PERMUTATION AND COMBINATION

1. Five students are to be arranged on five chairs for a photograph. Three of these are girls and the rest are boys.

Find out the total number of ways in which three girls are together.

A. 36
B. 84
C. 100
D. 120

2. Using all the letters of the word LINEAR.

How many different words can be formed that start and end with vowel?

A. 126
B. 108
C. 144
D. 216

3. In a badminton competition involving some men and women of a society, every person had to play exactly one game with every other person. It was found that in 36 games both the players were men and in 78 games both the players were women. Find the number of games in which one player was a man and other was a woman.

A. 127
B. 117
C. 138
D. 146

4. A forgetful teacher had the test papers and the marksheets of 5 students. But, he entered someone else's marks for each of the 5 students. In how many ways could he have entered the correct marks for at least one student?

A. 36
B. 44
C. 84
D. 76

5. There are 5 singers, 8 dancers and 7 actors in a movie. A group of 6 members will be chosen to nominate in an award show.

How many combinations of members are possible if the group is to consist of all members of the same profession?

A. 20
B. 25
C. 30
D. 35

6. How many 3 - letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?

A. 720
B. 420
C. 5040
D. 120

7. The number of ways in which 8 different books can be arranged on a shelf so that 3 particular books shall not be together:

A. $11! - 3!$
B. 361000
C. $8! \times 3! - 5!$
D. 36000

8. There are 2 shirts, 3 jeans, 3 socks and 2 skirts. In how many ways a shopkeeper can arrange these things so that all the socks come together and all the skirts come together?

A. 2520
B. 5040
C. 4690
D. 3260

9. How many numbers are there in between 100 and 1000 such that exactly one of their digits is 3 if repetition is not allowed?

A. 100
B. 200
C. 300
D. 525

10. In how many ways a group of 3 students can be selected from 7 men and 5 women consisting of 1 man and 2 women?

A. 7
B. 110
C. 60

D. 70

11. 16 persons shake hands with one another in a party. How many shake hands took place?

A. 124
B. 120
C. 165
D. 150

12. A team of 7 children is to be selected out of 7 girls and 5 boys such that it contains at least 5 girls. In how many different ways can the selection be made?

A. 105
B. 246
C. 100
D. 128

13. How many five digit numbers can be formed with the numbers 1, 2, 3, 4, 5 and 6 which are divisible by 4 if repetition is allowed?

A. 1728
B. 1444
C. 1600
D. 1344

14. In how many different ways can the letters of word RABBIT be arranged?

A. 360
B. 240
C. 300
D. 275

15. In an auditorium the chairs were arranged such that the number of rows were 3 more than the number of columns. The chairs are rearranged by removing 3 columns and adding 6 rows without adding or removing any chair. How many people can sit in that auditorium at a time?

A. 124
B. 96
C. 108
D. 98

16. Five people out of whom only two can drive are to be seated in a five seater car with two seats in front and three in the rear. The people who know driving don't sit together.

Only someone who knows driving can sit on the driver's seat. Find the number of ways the five people can be seated.

A. 40
B. 60
C. 48
D. 36

17. A boy is playing a Snake & Ladder game; he is on 91 and has to get to 100 to complete the game. There is a snake on 93 and 96. In how many ways he can complete the game, if he doesn't want to roll the dice more than three times.

A. 20
B. 15
C. 16
D. 18

18. A chess board has rows and columns marked A to H and 1-8. Aman has a knight and a rook which he has to place on the board such that the two pieces are not in same row or column, what is total number of ways he can place the two pieces?

A. 3072
B. 3136
C. 6272
D. 6144

19. An objective test with all the questions mandatory to be answered can be attempted in 127 ways such that the student gets at least one question right. Find the number of ways in which he can answer 4 questions correctly.

A. 44
B. 35
C. 28
D. Can't be determined

20. A postmaster wants to get delivered 6 letters at six different addresses. In the post office there are 2 postmen then in how many ways can the postmaster send the letters at different addresses through the postmen?

A. $6! / 2!$
B. $6! \times 2!$
C. 64
D. 36

Answers:

1. A

Solution: As per the question, three girls can't occupy consecutive seats but two can.

Therefore, if we find the number of ways in which all three girls occupy consecutive seats and subtract this number from the total number of ways in which the five people can be arranged among themselves, we will get the required answer.

5 students can be arranged among themselves in $^5P_5$ ways = 120 ways.

Assume that the 3 girls are one entity. The total number of ways in which they can be arranged among themselves = 3! = 6

Also, the set of three girls and the other students can be arranged among themselves in 3! = 6 ways.

Thus, total number of ways in which three girls are together = 6 × 6 = 36

Thus, number of ways in which all 3 girls will not occupy consecutive seats = 120 – 36 = 84

2. C

Solution: The word LINEAR has three vowels - I, E and A. If a word starts and ends with a vowel, the two letters to occupy the first and the last positions can be selected and arranged in $^3P_2$ = 6 ways.

The remaining 4 letters can be arranged among themselves in $^4P_4$ = 4! = 24 ways.

∴ The number of words that start and end with a vowel = 24 × 6 = 144.

If a word starts with a vowel but ends with a consonant, its first letter can be selected from I, E and A in 3 ways. Its last letter can be selected from L, N and R in 3 ways. The remaining three letters can be arranged in 4! ways.

∴ The number of words that start with a vowel but end with a consonant = 3 × 3 × 4! = 9 × 24 = 216.

3. B

Solution: Let the number of men be x and women be y

In badminton two person can play at a time,

Therefore, no of games played between men is $^XC_2$=36

x (x – 1)/2 = 36

$x(x-1) = 72$

$x = 9$

Which means total number of men playing badminton are 9

Now, no of games played between women is ${}^{Y}C_2=78$

$y(y-1)/2 = 78$

$y(y-1) = 156$

$y = 13$

This means total number of women playing badminton are 13

Therefore, no of games in which one player is man and one is woman is,

${}^{9}C_1 \times {}^{13}C_1 = 117$

4. D

Solution: Applying the derangement formula, the number of ways in which this error could have been done is given by:

$D5 = 5! \times ((1/2!)-(1/3!)+(1/4!)-(1/5!))$

$\Rightarrow 120 \times ((1/2)-(1/6)+(1/24)-(1/120))$

$\Rightarrow 120 \times (44/120) = 44$

The teacher could have made this error in 44 different ways.

Total number of ways in which 5 mark sheets can be given to 5 students = ${}^{5}P_5 = 5! =$ 120 ways.

Of these, in 44 ways, all the marks entered would be incorrect.

So, the number of ways in which he could have entered correct marks for at least one student = 120 – 44 = 76.

5. D

Solution: Here we want 6 singers or 6 dancers or 6 actors.

The group cannot be of all singers since there are only 5 singers.

Therefore, the group can be a group of 6 dancers or 6 actors.

Number of groups of dancers = ${}^{8}C_6 = 28$

Number of groups of actors = ${}^{7}C_{6}$ =7

Since we want the number of groups of 6 dancers or 6 actors, we want the sum of each of these possibilities = 28 + 7 = 35

6. A

Solution: The word 'LOGARITHMS'' has 10 different alphabets.

Hence, the number of 3-letter words (with or without meaning) formed by using these letters = $({}^{10}P_{3})$ = 10 × 9 × 8 = 720

7. D

Solution: Number of ways in which 8 books can be arranged = 8!

Number of ways when three particular books are together = 6! X 3!

Therefore Number of ways when three particular books are not together = 8! – 6! X 3!

= 6! (7 x 8 – 3 x 2)

= 6! X 50 = 720 × 50 = 36000

8. B

Solution: We will count 3 socks as 1 socks and 2 skirts as 1 skirt.

Total ways = 7!

= 7 × 6 × 5 × 4 × 3 × 2 × 1 = 5040

9. B

Solution: Surely 3 can occur at either hundreds place or tens place or units place. So three cases arise.

a) If 3 occurs at hundredths place then the digit at tens place can be chosen in only nine ways (all ten digits leaving only 3 so we are left with 9 digits) and digit at units place can be chosen in only 8 ways (as 3 and digit at tens place cannot be used again)

So total such numbers = 1×9×8 = 72

b) If 3 occurs at tens place then its hundreds place can be only chosen in only 8 ways (because use of 3 is not allowed and if we use 0 out of the remaining 9 digits it will be a 2-digit number which is not allowed) and unit place can be chosen only in 8 ways (since digit at hundredths place and 3 is not allowed)

So total such numbers = 8 × 1 × 8 = 64

c) If 3 occurs at units place then its hundreds place can be chosen in only 8 ways (because use of 3 is not allowed and if we use 0 out of the remaining 9 digits it will be a 2-digit number which is not allowed) and tens place can be chosen only in 8 ways (since digit at hundredths place and 3 is not allowed)

So total such numbers = $8 \times 8 \times 1 = 64$

Hence total such numbers = $72 + 64 + 64 = 200$

10. D

Solution: No. of ways of selecting one man out of 7 women = ${}^7C_1 = 7$

No of ways of selecting 2 women out of 5 women = ${}^5C_2 = 10$

Required ways = ${}^7C_1 \times {}^5C_2 = 7 \times 10 = 70$

11. B

Solution: Total possible ways = ${}^{16}C_2$

$= (16 \times 15) / (2 \times 1) = 120$

12. B

Solution: A team of seven children consisting of at least five girls can be formed in the following ways

Case 1: When five girls and two boys are selected 7C5 x 5C2

Case II: When six girls and one boys are selected 7C6 x 5C1

Case III: When seven girls and no boy are selected 7C7

Regd. no. of selections = 7C5 x 5C2 + 7C6 x 5C1 + 7C7

$= ((7 \times 6)/2) \times ((5 \times 4)/2) + (7 \times (5/1)) + 1$

$= 21 \times 10 + 7 \times 5 + 1 = 210 + 35 + 1 = 246$

13. A

Solution: A number is divisible by 4 if its last two digits are divisible by 4.

Since the divisibility of 4 depends upon only the last two number, the first three numbers can be anything. So the combination of first three numbers can be $6 \times 6 \times 6 = 216$

Two word combinations which are divisible by 4 are - 12, 16, 24, 32, 36, 44, 56 and 64. So the last 2 digits can be of 8 combinations.

Total numbers that can be formed are 216 × 8= 1728

14. A

Solution: Required no. of ways = 6! / 2!

= 360 ways

15. C

Solution: Since no chair was added or removed, the capacity of the auditorium remains constant.

The capacity of the auditorium is the product of the number of rows and number of columns.

Let there be x columns and x + 3 rows, then

$x (x + 3) = (x - 3) (x + 9)$

$\therefore x^2 + 3x = x^2 + 6x - 27$

$\therefore x = 9$

Thus there were 9 columns and 12 rows, i.e., 9 × 12 = 108 people can sit in the auditorium at a time

16. D

Solution: Number of people who can drive = 2

Number of ways of selecting driver = $^2C_1$

The other person who knows driving can be seated only in the rear three seats in 3 ways

Total number of ways of seating the two persons = $^2C_1 \times 3$

Number of ways of seating remaining = 3!

Total number of all five can be seated = $^2C_1 \times 3 \times 3! = 36$

Hence, correct answer is 36

17. C

Solution: 91 --92 — 93 — 94 — 95 — 96 — 97 — 98 — 99 — 100

Total position advance needed = 100 – 91 = 9

One roll of dice can’t complete the game.

If he completes in two roll of dice.

Possible dice throws are – (3&6), (4&5), (5&4), (6&3)

But (5&4) will bring the token on 96, so this is rejected.

If he completes the game in three roll of dices

First dice reading options are 1,3,4,6

After checking all option and rejecting those in which token reaches on 93 or 96

Possible dice throws are (1,2,6), (1,3,5),(1,5,3), (1,6,2) ;

(3,1,5),(3,3,3),(3,4,2),(3,5,1);

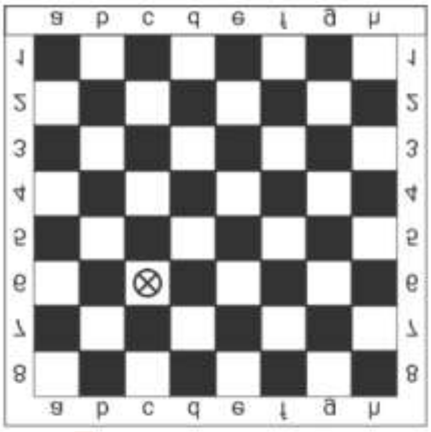
(4,2,3),(4,3,2),(4,4,1)

(6, 1, 2), (6, 2, 1)

Total number of ways = 16

18. B

Solution: As shows in the image a knight and a rook has to be placed, but not in the same row or column.



Let us select any box out of 64 for placing knight, no of ways = ${}^{64}C_1$

Now, row 6 and column c can't be used to place rook. Remaining boxes = 64 – (8 + 7) = 49

The rook can be place in any of 49 boxes, no of ways = ${}^{49}C_1$

Total number of possible ways = ${}^{49}C_1 \times {}^{64}C_1 = 3136$

19. B

Solution: Any question can be answered in 2 ways (right or wrong)

Let the number of questions be N

$2^N - 1 = 127$

Therefore N = 7

Number of ways in answering 4 answers correctly = ${}^{7}C_4 = 35$

20. C

Solution: Each letter can be delivered at the six different addresses in 2 different ways

Hence, the required number of ways = $2^6 = 64$