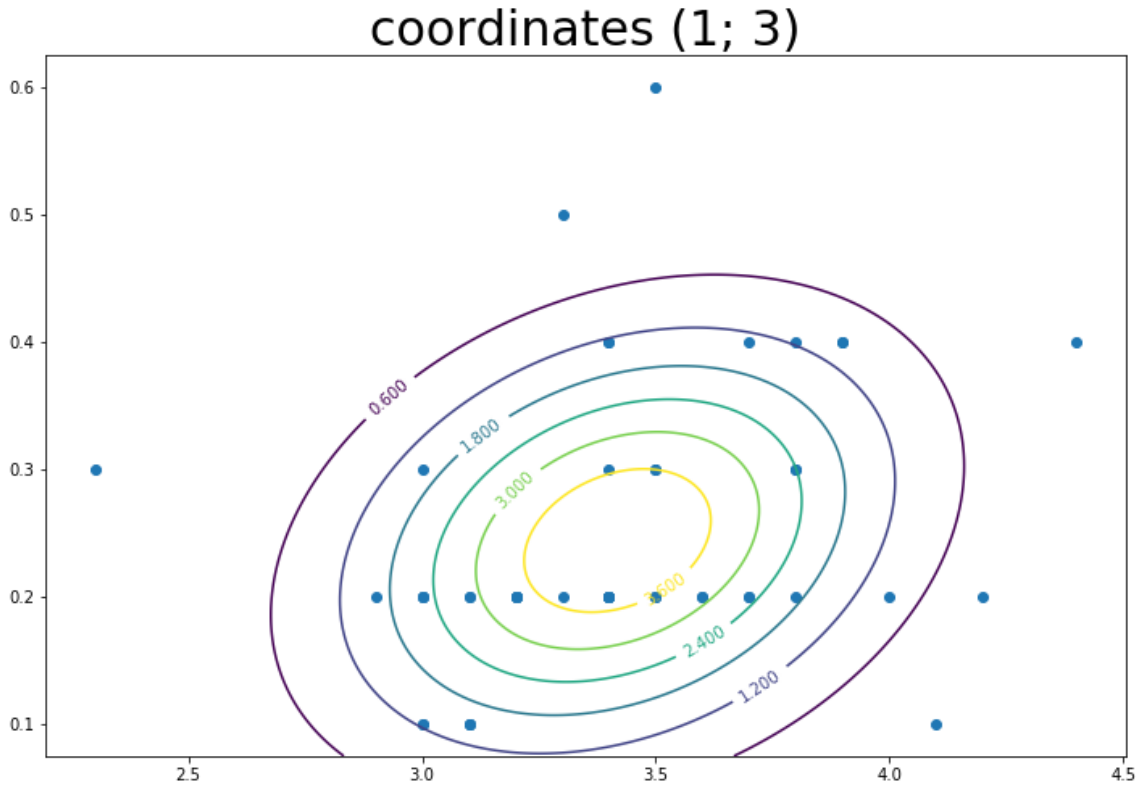
```
from scipy.stats import multivariate_normal
import matplotlib.cm as cm
import matplotlib.mlab as mlab

def countur(pdf, data, i, j):
    imin, imax, jmin, jmax = np.min(data[:,i]), np.max(data[:,i]),
np.min(data[:,j]), np.max(data[:,j])
    imin, imax = imin - 0.05*(imax-imin), imax + 0.05*(imax-imin)
    jmin, jmax = jmin - 0.05*(jmax-jmin), jmax + 0.05*(jmax-jmin)
    X, Y = np.meshgrid(np.linspace(imin, imax, 1000), np.linspace(jmin, jmax, 10
00))
    Z = np.zeros_like(X)
    for k in range(X.shape[0]):
        for l in range(Y.shape[1]):
            Z[k][l] = pdf((X[k][l], Y[k][l]))

    plt.figure()
    plt.scatter(data[:, i], data[:, j])
    CS = plt.contour(X, Y, Z)
    plt.clabel(CS, inline=1, fontsize=10)
    plt.title('coordinates (' + str(i) + '; ' + str(j) + ')', fontsize=30)
    plt.show()

def coord_pdf(a, cov_m, i, j):
    ij_a = np.array([a[i], a[j]])
    ij_cov = np.array([[cov_m[i][i], cov_m[i][j]], [cov_m[j][i], cov_m[j][j]]])
    rv = multivariate_normal(ij_a, ij_cov)
    return rv.pdf

def deal_with_component(data, a, cov_m):
    pdf1 = coord_pdf(a, cov_m, 0, 1)
    pdf2 = coord_pdf(a, cov_m, 1, 3)
    pdf3 = coord_pdf(a, cov_m, 2, 3)
    countur(pdf1, data, 0, 1)
    countur(pdf2, data, 1, 3)
    countur(pdf3, data, 2, 3)
```

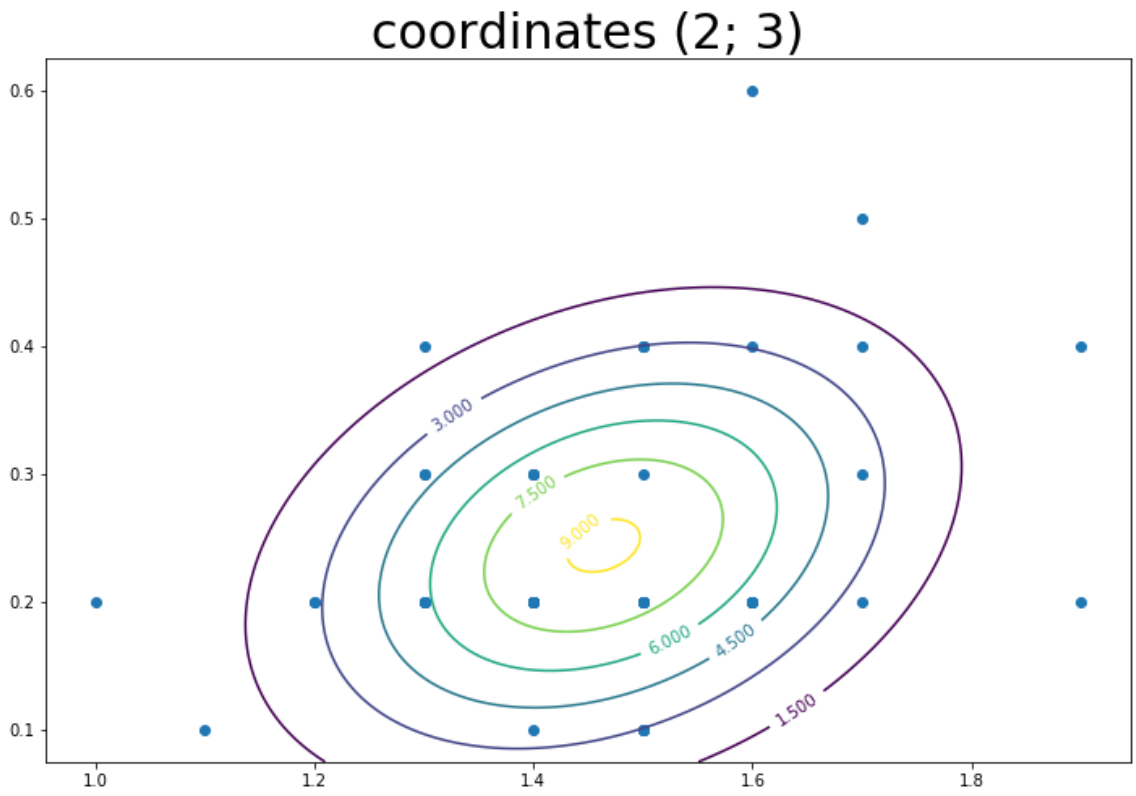
Компонента 0

In [10]:

```
deal_with_component(c1, a1, cov1)
```


coordinates (0; 1)
0.500
1.000
1.750
1.500
1.250
0.750
0.250
4.5
4.0
3.5
3.0
2.5
4.4
4.6
4.8
5.0
5.2
5.4
5.6
5.8

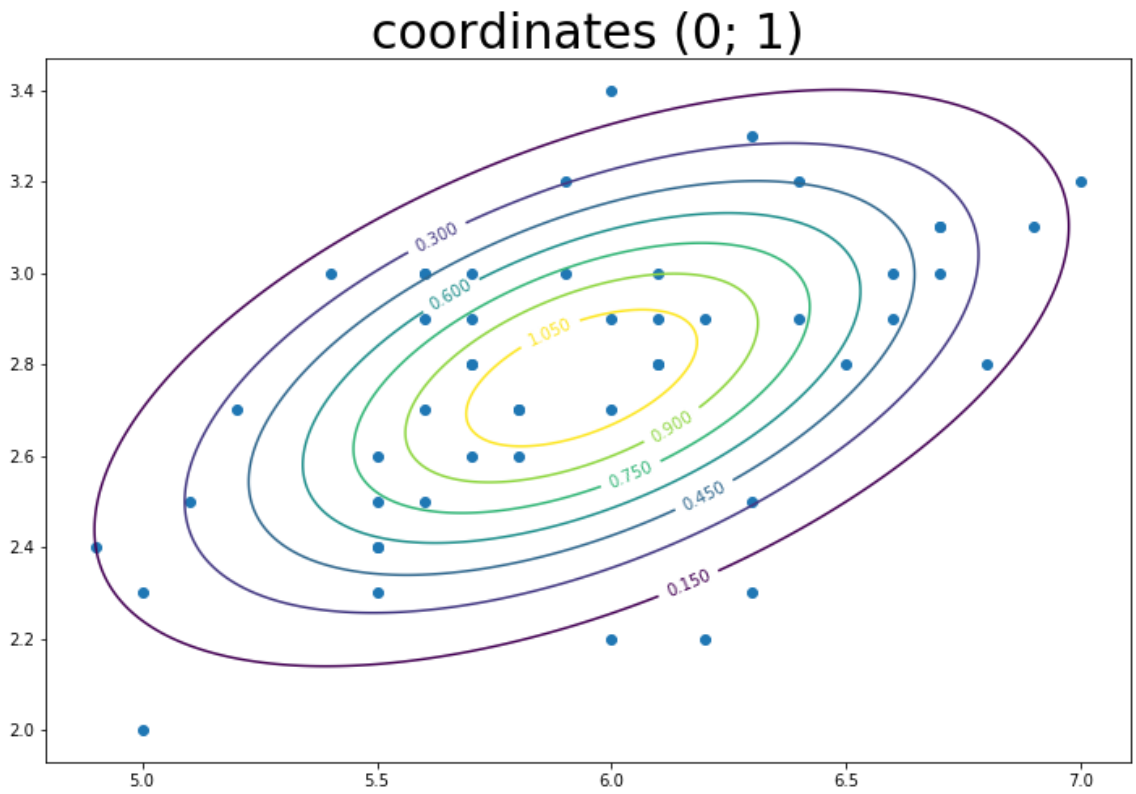

coordinates (1; 3)
0.600
1.800
3.000
2.400
1.200
0.6
0.5
0.4
0.3
0.2
0.1
2.5
3.0
3.5
4.0
4.5

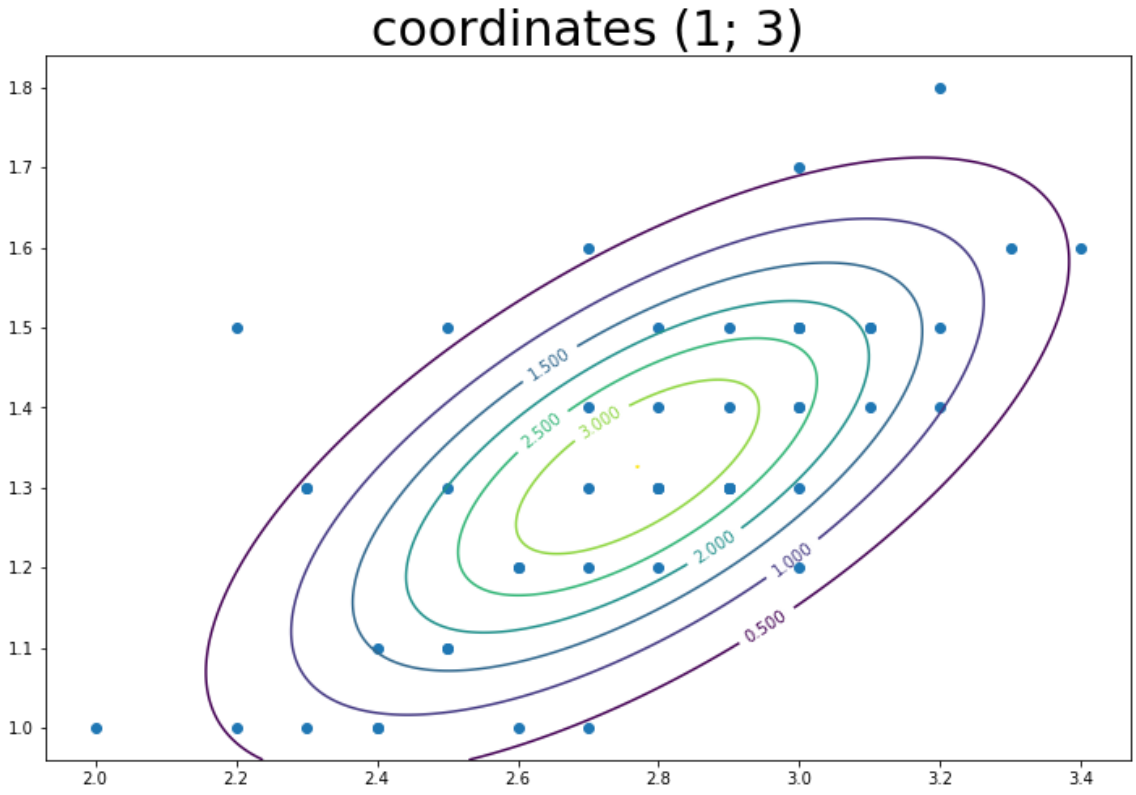


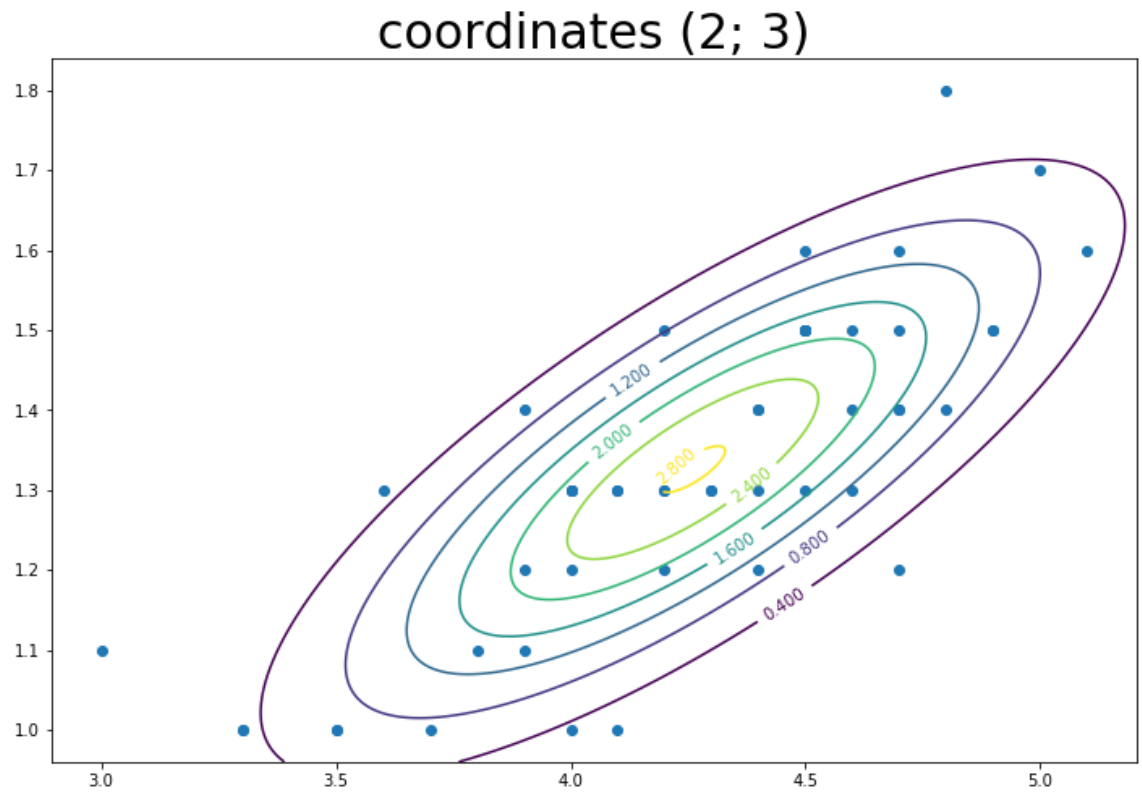
Компонента 1

In [11]:

```
deal_with_component(c2, a2, cov2)
```


coordinates (0; 1)
0.300
0.600
1.050
0.900
0.750
0.450
0.150
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
5.0
5.5
6.0
6.5
7.0

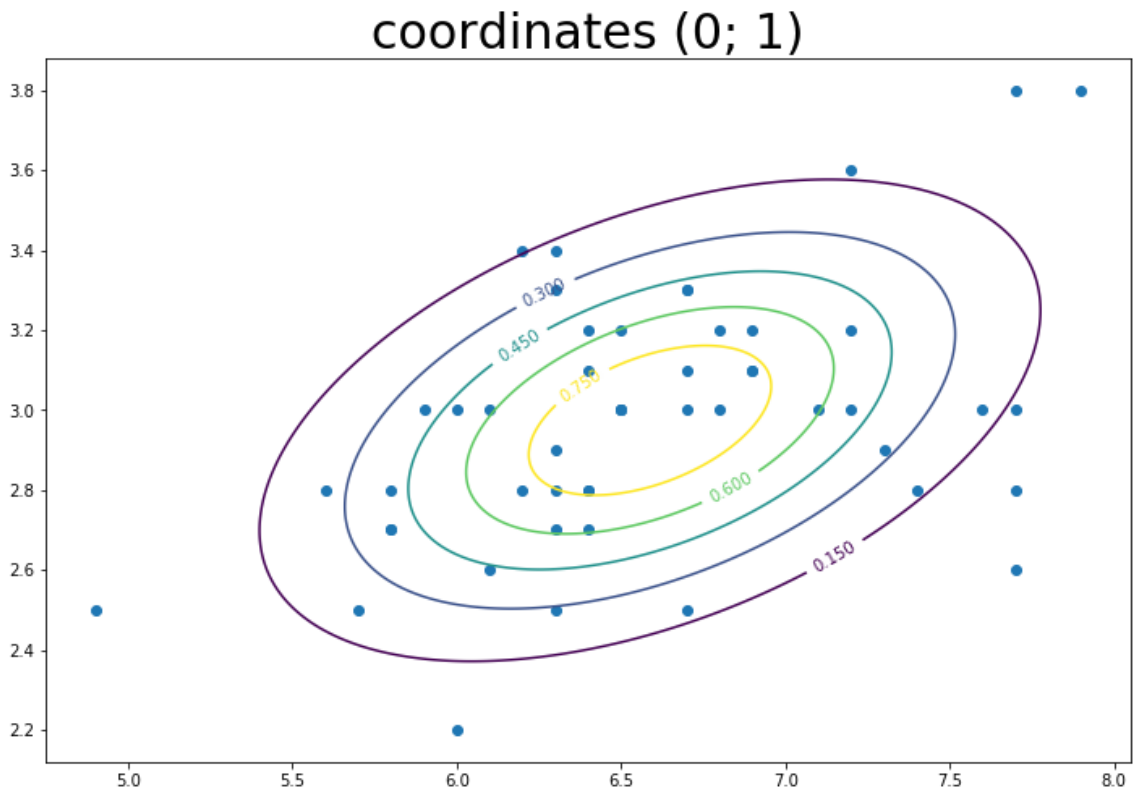

coordinates (1; 3)
1.500
2.500
3.000
2.000
1.000
0.500
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4

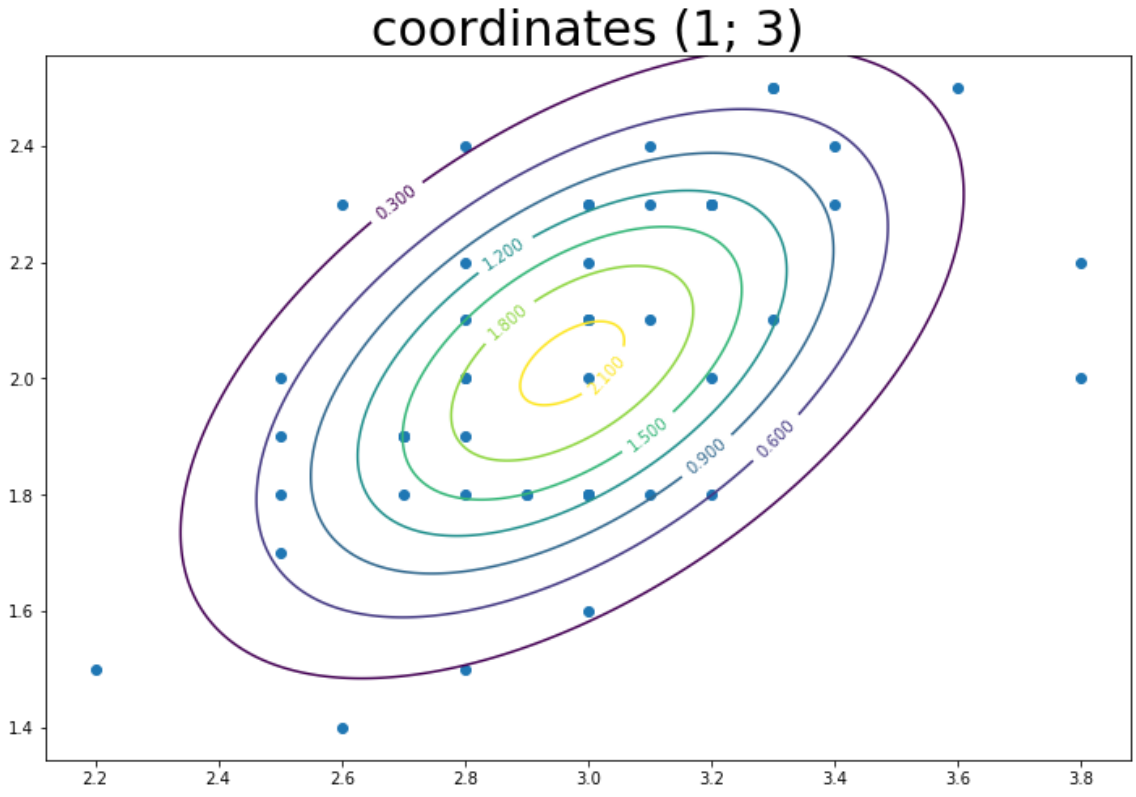


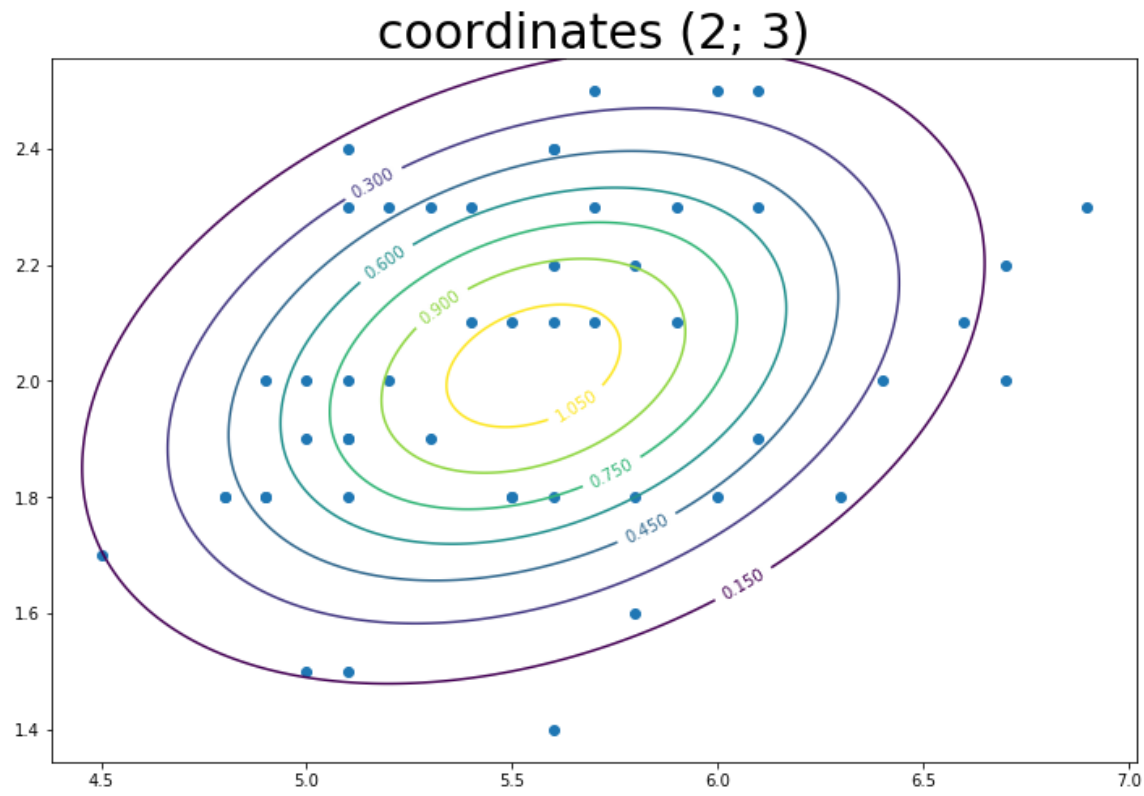
Компонента 2

In [12]:

```
deal_with_component(c3, a3, cov3)
```


coordinates (0; 1)
0.300
0.450
0.750
0.600
0.150


coordinates (1; 3)
0.300
1.200
1.800
2.100
1.500
0.900
0.600



Оценка P(t=k)

In [9]:

```
p1, p2, p3 = float(len(c1))/len(X), float(len(c2))/len(X), float(len(c3))/len(X)
```

$p(x|T \neq l) = \frac{p(x, T \neq l)}{p(T \neq l)}$

$p(x, T \neq l) = \sum_{k=1, k \neq l}^{3} p_k(x) P(T = k)$

$p(T \neq l) = \sum_{k=1, k \neq l}^{3} P(T = k)$

$E(x|T \neq l) = \frac{1}{p(T \neq l)} \sum_{k=1, k \neq l}^{3} a_k P(T = k)$, где a_k - матожидание соответствующей компоненты

$E(x|T \neq 0)$

In [10]:

```
print 1.0/(p2 + p3)*(a2*p2 + a3*p3)
```

```
[ 6.262  2.872  4.906  1.676]
```

$E(x|T \neq 1)$

In [11]:

```
print 1.0/(p1 + p3)*(a1*p1 + a3*p3)
```

```
[ 5.797  3.196  3.508  1.135]
```

$E(x|T \neq 2)$

In [12]:

```
print 1.0/(p1 + p2)*(a1*p1 + a2*p2)
```

```
[ 5.471  3.094  2.862  0.785]
```

Условные распределения

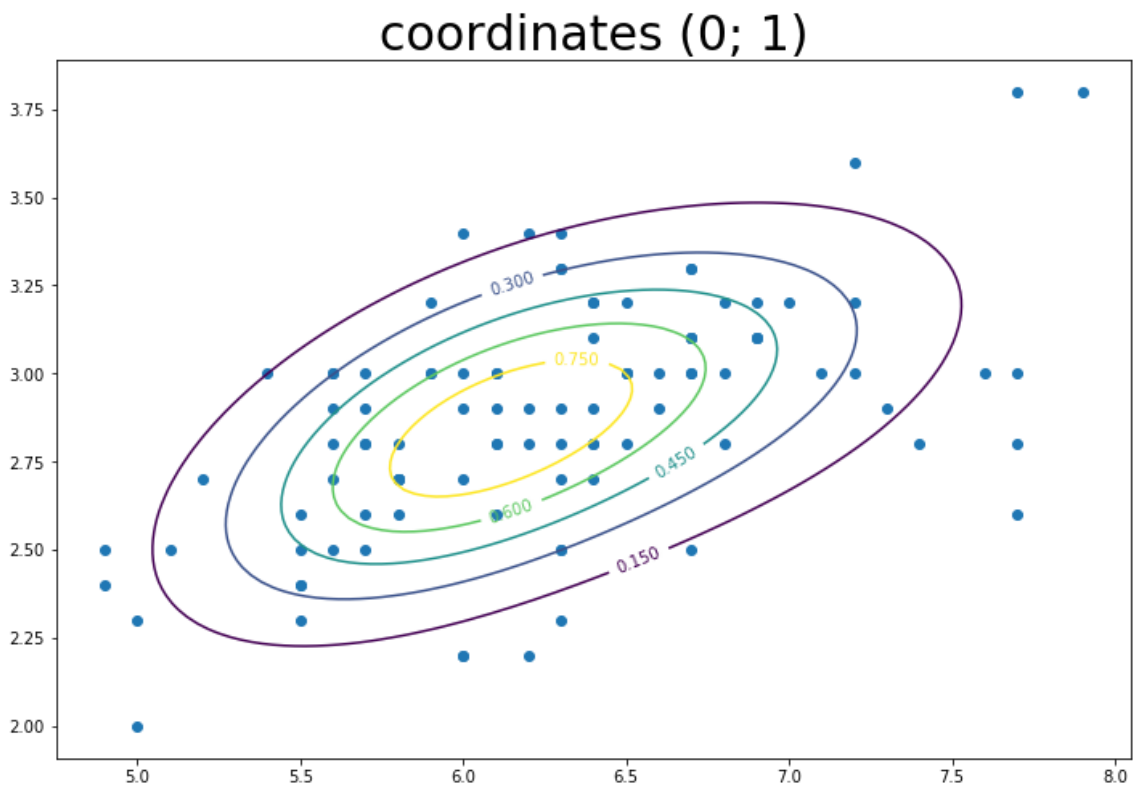
In [13]:

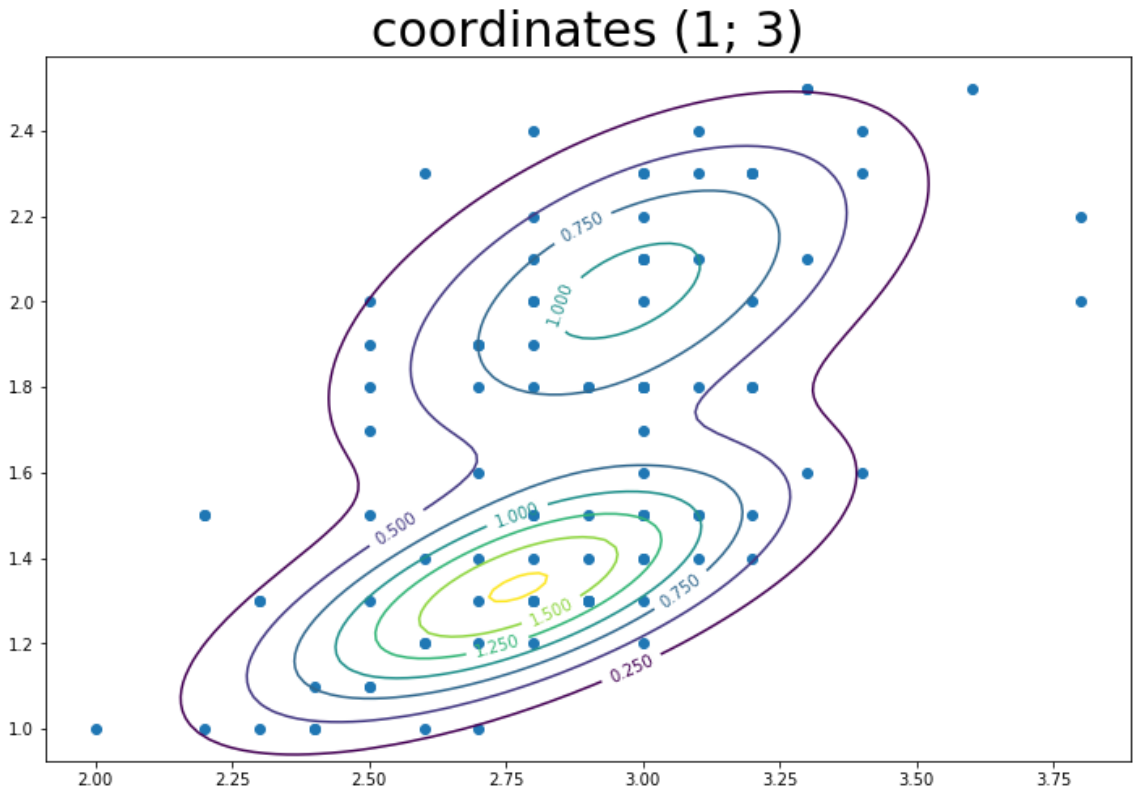
```
def countur(pdf, data, i, j):
    imin, imax, jmin, jmax = np.min(data[:,i]), np.max(data[:,i]),
np.min(data[:,j]), np.max(data[:,j])
    imin, imax = imin - 0.05*(imax-imin), imax + 0.05*(imax-imin)
    jmin, jmax = jmin - 0.05*(jmax-jmin), jmax + 0.05*(jmax-jmin)
    X, Y = np.meshgrid(np.linspace(imin, imax, 100), np.linspace(jmin, jmax,
100))
    Z = np.zeros_like(X)
    for k in range(X.shape[0]):
        for l in range(Y.shape[1]):
            Z[k][l] = pdf((X[k][l], Y[k][l]))

    plt.figure()
    CS = plt.contour(X, Y, Z)
    plt.scatter(data[:, i], data[:, j])
    plt.clabel(CS, inline=1, fontsize=10)
    plt.title('coordinates (' + str(i) + '; ' + str(j) + ')', fontsize=30)
    plt.show()

arra, arrcov, arrp, arrdata = [a1, a2, a3], [cov1, cov2, cov3], [p1, p2, p3], [c
1, c2, c3]
def deal_with_k(k):
    i, j = [n for n in range(3) if n != k-1]
    pdf1 = lambda vec: 1./(arrp[i] + arrp[j])*(arrp[i]*coord_pdf(arra[i],
arrcov[i], 0, 1)(vec) + arrp[j]*coord_pdf(arra[j], arrcov[j], 0, 1)(vec))
    pdf2 = lambda vec: 1./(arrp[i] + arrp[j])*(arrp[i]*coord_pdf(arra[i],
arrcov[i], 1, 3)(vec) + arrp[j]*coord_pdf(arra[j], arrcov[j], 1, 3)(vec))
    pdf3 = lambda vec: 1./(arrp[i] + arrp[j])*(arrp[i]*coord_pdf(arra[i],
arrcov[i], 2, 3)(vec) + arrp[j]*coord_pdf(arra[j], arrcov[j], 2, 3)(vec))
    countur(pdf1, np.concatenate([arrdata[i], arrdata[j]], axis=0), 0, 1)
    countur(pdf2, np.concatenate([arrdata[i], arrdata[j]], axis=0), 1, 3)
    countur(pdf3, np.concatenate([arrdata[i], arrdata[j]], axis=0), 2, 3)
```

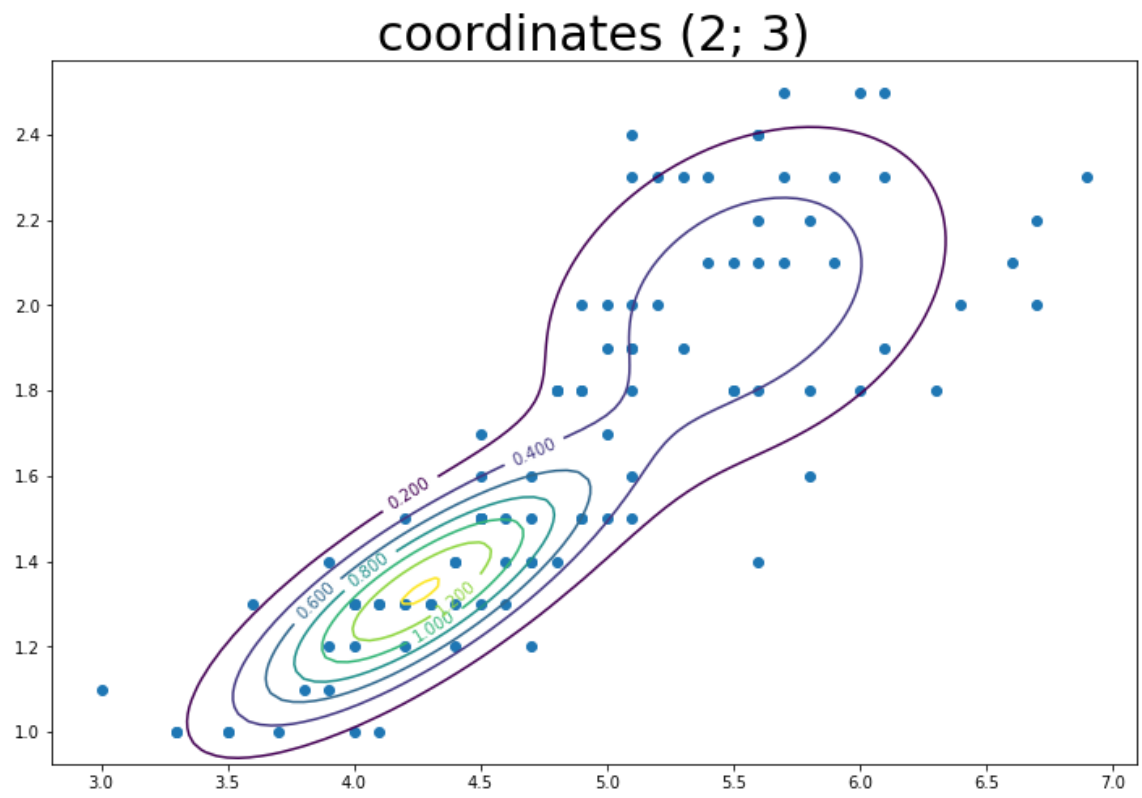
$T \neq 1$

In [35]:

```
deal_with_k(1)
```


coordinates (0; 1)
0.300
0.750
0.450
0.600
0.150

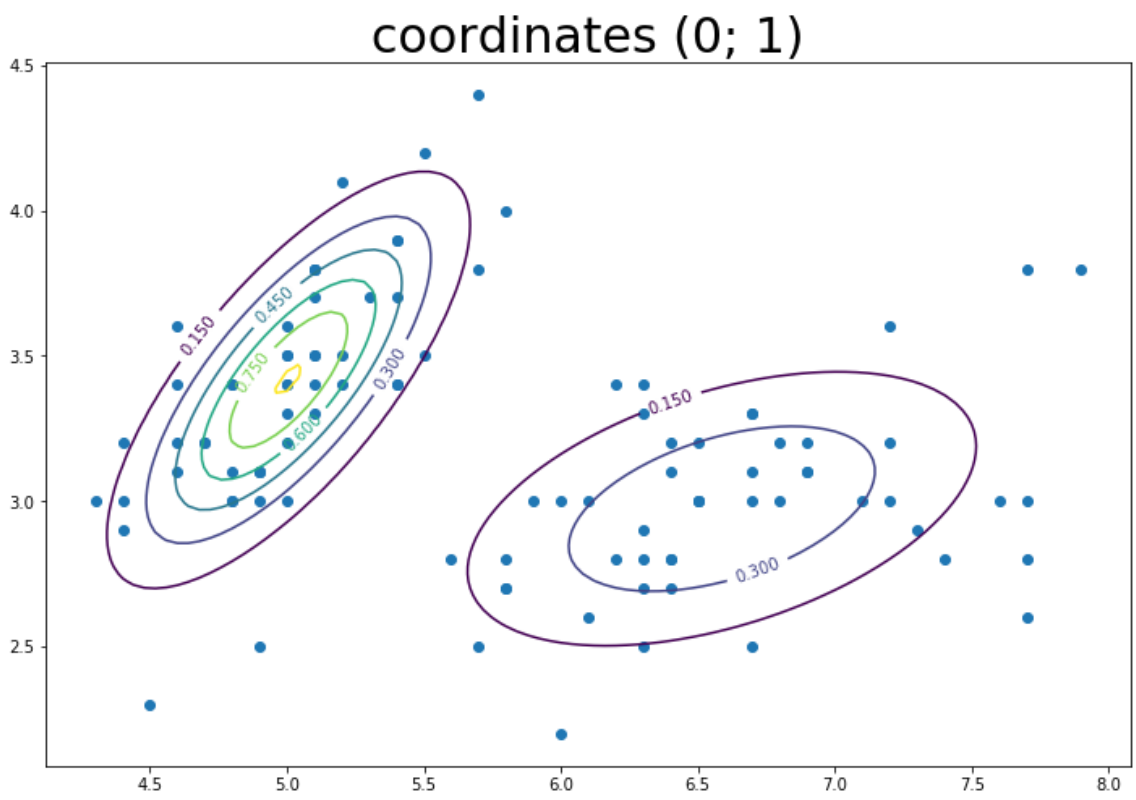

coordinates (1; 3)
0.750
1.000
0.500
1.000
1.500
0.750
1.250
0.250

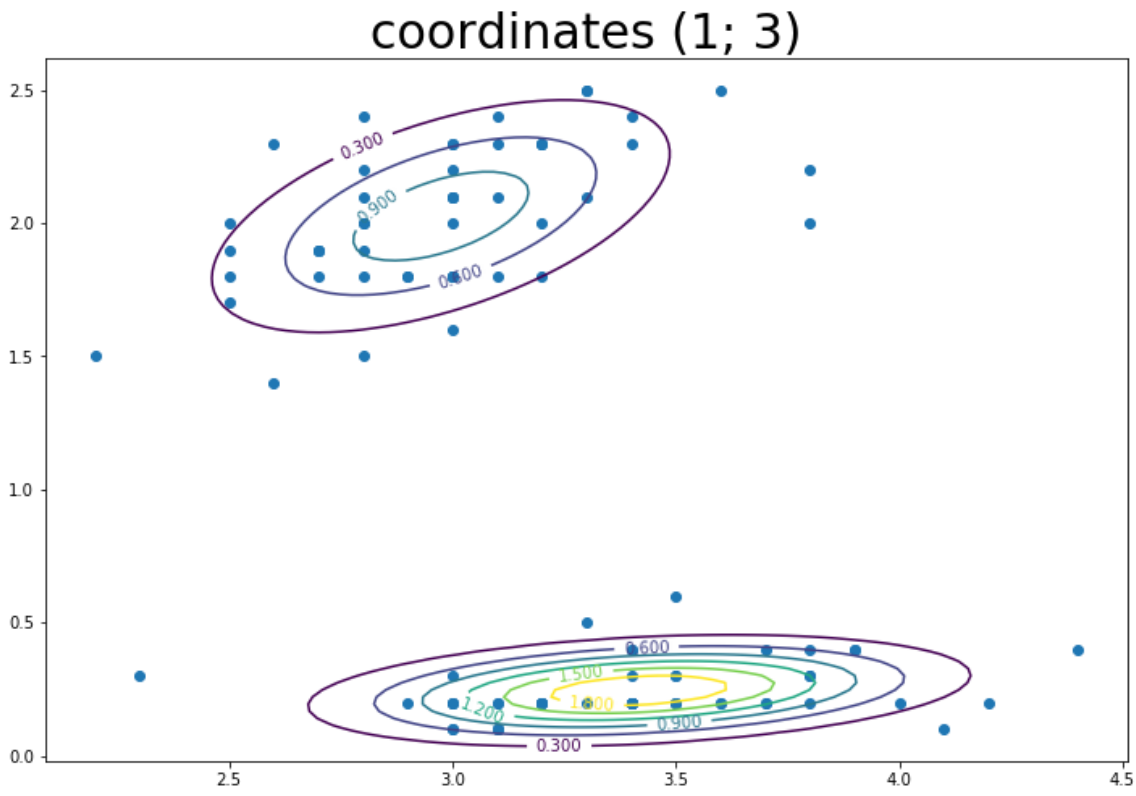

coordinates (2; 3)
0.200
0.400
0.600
0.800
1.000
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0

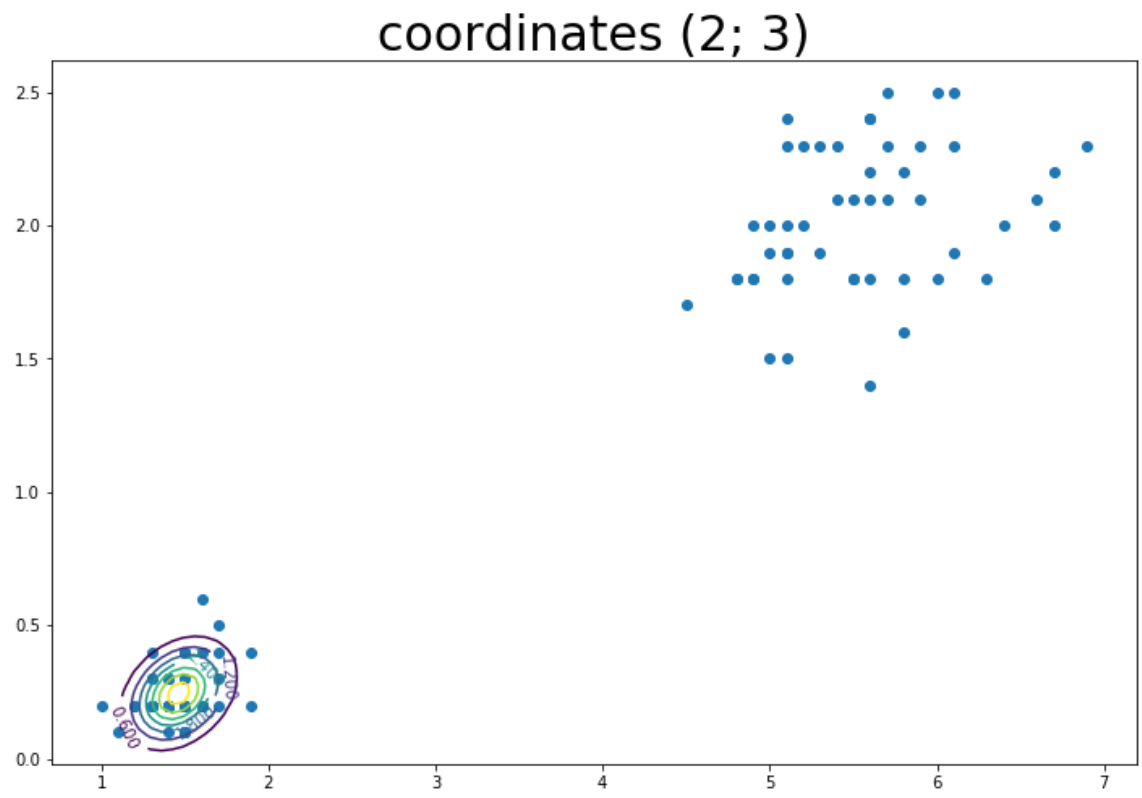
$T \neq 2$

In [36]:

```
deal_with_k(2)
```


coordinates (0; 1)
0.150
0.450
0.750
0.600
0.300
0.150
0.300

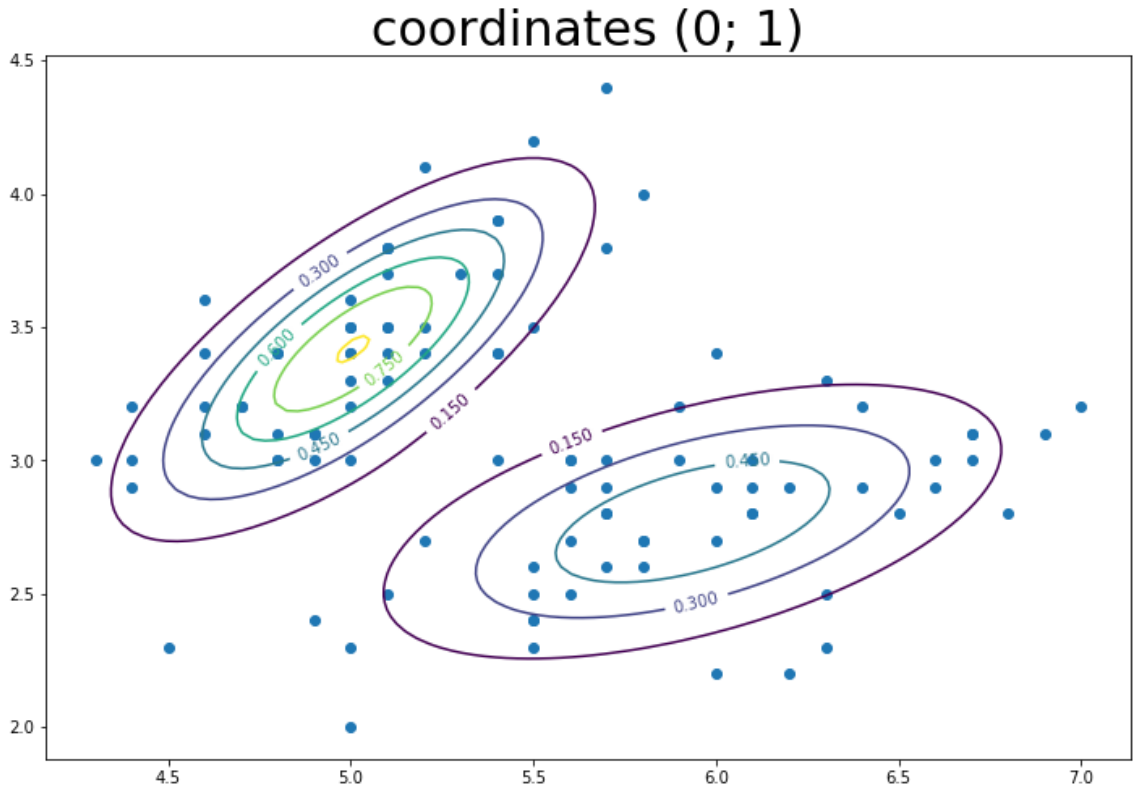

coordinates (1; 3)
0.300
0.900
0.600
1.500
1.200
0.900
0.300

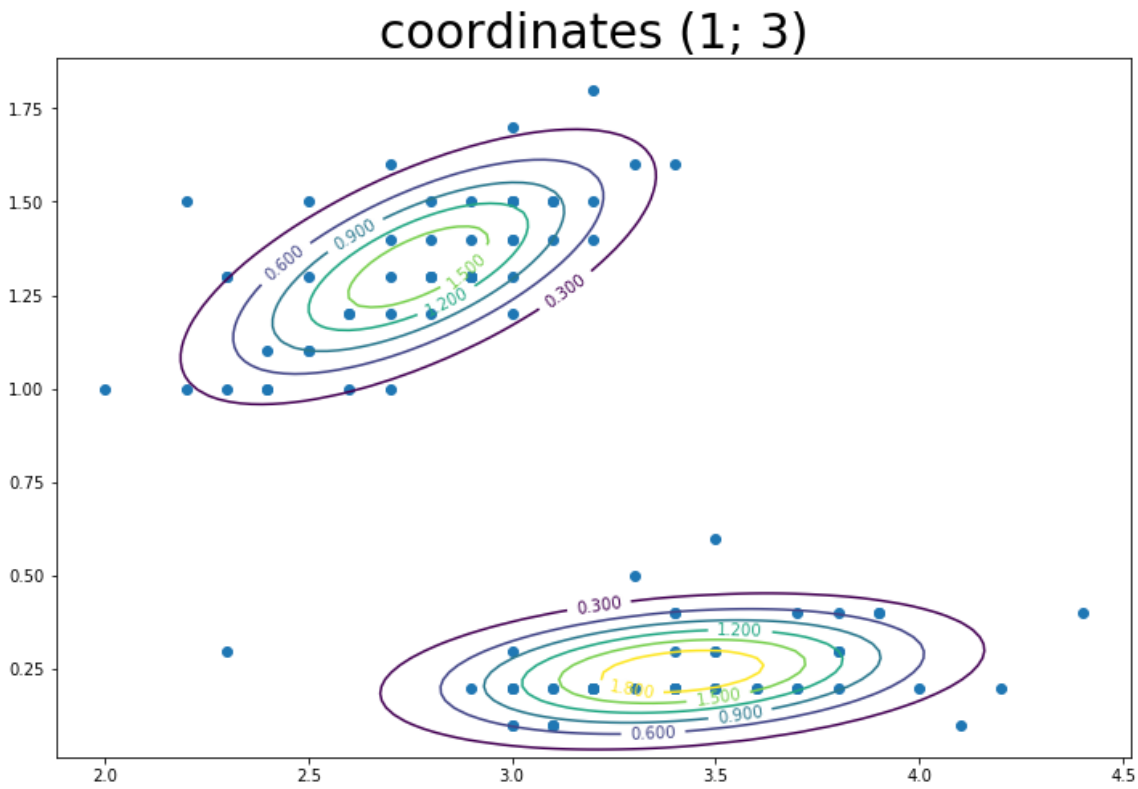


$T \neq 3$

In [37]:

```
deal_with_k(3)
```


coordinates (0; 1)
0.300
0.600
0.750
0.450
0.150
0.150
0.450
0.300


coordinates (1; 3)
0.600
0.900
1.500
1.200
0.300
0.300
1.200
0.900
0.600