Topic 2

Resistive Circuit calculations

Lesson objectives

By the end of the lesson, the learner should be able to

(i) Use Ohms law to calculate resistance, current and voltage drop in a resistive circuit

(ii) Justify the accuracy of resistor calculation results using Kirchoff's law of current and voltage

(iii) Apply superposition theorem to solve resistive circuits with two or more power sources

Ohms law

Ohm's Law defines the relationships between (P) power, (E) voltage, (I) current, and (R) resistance. One ohm is the resistance value through which one volt will maintain a current of one ampere.

( I ) Current is what flows on a wire or conductor like water flowing down a river. Current flows from positive to negative on the surface of a conductor. Current is measured in (A) amperes or amps.

( E ) Voltage is the difference in electrical potential between two points in a circuit. It's the push or pressure behind current flow through a circuit, and is measured in (V) volts.

( R ) Resistance determines how much current will flow through a component. Resistors are used to control voltage and current levels. A very high resistance allows a small amount of current to flow. A very low resistance allows a large amount of current to flow. Resistance is measured in Ωohms.

( P ) Power is the amount of current times the voltage level at a given point measured in wattage or watts.

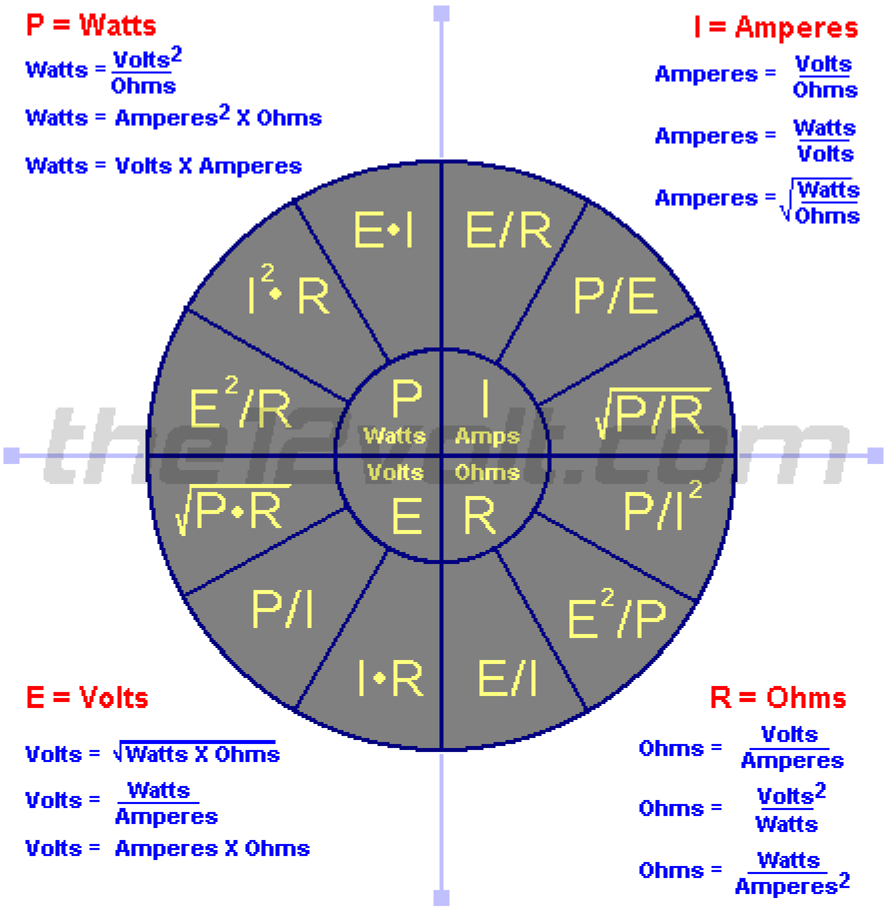



Fig 2.1 Ohms law relationship between P, I, R, and E

**Kirchhoff's Current Law**

Kirchhoff's Current Law, also known as Kirchhoff's Junction Law and Kirchhoff's First Law, defines the way that electrical current is distributed when it crosses through a junction - a point where three or more conductors meet. Specifically, the law states that: *The algebraic sum of current into any junction is zero.*

Since current is the flow of electrons through a conductor, it cannot build up at a junction, meaning that current is conserved: what comes in must come out. When performing calculations, current flowing into and out of the junction typically have opposite signs. This allows Kirchhoff's Current Law to be restated as:

*The sum of current into a junction equals the sum of current out of the junction.*

**Kirchhoff's Current Law in action**

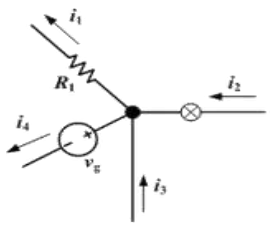



**Fig 2.2 KCL**

In the picture above, a junction of four conductors (i.e. wires) is shown. The currents $i_2$ and $i_3$ are flowing into the junction, while $i_1$ and $i_4$ flow out of it. In this example, Kirchhoff's Junction Rule yields the following equation:
$i_2 + i_3 = i_1 + i_4$

**Kirchhoff's Voltage Law**

Kirchhoff's Voltage Law describes the distribution of voltage within a loop, or closed conducting path, of an electrical circuit. Specifically, Kirchhoff's Voltage Law states that: *The algebraic sum of the voltage (potential) differences in any loop must equal zero.*

The voltage differences include those associated with electromagnetic fields (emfs) and resistive elements, such as resistors, power sources (i.e. batteries) or devices (i.e. lamps, televisions, blenders, etc.) plugged into the circuit.

Kirchhoff's Voltage Law comes about because the electrostatic field within an electric circuit is a conservative force field. As you go around a loop, when you arrive at the starting point has the same potential as it did when you began, so any increases and decreases along the loop have to cancel out for a total change of 0. If it didn't, then the potential at the start/end point would have two different values.

**Positive and Negative Signs in Kirchhoff's Voltage Law**

Using the Voltage Rule requires some sign conventions, which aren't necessarily as clear as those in the Current Rule. You choose a direction (clockwise or counter-clockwise) to go along the loop.

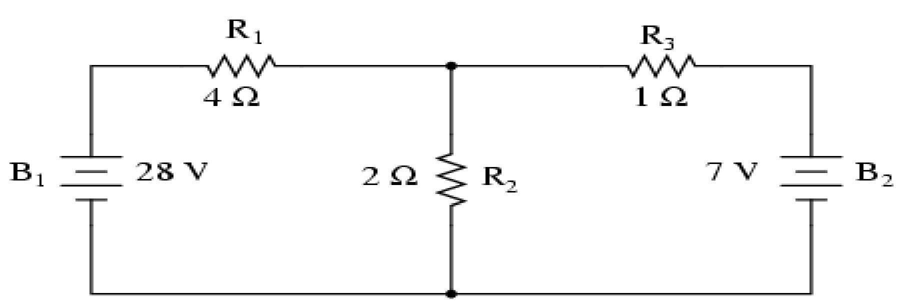
When travelling from positive to negative (+ to -) in an emf (power source) the voltage drops, so the value is negative. When going from negative to positive (- to +) the voltage goes up, so the value is positive.

When crossing a resistor, the voltage change is determined by the formula *I***R*, where *I* is the value of the current and *R* is the resistance of the resistor. Crossing in the same direction as the current means the voltage goes down, so its value is negative. When crossing a resistor in the direction opposite the current, the voltage value is positive (the voltage is increasing).

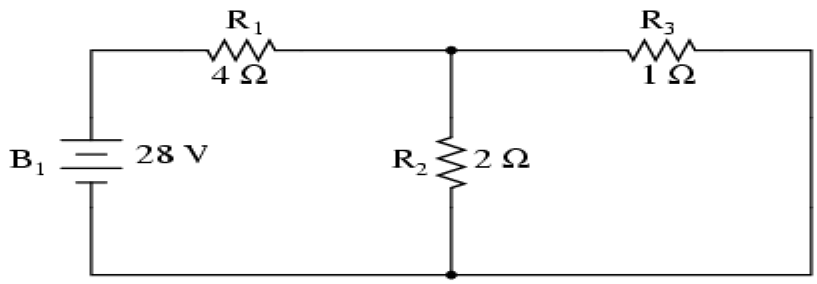
**Superposition theorem**

Superposition theorem is one of those strokes of genius that takes a complex subject and simplifies it in a way that makes perfect sense.

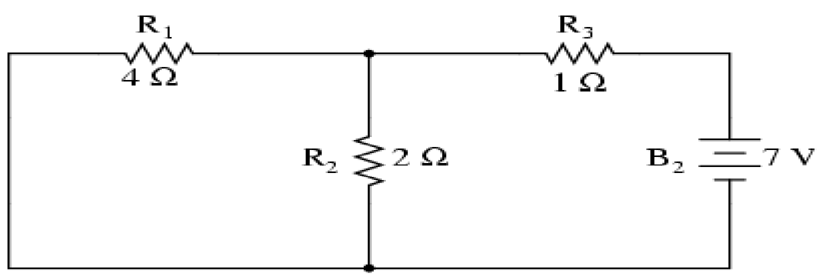
The strategy used in the Superposition Theorem is to eliminate all but one source of power within a network at a time, using series/parallel analysis to determine voltage drops (and/or currents) within the modified network for each power source separately. Then, once voltage drops and/or currents have been determined for each power source working separately, the values are all "superimposed" on top of each other (added algebraically) to find the actual voltage drops/currents with all sources active. Let's look at an example circuit and apply Superposition Theorem to it:




Since we have two sources of power in this circuit, we will have to calculate two sets of values for voltage drops and/or currents, one for the circuit with only the 28 volt battery in effect. . .



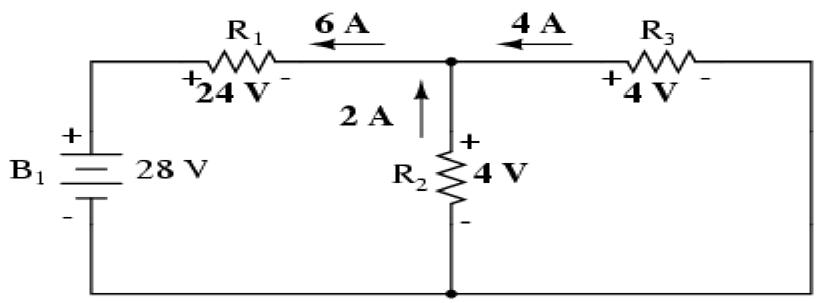

. . . and one for the circuit with only the 7 volt battery in effect:




When re-drawing the circuit for series/parallel analysis with one source, all other voltage sources are replaced by wires (shorts), and all current sources with open circuits (breaks). Since we only have voltage sources (batteries) in our example circuit, we will replace every inactive source during analysis with a wire.

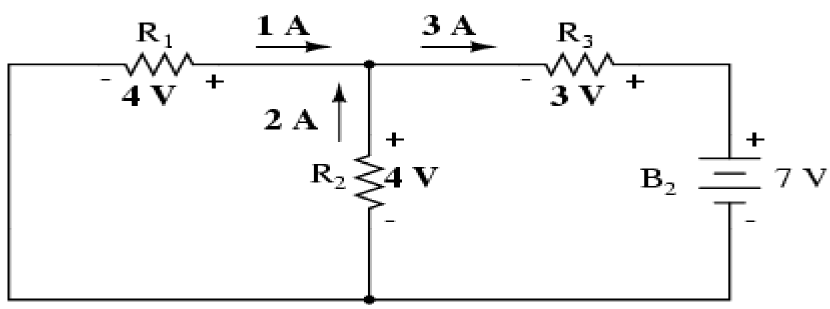
Analyzing the circuit with only the 28 volt battery, we obtain the following values for voltage and current:

| | $R_1$ | $R_2$ | $R_3$ | $R_2//R_3$ | $R_1 + R_2//R_3$ Total | |
|---|---|---|---|---|---|---|
| E | 24 | 4 | 4 | 4 | 28 | Volts |
| I | 6 | 2 | 4 | 6 | 6 | Amps |
| R | 4 | 2 | 1 | 0.667 | 4.667 | Ohms |




Analyzing the circuit with only the 7 volt battery, we obtain another set of values for voltage and current:

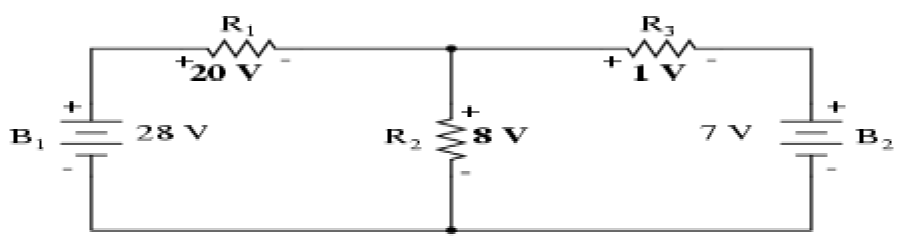
| | $R_1$ | $R_2$ | $R_3$ | $R_1//R_2$ | $R_3 + R_1//R_2$ Total | |
|---|---|---|---|---|---|---|
| E | 4 | 4 | 3 | 4 | 7 | Volts |
| I | 1 | 2 | 3 | 3 | 3 | Amps |
| R | 4 | 2 | 1 | 1.333 | 2.333 | Ohms |




When superimposing these values of voltage and current, we have to be very careful to consider polarity (voltage drop) and direction (electron flow), as the values have to be added *algebraically*.

| With 28 V battery | With 7 V battery | With both batteries |
|---|---|---|
| 24 V<br>+ −<br>$E_{R1}$ | 4 V<br>− +<br>$E_{R1}$ | **20 V**<br>$E_{R1}$ + −<br>*24 V - 4 V = 20 V* |
| $E_{R2}$ + 4 V − | $E_{R2}$ + 4 V − | $E_{R2}$ + **8 V** −<br>*4 V + 4 V = 8 V* |
| 4 V<br>+ −<br>$E_{R3}$ | 3 V<br>− +<br>$E_{R3}$ | **1 V**<br>$E_{R3}$ + −<br>*4 V - 3 V = 1 V* |

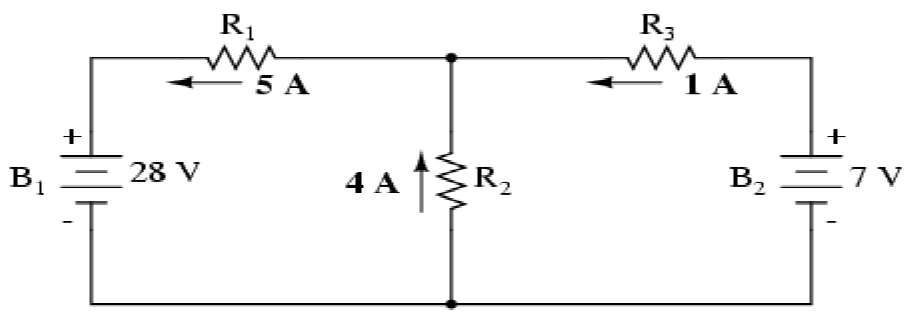
Applying these superimposed voltage figures to the circuit, the end result looks something like this:




Currents add up algebraically as well, and can either be superimposed as done with the resistor voltage drops, or simply calculated from the final voltage drops and respective resistances (I=E/R). Either way, the answers will be the same. Here we see the superposition method applied to current:

| With 28 V battery | With 7 V battery | With both batteries |
|---|---|---|
| ← 6 A<br>$I_{R1}$ | → 1 A<br>$I_{R1}$ | $I_{R1}$ ← **5 A**<br>$6\ A - 1\ A = 5\ A$ |
| $I_{R2}$ ↑ 2 A | $I_{R2}$ ↑ 2 A | $I_{R2}$ ↑ **4 A**<br>$2\ A + 2\ A = 4\ A$ |
| ← 4 A<br>$I_{R3}$ | → 3 A<br>$I_{R3}$ | $I_{R3}$ ← **1 A**<br>$4\ A - 3\ A = 1\ A$ |

Once again applying these superimposed figures to our circuit:




Quite simple and elegant, don't you think? It must be noted, though, that the Superposition Theorem works only for circuits that are reducible to series/parallel combinations for each of the power sources at a time (thus, this theorem is useless for analyzing an unbalanced bridge circuit), and it only works where the underlying equations are linear (no mathematical powers or roots). The requisite of linearity means that Superposition Theorem is only applicable for determining voltage and current, *not power!!!* Power dissipations, being nonlinear functions, do not algebraically add to an accurate total when only one source is considered at a time. The need for linearity also means this Theorem cannot be applied in circuits where the resistance of a component changes with voltage or current. Hence, networks containing components like lamps (incandescent or gas-discharge) or varistors could not be analyzed.

Another prerequisite for Superposition Theorem is that all components must be "bilateral," meaning that they behave the same with electrons flowing either direction through them. Resistors have no polarity-specific behavior, and so the circuits we've been studying so far all meet this criterion. We can assume the current to be moving from the +ve terminal of the battery and going back through the – ve terminal of the battery or vice versa as long as we remain consistent for each case.

The Superposition Theorem finds use in the study of alternating current (AC) circuits, and semiconductor (amplifier) circuits, where sometimes AC is often mixed (superimposed) with DC. Because AC voltage and current equations (Ohm's Law) are linear just like DC, we can use Superposition to analyze the circuit with just the DC power source, then just the AC power source, combining the results to tell what will happen with both AC and DC sources in effect. For now, though, Superposition will suffice as a break from having to do simultaneous equations to analyze a circuit.

- **REVIEW:**
- The Superposition Theorem states that a circuit can be analyzed with only one source of power at a time, the corresponding component voltages and currents algebraically added to find out what they'll do with all power sources in effect.
- To negate all but one power source for analysis, replace any source of voltage (batteries) with a wire; replace any current source with an open (break).

http://www.allaboutcircuits.com/vol_1/chpt_10/8.html

https://www.allaboutcircuits.com/textbook/direct-current/chpt-10/superposition-theorem/