Session 7 Logic Gates

1. Introduction

A *logic gate* is an electronic circuit which makes logic decisions. It has one output and one or more inputs. The output signal appears only for certain combinations of input signals. Logic gates are the building blocks from which most of the digital systems are built.

- They implement the hardware logic function based on the logical algebra developed by George Boole which is called Boolean Algebra in his honour.
- A unique characteristic of the Boolean Algebra is that variables used in it can only have one of the two values. i.e. either 0 or 1.
- These gates are available to-day in form of various Integrated Circuits (ICs) families. The most popular families are:
    - Transistor-Transistor Logic (TTL)
    - Emitter-Coupled Logic (ECL)
    - Metal-Oxide-Semiconductor (MOS)
    - Complementary Metal-Oxide-Semiconductor (CMOS)
- We will consider OR, AND, XOR, NOT, NOR, NAND and XNOR gates along with their *Truth Tables*.

2. Positive and Negative Logic

In computing systems, the number symbols 0 or 1 represent two possible states of a circuit or device. They symbolise two opposite conditions:

- ON and OFF
- Closed and High
- Plus and Minus
- True and False

In the positive logic, a 1 represents :

1. an ON circuit
2. a CLOSED switch
3. a HIGHvoltage
4. a PLUS sign
5. a TRUE statement

Consequently, o represents:

1. an OFF circuit
2. a OPEN switch
3. a LOW voltage
4. a MINUS sign
5. a FALSE statement

In the negative logic, just the opposite conditions prevail.

Example

Suppose a digital system has two voltage levels 0V and 5V. if we say that symbol 1stands for 5V and symbol 0 for 0V, then we have positive system. If we decide

that a 0 should represent 5V and a 1 represents 0V, then we will get negative logic system.
In positive logic, the more positive of the two voltage levels represent the 1 while in negative logic, the more negative voltage represents the 1.
Remark
Unless stated otherwise, we will be using the positive logic throughout.

3. The OR Gate
The electronic symbol for a two-input OR gate is

A + B = C

The equivalent switching circuit is

The two inputs have been marked as A and B and the output as C.

Logic Operation
The OR gate has an output of 1 when either A or B or BOTH are 1. The lamp will light up (Logic 1) when either switch A or B or BOTH are CLOSED. Obviously, the output would be 0 if and only if BOTH of its inputs are 0.
- In switching conditions, it means that lamp would be OFF (LOGIC 0) only when BOTH switches A or B are OFF.
The OR gate represents the Boolean equation A + B = C.
The meaning of this equation is that C is true when either A is True or B is true or both are true. Alternatively, it means that output C is 1 when either A or B or both are 1.

The above logic operation of the OR gate can be summarised with the help of a TRUTH TABLE.
Definition:
A *TRUTH TABLE* is a table which gives the output state for all possible input combinations of a logic gate.

For the OR gate, the Truth Table is:

A	**B**	**C**
0	0	0
0	1	1
1	0	1
1	1	1

Interpretation

When both inputs are 0 (switches are OPEN), the output C is 0 (lamp OFF). When A is in the logic state 0 (switch A is OPEN) but B is in logic state 1(switch B is CLOSED), the output C is logic state 1 (lamp is ON). Lamp would be also ON when A is CLOSED and B is OPEN. Of course, lamp would be ON when both switches are CLOSED. It is so because an OR gate is equivalent to a parallel circuit in its logic function.

Remember
The OR gate is called INCLUSIVE OR gate because it includes the case when both inputs are TRUTH.

OR Gate Symbolised Logic Addition
According to Boolean algebra, OR gate performs LOGIC ADDITION. The truth table can be written as given below:
0 + 0 = 0
0 + 1 = 1
1 + 0 = 1
1 + 1 = 1

It must be clearly understood that "+" sign Boolean algebra does not stand for the addition as understand in the ordinary or numerical algebra. In symbol logic, the "+" sign indicates OR operation whose rules are given above.
In the logic algebra, A + B = C means that A is TRUE or B is TRUE, then C will be TRUE. It does not mean here that sum of A and B equals C. the other symbol used for "+" signs are UNION U and V. hence, the above equation could also be written as A U B = C or A V B = C.
The OR laws are :
i. A + 1 = 1
ii. A + 0 = A
iii. A + A = A

Activity
Prove these!!!

Example

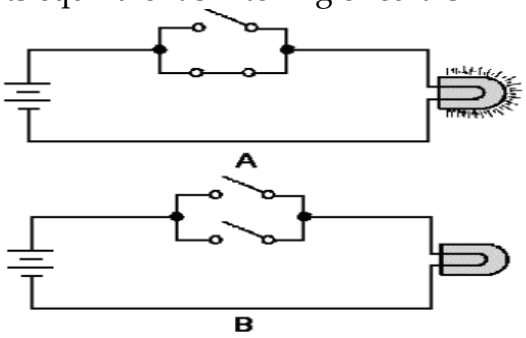
For a three-inputs OR Gate draw a logic gate, write the Boolean equation, and a truth table.

Solution
Boolean equation is **A + B + C = D**
The truth table is

A	B	C	D
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

4. The Exclusive OR Gate (XOR)
The electronic symbol is

The Boolean equation is A ⊕ B = C.
Its equivalent switching circuit is



In this gate, output is 1 if its either input BUT NOT BOTH, is 1. In other words, it has an output 1 when its inputs are different. The output is 0 only when inputs are SAME.
Note
This logic gate has 0 when inputs are either all 0 or all 1.
The truth table is

A	B	C
0	0	0

0	1	1
1	0	1
1	1	0

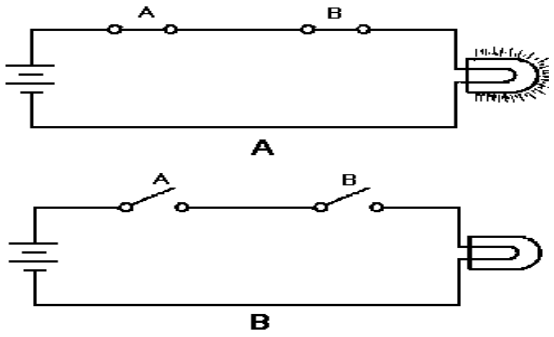
"You can be rich OR you can be poor"

5. The AND Gate

The electronic symbol is

The Boolean equation is A X B = C, or A.B=C or AB = C.

Its equivalent switching circuit is



Logic Operation

- The AND gate gives an output only when all its inputs are present.

The AND gate has a 1 output when BOTH A and B are 1. Hence this gate is an all-or-nothing gate whose output occurs only when all its inputs are present. The True / False terminology, the output of an AND gate will be TRUE only if all its inputs are true. Its output would be false if any of its inputs is FALSE.

The AND gate works on the Boolean algebra A X B = C, or A.B=C or AB = C.
It is a logical multiplication is different from the arithmetic multiplication.
The logical meaning of the above equation is that:
i. output C is 1 only when both A and B are 1
ii. Output C is true only when both A and B are true.
The truth table for Two-input AND gate is:
A X B = C, or A.B=C or AB = C.

A	**B**	**C**

0	0	0
0	1	0
1	0	0
1	1	1

AND Gate Symbolises Logic Multiplication
According to Boolean, the AND gate performs LOGICAL multiplication on its inputs. i.e.
0.0 = 0
0.1= 0
1.0 = 0
1.1 = 1

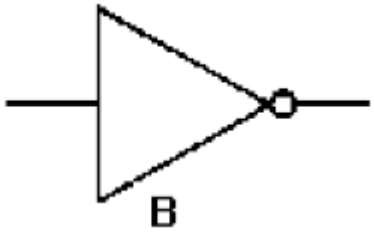
The Boolean laws of multiplication are:
i. A.1 = 1
ii. A.0 = 0
iii. A.A = A
Verify these!!!

6. The NOT gate

It is so called because its output is NOT the same as its input. It is also called an inverter because it inverts the input signal.

It has one input and one output.

The electronic / logic symbol is



The Truth Table is

Input A	**Output B**
0	1
1	0

The Boolean equation is B = Ā.

The schematic symbol for inversion is a small circle as above. The logical symbol for *inversion or negation or complementation* is a bar over (*or overbar*) the function to indicate the opposite state. Sometimes a primer À is also used. E.g. Ā means not-A. Similarly, (A + B) means the complement of (A + B).

The NOT operation

It is a complementation operation and its symbol is an overbar.

Definition:

0 means taking the negation or complement of 0 which is 1. It should also be noted that complement of a value can be taken repeated. E.g.

1 = 0 = 1 or 0 = 1 = 0.

The double complementation gives the original value as shown below.

Examples

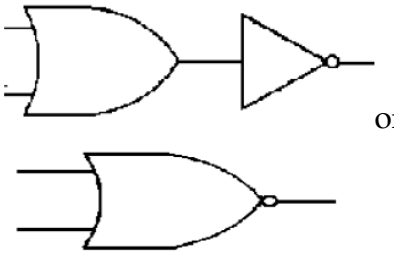
Find the Boolean equation for the output D of the figure below.

Evaluate D when

i) A = 0, B = 1, C = 1
ii) A = 1, B = 1, C = 1.

7. The NOR gate

It is a NOT-OR gate. It is can be made out of an OR gate by connecting an inverter in its output as shown:



or

The Boolean equation for the output is $C = \overline{(A + B)}$

A NOR function is just the t=reverse of the OR function.

The Logic operation

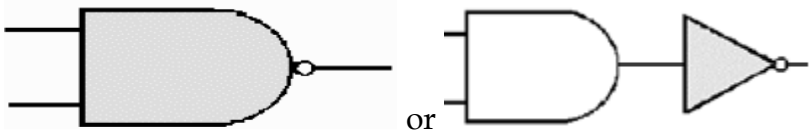
A NOR gate will have an output of 1 only when all its inputs are 0. In any input is 1, the output will be 0. Alternatively, in a NOR gate, output is *true*, only when all inputs are *false.*

The Truth Table for a 2–input NOR gate is

A	**B**	**C**
0	0	1
0	1	0
1	0	0
1	1	0

8. The NAND gate

It is a NOT–AND gate. It can be obtained by connecting a NOT gate in the output of an AND gate as show



or

The Boolean equation (or output equation) is $\overline{C} = AB$.

This gate gives an output of 1 if its both inputs are NOT 1. In other words, it gives an output 1 if either A or B or BOTH are 0.

The Truth Table:

A	**B**	**C**
0	0	1
0	1	1
1	0	1
1	1	0

9. The XNOR gate

It is known as NOT – XOR gate i.e. $\overline{\text{XOR}}$.

Its logic symbol is

The Truth Table is

A	B	C
0	0	1
0	1	0
1	0	0
1	1	1

The logic function and truth table are just the reverse of those for XOR gate.

This gate has an output 1 if its both inputs are either 0 or 1.

10. LOGIC GATES IN COMBINATION

When you look at logic circuit diagrams for digital equipment, you are not going to see just a single gate, but many combinations of gates. At first it may seem confusing and complex. If you interpret *one gate at a time,* you can work your way through *any network*. In this section, we will analyze several combinations of gates and then provide you with some practice problems.

Combinational circuits can be constructed using a combination of INVERTER, OR, AND, XOR, NOR, XNOR and NAND gates. When combinations of circuits are formed, some gates may share inputs. E.g.