

WEEK 8

Dataset Overview

SNo.	Weather Condition	Road Condition	Traffic Condition	Engine Problem	Accident
1	Rain	Bad	High	No	Yes
2	Snow	Average	Normal	Yes	Yes
3	Clear	Bad	Light	No	No
4	Clear	Good	Light	Yes	Yes
5	Snow	Good	Normal	No	No
6	Rain	Average	Light	No	No
7	Rain	Good	Normal	No	No
8	Snow	Bad	High	No	Yes
9	Clear	Good	High	Yes	No
10	Clear	Bad	High	Yes	Yes

Student Exercises

1. Calculate Entropy and Information Gain:
 - Compute the Information Gain for each attribute in the dataset.

Entropy of Dataset D (Training set of instances with class labeled)

Formula to calculate entropy, $Ent(D) = - \sum (p_i * \log_2(p_i))$

Where p_i is the probability of each class.

Total Instances = 10

Accident = Yes → 5

Accident = No → 5

Therefore probabilities accident equals to yes and no ;

$p(\text{Yes}) = 5/10 = 0.5$

$p(\text{No}) = 5/10 = 0.5$

Using formula to calculate entropy;

$$\begin{aligned}\text{Entropy}(D) &= -(0.5 * \log_2(0.5) + 0.5 * \log_2(0.5)) \\ &= -(0.5 * (-1) + 0.5 * (-1)) \\ &= -(-0.5 - 0.5) \\ &= 1\end{aligned}$$

$$\text{Entropy}(D) = 1.000 \text{ bit}$$

1.2. Calculating entropy for each attributes in a dataset:

- **Weather Condition**

For Rain: instance=3, Yes=1, No =2,

$$\begin{aligned}\text{Using formula, Ent(rain)} &= -(1/3 * \log_2(1/3) + 2/3 * \log_2(2/3)) \\ &= 0.9183\end{aligned}$$

For Snow : instance=3, Yes=2, No=1;

$$\begin{aligned}\text{Using formula, Ent(snow)} &= -(1/3 * \log_2(1/3) + 2/3 * \log_2(2/3)) \\ &= 0.9183\end{aligned}$$

For Clear : instance=4, Yes=2, No=2;

$$\begin{aligned}\text{Using formula, Ent(clear)} &= -(2/4 * \log_2(2/4) + 2/4 * \log_2(2/4)) \\ &= 1\end{aligned}$$

Calculating weight average entropy of weather condition:

$$\begin{aligned}\text{Using formula, Ent(weather)} &= 3/10 * 0.9183 + 3/10 * 0.9183 + 4/10 * 1 \\ &= 0.951\end{aligned}$$

Calculating information gain of weather condition;

$$\text{Using formula, Gain(weather)} = 1 - 0.951 = 0.049$$

- **Road Condition**

For Good; instance=4, Yes=1, No=3;

Using formula, $\text{Ent}(\text{good}) = -(1/4 \cdot \log_2(1/4) + 3/4 \cdot \log_2(3/4))$
 $= 0.811$

For Bad; instance=4, Yes=3, No=1;

Using formula, $\text{Ent}(\text{bad}) = -(3/4 \cdot \log_2(3/4) + 1/4 \cdot \log_2(1/4))$
 $= 0.811$

For Average; instance=2, Yes=1, No=1;

Using formula, $\text{Ent}(\text{average}) = -(1/2 \cdot \log_2(1/2) + 1/2 \cdot \log_2(1/2))$
 $= 1.0000$

Calculating weight average entropy of Road condition:

Using formula, $\text{Ent}(\text{road}) = 4/10 \cdot 0.811 + 4/10 \cdot 0.811 + 2/10 \cdot 1$
 $= 0.849$

Calculating information gain of Road condition;

Using formula, $\text{Gain}(\text{road}) = 1 - 0.849 = 0.151$

- **Traffic Condition**

For High; instance=4, Yes=3, No=1;

Using formula, $\text{Ent}(\text{high}) = -(3/4 \cdot \log_2(3/4) + 1/4 \cdot \log_2(1/4))$
 $= 0.811$

For Normal; instance=3, Yes=1, No=2;

Using formula, $\text{Ent}(\text{normal}) = -(1/3 \cdot \log_2(1/3) + 2/3 \cdot \log_2(2/3))$
 $= 0.918$

For Light; instance=3, Yes=1, No=2;

Using formula, $\text{Ent}(\text{light}) = -(1/3 \cdot \log_2(1/3) + 2/3 \cdot \log_2(2/3))$
 $= 0.918$

Calculating weight average entropy of Traffic condition:

Using formula, $\text{Ent}(\text{traffic}) = 4/10 \cdot 0.811 + 3/10 \cdot 0.918 + 3/10 \cdot 0.918$
 $= 0.875$

Calculating information gain of weather condition;

Using formula, $\text{Gain}(\text{traffic}) = 1 - 0.875 = 0.125$

- **Engine Problem**

For YES; instance=4, Yes=3, No=1;

Using formula, $\text{Ent}(\text{yes}) = -(3/4 \cdot \log_2(3/4) + 1/4 \cdot \log_2(1/4))$
 $= 0.811$

For NO; instance=6, Yes=2, No=4;

Using formula, $\text{Ent}(\text{no}) = -(2/6 \cdot \log_2(2/6) + 4/6 \cdot \log_2(4/6))$
 $= 0.918$

Calculating weight average entropy of Traffic condition:

Using formula, $\text{Ent}(\text{engine}) = 4/10 \cdot 0.811 + 6/10 \cdot 0.918$
 $= 0.875$

Calculating information gain of weather condition;

Using formula, $\text{Gain}(\text{engine}) = 1 - 0.875 = 0.125$

Determine the root of the decision tree.

=> The root of the decision tree is Road condition with the highest information gain of 0.151

2. Calculate Gini Impurity:

- Perform the same split analysis using Gini Impurity.

Gini Impurity Formula:

$$\text{Gini}(D) = 1 - \sum (p_i^2)$$

For Full Dataset :

$p(\text{Yes})=0.5$, $p(\text{No})=0.5$,

$$\text{Gini}(D) = 1 - (0.5*0.5 + 0.5*0.5)$$

$$= 1 - (0.25 + 0.25)$$

$$= 1 - 0.5 = 0.5$$

Gini Impurity of all subsets:

- Weather

· Rain: Yes=1 No=2

$$\rightarrow \text{Gini}(\text{rain}) = 1 - [(1/3*1/3) + (2/3*2/3)]$$

$$= 0.4444$$

· Snow: Yes=2 No=1

$$\rightarrow \text{Gini}(\text{snow}) = 1 - [(2/3*2/3) + (1/3*1/3)]$$

$$= 0.4444$$

· Clear: Yes=2 No=2

$$\rightarrow \text{Gini}(\text{clear}) = 1 - [(2/4*2/4) + (2/4*2/4)]$$

$$= 0.5$$

$$\text{Weight gini(weather)} = 3/10 \cdot (0.444) + 3/10 \cdot (0.444) + 4/10 \cdot (0.5) = 0.4667$$

$$\text{Gain (weather)} = \text{gini(D)} - \text{weight gini(weather)} = 0.0333$$

- Road Condition

$$\cdot \text{ Bad: Yes}=3, \text{ No}=1 \rightarrow \text{Gini(bad)} = 1 - [(3/4 \cdot 3/4) + (1/4 \cdot 1/4)] = 0.375$$

$$\cdot \text{ Average: Yes}=1, \text{ No}=1 \rightarrow \text{Gini(average)} = 1 - [(1/2 \cdot 1/2) + (1/2 \cdot 1/2)] = 0.5$$

$$\cdot \text{ Good: Yes}=1, \text{ No}=3 \rightarrow \text{Gini (good)} = 1 - [(1/4 \cdot 1/4) + (3/4 \cdot 3/4)] = 0.375$$

$$\text{Weight gini(road)} = 0.4(0.375) + 0.2(0.5) + 0.4(0.375) = 0.40$$

$$\text{Gain (road)} = \text{gini(D)} - \text{weight gini(road)} = 0.5 - 0.40 = 0.10$$

- Traffic Condition

$$\cdot \text{ High: Yes}=2, \text{ No}=2 \rightarrow \text{Gini(high)} = 1 - [(2/4 \cdot 2/4) + (2/4 \cdot 2/4)] = 0.5$$

$$\cdot \text{ Normal: Yes}=1, \text{ No}=2 \rightarrow \text{Gini(normal)} = 1 - [(1/3 \cdot 1/3) + (2/3 \cdot 2/3)] = 0.4444$$

$$\cdot \text{ Light: Yes}=1, \text{ No}=2 \rightarrow \text{Gini (light)} = 1 - [(1/3 \cdot 1/3) + (2/3 \cdot 2/3)] = 0.444$$

$$\text{Weight gini(traffic)} = 0.4(0.5) + 0.3(0.4444) + 0.3(0.4444)$$

$$= \mathbf{0.4667}$$

$$\text{Gain (traffic)} = \text{gini(D)} - \text{weight gini(traffic)} = 0.0333$$

- **Engine Problem**

YES: Yes=3, No=1 \rightarrow Gini(high) = $1 - [(3/4 * 3/4) + (1/4 * 1/4)] = 0.375$

NO: Yes=2, No=4 \rightarrow Gini(normal) = $1 - [(2/6 * 2/6) + (4/6 * 4/6)] = 0.4444$

Weight gini(engine) = $0.4(0.375) + 0.3(0.4444)$
 $= 0.417$

Gain (engine) = gini(D) - weight gini(engine) = 0.083