In [1]:

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
plt.rcParams['figure.figsize'] = (15, 10)
```

In [3]:

```
data = pd.read_csv('Regression.csv')
```

In [29]:

```
X = np.concatenate([np.array([20.6165]), np.array(data.values).reshape(1, -1)
[0]], axis=0)
```

Введем Y_i:

$Y_0 = X_0 = \beta_1 + \varepsilon_0$

$Y_i = X_i - X_{i-1} = \beta_2 + \varepsilon_i$, для для $i = 1 \dots n$

Рассмотрим линейную модель для Y

где

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ \dots & \dots \\ 0 & 1 \end{pmatrix}, \quad Z\hat{\beta} = Y$$

Получаем оценки

$\hat{\beta} = \binom{\hat{\beta}_1}{\hat{\beta}_2} = (Z^T Z)^{-1} Z^T Y = \binom{X_0}{\frac{X_n - X_1}{n}}$

$\hat{\sigma^2} = \frac{1}{n-1}|Y - Z\hat{\beta}| = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - X_{i-1} - \frac{X_n - X_1}{n})^2$

$\hat{\sigma_t^2} = \frac{\hat{\sigma^2}}{\beta_2^2}$

Оценки:

$\binom{\hat{\beta}_1}{\hat{\beta}_2} = (Z^T Z)^{-1} Z^T Y = \binom{X_0}{\frac{X_n - X_1}{n}}$

In [31]:

```
beta1, beta2 = X[0], (X[-1] - X[1])/float(len(X) - 1)
print 'beta1 =', beta1, ' ||  beta2 =', beta2
```

```
beta1 = 20.6165  ||  beta2 = 12.0239843844
```

$\hat{\sigma^2} = \frac{1}{n-1}|Y - Z\hat{\beta}| = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - X_{i-1} - \frac{X_n - X_1}{n})^2$

In [33]:

```
n = len(X) - 1
sigma = 1.0/(n-1)*np.sum(map(lambda i: (X[i] - X[i-1] - (X[-1] -
X[1])/float(n))**2, range(1, len(X))))
print sigma
```

4.26325954594

$\hat{\sigma_t^2} = \frac{\hat{\sigma^2}}{\beta_2^2}$

In [35]:

```
print sigma/beta2**2
```

0.0294879761116

Видим, что линейная модель дает достаточно точные оценки (относительная ошибка меньше 3%).

In [ ]:

```
```