NON-CONTEXT-FREE LANGUAGES

INTRODUCTION

- Non-context-free languages are those languages that cannot be generated by any context-free grammar (CFG).
- These languages are more complex than context-free languages and require more powerful computational models, such as context-sensitive grammars or Turing machines, to describe them.
- We again use the pumping lemma to verify that a language is non-context-free.

PUMPING LEMMA FOR CONTEXT-FREE LANGUAGES

- If ***A*** is a context-free language, then there is a number p (the pumping length) where, if ***s*** is any string in ***A*** of length at least p, then ***s*** may be divided into five pieces s = uvxyz satisfying the conditions

  1. for each $i \geq 0$, $uv^ixy^iz \in A$,

  2. $|vy| > 0$, and

  3. $|vxy| \leq p$.

- When ***s*** is being divided into uvxyz, condition 2 says that either u or y is not the empty string. Otherwise the theorem would be trivially true. Condition 3 states that the pieces x, and y together have length at most p. This technical condition sometimes is useful in proving that certain languages are not context free.

CHARACTERISTICS OF NON CONTEXT-FREE LANGUAGES

Pumping Lemma for Context-Free Languages

- One of the methods used to prove that a language is non context-free.
- It provides a property that all context-free languages must satisfy.
- If a language does not satisfy this property, it is not context-free.

Closure Properties

- Context-free languages are closed under certain operations, such as union, concatenation, and Kleene star, but not under intersection or complementation.
- Non-context-free languages often arise in contexts where these closure properties do not hold.

PROOF OF NON CONTEXT-FREE LANGUAGES

1. Assume for contradiction that L is context-free.

2. Pumping length: According to the pumping lemma, there exists a pumping length p such that any string $s \in L$ with $|s| \geq p$ can be decomposed into five parts $s = uvwxy$ with the following conditions:

   - $|vwx| \leq p$
   - $vx \neq \epsilon$ (either v or x or both are non-empty)
   - $uv^n wx^n y \in L$ for all $n \geq 0$

3. Choose a specific string $s \in L$ with $|s| \geq p$.

4. Decompose s into $uvwxy$ as described above.

5. Show that for some n, $uv^n wx^n y$ does not belong to L, thereby contradicting the pumping lemma.

EXAMPLES

1. $L=\{a^n b^n c^n | n \geq 0\}$

This language consists of strings with equal numbers of a's, b's, and c's in that order. It is non context-free because a context-free grammar cannot enforce the equal counts of three different symbols.

2. $L=\{ww | w \in \{a,b\}^*\}$

L consists of strings that are the concatenation of some string w with itself. It is non context-free because a context-free grammar cannot ensure that the second half of the string is an exact duplicate of the first half.

3. $L=\{a^i b^j c^k |\ i,j,k \geq 0$ and $i=j$ or $j=k\}$

L requires balancing between two different pairs of symbols, which cannot be done with a context-free grammar.