Chapter 6

The Relational Algebra

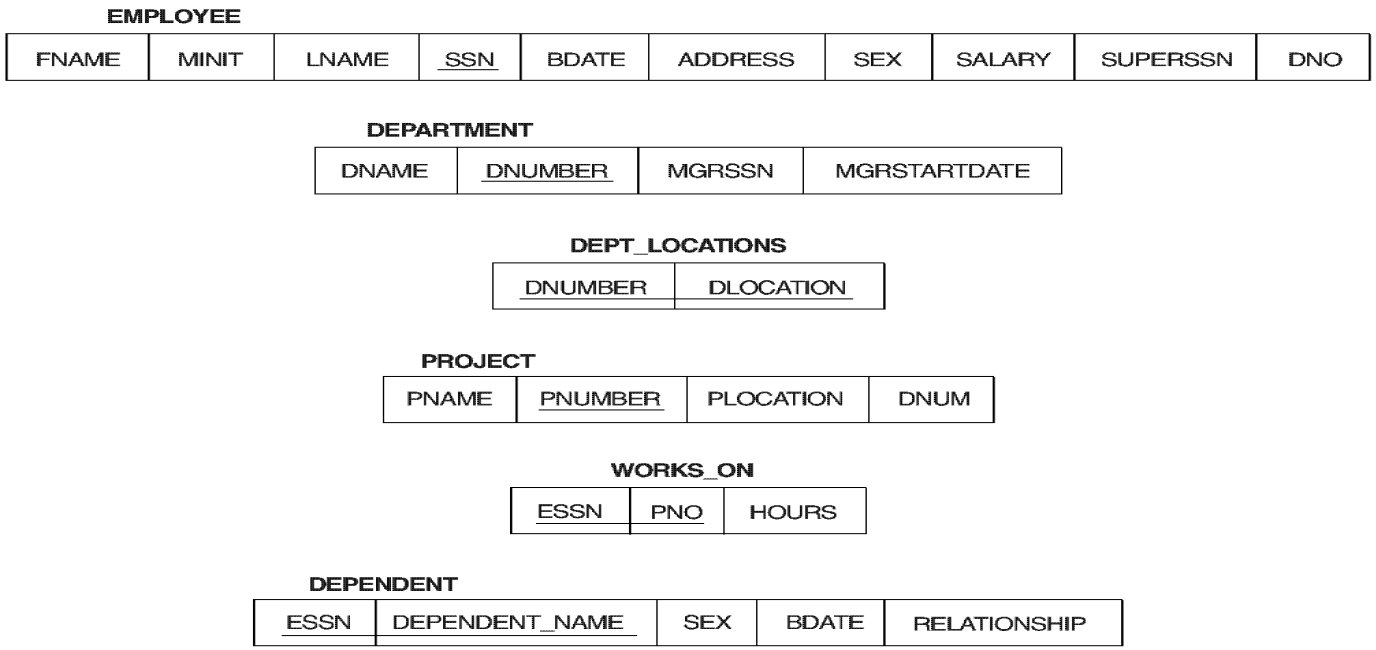
Chapter Outline

- Example Database Application (COMPANY)
- Relational Algebra
  - Unary Relational Operations
  - Relational Algebra Operations From Set Theory
  - Binary Relational Operations
  - Additional Relational Operations
  - Examples of Queries in Relational Algebra

Database State for COMPANY

All examples discussed below refer to the COMPANY database shown here.

Figure 7.5 Schema diagram for the COMPANY relational database schema; the primary keys are underlined.



Relational Algebra

- The basic set of operations for the relational model is known as the relational algebra. These operations enable a user to specify basic retrieval requests.

- The result of a retrieval is a new relation, which may have been formed from one or more relations. The **algebra operations** thus produce new relations, which can be further manipulated using operations of the same algebra.

- A sequence of relational algebra operations forms a **relational algebra expression**, whose result will also be a relation that represents the result of a database query (or retrieval request).

Unary Relational Operations

- **SELECT Operation**

SELECT operation is used to select a *subset* of the tuples from a relation that satisfy a **selection condition**. It is a filter that keeps only those tuples that satisfy a qualifying condition – those satisfying the condition are selected while others are discarded.

Example: To select the EMPLOYEE tuples whose department number is four or those whose salary is greater than $30,000 the following notation is used:

$$\sigma_{DNO = 4}(EMPLOYEE)$$

$$\sigma_{SALARY > 30,000}(EMPLOYEE)$$

In general, the select operation is denoted by $\sigma_{<\text{selection condition}>}(R)$ where the symbol σ (sigma) is used to denote the select operator, and the selection condition is a Boolean expression specified on the attributes of relation R

Unary Relational Operations

SELECT Operation Properties

- The SELECT operation $\sigma_{<\text{selection condition}>}(R)$ produces a relation S that has the same schema as R
- The SELECT operation σ is **commutative**; i.e.,
  $\sigma_{<\text{condition1}>}(\sigma_{<\text{condition2}>}(R)) = \sigma_{<\text{condition2}>}(\sigma_{<\text{condition1}>}(R))$
- A cascaded SELECT operation **may be applied in any order**; i.e.,
  $\sigma_{<\text{condition1}>}(\sigma_{<\text{condition2}>}(\sigma_{<\text{condition3}>}(R))$
  $= \sigma_{<\text{condition2}>}(\sigma_{<\text{condition3}>}(\sigma_{<\text{condition1}>}(R)))$
- A cascaded SELECT operation may be replaced by a single selection with a conjunction of all the conditions; i.e.,
  $\sigma_{<\text{condition1}>}(\sigma_{<\text{condition2}>}(\sigma_{<\text{condition3}>}(R))$
  $= \sigma_{<\text{condition1}> \text{ AND } <\text{condition2}> \text{ AND } <\text{condition3}>}(R)))$

Unary Relational Operations (cont.)

Figure 7.8 Results of SELECT and PROJECT operations.

(a) $\sigma_{(DNO=4\ AND\ SALARY>25000)\ OR\ (DNO=5\ AND\ SALARY>30000)}(EMPLOYEE)$.

(b) $\pi_{LNAME,\ FNAME,\ SALARY}(EMPLOYEE)$. (c) $\pi_{SEX,\ SALARY}(EMPLOYEE)$

(a)

FNAME	MINIT	LNAME	SSN	BDATE	ADDRESS	SEX	SALARY	SUPERSSN	DNO
Franklin	T	Wong	333445555	1955-12-08	638 Voss,Houston,TX	M	40000	888665555	5
Jennifer		Wallace	987654321	1941-06-20	291 Berry,Bellaire,TX	F	43000	888665555	4
Ramesh		Narayan	666884444	1962-09-15	975 FireOak,Humble,TX	M	38000	333445555	5

(b)

LNAME	FNAME	SALARY
Smith	John	30000
Wong	Franklin	40000
Zelaya	Alicia	25000
Wallace	Jennifer	43000
Narayan	Ramesh	38000
English	Joyce	25000
Jabbar	Ahmad	25000
Borg	James	55000

(c)

SEX	SALARY
M	30000
M	40000
F	25000
F	43000
M	38000
M	25000
M	55000

Unary Relational Operations (cont.)

- **PROJECT Operation**

  This operation selects certain *columns* from the table and discards the other columns. The PROJECT creates a vertical partitioning – one with the needed columns (attributes) containing results of the operation and other containing the discarded Columns.

  Example: To list each employee's first and last name and salary, the following is used:

  $$\pi_{\text{LNAME, FNAME,SALARY}}(\text{EMPLOYEE})$$

  The general form of the project operation is π<attribute list>(R) where π (pi) is the symbol used to represent the project operation and <attribute list> is the desired list of attributes from the attributes of relation R.

  The project operation *removes any duplicate tuples,* so the result of the project operation is a set of tuples and hence a valid relation.

Unary Relational Operations (cont.)

PROJECT Operation Properties

- The number of tuples in the result of projection $\pi_{<list>}(R)$ is always less or equal to the number of tuples in R.
- If the list of attributes includes a key of R, then the number of tuples is equal to the number of tuples in R.
- $\pi_{<list1>}(\pi_{<list2>}(R)) = \pi_{<list1>}(R)$ as long as <list2> contains the attributes in <list2>

Unary Relational Operations (cont.)

Figure 7.8 Results of SELECT and PROJECT operations.

(a) $\sigma_{(DNO=4\ AND\ SALARY>25000)\ OR\ (DNO=5\ AND\ SALARY>30000)}(EMPLOYEE)$.

(b) $\pi_{LNAME,\ FNAME,\ SALARY}(EMPLOYEE)$. (c) $\pi_{SEX,\ SALARY}(EMPLOYEE)$

(a)

FNAME	MINIT	LNAME	SSN	BDATE	ADDRESS	SEX	SALARY	SUPERSSN	DNO
Franklin	T	Wong	333445555	1955-12-08	638 Voss,Houston,TX	M	40000	888665555	5
Jennifer		Wallace	987654321	1941-06-20	291 Berry,Bellaire,TX	F	43000	888665555	4
Ramesh		Narayan	666884444	1962-09-15	975 FireOak,Humble,TX	M	38000	333445555	5

(b)

LNAME	FNAME	SALARY
Smith	John	30000
Wong	Franklin	40000
Zelaya	Alicia	25000
Wallace	Jennifer	43000
Narayan	Ramesh	38000
English	Joyce	25000
Jabbar	Ahmad	25000
Borg	James	55000

(c)

SEX	SALARY
M	30000
M	40000
F	25000
F	43000
M	38000
M	25000
M	55000

Unary Relational Operations (cont.)

- **Rename Operation**

  We may want to apply several relational algebra operations one after the other. Either we can write the operations as a single **relational algebra expression** by nesting the operations, or we can apply one operation at a time and create **intermediate result relations**. In the latter case, we must give names to the relations that hold the intermediate results.

  Example: To retrieve the first name, last name, and salary of all employees who work in department number 5, we must apply a select and a project operation. We can write a single relational algebra expression as follows:

  $\pi_{FNAME, LNAME, SALARY}(\sigma_{DNO=5}(EMPLOYEE))$

  OR We can explicitly show the sequence of operations, giving a name to each intermediate relation:

  $DEP5\_EMPS \leftarrow \sigma_{DNO=5}(EMPLOYEE)$

  $RESULT \leftarrow \pi_{FNAME, LNAME, SALARY}(DEP5\_EMPS)$

Unary Relational Operations (cont.)

- **Rename Operation (cont.)**

  The rename operator is ρ

  The general Rename operation can be expressed by any of the following forms:

- $\rho_{S(B_1, B_2, \ldots, B_n)}(R)$ is a renamed relation S based on R with column names $B_1, B_1, \ldots..B_n$.
- $\rho_S(R)$ is a renamed relation S based on R (which does not specify column names).
- $\rho_{(B_1, B_2, \ldots, B_n)}(R)$ is a renamed relation with column names $B_1, B_1, \ldots..B_n$ which does not specify a new relation name.

Unary Relational Operations (cont.)

Figure 7.9 Results of relational algebra expressions.
(a) $\pi_{\text{LNAME, FNAME, SALARY}}(\sigma_{\text{DNO=5}}(\text{EMPLOYEE}))$. (b) The same expression using intermediate relations and renaming of attributes.

(a)

FNAME	LNAME	SALARY
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

(b)

TEMP	FNAME	MINIT	LNAME	SSN	BDATE	ADDRESS	SEX	SALARY	SUPERSSN	DNO
	John	B	Smith	123456789	1965-01-09	731 Fondren,Houston,TX	M	30000	333445555	5
	Franklin	T	Wong	333445555	1955-12-08	638 Voss,Houston,TX	M	40000	888665555	5
	Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak,Humble,TX	M	38000	333445555	5
	Joyce	A	English	453453453	1972-07-31	5631 Rice,Houston,TX	F	25000	333445555	5

	FIRSTNAME	LASTNAME	SALARY
	John	Smith	30000
	Franklin	Wong	40000
	Ramesh	Narayan	38000
	Joyce	English	25000

Relational Algebra Operations From Set Theory

- **UNION Operation**

  The result of this operation, denoted by R ∪ S, is a relation that includes all tuples that are either in R or in S or in both R and S. Duplicate tuples are eliminated.

  Example: To retrieve the social security numbers of all employees who either work in department 5 or directly supervise an employee who works in department 5, we can use the union operation as follows:

  DEP5_EMPS ← $\sigma_{DNO=5}$ (EMPLOYEE)

  RESULT1 ← π_{SSN}(DEP5_EMPS)

  RESULT2(SSN) ← $\pi_{SUPERSSN}$(DEP5_EMPS)

  RESULT ← RESULT1 ∪ RESULT2

  The union operation produces the tuples that are in either RESULT1 or RESULT2 or both. The two operands must be “type compatible”.

Relational Algebra Operations From Set Theory

- **Type Compatibility**
  - The operand relations $R_1(A_1, A_2, ..., A_n)$ and $R_2(B_1, B_2, ..., B_n)$ must have the same number of attributes, and the domains of corresponding attributes must be compatible; that is, $dom(A_i)=dom(B_i)$ for i=1, 2, ..., n.
  - The resulting relation for $R_1 \cup R_2$, $R_1 \cap R_2$, or $R_1 - R_2$ has the same attribute names as the *first* operand relation R1 (by convention).

Relational Algebra Operations From Set Theory

- **UNION Example**

STUDENT	FN	LN
	Susan	Yao
	Ramesh	Shah
	Johnny	Kohler
	Barbara	Jones
	Amy	Ford
	Jimmy	Wang
	Ernest	Gilbert

INSTRUCTOR	FNAME	LNAME
	John	Smith
	Ricardo	Browne
	Susan	Yao
	Francis	Johnson
	Ramesh	Shah

(b)

FN	LN
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert
John	Smith
Ricardo	Browne
Francis	Johnson

FN	LN
Susan	Yao
Ramesh	Shah

(d)

FN	LN
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

(e)

FNAME	LNAME
John	Smith
Ricardo	Browne
Francis	Johnson

STUDENT∪INSTRUCTOR

Relational Algebra Operations From Set Theory (cont.) – use Fig. 6.4

Figure 7.11 Illustrating the set operations union, intersection, and difference. (a) Two union compatible relations. (b) STUDENT ∪ INSTRUCTOR. (c) STUDENT ∪ INSTRUCTOR. (d) STUDENT − INSTRUCTOR. (e) INSTRUCTOR − STUDENT.

(a)

STUDENT	FN	LN
	Susan	Yao
	Ramesh	Shah
	Johnny	Kohler
	Barbara	Jones
	Amy	Ford
	Jimmy	Wang
	Ernest	Gilbert

INSTRUCTOR	FNAME	LNAME
	John	Smith
	Ricardo	Browne
	Susan	Yao
	Francis	Johnson
	Ramesh	Shah

(b)

FN	LN
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert
John	Smith
Ricardo	Browne
Francis	Johnson

(c)

FN	LN
Susan	Yao
Ramesh	Shah

(d)

FN	LN
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

(e)

FNAME	LNAME
John	Smith
Ricardo	Browne
Francis	Johnson

Relational Algebra Operations From Set Theory (cont.)

- **INTERSECTION OPERATION**

The result of this operation, denoted by R ∩ S, is a relation that includes all tuples that are in both R and S. The two operands must be "type compatible"

Example: The result of the intersection operation (figure below) includes only those who are both students and instructors.

FN	LN
Susan	Yao
Ramesh	Shah

(d)

FN	LN
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

STUDENT ∩ INSTRUCTOR

Relational Algebra Operations From Set Theory (cont.)

- **Set Difference (or MINUS) Operation**

  The result of this operation, denoted by R - S, is a relation that includes all tuples that are in R but not in S. The two operands must be "type compatible".

  Example: The figure shows the names of students who are not instructors, and the names of instructors who are not students.

STUDENT	FN	LN
	Susan	Yao
	Ramesh	Shah
	Johnny	Kohler
	Barbara	Jones
	Amy	Ford
	Jimmy	Wang
	Ernest	Gilbert

FN	LN
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

STUDENT–INSTRUCTOR

FNAME	LNAME
John	Smith
Ricardo	Browne
Francis	Johnson

INSTRUCTOR–STUDENT

Relational Algebra Operations From Set Theory (cont.)

- Notice that both union and intersection are *commutative operations;* that is

  $R \cup S = S \cup R$, and $R \cap S = S \cap R$

- Both union and intersection can be treated as n-ary operations applicable to any number of relations as both are *associative operations;* that is

  $R \cup (S \cup T) = (R \cup S) \cup T$, and $(R \cap S) \cap T = R \cap (S \cap T)$

- The minus operation is *not commutative;* that is, in general

  $R - S \neq S - R$

Relational Algebra Operations From Set Theory (cont.)

- **CARTESIAN (or cross product) Operation**
  - This operation is used to combine tuples from two relations in a combinatorial fashion. In general, the result of $R(A_1, A_2, \ldots, A_n)$ x $S(B_1, B_2, \ldots, B_m)$ is a relation Q with degree n + m attributes $Q(A_1, A_2, \ldots, A_n, B_1, B_2, \ldots, B_m)$, in that order. The resulting relation Q has one tuple for each combination of tuples—one from R and one from S.
  - Hence, if R has n_R tuples (denoted as $|R| = n_R$), and S has n_S tuples, then
    $|R \times S|$ will have $n_R * n_S$ tuples.
  - The two operands do NOT have to be "type compatible"

Example:

$$FEMALE\_EMPS \leftarrow \sigma_{SEX='F'}(EMPLOYEE)$$
$$EMPNAMES \leftarrow \pi_{FNAME, LNAME, SSN}(FEMALE\_EMPS)$$
$$EMP\_DEPENDENTS \leftarrow EMPNAMES \times DEPENDENT$$

Relational Algebra Operations From Set Theory (cont.)

Figure 7.12 An illustration of the CARTESIAN PRODUCT operation.

FEMALE_EMPS

FNAME	MINIT	LNAME	SSN	BDATE	ADDRESS	SEX	SALARY	SUPERSSN	DNO
Alicia	J	Zelaya	999887777	1968-07-19	3321 Castle,Spring,TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry,Bellaire,TX	F	43000	888665555	4
Joyce	A	English	453453453	1972-07-31	5631 Rice,Houston,TX	F	25000	333445555	5

EMPNAMES

FNAME	LNAME	SSN
Alicia	Zelaya	999887777
Jennifer	Wallace	987654321
Joyce	English	453453453

EMP_DEPENDENTS

FNAME	LNAME	SSN	ESSN	DEPENDENT_NAME	SEX	BDATE	• • •
Alicia	Zelaya	999887777	333445555	Alice	F	1986-04-05	• • •
Alicia	Zelaya	999887777	333445555	Theodore	M	1983-10-25	• • •
Alicia	Zelaya	999887777	333445555	Joy	F	1958-05-03	• • •
Alicia	Zelaya	999887777	987654321	Abner	M	1942-02-28	• • •
Alicia	Zelaya	999887777	123456789	Michael	M	1988-01-04	• • •
Alicia	Zelaya	999887777	123456789	Alice	F	1988-12-30	• • •
Alicia	Zelaya	999887777	123456789	Elizabeth	F	1967-05-05	• • •
Jennifer	Wallace	987654321	333445555	Alice	F	1986-04-05	• • •
Jennifer	Wallace	987654321	333445555	Theodore	M	1983-10-25	• • •
Jennifer	Wallace	987654321	333445555	Joy	F	1958-05-03	• • •
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	• • •
Jennifer	Wallace	987654321	123456789	Michael	M	1988-01-04	• • •
Jennifer	Wallace	987654321	123456789	Alice	F	1988-12-30	• • •
Jennifer	Wallace	987654321	123456789	Elizabeth	F	1967-05-05	• • •
Joyce	English	453453453	333445555	Alice	F	1986-04-05	• • •
Joyce	English	453453453	333445555	Theodore	M	1983-10-25	• • •
Joyce	English	453453453	333445555	Joy	F	1958-05-03	• • •
Joyce	English	453453453	987654321	Abner	M	1942-02-28	• • •
Joyce	English	453453453	123456789	Michael	M	1988-01-04	• • •
Joyce	English	453453453	123456789	Alice	F	1988-12-30	• • •
Joyce	English	453453453	123456789	Elizabeth	F	1967-05-05	• • •

ACTUAL_DEPENDENTS

FNAME	LNAME	SSN	ESSN	DEPENDENT_NAME	SEX	BDATE
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28

RESULT

FNAME	LNAME	DEPENDENT_NAME
Jennifer	Wallace	Abner

In general, the CARTESIAN PRODUCT operation applied by itself is generally meaningless. It is mostly useful when followed by a selection that matches values of attributes coming from the component relations. For example, suppose that we want to retrieve a list of names of each female employee's dependents. We can do this as follows:

$$\text{FEMALE\_EMPS} \leftarrow \sigma_{\text{Sex='F'}}(\text{EMPLOYEE})$$
$$\text{EMPNAMES} \leftarrow \pi_{\text{Fname, Lname, Ssn}}(\text{FEMALE\_EMPS})$$
$$\text{EMP\_DEPENDENTS} \leftarrow \text{EMPNAMES} \times \text{DEPENDENT}$$
$$\text{ACTUAL\_DEPENDENTS} \leftarrow \sigma_{\text{Ssn=Essn}}(\text{EMP\_DEPENDENTS})$$
$$\text{RESULT} \leftarrow \pi_{\text{Fname, Lname, Dependent\_name}}(\text{ACTUAL\_DEPENDENTS})$$

Binary Relational Operations

- **JOIN Operation**
  - The sequence of cartesian product followed by select is used quite commonly to identify and select related tuples from two relations, a special operation, called **JOIN**. It is denoted by a ⋈
  - This operation is very important for any relational database with more than a single relation, because it allows us to process relationships among relations.
  - The general form of a join operation on two relations $R(A_1, A_2, \ldots, A_n)$ and $S(B_1, B_2, \ldots, B_m)$ is:

    $R \bowtie_{\text{<join condition>}} S$

    where R and S can be any relations that result from general *relational algebra expressions*.

Binary Relational Operations (cont.)

Example: Suppose that we want to retrieve the name of the manager of each department. To get the manager's name, we need to combine each DEPARTMENT tuple with the EMPLOYEE tuple whose SSN value matches the MGRSSN value in the department tuple. We do this by using the join ⋈ operation.

DEPT_MGR ← DEPARTMENT $\underset{\text{MGRSSN=SSN}}{\bowtie}$ EMPLOYEE

DEPT_MGR	DNAME	DNUMBER	MGRSSN	. . .	FNAME	MINIT	LNAME	SSN	. . .
	Research	5	333445555	. . .	Franklin	T	Wong	333445555	. . .
	Administration	4	987654321	. . .	Jennifer	S	Wallace	987654321	. . .
	Headquarters	1	888665555	. . .	James	E	Borg	888665555	. . .

FIGURE 6.6 Result of the JOIN operation DEPT_MGR ← DEPARTMENT $\underset{\text{MGRSSN=SSN}}{\bowtie}$ EMPLOYEE.

Binary Relational Operations (cont.)

- **EQUIJOIN Operation**

  The most common use of join involves join conditions with equality comparisons only. Such a join, where the only comparison operator used is =, is called an EQUIJOIN. In the result of an EQUIJOIN we always have one or more pairs of attributes (whose names need not be identical) that have *identical values* in every tuple.

  The JOIN seen in the previous example was EQUIJOIN.

- **NATURAL JOIN Operation**

  Because one of each pair of attributes with identical values is superfluous, a new operation called natural join—denoted by *—was created to get rid of the second (superfluous) attribute in an EQUIJOIN condition.

  The standard definition of natural join requires that the two join attributes, or each pair of corresponding join attributes, have the **same name** in both relations. If this is not the case, a renaming operation is applied first.

Binary Relational Operations (cont.)

Example: To apply a natural join on the DNUMBER attributes of DEPARTMENT and DEPT_LOCATIONS, it is sufficient to write:

DEPT_LOCS ← DEPARTMENT * DEPT_LOCATIONS

(a)

PROJ_DEPT	PNAME	PNUMBER	PLOCATION	DNUM	DNAME	MGRSSN	MGRSTARTDATE
	ProductX	1	Bellaire	5	Research	333445555	1988-05-22
	ProductY	2	Sugarland	5	Research	333445555	1988-05-22
	ProductZ	3	Houston	5	Research	333445555	1988-05-22
	Computerization	10	Stafford	4	Administration	987654321	1995-01-01
	Reorganization	20	Houston	1	Headquarters	888665555	1981-06-19
	Newbenefits	30	Stafford	4	Administration	987654321	1995-01-01

(b)

DEPT_LOCS	DNAME	DNUMBER	MGRSSN	MGRSTARTDATE	LOCATION
	Headquarters	1	888665555	1981-06-19	Houston
	Administration	4	987654321	1995-01-01	Stafford
	Research	5	333445555	1988-05-22	Bellaire
	Research	5	333445555	1988-05-22	Sugarland
	Research	5	333445555	1988-05-22	Houston

FIGURE 6.7 Results of two NATURAL JOIN operations. (a) PROJ_DEPT ← PROJECT * DEPT. (b) DEPT_LOCS ← DEPARTMENT * DEPT_LOCATIONS.

Complete Set of Relational Operations

- The set of operations including **select σ, project π , union ∪, set difference - , and cartesian product X** is called a complete set because any other relational algebra expression can be expressed by a combination of these five operations.

- For example:

  $$\mathbf{R \cap S = (R \cup S) - ((R - S) \cup (S - R))}$$

  $$R \bowtie_{\text{<join condition>}} S = \sigma_{\text{<join condition>}} (R \times S)$$

Binary Relational Operations (cont.)

- **DIVISION Operation**
  - The division operation is applied to two relations $R(Z) \div S(X)$, where X subset Z. Let $Y = Z - X$ (and hence $Z = X \cup Y$); that is, let Y be the set of attributes of R that are not attributes of S.
  - The result of DIVISION is a relation T(Y) that includes a tuple t if tuples t_R appear in R with $t_R[Y] = t$, and with
    $t_R[X] = t_s$ *for every tuple* t_s in S.
  - For a tuple t to appear in the result T of the DIVISION, the values in t must appear in R in combination with *every* tuple in S.

Binary Relational Operations (cont.)

The DIVISION operation, denoted by ÷, is useful for a special kind of query that sometimes occurs in database applications. An example is *Retrieve the names of employees who work on* ***all*** *the projects that 'John Smith' works on*. To express this query using the DIVISION operation, proceed as follows. First, retrieve the list of project numbers that 'John Smith' works on in the intermediate relation SMITH_PNOS:

$$\text{SMITH} \leftarrow \sigma_{\text{Fname='John' AND Lname='Smith'}}(\text{EMPLOYEE})$$
$$\text{SMITH\_PNOS} \leftarrow \pi_{\text{Pno}}(\text{WORKS\_ON} \bowtie_{\text{Essn=Ssn}} \text{SMITH})$$

Next, create a relation that includes a tuple <Pno, Essn> whenever the employee whose Ssn is Essn works on the project whose number is Pno in the intermediate relation SSN_PNOS:

$$\text{SSN\_PNOS} \leftarrow \pi_{\text{Essn, Pno}}(\text{WORKS\_ON})$$

Finally, apply the DIVISION operation to the two relations, which gives the desired employees' Social Security numbers:

$$\text{SSNS(Ssn)} \leftarrow \text{SSN\_PNOS} \div \text{SMITH\_PNOS}$$
$$\text{RESULT} \leftarrow \pi_{\text{Fname, Lname}}(\text{SSNS} * \text{EMPLOYEE})$$

Binary Relational Operations (cont.)

The DIVISION operation. (a) Dividing SSN_PNOS by SMITH_PNOS. (b) $T \leftarrow R \div S$.

(a)

SSN_PNOS

Essn	Pno
123456789	1
123456789	2
666884444	3
453453453	1
453453453	2
333445555	2
333445555	3
333445555	10
333445555	20
999887777	30
999887777	10
987987987	10
987987987	30
987654321	30
987654321	20
888665555	20

SMITH_PNOS

Pno
1
2

SSNS

Ssn
123456789
453453453

(b)

R

A	B
a1	b1
a2	b1
a3	b1
a4	b1
a1	b2
a3	b2
a2	b3
a3	b3
a4	b3
a1	b4
a2	b4
a3	b4

S

A
a1
a2
a3

T

B
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Recap of Relational Algebra Operations

TABLE 6.1 OPERATIONS OF RELATIONAL ALGEBRA

Operation	Purpose	Notation
SELECT	Selects all tuples that satisfy the selection condition from a relation R.	$\sigma_{\text{<SELECTION CONDITION>}}(R)$
PROJECT	Produces a new relation with only some of the attributes of R, and removes duplicate tuples.	$\pi_{\text{<ATTRIBUTE LIST>}}(R)$
THETA JOIN	Produces all combinations of tuples from R_1 and R_2 that satisfy the join condition.	$R_1 \bowtie_{\text{<JOIN CONDITION>}} R_2$
EQUIJOIN	Produces all the combinations of tuples from R_1 and R_2 that satisfy a join condition with only equality comparisons.	$R_1 \bowtie_{\text{<JOIN CONDITION>}} R_2$, OR $R_1 \bowtie (\text{<JOIN ATTRIBUTES 1>}), (\text{<JOIN ATTRIBUTES 2>}) R_2$
NATURAL JOIN	Same as EQUIJOIN except that the join attributes of R_2 are not included in the resulting relation; if the join attributes have the same names, they do not have to be specified at all.	$R_1 *_{\text{<JOIN CONDITION>}} R_2$, OR $R_1 * (\text{<JOIN ATTRIBUTES 1>}), (\text{<JOIN ATTRIBUTES 2>}) R_2$ OR $R_1 * R_2$
UNION	Produces a relation that includes all the tuples in R_1 or R_2 or both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cup R_2$
INTERSECTION	Produces a relation that includes all the tuples in both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cap R_2$
DIFFERENCE	Produces a relation that includes all the tuples in R_1 that are not in R_2; R_1 and R_2 must be union compatible.	$R_1 - R_2$
CARTESIAN PRODUCT	Produces a relation that has the attributes of R_1 and R_2 and includes as tuples all possible combinations of tuples from R_1 and R_2.	$R_1 \times R_2$
DIVISION	Produces a relation $R(X)$ that includes all tuples $t[X]$ in $R_1(Z)$ that appear in R_1 in combination with every tuple from $R_2(Y)$, where $Z = X \cup Y$.	$R_1(Z) \div R_2(Y)$

Notation for Query Trees

as a *query evaluation tree* or *query execution tree*. It includes the relational algebra operations being executed and is used as a possible data structure for the internal representation of the query in an RDBMS.

A **query tree** is a tree data structure that corresponds to a relational algebra expression. It represents the input relations of the query as *leaf nodes* of the tree, and represents the relational algebra operations as internal nodes. An execution of the query tree consists of executing an internal node operation whenever its operands (represented by its child nodes) are available, and then replacing that internal node by the relation that results from executing the operation. The execution terminates when the root node is executed and produces the result relation for the query.

Notation for Query Trees

Figure 6.9 shows a query tree for Query 2 (see Section 4.3.1): *For every project located in 'Stafford', list the project number, the controlling department number, and the department manager's last name, address, and birth date.* This query is specified on the relational schema of Figure 3.5 and corresponds to the following relational algebra expression:

$$\pi_{\text{Pnumber, Dnum, Lname, Address, Bdate}}(((\sigma_{\text{Plocation='Stafford'}}(\text{PROJECT})) \bowtie_{\text{Dnum=Dnumber}}(\text{DEPARTMENT})) \bowtie_{\text{Mgr\_ssn=Ssn}}(\text{EMPLOYEE}))$$

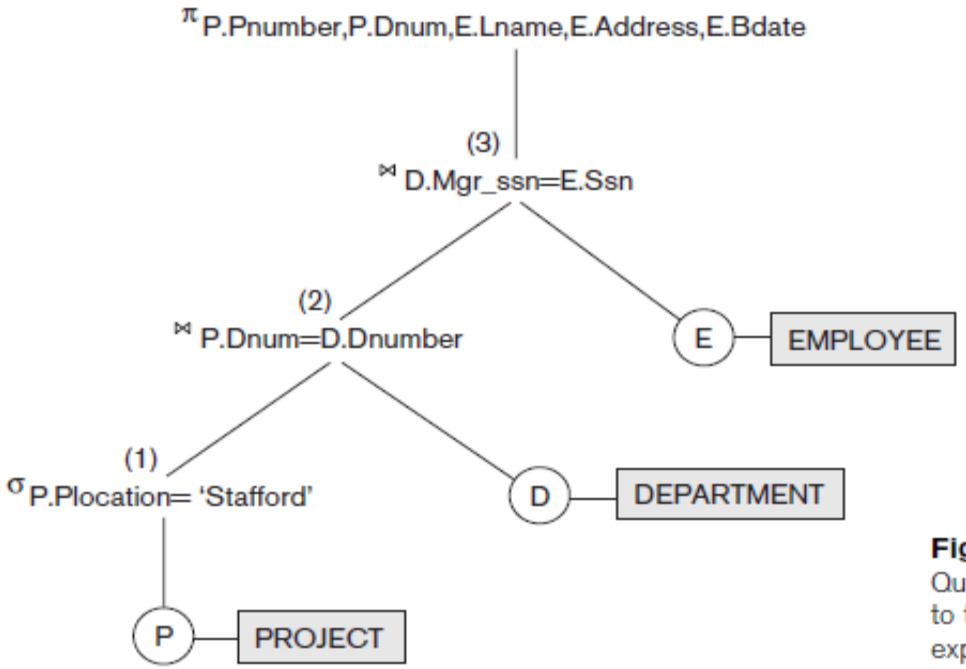

Figure 6.9
Query tree corresponding to the relational algebra expression for Q2.

Notation for Query Trees

In Figure 6.9, the three leaf nodes P, D, and E represent the three relations PROJECT, DEPARTMENT, and EMPLOYEE. The relational algebra operations in the expression are represented by internal tree nodes. The query tree signifies an explicit order of execution in the following sense. In order to execute Q2, the node marked (1) in Figure 6.9 must begin execution before node (2) because some resulting tuples of operation (1) must be available before we can begin to execute operation (2). Similarly, node (2) must begin to execute and produce results before node (3) can start execution, and so on. In general, a query tree gives a good visual representation and understanding of the query in terms of the relational operations it uses and is recommended as an additional means for expressing queries in relational algebra.

Additional Relational Operations

- **Aggregate Functions and Grouping**
  - A type of request that cannot be expressed in the basic relational algebra is to specify mathematical **aggregate functions** on collections of values from the database.
  - Examples of such functions include retrieving the average or total salary of all employees or the total number of employee tuples. These functions are used in simple statistical queries that summarize information from the database tuples.
  - Common functions applied to collections of numeric values include SUM, AVERAGE, MAXIMUM, and MINIMUM. The COUNT function is used for counting tuples or values.

$$\langle \text{grouping attributes} \rangle \, \Im \, \langle \text{function list} \rangle \, (R)$$

Additional Relational Operations (cont.)

The aggregate function operation.

a. $\rho_{R(\text{Dno, No\_of\_employees, Average\_sal})}(\text{Dno } \mathfrak{I}_{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE}))$.

b. $\text{Dno } \mathfrak{I}_{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE})$.

c. $\mathfrak{I}_{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE})$.

(a)

R	DNO	NO_OF_EMPLOYEES	AVERAGE_SAL
	5	4	33250
	4	3	31000
	1	1	55000

(b)

DNO	COUNT_SSN	AVERAGE_SALARY
5	4	33250
4	3	31000
1	1	55000

(c)

COUNT_SSN	AVERAGE_SALARY
8	35125

Additional Relational Operations (cont.)

Use of the Functional operator $\mathcal{F}$

$\mathcal{F}_{\text{MAX } Salary}$ **(Employee)** retrieves the maximum salary value from the Employee relation

$\mathcal{F}_{\text{MIN } Salary}$ **(Employee)** retrieves the minimum Salary value from the Employee relation

$\mathcal{F}_{\text{SUM } Salary}$ **(Employee)** retrieves the sum of the Salary from the Employee relation

DNO $\mathcal{F}_{COUNT\ SSN,\ AVERAGE\ Salary}$ **(Employee)** groups employees by DNO (department number) and computes the count of employees and average salary per department.[Note: count just counts the number of rows, without removing duplicates]

Additional Relational Operations (cont.)

- **Recursive Closure Operations**
  - Another type of operation that, in general, cannot be specified in the basic original relational algebra is **recursive closure.** This operation is applied to a **recursive relationship.**
  - An example of a recursive operation is to retrieve all SUPERVISEES of an EMPLOYEE e at all levels—that is, all EMPLOYEE e' directly supervised by e; all employees e'' directly supervised by each employee e'; all employees e''' directly supervised by each employee e''; and so on .
  - Although it is possible to retrieve employees at each level and then take their union, we cannot, in general, specify a query such as "retrieve the supervisees of 'James Borg' at all levels" without utilizing a looping mechanism.
  - The SQL3 standard includes syntax for recursive closure.

Recursive Closure Operations (contd…)

$$\text{BORG\_SSN} \leftarrow \pi_{\text{Ssn}}(\sigma_{\text{Fname='James' AND Lname='Borg'}}(\text{EMPLOYEE}))$$
$$\text{SUPERVISION}(\text{Ssn1}, \text{Ssn2}) \leftarrow \pi_{\text{Ssn,Super\_ssn}}(\text{EMPLOYEE})$$
$$\text{RESULT1}(\text{Ssn}) \leftarrow \pi_{\text{Ssn1}}(\text{SUPERVISION} \bowtie_{\text{Ssn2=Ssn}} \text{BORG\_SSN})$$

To retrieve all employees supervised by Borg at level 2—that is, all employees e'' supervised by some employee e' who is directly supervised by Borg—we can apply another **JOIN** to the result of the first query, as follows:

$$\text{RESULT2}(\text{Ssn}) \leftarrow \pi_{\text{Ssn1}}(\text{SUPERVISION} \bowtie_{\text{Ssn2=Ssn}} \text{RESULT1})$$

To get both sets of employees supervised at levels 1 and 2 by 'James Borg', we can apply the UNION operation to the two results, as follows:

$$\text{RESULT} \leftarrow \text{RESULT2} \cup \text{RESULT1}$$

Additional Relational Operations (cont.)

SUPERVISION

(Borg's Ssn is 888665555)

(Ssn)	(Super_ssn)
Ssn1	Ssn2
123456789	333445555
333445555	888665555
999887777	987654321
987654321	888665555
666884444	333445555
453453453	333445555
987987987	987654321
888665555	null

RESULT1

Ssn
333445555
987654321

(Supervised by Borg)

RESULT2

Ssn
123456789
999887777
666884444
453453453
987987987

(Supervised by Borg's subordinates)

RESULT

Ssn
123456789
999887777
666884444
453453453
987987987
333445555
987654321

(RESULT1 ∪ RESULT2)

Additional Relational Operations (cont.)

- **The OUTER JOIN Operation**
  - In NATURAL JOIN tuples without a *matching* (or *related*) tuple are eliminated from the join result. Tuples with null in the join attributes are also eliminated. This amounts to loss of information.
  - A set of operations, called outer joins, can be used when we want to keep all the tuples in R, or all those in S, or all those in both relations in the result of the join, regardless of whether or not they have matching tuples in the other relation.
  - The left outer join operation keeps every tuple in the *first* or *left* relation R in R ⟕ S; if no matching tuple is found in S, then the attributes of S in the join result are filled or "padded" with null values.
  - A similar operation, right outer join, keeps every tuple in the *second* or right relation S in the result of R ⟖ S.
  - A third operation, full outer join, denoted by ⟗ keeps all tuples in both the left and the right relations when no matching tuples are found, padding them with null values as needed.

Additional Relational Operations (cont.)

RESULT	FNAME	MINIT	LNAME	DNAME
	John	B	Smith	null
	Franklin	T	Wong	Research
	Alicia	J	Zelaya	null
	Jennifer	S	Wallace	Administration
	Ramesh	K	Narayan	null
	Joyce	A	English	null
	Ahmad	V	Jabbar	null
	James	E	Borg	Headquarters

The result of a LEFT OUTER JOIN operation.

Additional Relational Operations (cont.)

- **OUTER UNION Operations**
  - The outer union operation was developed to take the union of tuples from two relations if the relations are *not union compatible.*
  - This operation will take the union of tuples in two relations R(X, Y) and S(X, Z) that are **partially compatible**, meaning that only some of their attributes, say X, are union compatible.
  - The attributes that are union compatible are represented only once in the result, and those attributes that are not union compatible from either relation are also kept in the result relation T(X, Y, Z).
  - **Example**: An outer union can be applied to two relations whose schemas are STUDENT(Name, SSN, Department, Advisor) and INSTRUCTOR(Name, SSN, Department, Rank). Tuples from the two relations are matched based on having the same combination of values of the shared attributes—Name, SSN, Department. If a student is also an instructor, both Advisor and Rank will have a value; otherwise, one of these two attributes will be null.

    The result relation STUDENT_OR_INSTRUCTOR will have the following attributes:

    STUDENT_OR_INSTRUCTOR (Name, SSN, Department, Advisor, Rank)

Examples of Queries in Relational Algebra

- **Q1: Retrieve the name and address of all employees who work for the 'Research' department.**

  $RESEARCH\_DEPT \leftarrow \sigma_{DNAME='Research'}(DEPARTMENT)$

  $RESEARCH\_EMPS \leftarrow (RESEARCH\_DEPT \bowtie_{DNUMBER=DNOEMPLOYEE} EMPLOYEE)$

  $RESULT \leftarrow \pi_{FNAME, LNAME, ADDRESS}(RESEARCH\_EMPS)$

- **Q6: Retrieve the names of employees who have no dependents.**

  $ALL\_EMPS \leftarrow \pi_{SSN}(EMPLOYEE)$

  $EMPS\_WITH\_DEPS(SSN) \leftarrow \pi_{ESSN}(DEPENDENT)$

  $EMPS\_WITHOUT\_DEPS \leftarrow (ALL\_EMPS - EMPS\_WITH\_DEPS)$

  $RESULT \leftarrow \pi_{LNAME, FNAME}(EMPS\_WITHOUT\_DEPS * EMPLOYEE)$