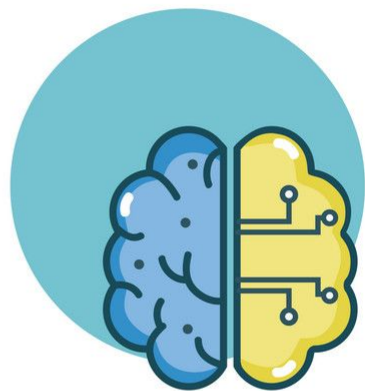


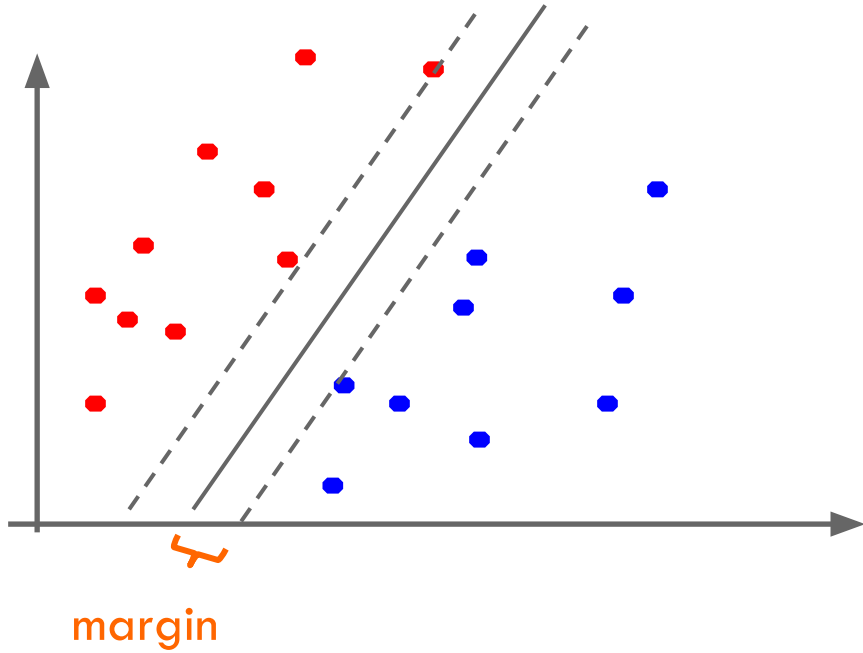
INTRODUCTION TO MACHINE LEARNING

SUPPORT VECTOR MACHINES



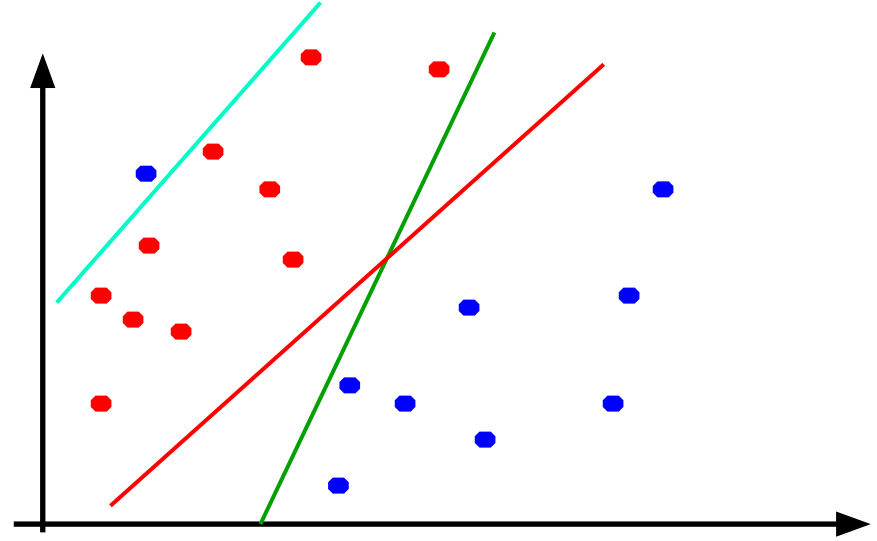
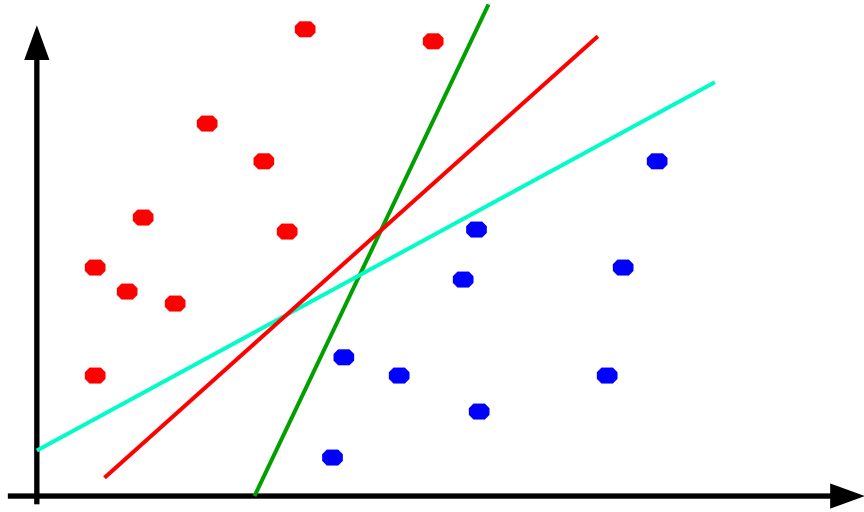
Elisa Ricci





SUPPORT VECTOR MACHINES

WHICH HYPERPLANE?



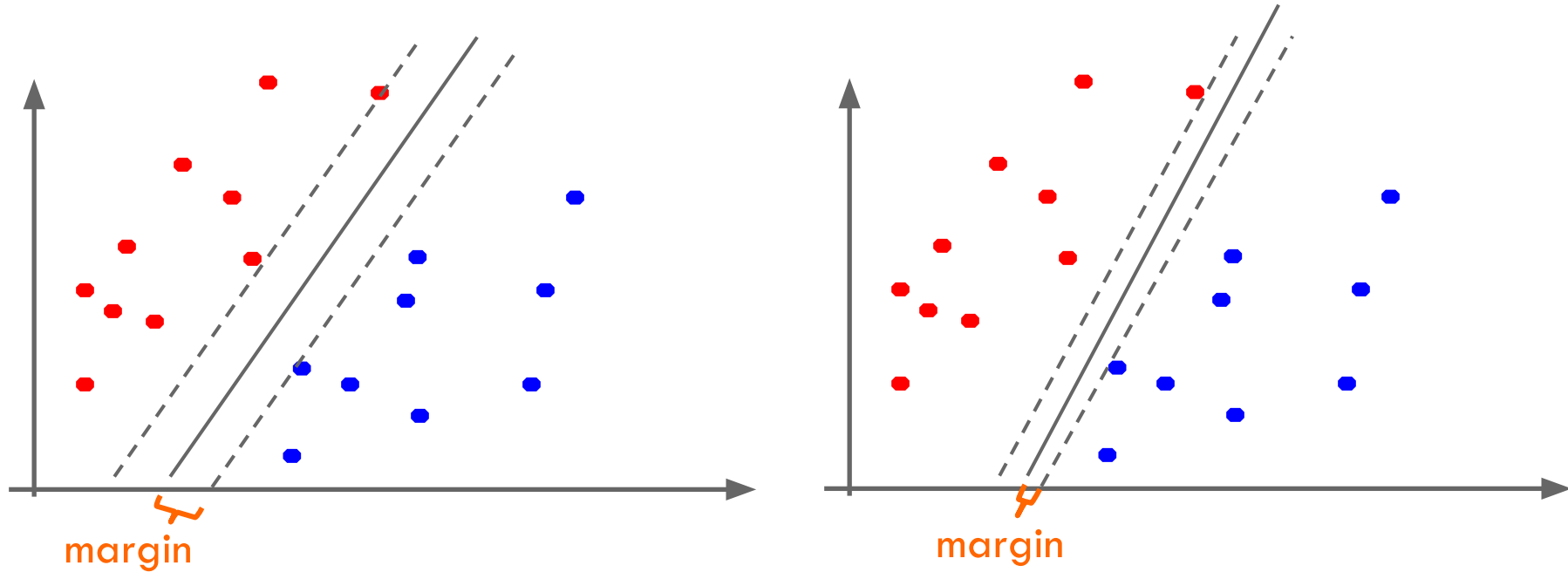
Two main variations in linear classifiers:

- which hyperplane they choose when the data is linearly separable
- how they handle data that is not linearly separable

LINEAR APPROACHES SO FAR

- Perceptron:
 - separable:
 - finds **some** hyperplane that separates the data
 - non-separable:
 - will continue to adjust as it iterates through the examples
 - final hyperplane will depend on which examples it saw recently

LARGE MARGIN CLASSIFIERS



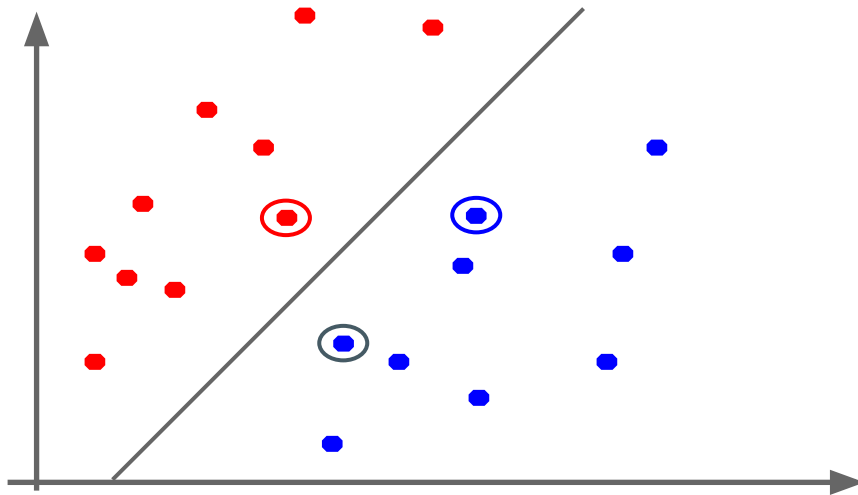
The **margin** of a classifier is the distance to the closest points of either class
Large margin classifiers attempt to maximize this

SUPPORT VECTORS

For any separating hyperplane, there exist some set of “closest points”

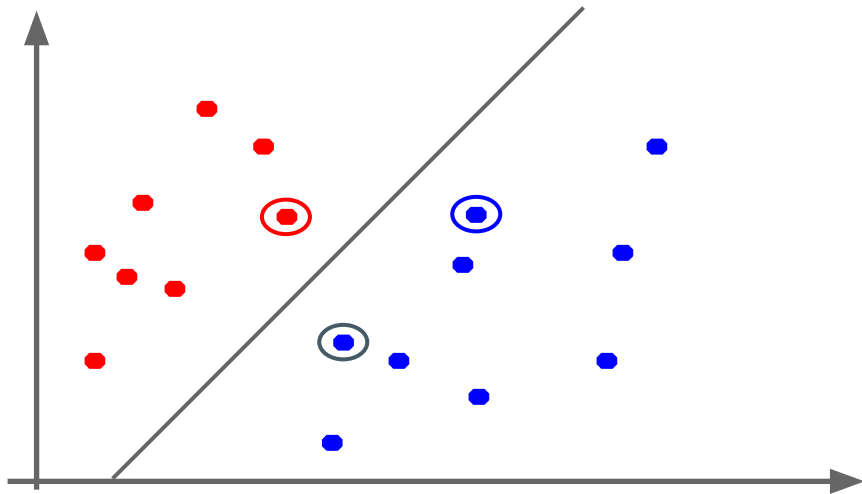
These are called the **support vectors**

For n dimensions, there will be at least $n+1$ support vectors



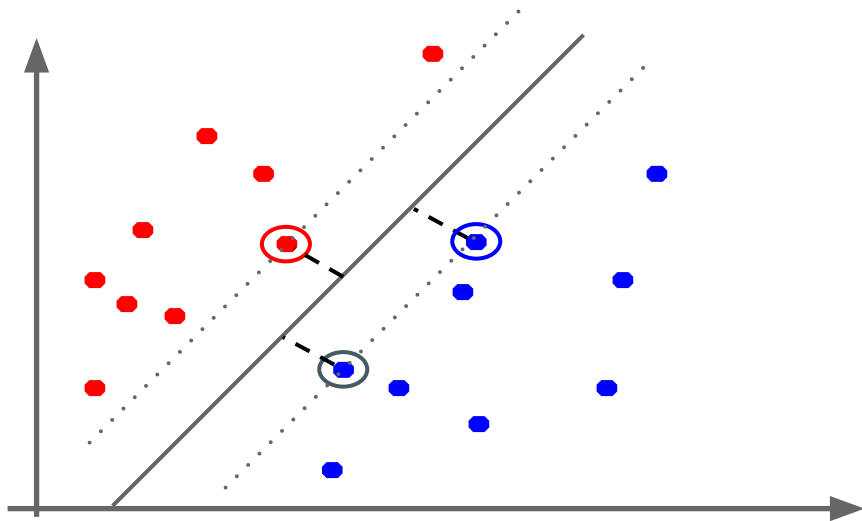
LARGE MARGIN CLASSIFIERS

Maximizing the margin is good since it implies that **only support vectors matter**, other training examples are ignorable.



MEASURING THE MARGIN

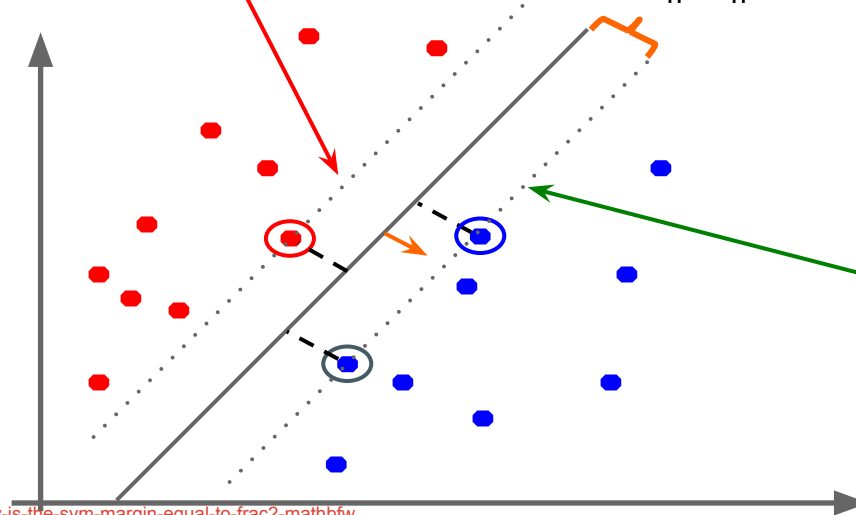
The margin is the distance to the support vectors, i.e. the “closest points”, on either side of the hyperplane



MEASURING THE MARGIN

$$w \cdot x_i + b = -1$$

$$\frac{w \cdot x_i + b}{\|w\|} = \frac{1}{\|w\|}$$



$$w \cdot x_i + b = 1$$

MAXIMIZING THE MARGIN

Select the hyperplane with the largest margin where the points are classified correctly and outside the margin!

Setup as a **constrained optimization problem**:

$$\begin{aligned} & \max_{w,b} \quad \text{margin}(w,b) \\ & \text{subject to:} \\ & \quad y_i(w \cdot x_i + b) \geq 1 \quad \forall i \end{aligned}$$

*what does
this mean?*

MAXIMIZING THE MARGIN

Select the hyperplane with the largest margin where the points are classified correctly and outside the margin!

Setup as a **constrained optimization problem**:

$$\max_{w,b} \frac{1}{\|w\|}$$

subject to:

$$y_i(w \cdot x_i + b) \geq 1 \quad \forall i$$

MAXIMIZING THE MARGIN

$$\begin{aligned} & \min_{w,b} \quad \|w\| \\ & \text{subject to:} \\ & \quad y_i(w \cdot x_i + b) \geq 1 \quad \forall i \end{aligned}$$

Maximizing the margin is equivalent to minimize the norm of the weights (subject to separating constraints).

MAXIMIZING THE MARGIN

The minimization criterion wants w to be as small as possible

$$\min_{w,b} \|w\|$$

subject to:

$$y_i(w \cdot x_i + b) \geq 1 \quad \forall i$$

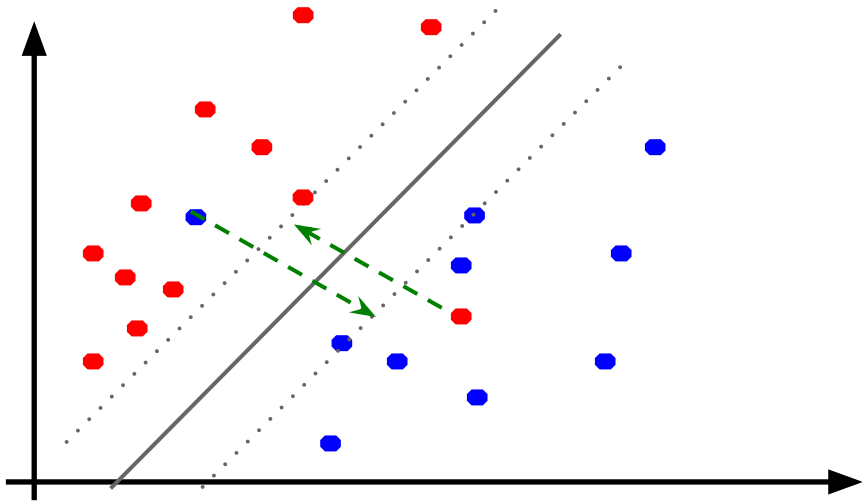
The constraints make sure that the data is separable

SUPPORT VECTOR MACHINE PROBLEM

$$\begin{aligned} &\min_{w,b} \quad \|w\|^2 \\ &\text{subject to:} \\ &\quad y_i(w \cdot x_i + b) \geq 1 \quad \forall i \end{aligned}$$

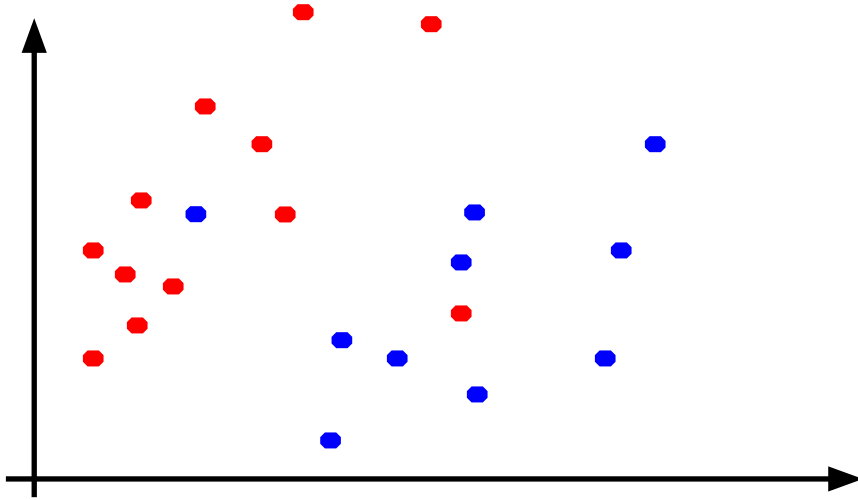
This is a version of a **quadratic optimization problem**

Maximize/minimize a quadratic function subject to a set of linear constraints



SOFT MARGIN
CLASSIFICATION

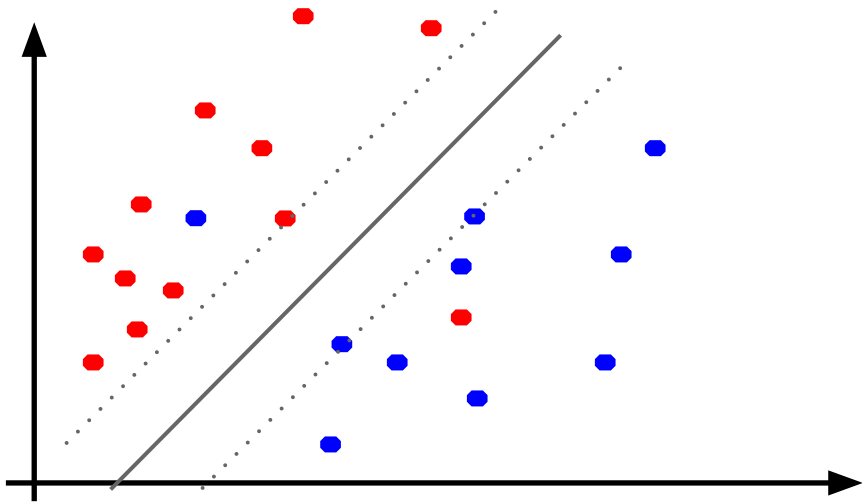
SOFT MARGIN CLASSIFICATION



$$\begin{aligned} & \min_{w,b} \|w\|^2 \\ & \text{subject to:} \\ & y_i(w \cdot x_i + b) \geq 1 \quad \forall i \end{aligned}$$

What about this problem?

SOFT MARGIN CLASSIFICATION

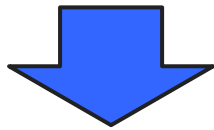


$$\begin{aligned} & \min_{w,b} \quad \|w\|^2 \\ & \text{subject to:} \\ & \quad y_i(w \cdot x_i + b) \geq 1 \quad \forall i \end{aligned}$$

We would like to learn something like this, but our constraints do not allow it...

SLACK VARIABLES

$$\begin{aligned} & \min_{w,b} \quad \|w\|^2 \\ & \text{subject to:} \\ & \quad y_i(w \cdot x_i + b) \geq 1 \quad \forall i \end{aligned}$$

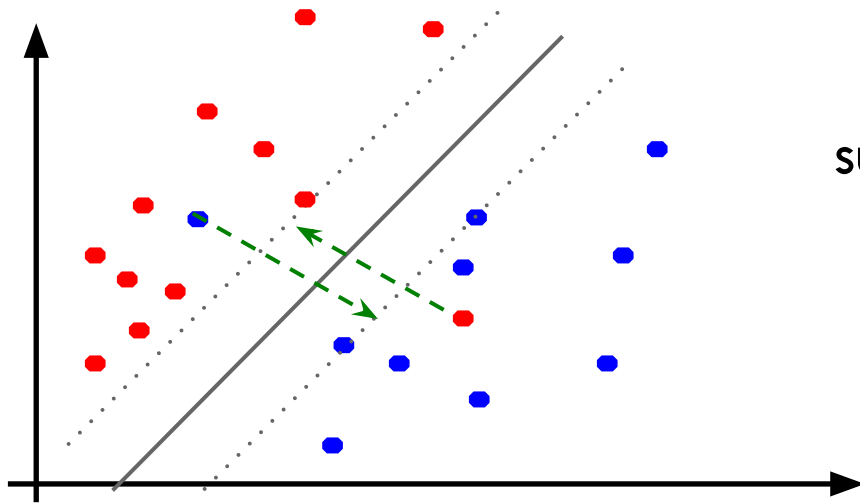


$$\begin{aligned} & \min_{w,b} \quad \|w\|^2 + C \sum_i \zeta_i \\ & \text{subject to:} \\ & \quad y_i(w \cdot x_i + b) \geq 1 - \zeta_i \quad \forall i \\ & \quad \zeta_i \geq 0 \end{aligned}$$

slack variables
(one for each example)

What effect do they have?

SLACK VARIABLES



slack penalties

$$\min_{w,b} \|w\|^2 + C \sum_i \zeta_i$$

subject to:

$$y_i(w \cdot x_i + b) \geq 1 - \zeta_i \quad \forall i$$

$$\zeta_i \geq 0$$

SLACK VARIABLES

margin

trade-off between margin
maximization and penalization

$$\min_{w,b} \|w\|^2 + C \sum_i \zeta_i$$

penalized by how far
from “correct”

subject to:

$$y_i(w \cdot x_i + b) \geq 1 - \zeta_i \quad \forall i$$

allowed to make a mistake

$$\zeta_i \geq 0$$

SOFT MARGIN SVM

$$\min_{w,b} \quad \|w\|^2 + C \sum_i \varsigma_i$$

subject to:

$$y_i(w \cdot x_i + b) \geq 1 - \varsigma_i \quad \forall i$$

$$\varsigma_i \geq 0$$

Still a **quadratic optimization problem!**

SOFT MARGIN SVM

$$\min_{w,b} \quad \|w\|^2 + C \sum_i \varsigma_i$$

subject to:

$$y_i(w \cdot x_i + b) \geq 1 - \varsigma_i \quad \forall i$$
$$\varsigma_i \geq 0$$

Parameter C can be viewed as a way to control **overfitting**:
it “trades off” the relative importance of maximizing the
margin and fitting the training data.

SOFT MARGIN SVM

$$\min_{w,b} \quad \|w\|^2 + C \sum_i \zeta_i$$

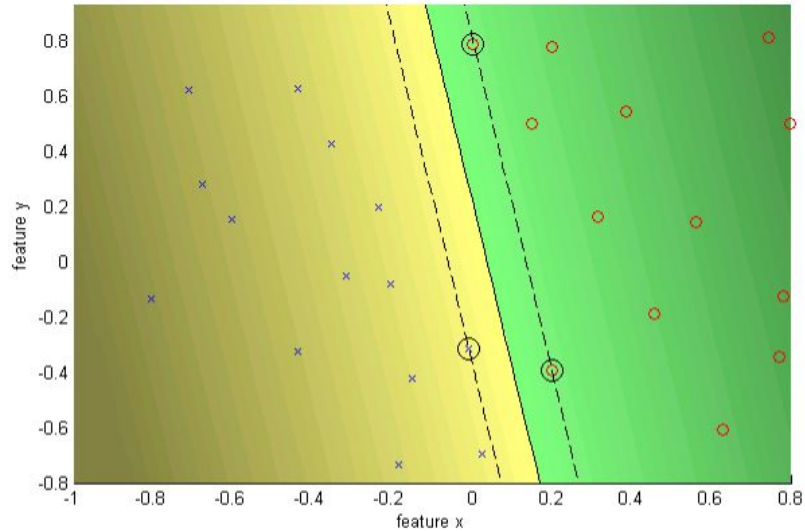
subject to:

$$y_i(w \cdot x_i + b) \geq 1 - \zeta_i \quad \forall i$$
$$\zeta_i \geq 0$$

C is a regularization parameter:

- small C allows constraints to be easily ignored \rightarrow large margin
- large C makes constraints hard to ignore \rightarrow narrow margin
- $C = \infty$ enforces all constraints: hard margin

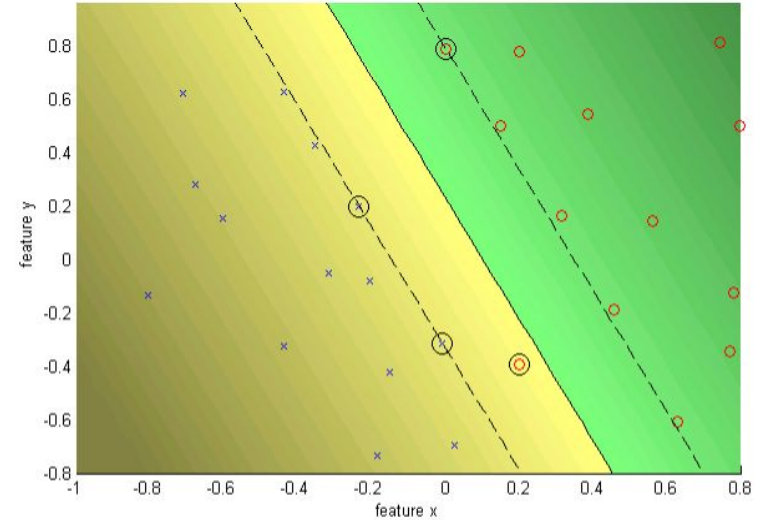
$C = \text{Infinity}$ hard margin



Comment Window

SVM (L1) by Sequential Minimal Optimizer
Kernel: linear (-), C: Inf
Kernel evaluations: 971
Number of Support Vectors: 3
Margin: 0.0966
Training error: 0.00%

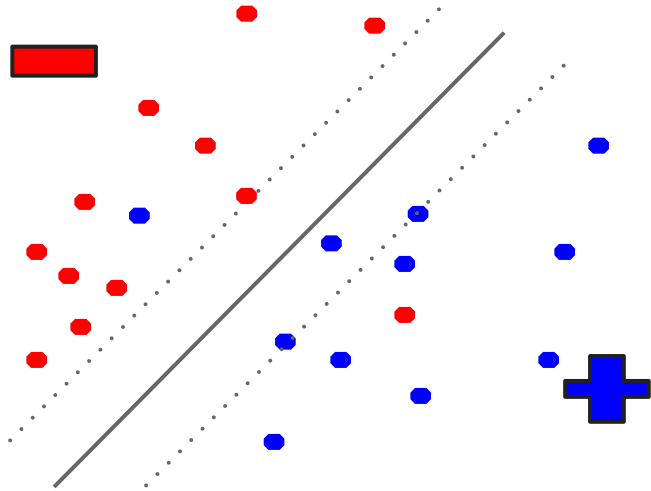
$C = 10$ soft margin



Comment Window

SVM (L1) by Sequential Minimal Optimizer
Kernel: linear (-), C: 10.0000
Kernel evaluations: 2645
Number of Support Vectors: 4
Margin: 0.2265
Training error: 3.70%

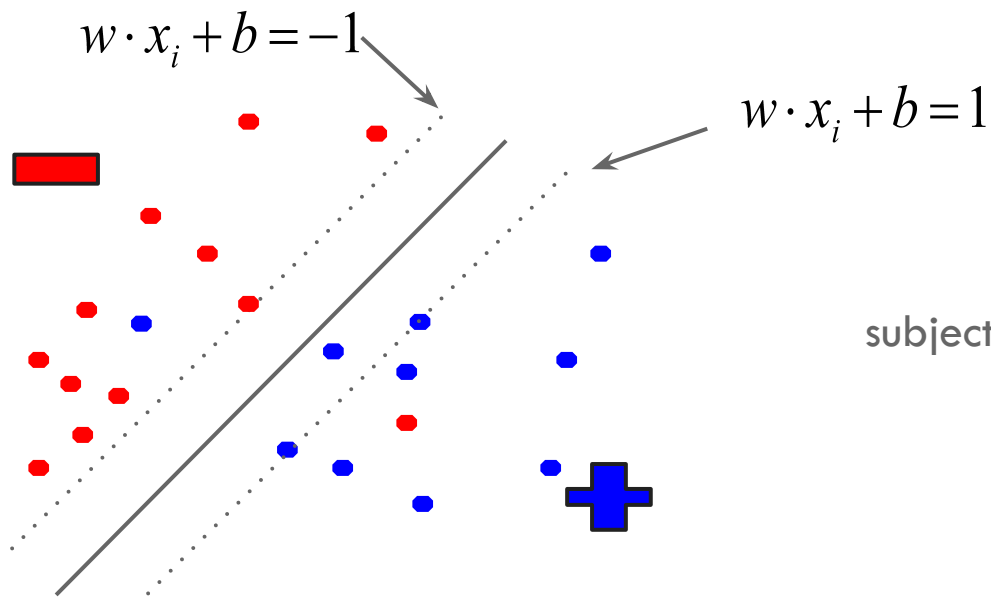
UNDERSTANDING THE SOFT MARGIN SVM



$$\begin{aligned} \min_{w,b} \quad & \|w\|^2 + C \sum_i \zeta_i \\ \text{subject to:} \quad & y_i(w \cdot x_i + b) \geq 1 - \zeta_i \quad \forall i \\ & \zeta_i \geq 0 \end{aligned}$$

Given the optimal solution (w, b) , can we figure out what the slack penalties are for each point?

UNDERSTANDING THE SOFT MARGIN SVM



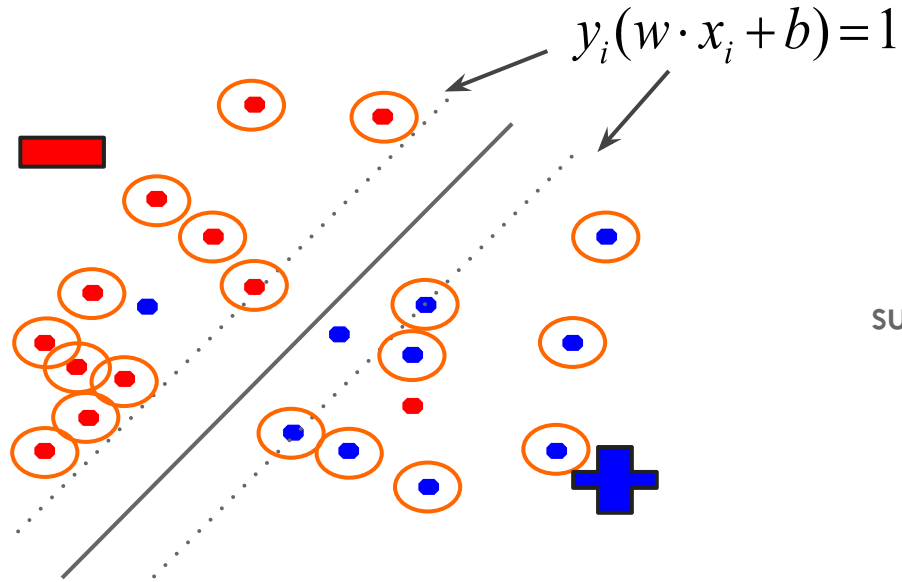
subject to:

$$\min_{w,b} \|w\|^2 + C \sum_i \zeta_i$$
$$y_i(w \cdot x_i + b) \geq 1 - \zeta_i \quad \forall i$$
$$\zeta_i \geq 0$$

or:

$$y_i(w \cdot x_i + b) = 1$$

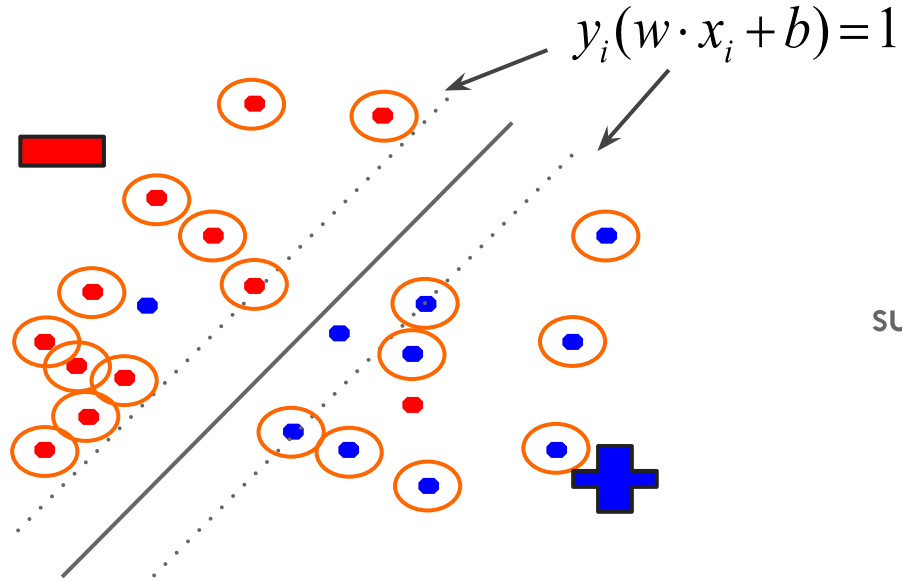
UNDERSTANDING THE SOFT MARGIN SVM



$$\begin{aligned} & \min_{w,b} \quad \|w\|^2 + C \sum_i \zeta_i \\ & \text{subject to:} \\ & y_i(w \cdot x_i + b) \geq 1 - \zeta_i \quad \forall i \\ & \zeta_i \geq 0 \end{aligned}$$

What are the slack values for points outside (or on) the margin AND correctly classified?

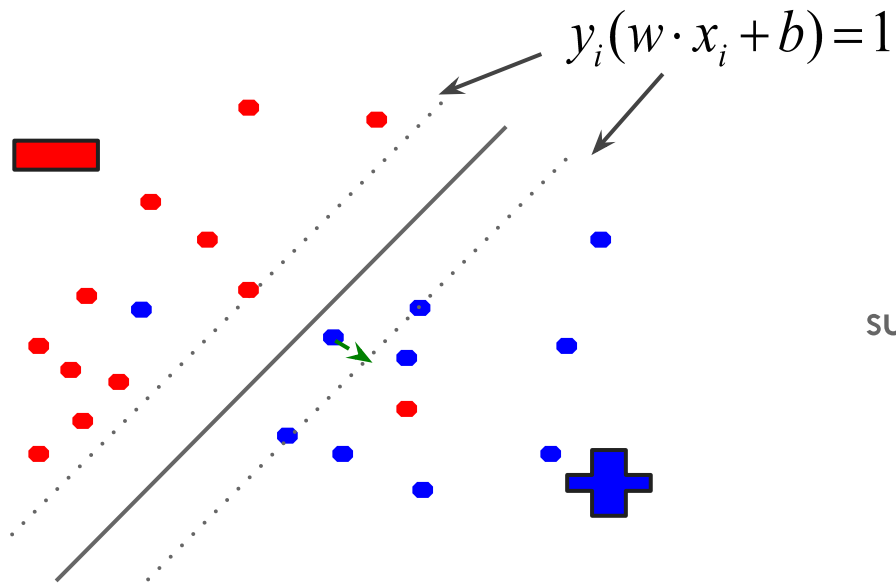
UNDERSTANDING THE SOFT MARGIN SVM



$$\begin{aligned} & \min_{w,b} \quad \|w\|^2 + C \sum_i \zeta_i \\ & \text{subject to:} \\ & \quad y_i(w \cdot x_i + b) \geq 1 - \zeta_i \quad \forall i \\ & \quad \zeta_i \geq 0 \end{aligned}$$

0! The slack variables have to be greater than or equal to zero and if they are on or beyond the margin then $y_i(w x_i + b) \geq 1$ already

UNDERSTANDING THE SOFT MARGIN SVM

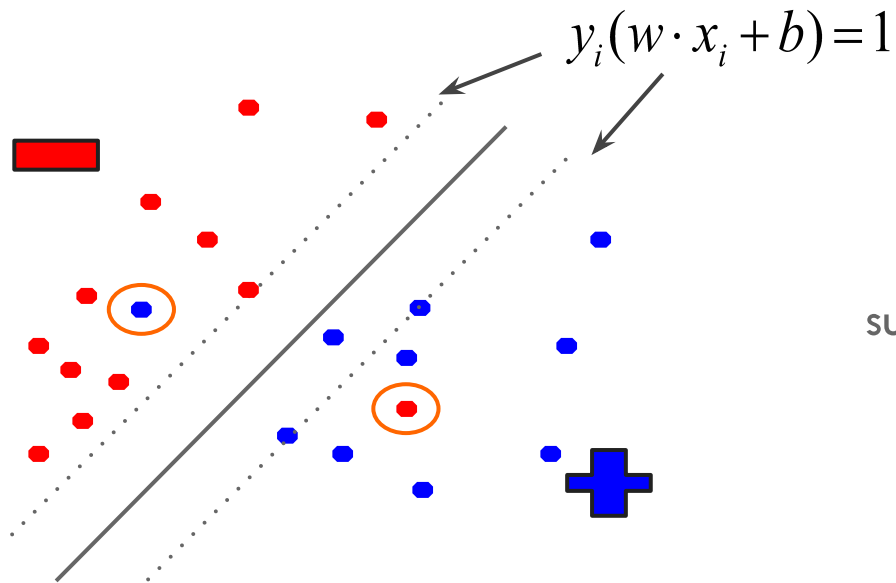


$$\begin{aligned} & \min_{w,b} \quad \|w\|^2 + C \sum_i \zeta_i \\ & \text{subject to:} \\ & y_i(w \cdot x_i + b) \geq 1 - \zeta_i \quad \forall i \\ & \zeta_i \geq 0 \end{aligned}$$

Difference from the point to the margin, i.e.

$$\zeta_i = 1 - y_i(w \cdot x_i + b)$$

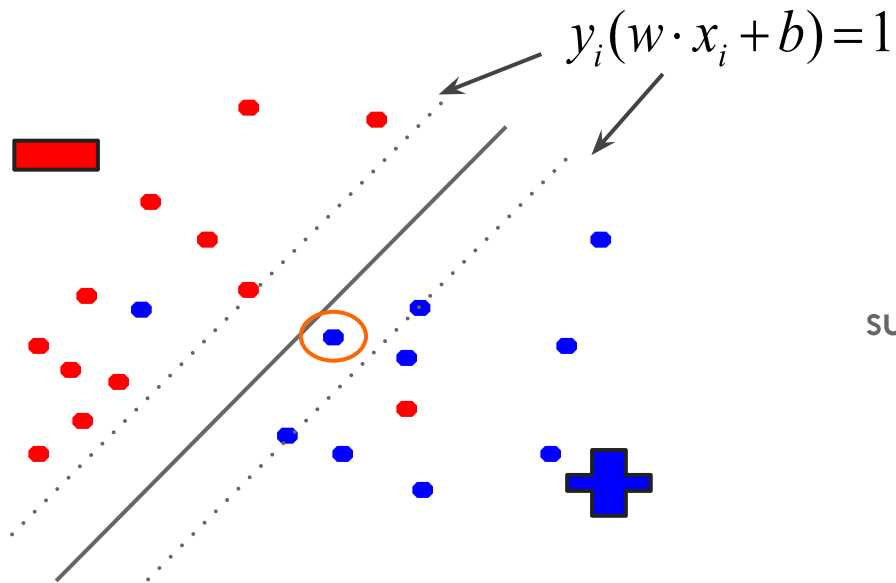
UNDERSTANDING THE SOFT MARGIN SVM



$$\begin{aligned} & \min_{w,b} \quad \|w\|^2 + C \sum_i \zeta_i \\ & \text{subject to:} \\ & y_i(w \cdot x_i + b) \geq 1 - \zeta_i \quad \forall i \\ & \zeta_i \geq 0 \end{aligned}$$

What are the slack values for points that are incorrectly classified?

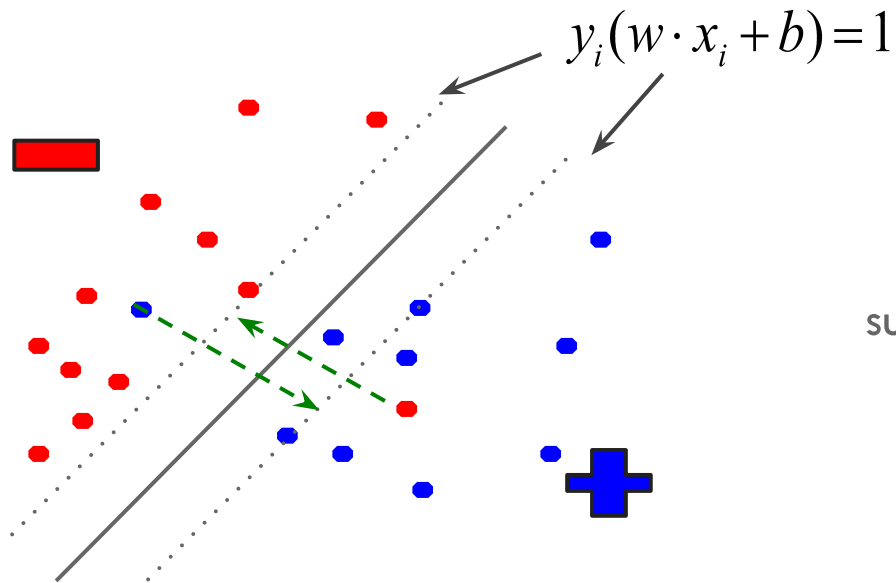
UNDERSTANDING THE SOFT MARGIN SVM



$$\begin{aligned} & \min_{w,b} \quad \|w\|^2 + C \sum_i \zeta_i \\ & \text{subject to:} \\ & y_i(w \cdot x_i + b) \geq 1 - \zeta_i \quad \forall i \\ & \zeta_i \geq 0 \end{aligned}$$

What are the slack values for points inside the margin AND classified correctly?

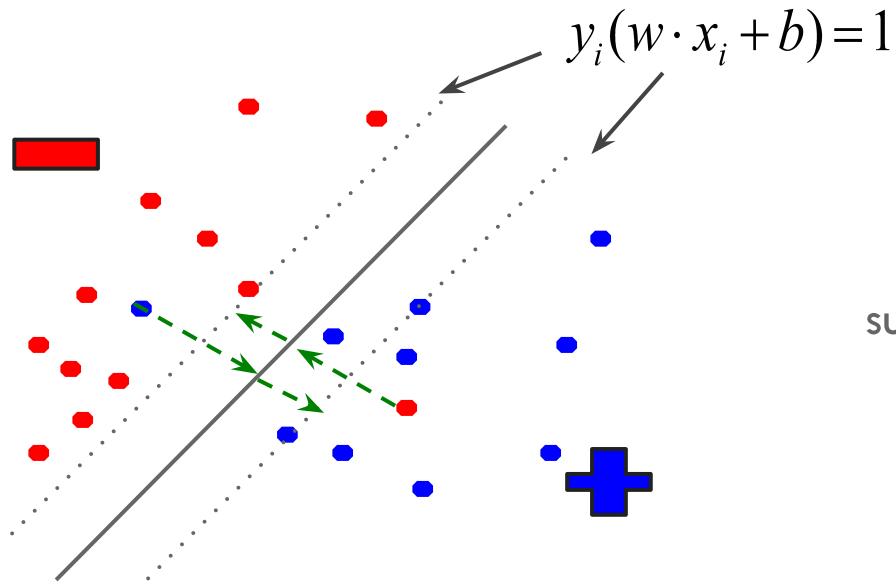
UNDERSTANDING THE SOFT MARGIN SVM



$$\begin{aligned} & \min_{w,b} \quad \|w\|^2 + C \sum_i \zeta_i \\ & \text{subject to:} \\ & y_i(w \cdot x_i + b) \geq 1 - \zeta_i \quad \forall i \\ & \zeta_i \geq 0 \end{aligned}$$

What are the slack values for points that are incorrectly classified?

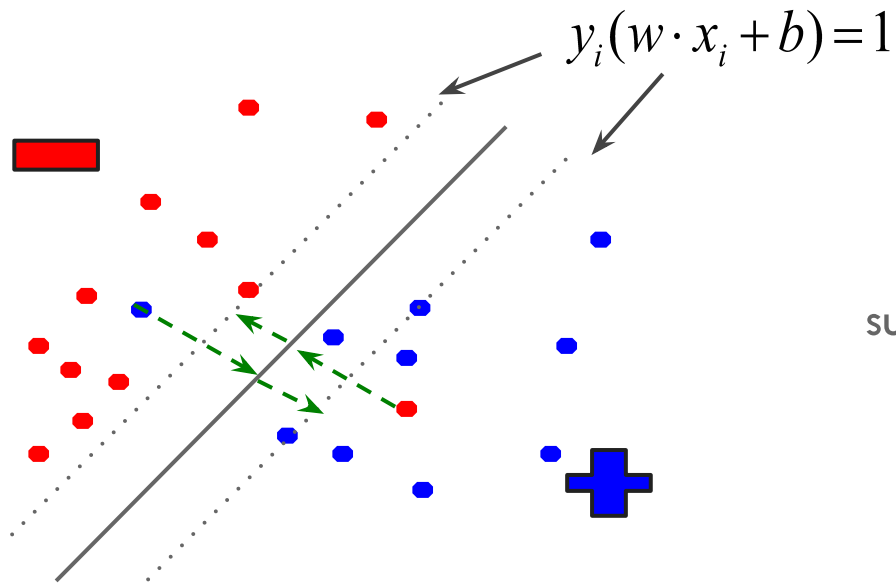
UNDERSTANDING THE SOFT MARGIN SVM



$$\begin{aligned} & \min_{w,b} \quad \|w\|^2 + C \sum_i \zeta_i \\ & \text{subject to:} \\ & \quad y_i(w \cdot x_i + b) \geq 1 - \zeta_i \quad \forall i \\ & \quad \zeta_i \geq 0 \end{aligned}$$

“distance” to the hyperplane *plus* the “distance” to the margin

UNDERSTANDING THE SOFT MARGIN SVM

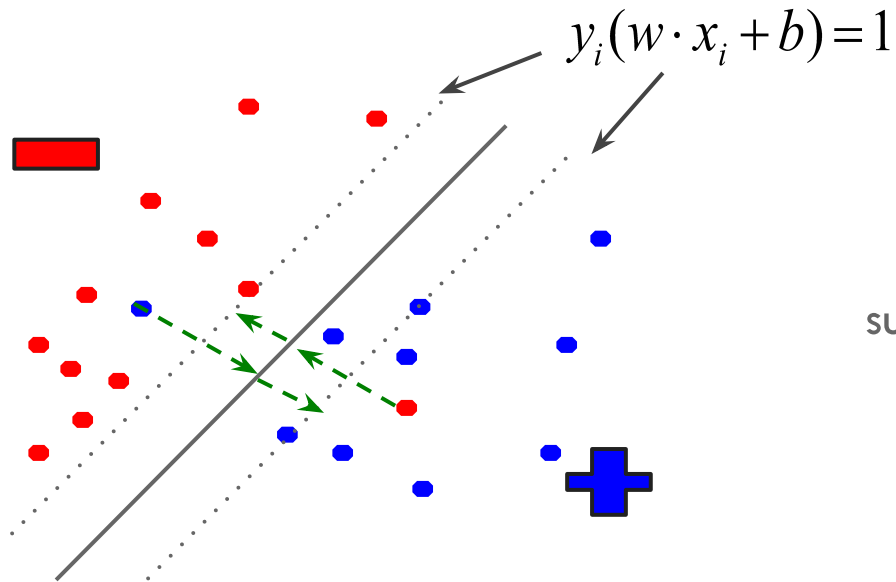


$$\begin{aligned} & \min_{w,b} \quad \|w\|^2 + C \sum_i \zeta_i \\ & \text{subject to:} \\ & \quad y_i(w \cdot x_i + b) \geq 1 - \zeta_i \quad \forall i \\ & \quad \zeta_i \geq 0 \end{aligned}$$

“distance” to the hyperplane *plus* the “distance” to the margin

$$-y_i(w \cdot x_i + b)$$

UNDERSTANDING THE SOFT MARGIN SVM



$$\begin{aligned} & \min_{w,b} \quad \|w\|^2 + C \sum_i \zeta_i \\ & \text{subject to:} \\ & \quad y_i(w \cdot x_i + b) \geq 1 - \zeta_i \quad \forall i \\ & \quad \zeta_i \geq 0 \end{aligned}$$

“distance” to the hyperplane *plus* the “distance” to the margin

$$\zeta_i = 1 - y_i(w \cdot x_i + b)$$

UNDERSTANDING THE SOFT MARGIN SVM

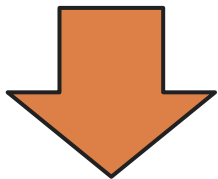
$$\begin{array}{ll} \min_{w,b} & \|w\|^2 + C \sum_i \varsigma_i \\ \text{subject to:} & \end{array}$$

$$\begin{array}{l} y_i(w \cdot x_i + b) \geq 1 - \varsigma_i \quad \forall i \\ \varsigma_i \geq 0 \end{array}$$

$$\varsigma_i = \begin{cases} 0 & \text{if } y_i(w \cdot x_i + b) \geq 1 \\ 1 - y_i(w \cdot x_i + b) & \text{otherwise} \end{cases}$$

UNDERSTANDING THE SOFT MARGIN SVM

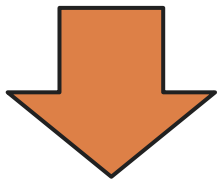
$$\zeta_i = \begin{cases} 0 & \text{if } y_i(w \cdot x_i + b) \geq 1 \\ 1 - y_i(w \cdot x_i + b) & \text{otherwise} \end{cases}$$



$$\begin{aligned} \zeta_i &= \max(0, 1 - y_i(w \cdot x_i + b)) \\ &= \max(0, 1 - yy') \end{aligned}$$

UNDERSTANDING THE SOFT MARGIN SVM

$$\zeta_i = \begin{cases} 0 & \text{if } y_i(w \cdot x_i + b) \geq 1 \\ 1 - y_i(w \cdot x_i + b) & \text{otherwise} \end{cases}$$



$$\begin{aligned} \zeta_i &= \max(0, 1 - y_i(w \cdot x_i + b)) \\ &= \max(0, 1 - yy') \end{aligned}$$

HINGE LOSS

Hinge:

$$l(y, y') = \max(0, 1 - yy')$$

Squared loss:

$$l(y, y') = (y - y')^2$$

0/1 loss:

$$l(y, y') = 1[yy' \leq 0]$$

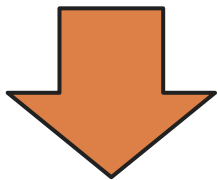
UNDERSTANDING THE SOFT MARGIN SVM

$$\min_{w,b} \quad \|w\|^2 + C \sum_i \varsigma_i$$

subject to:

$$y_i(w \cdot x_i + b) \geq 1 - \varsigma_i \quad \forall i$$
$$\varsigma_i \geq 0$$

$$\varsigma_i = \max(0, 1 - y_i(w \cdot x_i + b))$$



$$\min_{w,b} \quad \|w\|^2 + C \sum_i \max(0, 1 - y_i(w \cdot x_i + b))$$

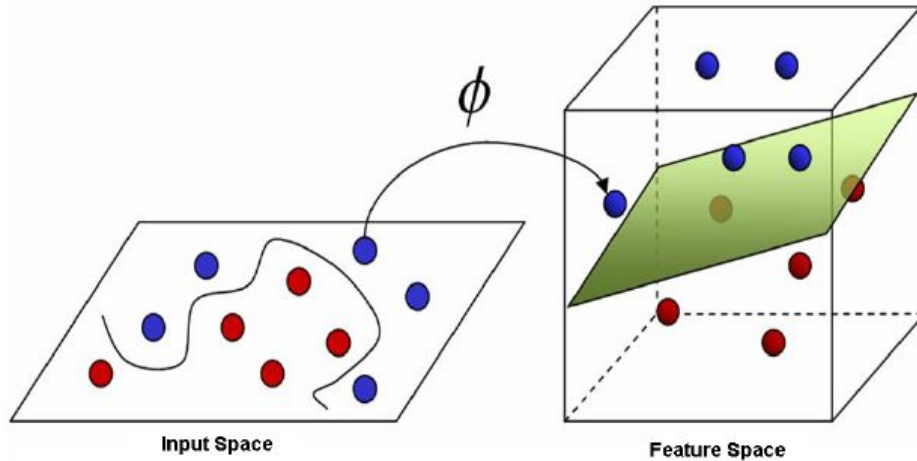
Unconstrained problem!

UNDERSTANDING THE SOFT MARGIN SVM

$$\min_{w,b} \quad \|w\|^2 + C \sum_i \text{loss}_{\text{hinge}}(y_i, y_i')$$

Does this look like something we have seen before?

$$\operatorname{argmin}_{w,b} \sum_{i=1}^n \text{loss}(yy') + \lambda \text{ regularizer}(w, b)$$



NON LINEARLY
SEPARABLE DATA

SUPPORT VECTOR MACHINE PROBLEM

$$\begin{aligned} & \min_{w,b} \quad \|w\|^2 \\ & \text{subject to:} \\ & \quad y_i(w \cdot x_i + b) \geq 1 \quad \forall i \end{aligned}$$

This is a version of a **quadratic optimization problem**

Maximize/minimize a quadratic function subject to a set of linear constraints

This is typically referred as **primal problem**

RECAP: CLASSES OF OPTIMIZATION PROBLEMS

Linear programming (LP): linear problem, linear constraints

$$\begin{array}{ll} \min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{Ax} = \mathbf{b}, \quad \mathbf{x} \geq 0 \end{array}$$

Quadratic programming (QP): quadratic objective and linear constraints, it is convex if Q is positive semidefinite

$$\begin{array}{ll} \min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} + \frac{1}{2} \mathbf{x}^T \mathbf{Q} \mathbf{x} \\ \text{s.t.} & \mathbf{Ax} = \mathbf{b}, \quad \mathbf{Cx} \geq \mathbf{d} \end{array}$$

Nonlinear programming problem (NLP): in general non-convex

$$\begin{array}{ll} \min_{\mathbf{x}} & f(\mathbf{x}) \\ \text{s.t.} & g(\mathbf{x}) = 0, \quad h(\mathbf{x}) \geq 0 \end{array}$$

DUAL PROBLEM

- Quadratic optimization problems are a well-known class of mathematical programming problems for which several (non-trivial) algorithms exist.
- One possible solution involves constructing a dual problem where a Lagrange multiplier α_i is associated with every inequality constraint in the primal (original) problem:

$$\begin{aligned} \max_{\alpha} \quad & \sum_i \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j y_i y_j \mathbf{x}_i^T \mathbf{x}_j \\ \text{s.t.} \quad & \sum_i \alpha_i y_i = 0, \quad \alpha_i \geq 0, \forall i \end{aligned}$$

THE SOLUTION

Given a solution $\alpha_1 \dots \alpha_n$ to the dual problem, the solution to the primal is:

$$\mathbf{w} = \sum_i \alpha_i y_i \mathbf{x}_i$$

$$b = y_k - \sum_i \alpha_i y_i \mathbf{x}_i^T \mathbf{x}_k$$

Each non-zero α_i indicates that corresponding \mathbf{x}_i is a support vector.
Then the classifying function is (note that we don't need \mathbf{w} explicitly):

$$f(\mathbf{x}) = \sum_i \alpha_i y_i \mathbf{x}_i^T \mathbf{x} + b$$

THE SOLUTION

$$f(\mathbf{x}) = \sum_i \alpha_i y_i \mathbf{x}_i^T \mathbf{x} + b$$

- Two important observations
 - The solution relies on an inner product between the test point \mathbf{x} and the support vectors \mathbf{x}_i .
 - Solving the optimization problem involves computing the inner products between all training points.

DUAL PROBLEM WITH SOFT MARGIN

- Dual problem is similar in the non separable case but notice the constraints.

$$\begin{aligned} \max_{\alpha} \quad & \sum_i \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j y_i y_j \mathbf{x}_i^T \mathbf{x}_j \\ \text{s.t.} \quad & \sum_i \alpha_i y_i = 0, \quad 0 \leq \alpha_i \leq C, \forall i \end{aligned}$$

- Again, \mathbf{x}_i with non-zero α_i will be support vectors.

LINEAR SVM SUMMARY

- The classifier is a separating hyperplane.
- Most “important” training points are support vectors; they define the hyperplane.
- Quadratic optimization algorithms can identify which training points are support vectors with non-zero Lagrangian multipliers α_i .

LINEAR SVM SUMMARY

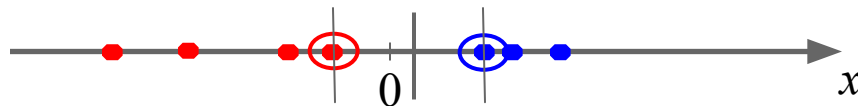
- Both in the dual formulation of the problem and in the solution training points appear only inside inner products:

$$\begin{aligned} \max_{\alpha} \quad & \sum_i \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j y_i y_j \mathbf{x}_i^T \mathbf{x}_j \\ \text{s.t.} \quad & \sum_i \alpha_i y_i = 0, \quad 0 \leq \alpha_i \leq C, \forall i \end{aligned}$$

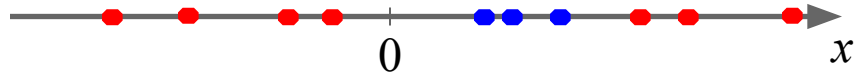
$$f(\mathbf{x}) = \sum_i \alpha_i y_i \mathbf{x}_i^T \mathbf{x} + b$$

NON LINEAR SVM

- Datasets that are linearly separable with some noise work out great:

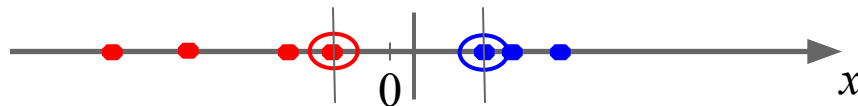


- But what are we going to do if the dataset is just too hard?

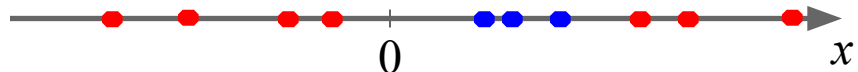


NON LINEAR SVM

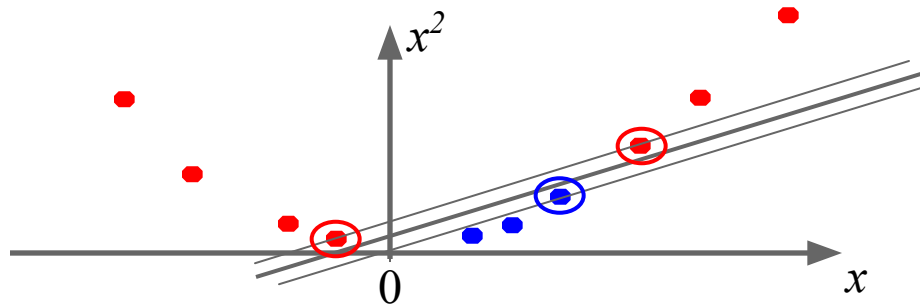
- Datasets that are linearly separable with some noise work out great:



- But what are we going to do if the dataset is just too hard?

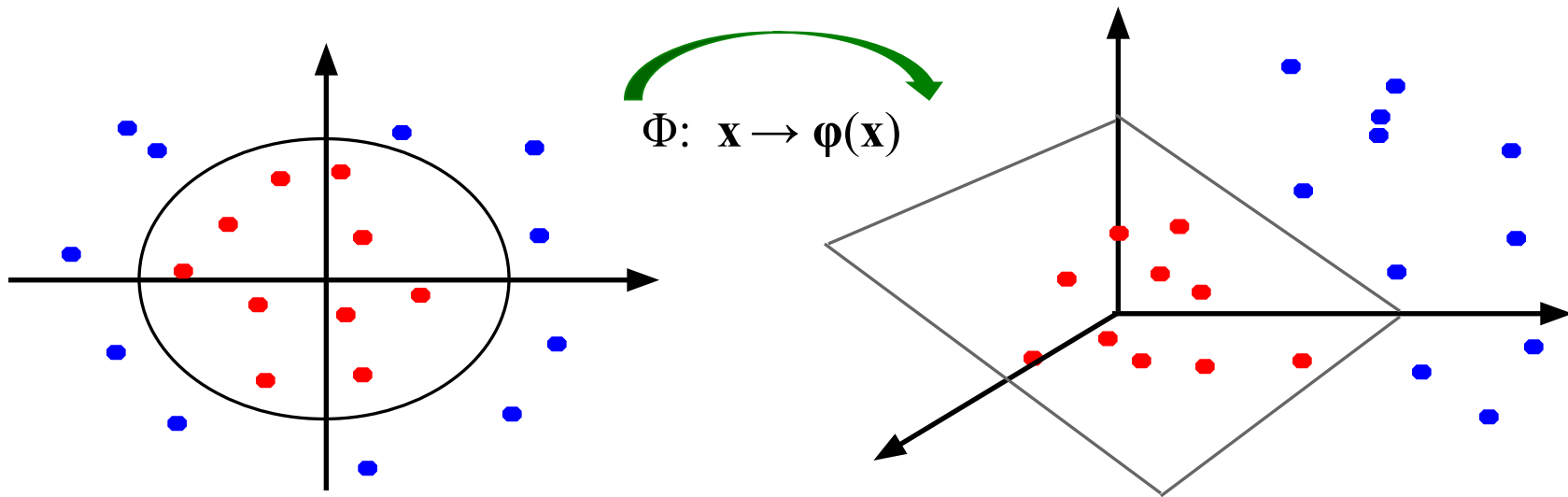


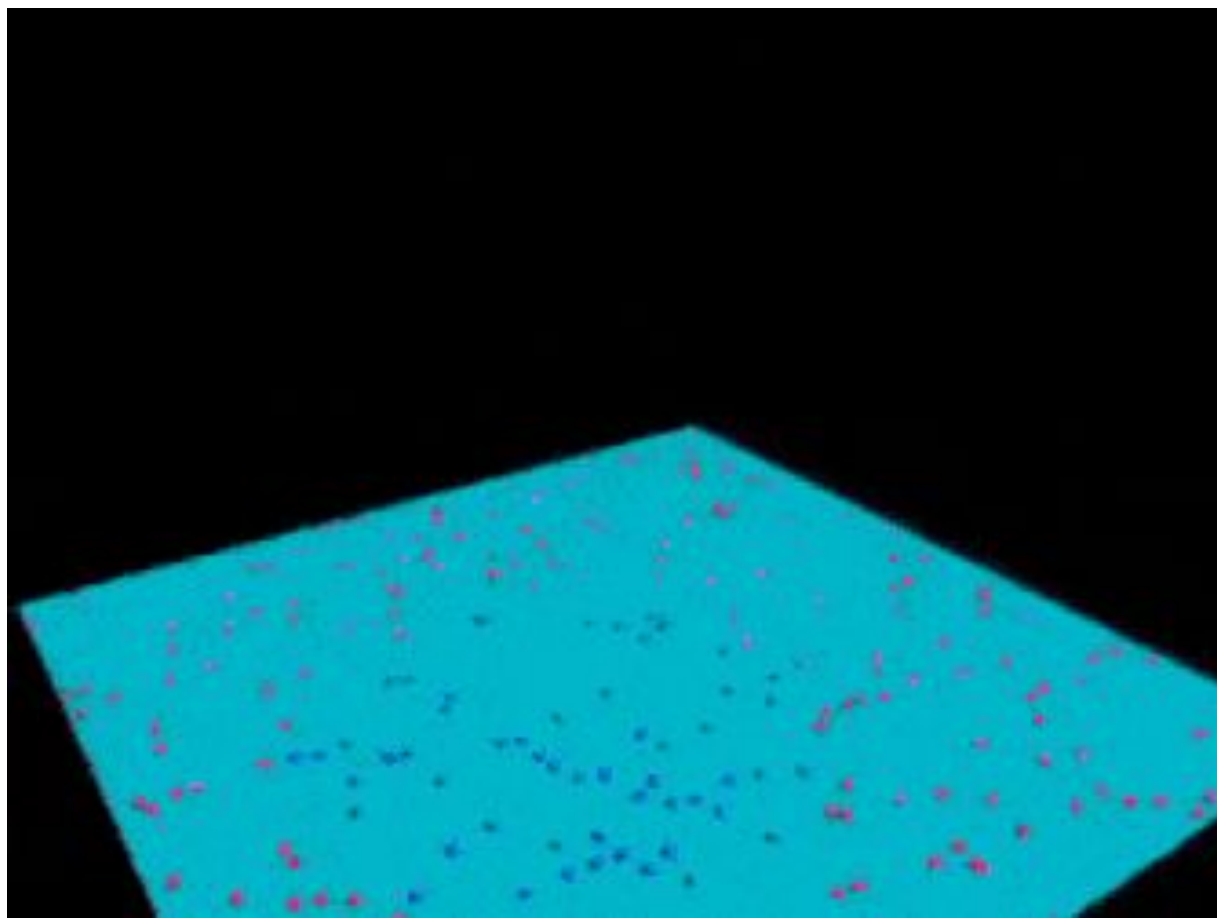
- How about... mapping data to a higher-dimensional space?



NON LINEAR SVM: FEATURE SPACES

- General idea: the original feature space can always be mapped to some higher-dimensional feature space where the training set is separable:





KERNEL TRICK

- The linear classifier relies on inner product between vectors $K(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{x}_i^T \mathbf{x}_j$
- If every datapoint is mapped into high-dimensional space via some , transformation $\Phi: \mathbf{x} \rightarrow \varphi(\mathbf{x})$, the inner product becomes:

$$K(\mathbf{x}_i, \mathbf{x}_j) = \varphi(\mathbf{x}_i)^T \varphi(\mathbf{x}_j)$$

- A kernel function is a function that is equivalent to an inner product in some feature space.

KERNEL TRICK

Example:

2-dimensional vectors $\mathbf{x}=[x_1 \ x_2]$; let $K(\mathbf{x}_i, \mathbf{x}_j)=(1 + \mathbf{x}_i^T \mathbf{x}_j)^2$,

Need to show that $K(\mathbf{x}_i, \mathbf{x}_j)=\varphi(\mathbf{x}_i)^T \varphi(\mathbf{x}_j)$:

$$\begin{aligned} K(\mathbf{x}_i, \mathbf{x}_j) &= (1 + \mathbf{x}_i^T \mathbf{x}_j)^2 = 1 + x_{i1}^2 x_{j1}^2 + 2 x_{i1} x_{j1} x_{i2} x_{j2} + x_{i2}^2 x_{j2}^2 + 2 x_{i1} x_{j1} + 2 x_{i2} x_{j2} = \\ &= [1 \ x_{i1}^2 \ \sqrt{2} x_{i1} x_{i2} \ x_{i2}^2 \ \sqrt{2} x_{i1} \ \sqrt{2} x_{i2}]^T [1 \ x_{j1}^2 \ \sqrt{2} x_{j1} x_{j2} \ x_{j2}^2 \ \sqrt{2} x_{j1} \ \sqrt{2} x_{j2}] = \\ &= \varphi(\mathbf{x}_i)^T \varphi(\mathbf{x}_j), \quad \text{where } \varphi(\mathbf{x}) = [1 \ x_1^2 \ \sqrt{2} x_1 x_2 \ x_2^2 \ \sqrt{2} x_1 \ \sqrt{2} x_2] \end{aligned}$$

- A kernel function **implicitly** maps data to a high-dimensional space (without the need to compute each $\varphi(\mathbf{x})$ explicitly).

KERNELS

- For some functions $K(\mathbf{x}_i, \mathbf{x}_j)$ checking that $K(\mathbf{x}_i, \mathbf{x}_j) = \varphi(\mathbf{x}_i)^T \varphi(\mathbf{x}_j)$ can be cumbersome.
- Mercer's theorem:
 - Every positive semidefinite symmetric function is a kernel
 - A positive semidefinite symmetric functions correspond to a positive semidefinite symmetric Gram matrix:

$K =$

$K(\mathbf{x}_1, \mathbf{x}_1)$	$K(\mathbf{x}_1, \mathbf{x}_2)$	$K(\mathbf{x}_1, \mathbf{x}_3)$...	$K(\mathbf{x}_1, \mathbf{x}_n)$
$K(\mathbf{x}_2, \mathbf{x}_1)$	$K(\mathbf{x}_2, \mathbf{x}_2)$	$K(\mathbf{x}_2, \mathbf{x}_3)$		$K(\mathbf{x}_2, \mathbf{x}_n)$
...
$K(\mathbf{x}_n, \mathbf{x}_1)$	$K(\mathbf{x}_n, \mathbf{x}_2)$	$K(\mathbf{x}_n, \mathbf{x}_3)$...	$K(\mathbf{x}_n, \mathbf{x}_n)$

- Recap: A symmetric matrix is positive semidefinite if and only if all eigenvalues are non-negative

KERNELS

- Linear: $K(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{x}_i^T \mathbf{x}_j$
- Polynomial of power p : $K(\mathbf{x}_i, \mathbf{x}_j) = (1 + \mathbf{x}_i^T \mathbf{x}_j)^p$
- Gaussian (radial-basis function): $K(\mathbf{x}_i, \mathbf{x}_j) = e^{-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{2\sigma^2}}$
 - Mapping $\Phi: \mathbf{x} \rightarrow \varphi(\mathbf{x})$, where $\varphi(\mathbf{x})$ is *infinite-dimensional*

NON LINEAR SVM PROBLEM

- Dual problem formulation:

$$\begin{aligned} \max_{\alpha} \quad & \sum_i \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j y_i y_j K(\mathbf{x}_i, \mathbf{x}_j) \\ \text{s.t.} \quad & \sum_i \alpha_i y_i = 0, \quad \alpha_i \geq 0, \forall i \end{aligned}$$

- The solution is:

$$f(\mathbf{x}) = \sum_i \alpha_i y_i K(\mathbf{x}_i, \mathbf{x}) + b$$

- Optimization techniques for finding α_i 's remain the same!

SVM REMARKS

- SVMs were originally proposed by Boser, Guyon and Vapnik in 1992 and gained increasing popularity in late 1990s.
- SVMs were successfully applied to a number of classification tasks ranging from text to genomic data.
- SVMs can be applied to complex data types beyond feature vectors (e.g. graphs, sequences, relational data) by designing kernel functions for such data.
- SVM techniques have been extended to a number of tasks such as regression [Vapnik et al. '97], principal component analysis [Schölkopf et al. '99], etc. .

SVM REMARKS

- Most popular optimization algorithms for SVMs use decomposition to hill-climb over a subset of α_i 's at a time, e.g. SMO [Platt '99] and [Joachims '99]
- Tuning SVMs remains a black art: selecting a specific kernel and parameters is usually done in a try-and-see manner (grid search)

SVM APPLICATIONS

Pedestrian detection in Computer Vision

Objective: detect (localize) standing humans in an image



- reduces object detection to binary classification
- does an image window contain a person or not?

SVM APPLICATIONS

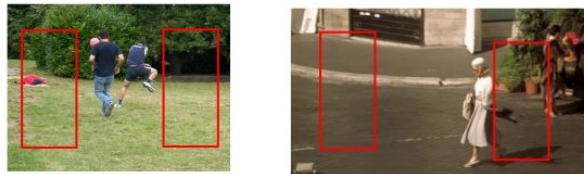
Pedestrian detection in Computer Vision

Training data and features

- Positive data – 1208 positive window examples



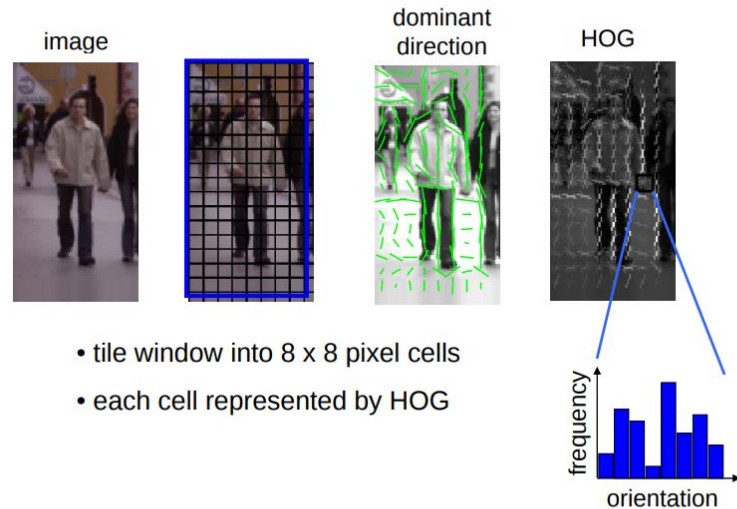
- Negative data – 1218 negative window examples (initially)



SVM APPLICATIONS

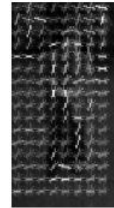
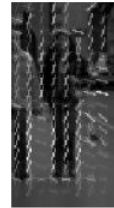
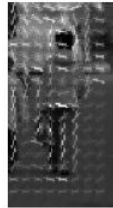
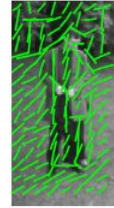
Pedestrian detection in Computer Vision

Training data and features



Feature vector dimension = 16×8 (for tiling) $\times 8$ (orientations) = 1024

SVM APPLICATIONS



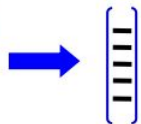
SVM APPLICATIONS

Pedestrian detection in Computer Vision

Algorithm

Training (Learning)

- Represent each example window by a HOG feature vector



$\mathbf{x}_i \in \mathbb{R}^d$, with $d = 1024$

- Train a SVM classifier

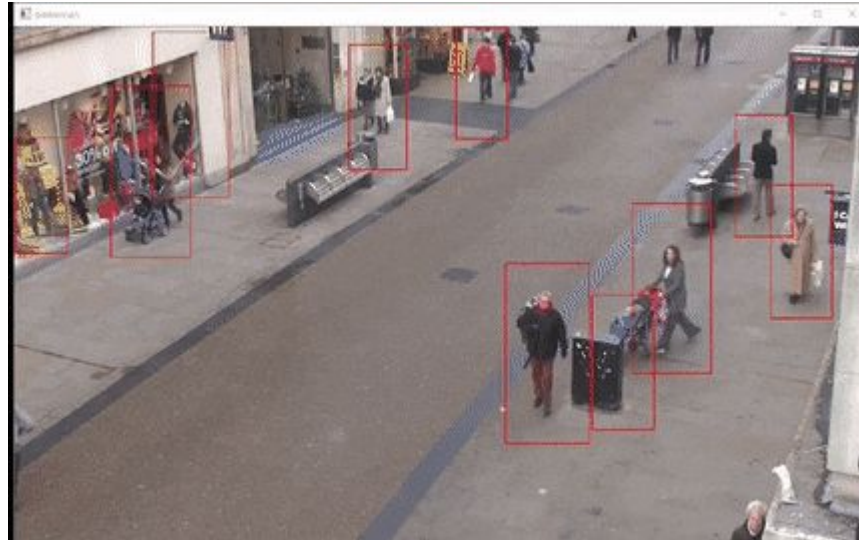
Testing (Detection)

- Sliding window classifier

$$f(x) = \mathbf{w}^\top \mathbf{x} + b$$

SVM APPLICATIONS

Pedestrian detection in Computer Vision



QUESTIONS?



Some slides are taken from David Kauchak