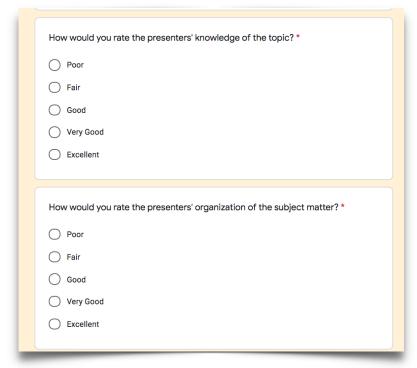
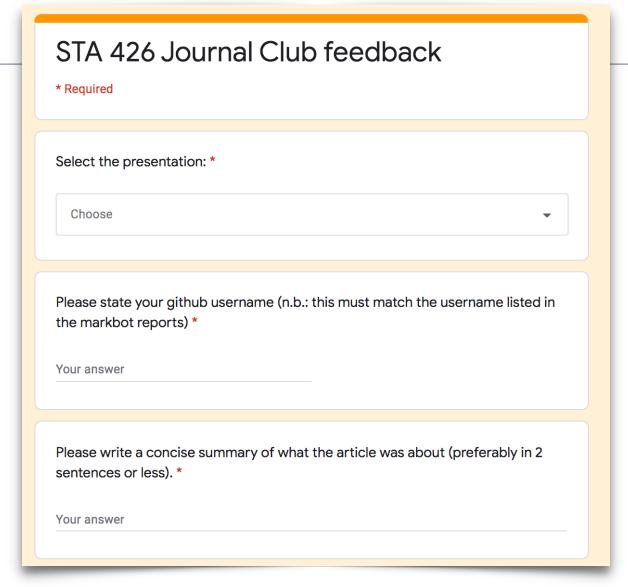


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Quick note on journal clubs

Giving feedback to your fellow students is part of your JC mark







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Goto journal clubs x2



Notes on Exercise 5 (limma fundamentals) .. general things

- please don't hide code (```\{r echo=FALSE} ..)! (even if it's the code given in the assignment)
- try to exclude package startup messages from report, at least for package startup messages (```r my_block, message=FALSE} ...)
- solutions are uploaded to solutions repo
- Ex 5 marks were sent by markbot



Notes on Exercise 5 (limma fundamentals) ..

general things

```
library("affy")
## Loading required package: BiocGenerics
## Loading required package: parallel
## Attaching package: 'BiocGenerics'
## The following objects are masked from 'package:parallel':
##
       clusterApply, clusterApplyLB, clusterCall, clusterEvalQ,
      clusterExport, clusterMap, parApply, parCapply, parLapply,
      parLapplyLB, parRapply, parSapply, parSapplyLB
## The following object is masked from 'package:limma':
##
       plotMA
## The following objects are masked from 'package:stats':
       IQR, mad, sd, var, xtabs
```

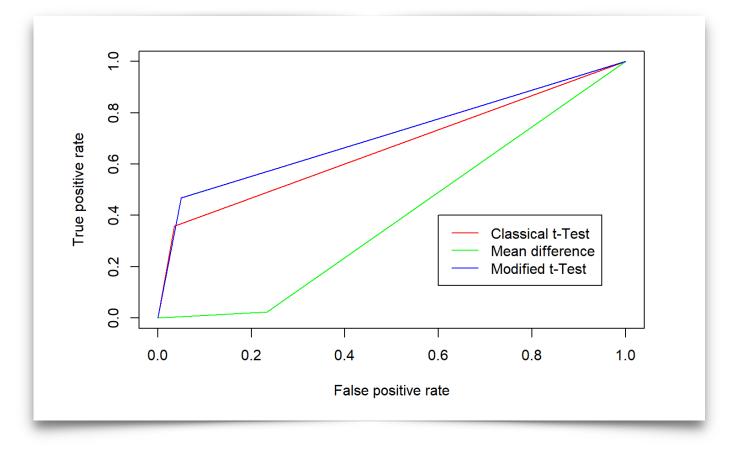


Notes on Exercise 5 (limma fundamentals) .. technical things

- residual variance
- Question asked for: t.test(..., var.equal=TRUE)
 (Welch's t-test is the default; in the setting here, it's not a paired t-test)
- ROC curve: absolute value of score .. or use P-value



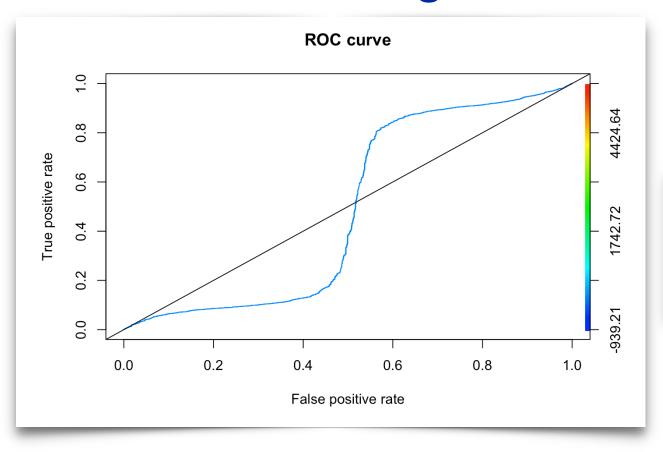
Notes on Exercise 5 (limma fundamentals) .. technical things

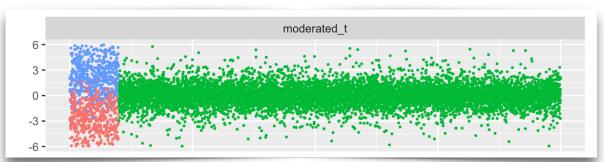


The point of the ROC curve is that you do NOT need too set a cutoff



Notes on Exercise 5 (limma fundamentals) .. technical things







Expectations: project

- ~10-15 page report, with R code in line (e.g. knitR / Rmarkdown)
- Describe the biological setting, statistical analysis, exploratory analysis with publication-quality graphics embedded
- Three possibilities:
 - Comparison of statistical methods (simulation / reference data + metrics)
 - Reproduce an analysis from a paper from the raw data
 - Real collaborative project with FGCZ or a local laboratory
- Be strategic: work on something related to your interests!
- Typically due at end of first working week of January

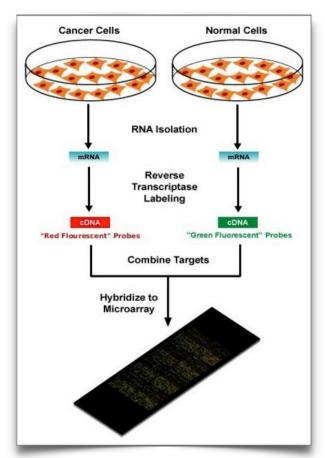


Statistical models for count data analysis (differential expression)

- simple counting (and new alternatives ..)
- edgeR, DESeq/DESeq2 —> why the negative binomial distribution?
- dispersion estimation and information sharing
- normalization considerations
- how about transformations of count data —> limma?



Abundance by Fluorescence Intensity



http://en.wikipedia.org/wiki/DNA_microarray

Abundance by Counting b 2× poly (A) selection 25-bp Add standards and shatter RNA Make cDNA and sequence Map 25-bp tags RNA-Seq onto genome graph 25-bp Calculate splices transcript prevalence Conservation 2 RPKM 1 RPKM 1 RPKM RepeatMasker

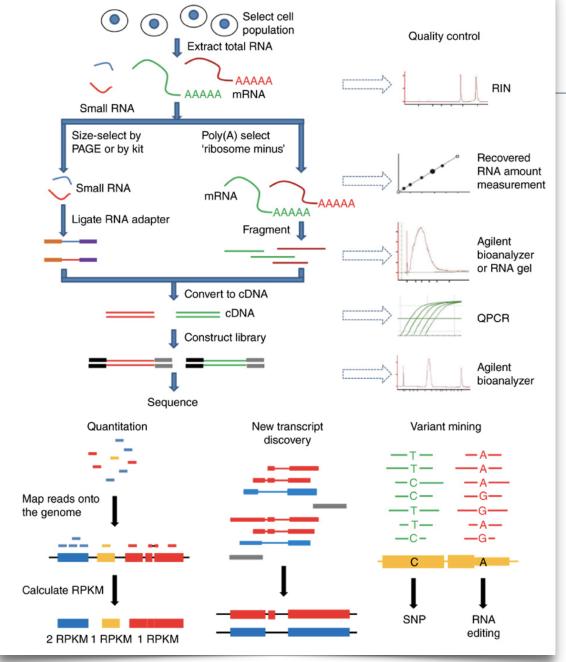
10



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RNA-seq differential expression analyses

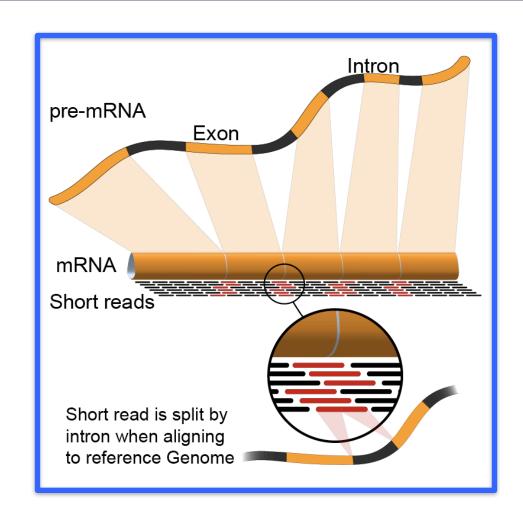
- **1. Map** the reads to reference sequences
- 2. "Count" reads that map to genes (quantify)
- 3. Compute DE Statistics

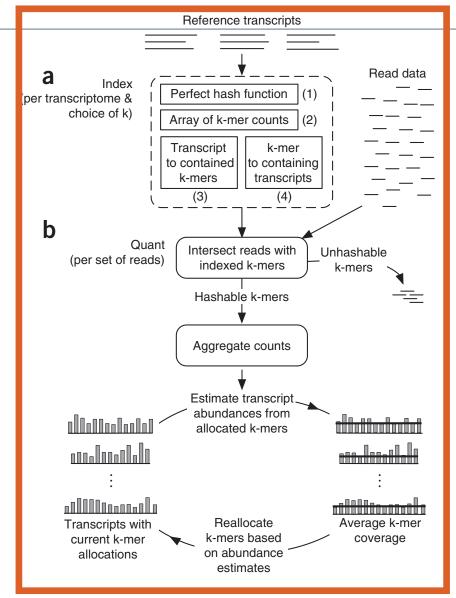




Alignment versus quasi-alignment

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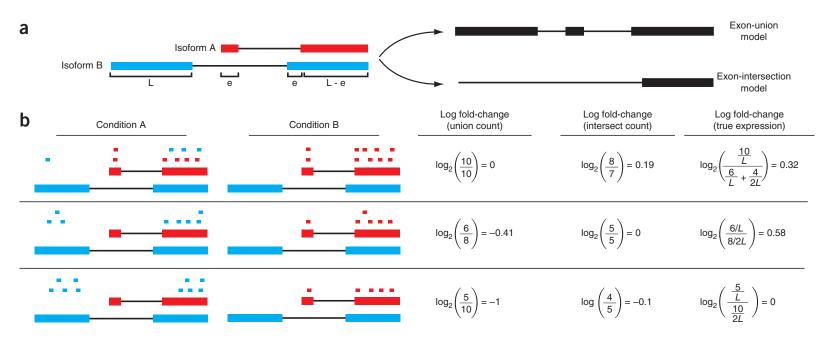




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Caveat: simple gene-level counting not perfect, but good first approximation

Trapnell et al. 2013 Nat Biotech



Transcriptome analysis of human tissues and cell lines reveals one dominant transcript per gene

Mar Gonzàlez-Porta¹, Adam Frankish², Johan Rung¹, Jennifer Harrow² and Alvis Brazma^{1*}

Mark D. Robinson, IMLS, UZH

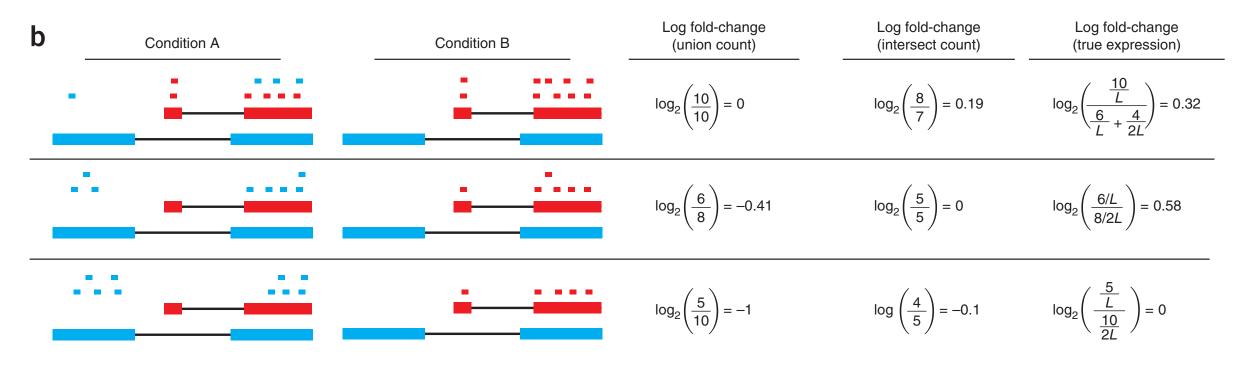


Counting/Quantification

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union counters —> transcript counters —>

simple sum of all reads sum of length-normalized reads (often unknown which reads map to which transcript —> portioning)





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How do all these methods of counting affect DE analyses?

You've been doing your RNA-Seq all wrong

⚠ Posted by: RNA-Seq Blog 🖿 in Expression and Quantification 🕚 November 12, 2015 💿 13,162 Views

In recent years, RNA-seq is emerging as a powerful technology in estimation of gene and/or transcript expression, and RPKM (Reads Per Kilobase per Million reads) is widely used to represent the relative abundance of mRNAs for a gene. In general, the methods for gene quantification can be largely divided into two categories: transcript-based approach and 'union exon'-based approach. Transcript-based approach is intrinsically more difficult because different isoforms of the gene typically have a high proportion of genomic overlap. On the other hand, 'union exon'-based approach method is much simpler and thus widely used in RNA-seq gene quantification. Biologically, a gene is expressed in one or more transcript isoforms. Therefore, transcript-based approach is logistically more meaningful than 'union exon'-based approach. Despite the fact that gene quantification is a fundamental task in most RNA-seq studies, however, it remains unclear whether 'union exon'-based approach for RNA-seq gene quantification is a good practice or not.

Researchers at Pfizer Worldwide Research & Development carried out a side-by-side comparison of 'union exon'-based approach and transcript-based method in RNA-seq gene quantification. It was found that the

F1000Research

F1000Research 2016, 4:1521 Last updated: 05 APR 2016



METHOD ARTICLE

REVISED Differential analyses for RNA-seq: transcript-level estimates improve gene-level inferences [version 2; referees: 2 approved]

Charlotte Soneson^{1,2}, Michael I. Love^{3,4}, Mark D. Robinson^{1,2}

¹Institute for Molecular Life Sciences, University of Zurich, Zurich, 8057, Switzerland

First published: 30 Dec 2015, 4:1521 (doi: 10.12688/f1000research.7563.1)
Latest published: 29 Feb 2016, 4:1521 (doi: 10.12688/f1000research.7563.2)

Open Peer Review







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³Department of Biostatistics and Computational Biology, Dana-Farber Cancer Institute, Boston, MA, 02210, USA

⁴Department of Biostatistics, Harvard TH Chan School of Public Health, Boston, MA, 02115, USA



Differential expression: why not use methods developed for microarrays?

Count data is discrete, not continuous.

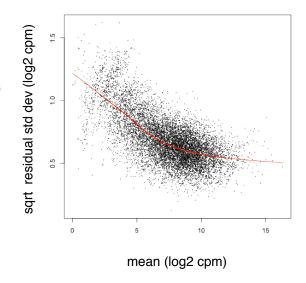
Methods designed for microarrays are not directly applicable and

suboptimal (more on this later)

Two options:

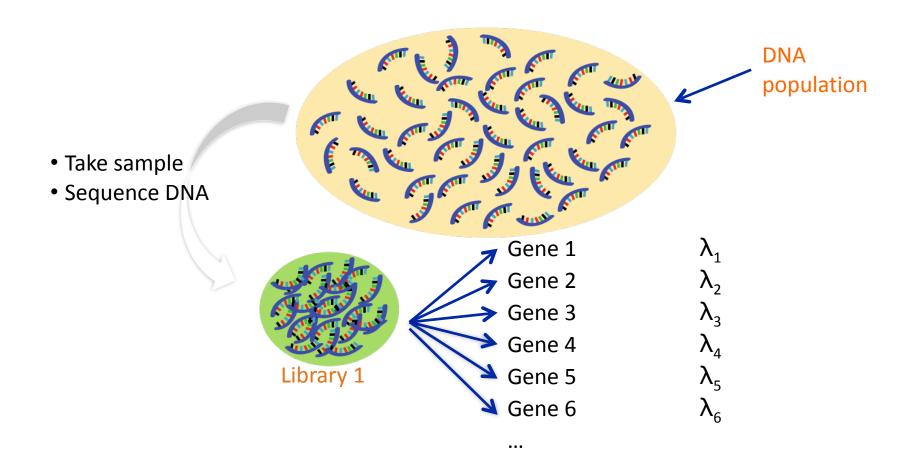
Transform count data and apply standard methodology

Analyze using models for count data





Sampling reads from population of DNA fragments is multinomial





For a single gene, it's a coin toss, i.e. Binomial



 $Y_i \sim Binomial(M, \lambda_i)$

Y_i - observed number of reads for gene i

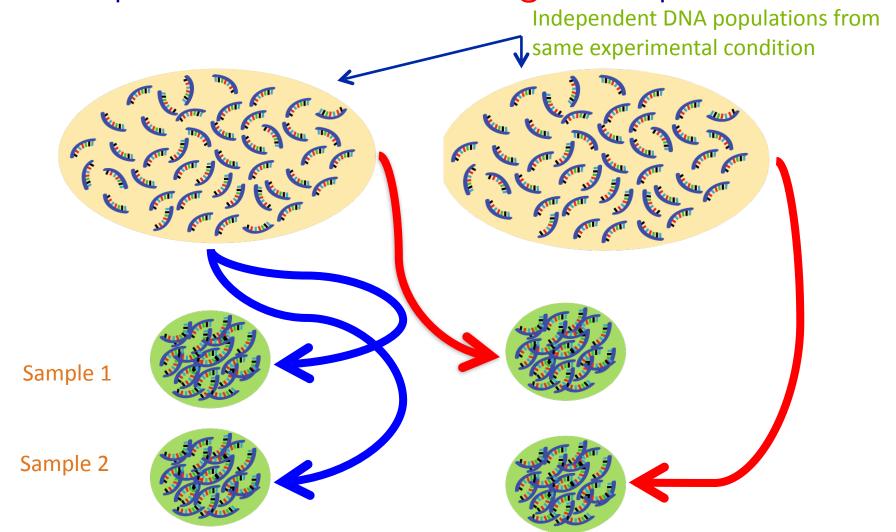
- total number of sequences

 $\lambda_{_{\mathsf{i}}}$ - proportion

Large M, small $\lambda_i \rightarrow$ approximated well by Poisson($\mu_i = M \cdot \lambda_i$)

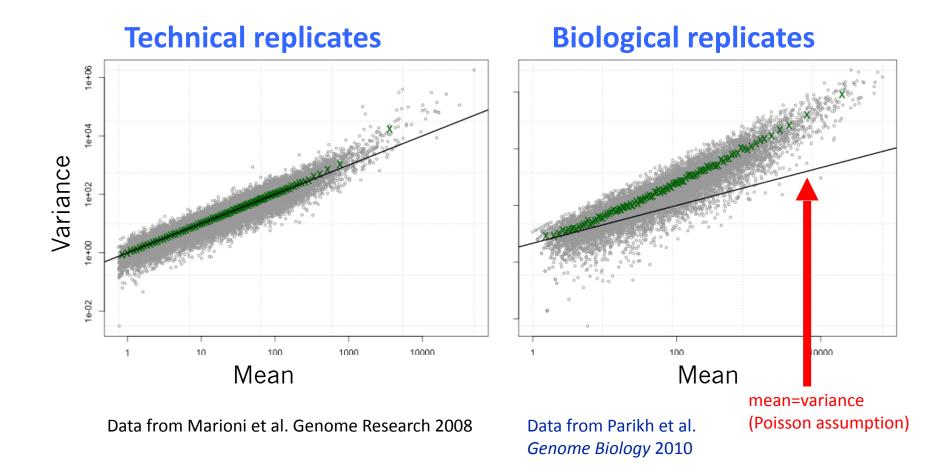


Technical replication versus biological replication





Mean-Variance plots: What we see in real data





Count data modeling assumptions

Poisson adequately describes technical variation

$$Y_i \sim Pois(M * \lambda_i)$$

 $mean(Y_i) = variance(Y_i) = M * \lambda_i$

Negative binomial (gamma-Poisson) model is a natural extension that allows **biological** variability:

$$Y_i \sim NB(\mu_i=M * \lambda_i, \phi_i)$$

Same mean, variance is quadratic in the mean:

variance(
$$Y_i$$
) = μ_i (1 + $\mu_i \phi_i$)

M = library size $\lambda_i = relative contribution of gene i$



Similar interpretation

$$Y_i \sim NB(\mu_i = Ni * \lambda_i, \phi_i)$$

$$E(y_{gi}) = \mu_{gi} = N_i \pi_{gi}.$$

(Coefficient of variation = standard deviation/mean)

$$\operatorname{var}(y_{gi}) = E_{\pi}[\operatorname{var}(y|\pi)] + \operatorname{var}_{\pi}[E(y|\pi)] = \mu_{gi} + \phi_g \mu_{gi}^2.$$

Dividing both sides by μ_{gi}^2 gives

$$CV^2(y_{gi}) = 1/\mu_{gi} + \phi_g.$$

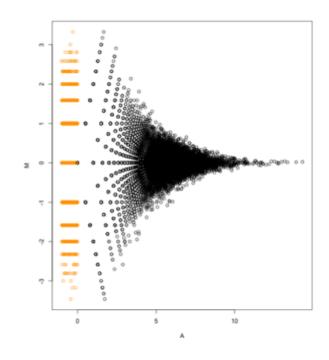
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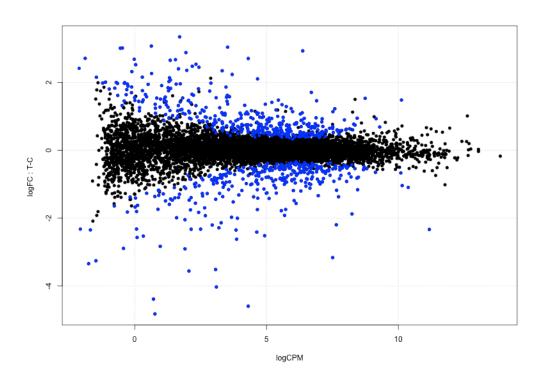
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A confirmation of what the theory states

Technical replicates (~Poisson)



Biological replicates



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Differential expression, small sample inference —> except now with counts

- Table of data (e.g., microarray gene expression data with replicates of each of condition A, condition B)
 - rows = features (e.g., genes), columns = experimental units (samples)
- Most common problem in statistical bioinformatics: want to infer whether there is a change in the response
 a statistical test for each row of the table.

What test might you use? Why is this hard? What issues arise? How much statistical power is there [1]?

<pre>> head(y)</pre>						
es Miles	group0	group0	group0	group1	group1	group1
gene1	-0.1874854	0.2584037	-0.05550717	-0.4617966	-0.3563024	-0.03271432
gene2	-3.5418798	-2.4540999	0.11750996	-4.3270442	-5.3462622	-5.54049106
gene3	-0.1226303	0.9354707	-1.10537767	-0.1037990	0.5221678	-1.72360854
gene4	-2.3394536	-0.3495697	-3.47742610	-3.2287093	6.1376670	-2.23871974
gene5	-3.7978820	1.4545702	-7.14796503	-4.0500796	4.7235714	10.00033769
gene6	1.4627078	-0.3096070	-0.26230124	-0.7903434	0.8398769	-0.96822312



What was successful with microarray data: classical/moderated/ shrunken t-tests

$$t_g = rac{\overline{y}_{
m mu} - \overline{y}_{
m wt}}{s_g \, c}$$
 $ilde{t}_g = rac{\overline{y}_{
m mu} - \overline{y}_{
m wt}}{\widetilde{s}_g \, u}$ $t_{g,
m pooled} = rac{\overline{y}_{
m mu} - \overline{y}_{
m wt}}{s_0 \, c}$

$$\tilde{t}_{\scriptscriptstyle g} = \frac{\overline{y}_{\scriptscriptstyle \rm mu} - \overline{y}_{\scriptscriptstyle \rm wt}}{\tilde{s}_{\scriptscriptstyle g} \, u}$$

$$t_{g, \mathrm{pooled}} = \frac{\overline{y}_{\mathrm{mu}} - \overline{y}_{\mathrm{wt}}}{s_0 \, c}$$

Feature-specific

Moderated

Common



Let's try the same strategy with counts

At one extreme, assume all genes have same dispersion (too strong)

At other extreme, estimate dispersion separately/independently for each gene (poor estimates)

Shrink individual estimates toward common/trend (how?)

No hierarchical model (e.g. limma) to do this —> approximations, weighted likelihood

No t-distribution theory to formulate statistical tests.

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Count data modeling assumptions

Poisson adequately describes technical variation

$$Y_i \sim Pois(M * \lambda_i)$$

$$mean(Y_i) = variance(Y_i) = M * \lambda_i$$

Negative binomial (gamma-Poisson) model is a natural extension that allows **biological** variability:

$$Y_i \sim NB(\mu_i=M * \lambda_i, \phi_i)$$

Same mean, variance is quadratic in the mean:

variance(
$$Y_i$$
) = μ_i (1 + $\mu_i \phi_i$)

M = library size $\lambda_i = relative contribution of gene i$



First challenge: getting good estimates of dispersion in small samples

Several choices here:

- Maximum Likelihood (MLE)
- Pseudo-Likelihood (PL)
- Quasi-Likelihood (QL)
- Conditional Maximum Likelihood (CML)
- Approximate Conditional Inference (Cox-Reid)
- quantile-adjusted Maximum Likelihood (qCML)

$$Y_{gij} \sim \text{NegBin}(\mu_{gi} = M_j \lambda_{gi}, \phi)$$

$$(\hat{\lambda}_{MLE}, \hat{\phi}_{MLE}) = \arg \max_{\lambda, \phi} l(\lambda, \phi)$$

$$X^{2} = \sum_{gij} \frac{(y_{gij} - \hat{\mu}_{gi})^{2}}{\hat{\mu}_{gi}(1 + \hat{\phi}_{PL}\hat{\mu}_{gi})} = G(n_{1} + n_{2} - 2)$$

$$D = 2\sum_{gij} \left\{ y_{gij} \log \left[\frac{y_{gij}}{\mu_{gi}} \right] - (y_{gij} + \phi_{QL}^{-1}) \log \left[\frac{y_{gij} + \phi_{QL}^{-1}}{\mu_{gi} + \phi_{QL}^{-1}} \right] \right\}$$



Conditional likelihood

Likelihood for single **negative binomial** observation:

$$f(y; \mu, \phi) = P(Y = y) = \frac{\Gamma(y + \phi^{-1})}{\Gamma(\phi^{-1})\Gamma(y + 1)} \left(\frac{1}{1 + \mu\phi}\right)^{\phi^{-1}} \left(\frac{\mu}{\phi^{-1} + \mu}\right)^{y}$$

If all libraries are the same size (i.e. $m_i \equiv m$), the sum $Z = Y_1 + \cdots + Y_n \sim NB(nm\lambda, \phi n^{-1})$

Thus, can form conditional likelihood:

$$l_{Y|Z=z}(\phi) = \left[\sum_{i=1}^{n} \log \Gamma(y_i + \phi^{-1})\right] + \log \Gamma(n\phi^{-1}) - \log \Gamma(z + n\phi^{-1}) - n \log \Gamma(\phi^{-1})$$

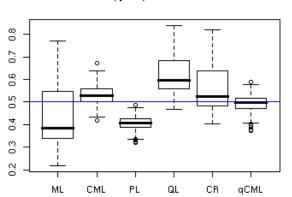
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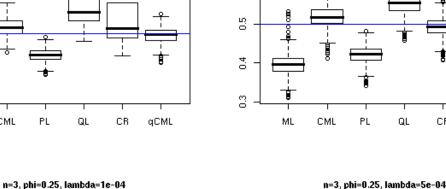
Comparison of Estimators (Common Dispersion)

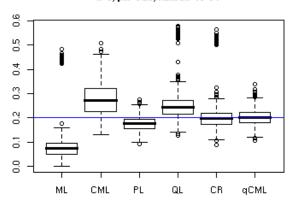
Horizontal blue line is TRUE value.

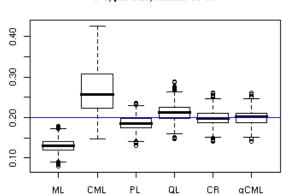
qCML performs best under a wide range of conditions.



n=3, phi=1, lambda=1e-04







n=3, phi=1, lambda=5e-04

qCML

30



<u>Likelihood</u> —> Weighted likelihood

Likelihood:
$$L(X;\theta) = \prod_{i=1}^{n} f(x_i;\theta)$$

log-likelihood:

$$l(X; \theta) = log(L(X; \theta)) = \sum_{i=1}^{n} log(f(x_i; \theta))$$

MLE:
$$\hat{\theta} = \arg \max_{\theta} l(X; \theta)$$

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Likelihood —> Weighted likelihood

$$WL(X;\theta) = \prod_{i}^{n} f(x_i;\theta)^{w_i}, \text{ where } w_i \text{ is weight.}$$

$$wl(X; \theta) = log(WL(X; \theta)) = \sum_{i=0}^{n} w_i log(f(x_i; \theta))$$

$$\hat{\theta} = \arg\max_{\theta} wl(X; \theta)$$

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Second challenge: Moderate dispersion estimate

Weighted likelihood -- individual loglikelihood plus a weighted version of the common log-likelihood:

$$WL(\phi_g) = l_g(\phi_g) + \alpha l_C(\phi_g)$$

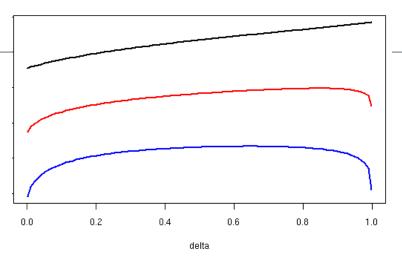
 I_g - quantile-adjusted conditional likelihood

Black: single tag

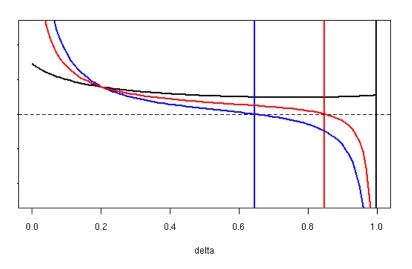
Blue: common dispersion

Red: Linear combination of the two

Log-Likelihood



Score (1st derivative of LL)



$$\delta = \frac{\phi}{\phi + 1}$$

How much to shrink?

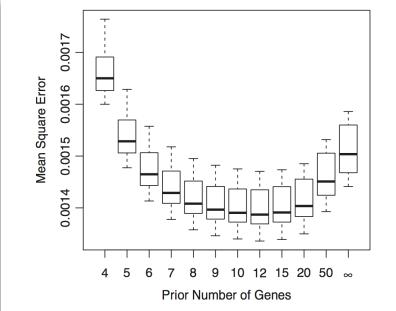


Figure 4. Mean-square error with which empirical Bayes genewise dispersions estimate the true dispersion (BCV²), when true dispersions are randomly generated. In this case, the optimal prior weight is 10–12 prior genes, equivalent to 20–24 prior degrees of freedom. The common BCV estimator is equivalent to using infinite weight for the prior. Boxplots show results for 10 simulations.

Simulations suggest there is an optimal amount to shrink.

Challenge: choosing/ estimating how much

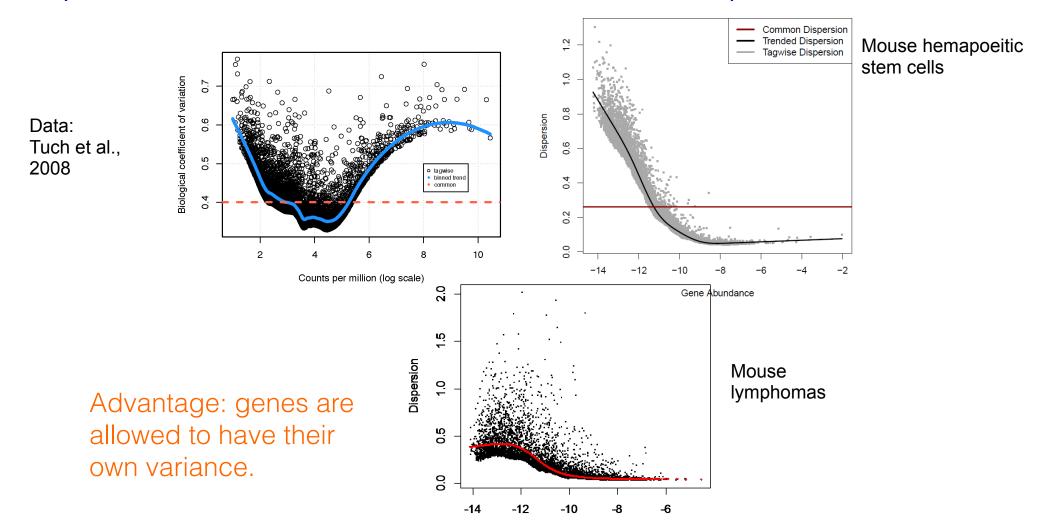
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McCarthy et al. NAR 2012



Dispersion varies with mean: moderate dispersion towards trend



Abundance



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Nature Reviews Genetics | AOP, published online 18 November 2008; doi:10.1038/nrg2484

INNOVATION

RNA-Seq: a revolutionary tool for transcriptomics

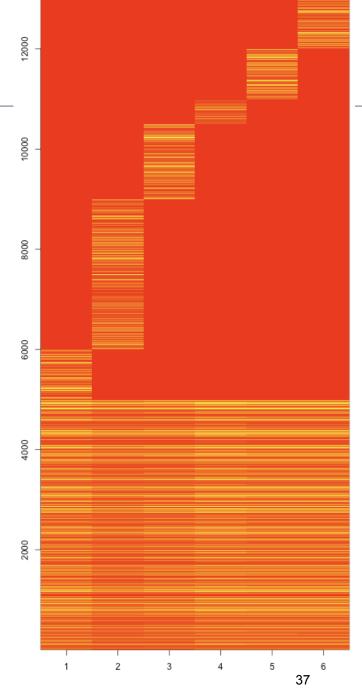
Zhong Wang, Mark Gerstein and Michael Snyder

One particularly powerful advantage of RNA-Seq is that it can capture transcriptome dynamics across different tissues or conditions without sophisticated normalization of data sets^{19,20,22}.

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"Composition" or "Diversity" can affect read depth

- Hypothetical example: Sequence 6 libraries to the same depth, with varying levels of unique-to-sample counts
- Read depth is affected not only by expression (and length), but also expression levels of other genes
- Composition can induce (sometimes significant) differences in counts





Kidney and Liver RNA have very different composition

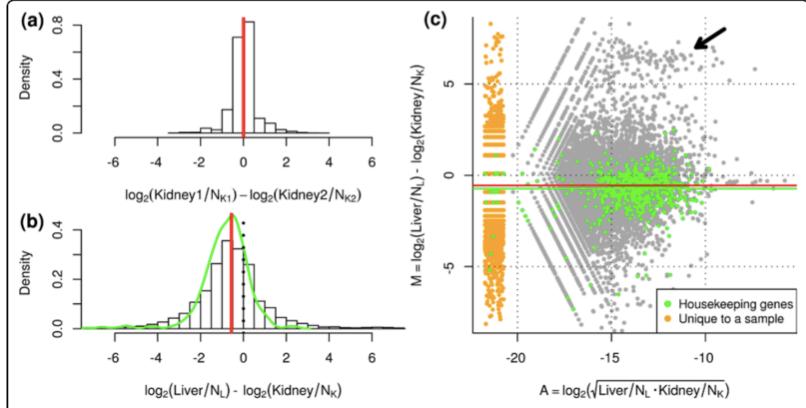


Figure 1 Normalization is required for RNA-seq data. Data from [6] comparing log ratios of **(a)** technical replicates and **(b)** liver versus kidney expression levels, after adjusting for the total number of reads in each sample. The green line shows the smoothed distribution of log-fold-changes of the housekeeping genes. **(c)** An M versus A plot comparing liver and kidney shows a clear offset from zero. Green points indicate 545 housekeeping genes, while the green line signifies the median log-ratio of the housekeeping genes. The red line shows the estimated TMM normalization factor. The smear of orange points highlights the genes that were observed in only one of the liver or kidney tissues. The black arrow highlights the set of prominent genes that are largely attributable for the overall bias in log-fold-changes.



Use scaling factor ("offset") in statistical model

Assumption: core set of genes/loci that do not change in expression.

Our Pick a reference sample, compute a weighted trimmed mean of M-values (TMM) to reference

Adjustment to statistical analysis:

- Use "effective" library size (edgeR)
- Use additional offset (GLM)

Note: count data is not modified



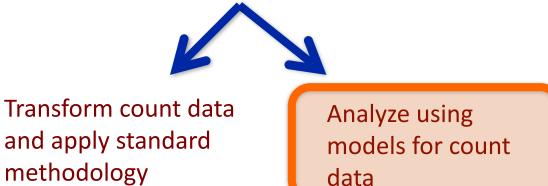
Differential expression: why not use methods developed for microarrays?

Count data is discrete, not continuous.

Methods designed for microarrays are not directly applicable and suboptimal

Transforming count data with logs, with some special treatment, can give very good results

Two options:





What does transformation do to M-V relationship?

For Poisson data, square-root should stabilize

Logarithm is too strong – variance decreases to asymptote (Neg Bin) or 0 (Poisson)

How to pick? Doesn't matter —> voom

voom: mean-variance modeling at the observational level

voom package:limma R Documentation

Transform RNA-Seq Data Ready for Linear Modelling

Description:

Transform count data to log2-counts per million, estimate the mean-variance relationship and use this to compute appropriate observational-level weights. The data are then ready for linear modeling.

Model log counts per million

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log counts per million:

$$z_{gi} = \log_2 \left(1e6 \frac{\text{count}_{gi} + 0.5}{\text{libsize}_{gi} + 1.0} \right) = \log_2 \left(1e6 \frac{y_{gi} + 0.5}{M_{gi} + 1.0} \right)$$

normalize libsize in advance or normalize z_{gi} as for microarrays.

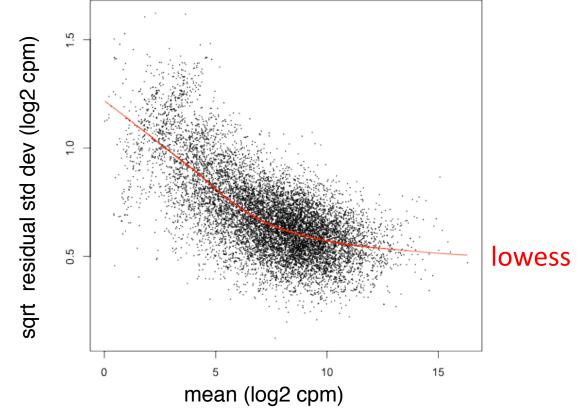
Linear modelling:

$$E(z_{gi}) = \mu_{gi} = x_i^T \beta_g$$

$$\operatorname{var}(z_{gi}) = s(\mu_{gi}) \sigma_g^2$$

Smooth function of mean

voom fits a lowess trend to the mean-variance relationship ...



—> Use weights (1/var) in limma analysis .. i.e., heteroscedastic regression



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