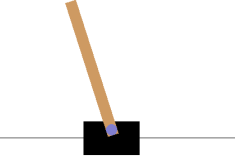
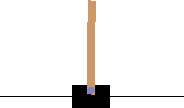
Balancing a CartPole System with Reinforcement Learning - A Tutorial

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***Abstract*— In this paper, we provide the details of imple- menting various reinforcement learning (RL) algorithms for controlling a Cart-Pole system. In particular, we describe vari- ous RL concepts such as Q-learning, Deep Q Networks (DQN), Double DQN, Dueling networks, (prioritized) experience replay and show their effect on the learning performance. In the process, the readers will be introduced to OpenAI/Gym and Keras utilities used for implementing the above concepts. It is observed that DQN with PER provides best performance among all other architectures being able to solve the problem within 150 episodes.**

1. Introduction

Reinforcement Learning (RL) is a machine learning paradigm where an agent learns the optimal action for a given task through its repeated interaction with a dynamic environment that either rewards or punishes the agent’s action. Reinforcement learning could be considered as a *semi-supervised* learning approach where the supervision signal required for training the model is made available in- directly in the form of rewards provided by the environment. Reinforcement learning is more suitable for learning dynamic behaviour of an agent interacting with an environment rather than learning static mappings between two sets of input and output variables. Over the years, a number of reinforcement learning methods and architectures have been proposed with varying success. However, the recent success of deep learn- ing algorithms has revived the field of reinforcement learning finding renewed interest among researchers who are now successfully applying this to solve very complex problems which were considered intractable earlier [1]. Events such as artificial agents like AlphaGo beating world chapmpion Lee Sedol [3] [9] or IBM Watson winning the game of Jeopardy

[5] [14] has attracted worldwide attention towards the rise of artificial intelligence which may surpass human intelligence in the near future [11] [4]. Reinforcement learning is a key paradigm to build such intelligent systems which can learn from its experience over time. Reinforcement algorithms are now being increasingly applied to Robotics, healthcare, rec- ommender system, data centres, smart grids, stock markets and transportation [13].

In this paper, we will provide the implementation de- tails of two well known reinforcement learning methods, namely, Q-learning [19] and Deep Q network (DQN) [15] for controlling a CartPole system. The objective is to provide a practical guide for implementing several reinforcement learning concepts by using using Python, OpenAI/Gym [16]

(a) (b)

**Fig. 1:** A Cart-Pole System: (a) Balanced state, (b) Unbalanced state

and Keras [7]. Some of these concepts are *ϵ*-greedy policy, Q-learning algorithm, Deep Q-learning, experience replay, Dueling networks etc. It will be useful to students and researchers willing to venture into this field. Theoretical details and mathematical analysis of these concepts have omitted to maintain the brevity of this paper. Readers are, instead, referred to the relevant literature for more in-depth understanding of these concepts.

The rest of this paper is organized as follows. Various deep learning concepts with their implementation details are provided in next section. The results of applying these concepts to solve the CartPole problem is discussed in Section [III.](#_bookmark5) The conclusion is provided in Section [IV.](#_bookmark7)

1. Methods
2. *The System*

We use OpenAI Gym [16] to simulate the Cart-Pole system. Few snapshots of Cart-Pole states are shown in Figure [1.](#_bookmark0) The left image shows the balanced state while the right image shows an imbalanced state. It consists of a cart (shown in black color) and a vertical bar attached to the cart using passive pivot joint. The cart can move left or right. The problem is to prevent the vertical bar from falling by moving the car left or right. One can see the animation of system behaviour under random action policy by executing the code given in Listing [1.](#_bookmark1) The state vector for this system **x**

is a four dimensional vector having components *{x, x*˙*, θ, θ*˙*}*.

The action has two states: left (0) and right (1). The episode terminates if (1) the pole angle is more than *±*12◦ from the vertical axis, or (2) the cart position is more than *±*2*.*4 cm from the centre, or (3) the episode length is greater than

200. The agent receives a reward of 1 for every step taken including the termination step. The problem is considered

solved, if the average reward is greater than or equal to 195 over 100 consecutive episodes.

**import gym**

env = gym.make("CartPole-v0") env.reset()

**for** i\_episode **in** range(100): obs = env.reset()

t = 0

**while not** done: env.render()

action = env.action\_space.sample() obs,reward,done,info = env.step(action) t += 1

**if** done:

**print**("Done after {} steps".format(t))

**break**; env.close()

**Listing 1:** Simple Code to visualize Cartpole Animation

1. *Q-Learning Algorithm*

Q-learning algorithm uses Bellman Equation to form a Q- function to quantify the expected discounted future rewards that can be obtained by taking an action *at* for a given state *st* at any time t. Mathematically, it can be written as:

*Qπ* (*st , at* ) = *E* [*R t*+1 + *γ R t*+2 + *γ* 2*R t*+3 + *···|*(*s t, at*)] (1) where *R i , i* = *t* + 1*, . . .* is the future rewards and *γ*

is the discount factor. The objective is to update these Q

function values through an iterative process by exploring all possible combinations of state and actions. Q-learning assumes a discrete observation space. Hence, the continuous state values are first discretized into fixed number of buckets by using bucketize() function as shown below:

**def** bucketize(state\_value): bucket\_indices = []

**for** i **in** range(len(state\_value)):

**if** state\_value[i] <= state\_value\_bounds[i][0]:

*# violates lower bound*

bucket\_index = 0

**elif** state\_value[i] >= state\_value\_bounds[i][1]:

*# violates upper bound # put in the last bucket*

bucket\_index = no\_buckets[i] - 1

**else**:

bound\_width = state\_value\_bounds[i][1] - \ state\_value\_bounds[i][0]

offset = (no\_buckets[i]-1) \* \ state\_value\_bounds[i][0] / bound\_width

scaling = (no\_buckets[i]-1) / bound\_width

bucket\_index = int(round(scaling\*state\_value[i] -offset)) bucket\_indices.append(bucket\_index)

**return**(tuple(bucket\_indices))

**Listing 2:** Discretizing continuous states into discrete states

The method involves creating a Q-table that stores rewards for all possible combinations of state and action choices. With a bucket size (1*,* 1*,* 6*,* 3) for states and two dimensional

action vector, the dimension of Q-table is 1 *×* 1 *×* 6 *×* 3 *×* 2.

The Q-learning algorithm is shown in Listing [4.](#_bookmark1) It consists of the following four major steps:

* 1. Select an action as per the *ϵ*-greedy policy where *ϵ* con- trols the balance between exploration and exploitation. A random action is selected during exploration. During exploitation however, an action is selected based on agent’s past experience. This is achieved by selecting an action that has maximum reward in the Q-table for the current state. Mathematically, we can write

*a*(*s*) = arg max *Q*(*s, a*′) (2)

*a'*

The exploration rate *ϵ* starts with a value of 1.0 at the beginning of the training and is reduced gradually over time. The corresponding code for selecting action is shown below:

**class DQNAgent**:

**def** select\_action(state\_value, explore\_rate):

**if** random.random() < explore\_rate:

action = env.action\_space.sample() *# explore*

**else**: *# exploit*

action = np.argmax(q\_value\_table[state\_value])

**return** action

* 1. Obtain new observations with the above action and collect reward from the environment.
  2. Update the Q-table using the following formulation:

*Q* (*s, a*) = *Q* (*s, a*)+ *α*[*R* (*s, a*)+ *γ* max *Q* (*s*′*, a*)*−Q* (*s, a*)]

*a*

(3)

where *α* is the learning rate which is reduce monoton-

ically from 1.0 to 0.1 as the training progresses.

* 1. Update the current state and repeat the above steps in the the next iteration.

The initial configurations and user-defined parameters for Q-learning algorithm is shown in Code Listing [3.](#_bookmark1) The actual steps involved in the update of Q-table during each iteration is provided in Code Listing [4.](#_bookmark1) These two parts could be executed together as a single python program.

**import gym**

**import numpy as np import random**, **math**

**import matplotlib.pyplot as plt**

env= gym.make(’CartPole-v0’) no\_buckets = (1,1,6,3) no\_actions = env.action\_space.n

state\_value\_bounds = list(zip(env.observation\_space.low, env.observation\_space.high)) state\_value\_bounds[1] = (-0.5, 0.5)

state\_value\_bounds[3] = (-math.radians(50), math.radians(50))

*# define q\_value\_table - it has a dimension of 1 x 1 x 6 x 3 x 2*

q\_value\_table = np.zeros(no\_buckets + (no\_actions,))

*# user-defined parameters*

min\_explore\_rate = 0.1; min\_learning\_rate = 0.1; max\_episodes = 1000

max\_time\_steps = 250; streak\_to\_end = 120; solved\_time = 199; discount = 0.99

no\_streaks = 0

*# Select an action using epsilon-greedy policy*

**def** select\_action(state\_value, explore\_rate): *# omitted*

*# change the exploration rate over time.*

**def** select\_explore\_rate(x):

**return** max(min\_learning\_rate, min(1.0, 1.0 - math.log10((x+1)/25)))

*# Change learning rate over time*

**def** select\_learning\_rate(x):

**return** max(min\_learning\_rate, min(1.0, 1.0 - math.log10((x+1)/25)))

*# Bucketize the state\_value*

**def** bucketize(state\_value):*# omitted*

**Listing 3:** Initial Configurations for Q-learning algorithm

*# train the system*

totaltime = 0

**for** episode\_no **in** range(max\_episodes): *#learning rate and explore rate diminishes # monotonically over time*

explore\_rate = select\_explore\_rate(episode\_no) learning\_rate = select\_learning\_rate(episode\_no) *# initialize the environment*

observation = env.reset()

start\_state\_value = bucketize\_state\_value(observation) previous\_state\_value = start\_state\_value

done = False time\_step = 0 **while not** done:

*#env.render()*

*# select action using epsilon-greedy policy*

action = select\_action(previous\_state\_value, explore\_rate)

*# record new observations*

observation, reward\_gain, done, info = env.step(action)

*#update q\_value\_table*

best\_q\_value = np.max(q\_value\_table[state\_value]) q\_value\_table[previous\_state\_value][action] += learning\_rate \* (

reward\_gain + discount \* best\_q\_value -

q\_value\_table[previous\_state\_value][action]) *# update the states for next iteration* state\_value = bucketize\_state\_value(observation) previous\_state\_value = state\_value

time\_step += 1

*# while loop ends here*

**if** time\_step >= solved\_time: no\_streaks += 1

**else**:

no\_streaks = 0

**if** no\_streaks > streak\_to\_end:

**print**(’CartPole problem is solved after {} episodes.’, episode\_no)

**break**

env.close()

**Listing 4:** Q-learning algorithm

1. *Deep Q Network (DQN) Algorithm*

Q-learning algorithm suffers from

*Dimensionality* problem as it requires

the *Curse-of-*

discrete states

network at regular intervals. This is shown in the code listing [7](#_bookmark2) where the ddqn flag needs to be set to false.

**class DQNAgent**:

**def** update\_target\_model(self, tau = 0.1):

*’’’Apply Polyak Averaging during weight update*

to form the Q-table. The computational complexity of Q- learning increases exponentially with increasing dimension of the state and action vector. Deep Q learning solves this problem by approximating the Q-value function *Q*(*s, a*) with an artificial neural network. This is achieved by the function build model() that uses Keras APIs to build a deep Q-network as shown below:

**from keras.layers import** Dense

**from keras.optimizers import** Adam **from keras.models import** Sequential **class DQNAgent**:

*# approximate Q-function with a Neural Network*

**def** build\_model(self): model = Sequential()

model.add(Dense(24, input\_dim=self.state\_size, activation=’relu’)) model.add(Dense(24, activation=’relu’)) model.add(Dense(self.action\_size, activation=’linear’)) model.summary()

model.compile(loss=’mse’, optimizer=Adam(lr=self.learning\_rate))

**return** model

**Listing 5:** Creating a DQN using Keras APIs

It is a 4-24-24-2 feed-forward network with 4 inputs, 2 outputs and two hidden layers each having 24 nodes. Hidden nodes use a RELU activation function while the output layer nodes use a linear activation function. Having a deep network to estimate Q values allows us to work directly with continuous state and action values. The Q network needs to be trained to estimate Q-values for a given state and action pair. This is done by using the following loss function:

*L i* (*θi* ) = *E* (*s,a*)∼*P* (*s,a*)[*Q* ∗(*s, a*) *− Q* (*s, a*; *θi* )]2

where the target Q value *Q*∗(*s, a*) for each iteration is given by

*make tau = 1.0 for normal update*

*’’’*

**if** self.ddqn: *# for DDQN*

weights = self.model.get\_weights() target\_weights = self.target\_model.get\_weights() **for** i **in** range(len(target\_weights)):

target\_weights[i] = weights[i] \* tau + target\_weights[i] \* (1-tau)

*# end of for loop*

self.target\_model.set\_weights(target\_weights)

**else**: *# for DQN*

self.target\_model.set\_weights(self.model.get\_weights())

**Listing 7:** Weight update for the target network at regular intervals. Polyak Averaging can be implementing by setting ddqn flag.

1. *Experience Replay*

It has been shown that the network trains faster with a batch update rather than with an incremental weight update method. In a batch update, the network weights are updated after applying a number of samples to the network whereas in incremental update, the network is updated after applying each sample to the network. In this context, DQN uses a concept called *experience replay* where a random sample of past experiences of the agent is used for training the

Q network. The experiences are stored in a fixed size replay memory in the form of tuples (*s, a, r, s*′) containing current state, current action, reward and next state after each iteration. Once a sufficient number of entries are stored in the replay memory, we can train the DQN by using a batch of samples selected randomly from the replay memory.

The exploration rate *ϵ* is reduced monotonically after each

iteration of training.

**class DQNAgent**:

**def** experience\_replay(self):

**if** len(self.memory) < self.train\_start:

**return**

batch\_size = min(self.batch\_size, len(self.memory))

*Q* (*s, a*) = *E s'* ∈*S*

∗

[*R* (*s, a*) + *γ* max *Q* (*s* ′*, a* ′; *θi* −1)*|s, a*] (4)

*a'*

mini\_batch = random.sample(self.memory, batch\_size) state\_batch, q\_values\_batch = [], []

**for** state, action, reward, next\_state, done **in** mini\_batch:

*# q-value prediction for a given state*

q\_values\_cs = self.model.predict(state)

where *R* ( *s, a*) is the reward for the current state-action pair

*# target q-value*

′ ′ max\_q\_value\_ns = self.get\_target\_q\_value(next\_state, reward)

(*s, a*) obtained from the environment and *Q*(*s , a , θi*−1) is

the Q-value for the next state obtained using the Q-network

weights from the last iteration. This is implemented using the code provided in the code listing [6.](#_bookmark2) It also shows the code for computing Q targets for DDQN architecture which will be explained later in this paper.

[H]

**class DQNAgent**:

**def** get\_target\_q\_value(self, next\_state, reward): *# max Q value among the next state’s action* **if** self.ddqn:

*# DDQN*

*# Current Q network selects the action # a’\_max = argmax\_a’ Q(s’,a’)*

action = np.argmax(self.model.predict(next\_state)[0])

*# target Q network evaluates the action # Q\_max = Q\_target(s’, a’\_max)*

max\_q\_value = self.target\_model.predict(next\_state)[0][action]

**else**:

*# DQN chooses the max Q value among next actions*

*# Selection and evaluation of action is on the target Q network*

*# Q\_max = max\_a’ Q\_target(s’, a’)*

max\_q\_value = np.amax(self.target\_model.predict(next\_state)[0])

**return** max\_q\_value

**Listing 6:** Obtaining the target Q values required for training DQN

and DDQN

Sometimes it is convenient to have a separate network to obtain target Q values. It is called a target Q network *Q*′ having same architecture as that of the original Q network. The weights for the target network is copied from the original

*# correction on the Q value for the action used*

**if** done:

q\_values\_cs[0][action] = reward

**else**:

q\_values\_cs[0][action] = reward + \

self.discount\_factor \* max\_q\_value\_ns state\_batch.append(state[0]) q\_values\_batch.append(q\_values\_cs[0])

*# train the Q network*

self.model.fit(np.array(state\_batch),

np.array(q\_values\_batch), batch\_size = batch\_size, epochs = 1, verbose = 0)

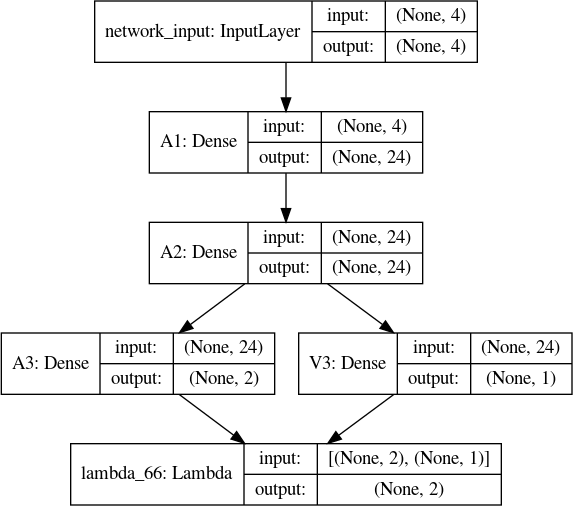
self.update\_epsilon()

**Listing 8:** Training DQN using Experience Replay

1. *Double DQN*

Taking the maximum of estimated Q value as the target value for training a DQN as per equation [4](#_bookmark2) may introduce a maximization bias in learning. Since Q learning involves *bootstrapping*, i.e., learning estimates from estimates, such overestimation may become problematic over time. This can be solved by using double Q learning [8] [18] which uses two Q-value estimators, each of which is used to update

the other. In this paper, we implement the version proposed in [18] that uses two models *Q* and *Q*′ sharing weights at regular intervals. The network *Q*′ is used for action selection while the network *Q* is used for action evaluation. That is,

the target value for network training is obtained by using the following equation:

∗ ′ ′

*Q* (*s, a*) *≈ rt* + *γQ* (*st*+1*,* arg max *Q* (*s t, at* )) (5)

*a*

We minimize the error between *Q* and *Q*∗, but have *Q*′ slowly copy the parameters of Q through Polyak averaging: *θ*′ = *τ θ* + (1 *− τ* )*θ*′. The code for computing target Q value and weight update is shown in code listings [6](#_bookmark2) and [7](#_bookmark2) respectively where the ddqn flag needs to be set to true.

1. *Dueling DQN*

The Q-value *Q*(*s, a*) tells us how good it is to take an action *a* being at state *s*. This Q-value can be decomposed as the sum of *V* (*s*), the value of being at that state, and *A*(*s, a*), the advantage of taking that action at the state (from all other possible actions). Mathematically, we can write this as

*Q* (*s, a*) = *V* (*s*) + *A* (*s, a*) (6)

Dueling DQN uses two separate estimators for these two components which are then combined together through a special aggregation layer to get an estimate of *Q*(*s, a*). By decoupling the estimation, intuitively the Dueling DQN can learn which states are (or are not) valuable without having to learn the effect of each action at each state. This is particularly useful for states where actions do not affect the environment in a meaningful way. In these cases, it is unnecessary to evaluate each action for such states and could be skipped to speed up the learning process.

Rather than directly adding individual components as shown in [(6),](#_bookmark3) the q-value estimate can be obtained by using the following two forms of aggregation:

*Q*(*s, a*) = *V* (*s, β* ) + *A*(*s, a, α*) *−* max *A*(*s, a*′*, α*) (7)

*a'*

1 Σ

′

*Q* (*s, a*) = *V* (*s, β* ) + *A* (*s, a, α*) *− |A| A*(*s, a , α*) (8)

*a'*

where *β* and *α* are the weights for the networks *V* (*s*) and *A*(*s, a*) respectively. The first equation [(7)](#_bookmark3) uses max advantage value and the second equation [(8)](#_bookmark3) uses the average advantage value to estimate *Q*(*s, a*) from *V* (*s*). This form of aggregation apparently solves the *issue of identifiability*, that is - given *Q*(*s, a*), it is difficult to find *A*(*s, a*) and *V* (*s*).

The implementation of Dueling DQN architecture involves replacing the build model() function provided in Code Listing [5](#_bookmark2) with the function provided in the listing [9.](#_bookmark3) A block- diagram visualization of the dueling architecture is shown in

Figure [2.](#_bookmark3) It uses Keras’ Lambda function utility to implement

**Fig. 2:** Dueling Model architecture for DQN

**elif** self.dueling\_option == ’naive’: network\_output = Lambda(**lambda** x: x[0] + [1],\

output\_shape=(self.action\_size,))([A3,V3])

**else**:

**raise Exception**(’Invalid Dueling Option’)

model = Model(network\_input, network\_output) model.compile(loss=’mse’, optimizer=Adam(lr=self.learning\_rate)) model.summary()

plot\_model(model, to\_file=’model.png’, show\_shapes=True,\

show\_layer\_names=True)

**return** model

**Listing 9:** Implementing Dueling DQN Architecture with Keras

1. *The DQN Agent*

The final DQN Agent class that implements both DQN, DDQN and Dueling versions of these architectures will appear something as shown in the code listing [10.](#_bookmark4) The im- plementation details of functions which have been discussed earlier have been omitted here. Please remember to change your build model() if you are implementing a dueling architecture. The main body of the program that uses this DQNAgent class to control the cart-pole system is provided in Code Listing [11.](#_bookmark4) It is important to set the reward to -

100 when the episode ends (or the done flag is set to

true). This penalizes actions that prematurely terminates the episode.

**import numpy as np import random**

**from collections import** deque

**class DQNAgent**:

**def** init ( self, state\_size, action\_size, ddqn\_flag=False): self.state\_size = state\_size

self.action\_size = action\_size *# hyper parameters for DQN* self.discount\_factor = 0.9

self.learning\_rate = 0.001

the final aggregation layer.

**class DQNAgent**:

**def** build\_model(self):

self.epsilon = 1.0

self.epsilon\_decay = 0.99

self.epsilon\_min = 0.01

self.batch\_size = 24

self.train\_start = 1000

*# explore rate*

*# Advantage network*

network\_input = Input(shape=(self.state\_size,), name=’network\_input’) A1 = Dense(24, activation=’relu’, name=’A1’)(network\_input)

A2 = Dense(24, activation=’relu’, name =’A2’)(A1)

A3 = Dense(self.action\_size, activation=’linear’, name=’A3’)(A2)

*# Value network*

V3 = Dense(1, activation=’linear’, name=’V3’)(A2)

*# Final aggregation layer to compute Q(s,a)*

**if** self.dueling\_option == ’avg’:

network\_output = Lambda(**lambda** x: x[0] - K.mean(x[0]) + x[1],\ output\_shape=(self.action\_size,))([A3,V3])

**elif** self.dueling\_option == ’max’:

network\_output = Lambda(**lambda** x: x[0] - K.max(x[0]) + x[1],\ output\_shape=(self.action\_size,))([A3,V3])

self.dueling\_option = ’avg’

*# create replay memory using deque*

self.memory = deque(maxlen=2000)

*# create main model and target model* self.model = self.build\_model() self.target\_model = self.build\_model() *# initialize target model*

self.target\_model.set\_weights(self.model.get\_weights())

*# approximate Q-function with a Neural Network*

**def** build\_model(self): *# omitted*

*# update target model at regular interval to match the main model*

**def** update\_target\_model(self): *# omitted*

*# get action from the main model using epsilon-greedy policy*

**def** select\_action(self, state):

**if** np.random.rand() <= self.epsilon:

**return** random.randrange(self.action\_size)

**else**:

q\_value = self.model.predict(state)

**return** np.argmax(q\_value[0])

*# save sample <s, a, r, s’>. into replay memory*

**def** add\_experience(self, state, action, reward, next\_state, done): self.memory.append((state,action,reward,next\_state,done))

*# Compute target Q value*

**def** get\_target\_q\_value(self, next\_state, reward):

*# Train the model*

**def** experience\_replay(self): *# omitted*

where *a* is a hyperparameter used to reintroduce some randomness in the experience selection for the replay buffer. *a* = 0 will lead to pure uniform randomness while *a* = 1 will select the experiences with highest priorities. Priority sampling, in general, will introduce a bias towards high- priority samples and may lead to over-fitting. To correct this bias, we use important sampling (S) weights that will adjust the updating by reducing the weights of the often seen samples. The weights for each sample is given by:

*# decrease exploration, increase exploitation*

**def** update\_epsilon(self):

**if** self.epsilon > self.epsilon\_min: self.epsilon \*= self.epsilon\_decay

*wi* =

1 1 *b*

*.* (11)

*N P*

**Listing 10:** The DQNAgent class implementation

**if **name == " main ":

*# create Gym Environment*

env = gym.make(’CartPole-v0’) env.seed(0)

state\_size = env.observation\_space.shape[0] action\_size = env.action\_space.n

*# create a DQN model*

agent = DQNAgent(state\_size, action\_size) score = []

**for** e **in** range(EPISODES): done = False

t = 0

state = env.reset()

state = np.reshape(state, [1, state\_size])

**while not** done:

action = agent.get\_action(state)

next\_state, reward, done, info = env.step(action) next\_state = np.reshape(next\_state, [1, state\_size]) reward = reward **if not** done **else** -100 *#important step # add <s,a,r,s’> to replay memory*

agent.append\_sample(state, action, reward, next\_state, done)

*# Train through experience replay*

agent.experience\_replay() t += 1

state = next\_state

**if** done:

*# update target model for each episode* agent.update\_target\_model() score.append(t)

**break**

*# if mean score for last 100 episode bigger than 195, stop training*

The role of hyperparamete(r*i* ) *b* is to control how much

these importance sampling weights affect the learning. In practice, *b* is selected to be 0 in the beginning and is annealed upto 1 over the duration of training, because these weights are more important in the end of learning when the Q-values begin to converge. To reduce the computational burden a sumTree data structure is used which provides *O*(log *n* ) time complexity for sampling experiences and updating their priorities. A sumTree is a Binary Tree, that is a tree with a maximum of two children for each node. The leaves contain the priority values and a data array containing experiences. The code for creating sumTree data structure and the corresponding replay memory is shown in the code block listing [12](#_bookmark5) and [13](#_bookmark5) respectively. The training function experience replay() will be slightly different from the one given in code listing [8](#_bookmark2) as it will make use of sumtree data structure and require updating priorities

**if** np.mean(score[-min(100, len(score)):]) >= (env.spec.max\_episode\_steps-5):with each iteration. The code for the modified version of

**print**(’Problem is solved in {} episodes.’.format(e))

**break**

env.close()

**Listing 11:** The main code for using DQNAgent for balancing the CartPole System.

1. *Prioritized Experience Replay*

Prioritize experience replay (PER) [17] is based on the idea that some experiences may be more important than others for training, but might occur less frequently. Hence, it will make more sense to change the sampling distribution by using a criterion to define the priority of each tuple of experience. PER will be more useful in cases where there is a big difference between the predicted Q-value and its TD target value, since it means that there is a lot to learn about it. The priority of an experience is therefore defined as:

*pt* = *|δt|* + *e* (9)

where *|δt|* is the magnitude of TD error and *e* is a con- stant that ensures that no experience has zero probability of getting selected. Hence, the experiences are stored in the replay memory along with their priorities as a tuple

*< st, at, rt, st*+1*, pt >* . However, one can not simply do a

greedy prioritization as it will lead to training with the same experiences (having bigger priority) and hence over-fitting. Hence, this priority is converted into stochastic probability

given by

*a*

experience replay() function is provided in code list- ing [14.](#_bookmark5) The main changes in the DQNAgent class definition is shown in the code listing [15.](#_bookmark5) The main program required for solving the Cartpole problem remains the same as given in code listing [11.](#_bookmark4) The effect of PER is discussed later in the experiment section.

**import numpy as np class SumTree**(object):

data\_pointer = 0

**def **init(self, capacity):

*# Number of leaf nodes (final nodes) that contains experiences*

self.capacity = capacity *#*

self.tree = np.zeros(2 \* capacity - 1) self.data = np.zeros(capacity, dtype=object)

**def** add(self, priority, data):

*# Look at what index we want to put the experience* tree\_index = self.data\_pointer + self.capacity - 1 self.data[self.data\_pointer] = data *# Update data frame* self.update (tree\_index, priority) *# Update the leaf* self.data\_pointer += 1 *# Add 1 to data\_pointer*

**if** self.data\_pointer >= self.capacity: *# If we’re above the capacity*

self.data\_pointer = 0 *# we go back to first index (overwrite)*

**def** update(self, tree\_index, priority):

*# Change = new priority score - former priority score* change = priority - self.tree[tree\_index] self.tree[tree\_index] = priority

**while** tree\_index != 0: *# propagate changes through the tree* tree\_index = (tree\_index - 1) // 2 self.tree[tree\_index] += change

**def** get\_leaf(self, v): parent\_index = 0 **while** True:

left\_child\_index = 2 \* parent\_index + 1 right\_child\_index = left\_child\_index + 1 *# If we reach bottom, end the search*

**if** left\_child\_index >= len(self.tree): leaf\_index = parent\_index

**break**

**else**: *# downward search, always search for a higher priority node*

**if** v <= self.tree[left\_child\_index]: parent\_index = left\_child\_index

*P* (*i* ) = Σ *pi*

*a*

*k pk*

(10)

data\_index = leaf\_index - self.capacity + 1

**return** leaf\_index, self.tree[leaf\_index], self.data[data\_index]

@property

**def** total\_priority(self):

**return** self.tree[0] *# Returns the root node*

**Listing 12:** The sum tree data structure for creating replay memory.

**import numpy as np class Memory**(object):

*# stored as ( state, action, reward, next\_state ) in SumTree*

PER\_e = 0.01 *# hyper parameter*

PER\_a = 0.6 *# hyper parameter*

PER\_b = 0.4 *# importance-sampling, from initial value increasing to 1*

PER\_b\_increment\_per\_sampling = 0.001 absolute\_error\_upper = 1. *# clipped abs error*

**def **init(self, capacity):

self.tree = SumTree(capacity) *# Making the tree*

**def** store(self, experience): *# Find the max priority* max\_priority = np.max(self.tree.tree[-self.tree.capacity:]) **if** max\_priority == 0:

max\_priority = self.absolute\_error\_upper self.tree.add(max\_priority, experience)

**def** sample(self, n): minibatch = []

b\_idx = np.empty((n,), dtype=np.int32)

priority\_segment = self.tree.total\_priority / n *# priority segment*

**for** i **in** range(n):

*# A value is uniformly sample from each range*

a, b = priority\_segment \* i, priority\_segment \* (i + 1) value = np.random.uniform(a, b)

*# Experience that correspond to each value is retrieved* index, priority, data = self.tree.get\_leaf(value) b\_idx[i]= index minibatch.append([data[0],data[1],data[2],data[3],data[4]])

**return** b\_idx, minibatch

**def** batch\_update(self, tree\_idx, abs\_errors):

abs\_errors += self.PER\_e *# convert to abs and avoid 0* clipped\_errors = np.minimum(abs\_errors, self.absolute\_error\_upper) ps = np.power(clipped\_errors, self.PER\_a)

**for** ti, p **in** zip(tree\_idx, ps): self.tree.update(ti, p)

**Listing 13:** The replay memory using sum tree data structure.

**class DQNAgent**:

**def** experience\_replay(self):

*’’’ Training on Mini-Batch with Prioritized Experience Replay ’’’*

*# create a minibatch through prioritized sampling*

tree\_idx, mini\_batch = self.memory.sample(self.batch\_size) current\_state = np.zeros((self.batch\_size, self.state\_size)) next\_state = np.zeros((self.batch\_size, self.state\_size)) qValues = np.zeros((self.batch\_size, self.action\_size)) *#action, reward, done = [], [], []*

action = np.zeros(self.batch\_size, dtype=int) reward = np.zeros(self.batch\_size)

done = np.zeros(self.batch\_size, dtype=bool)

**for** i **in** range(self.batch\_size):

current\_state[i] = mini\_batch[i][0] *# current\_state*

action[i] = mini\_batch[i][1] reward[i] = mini\_batch[i][2]

next\_state[i] = mini\_batch[i][3] *# next\_state*

done[i] = mini\_batch[i][4]

qValues[i] = self.model.predict(current\_state[i]\

.reshape(1,self.state\_size))[0]

max\_qvalue\_ns = self.get\_maxQvalue\_nextstate(next\_state[i]\

.reshape(1,self.state\_size))

**if** done[i]:

qValues[i][action[i]] = reward[i]

**else**:

qValues[i][action[i]] = reward[i] + \ self.discount\_factor \* max\_qvalue\_ns

*# update priority in the replay memory*

target\_old = np.array(self.model.predict(current\_state)) target = qValues

indices = np.arange(self.batch\_size, dtype=np.int32) absolute\_errors = np.abs(target\_old[indices, \

np.array(action)]- target[indices, np.array(action)]) self.memory.batch\_update(tree\_idx, absolute\_errors)

*# train the model*

self.model.fit(current\_state, qValues,

batch\_size = self.batch\_size, epochs=1, verbose=0)

*# update epsilon with each training step*

self.update\_epsilon()

**Listing 14:** The code for training with prioritized experience replay.

**from sumtree import** sumTree **and** Memory

**class DQNAgent**:

**def **init(self):

self.memory = Memory(memory\_size)

**def** add\_experience(self, state, action, reward, next\_state, done): experience = [state, action, reward, next\_state, done] self.memory.store(experience)

**def** experience\_replay(self): *# provided separately # rest of functions remain same as before*

**Listing 15:** Main changes to the DQNAgent class definition pro- vided in code listing [10](#_bookmark4)

|  |  |  |
| --- | --- | --- |
| S.No. | Parameter | Value |
| 1 | discount factor, *γ* | 0.9 |
| 2 | learning rate | 0.001 |
| 3 | Exploration rate, *є* | 1 |
| 4 | *єmin* | 0.01 |

**TABLE I:** Values of user-defined parameters used for simulation

1. Experiments and Results

This section provides the details of experiments carried out to evaluate the performance of various reinforcement learning models described in the previous sections. This is described next in the following subsections.

1. *Software and Hardware Setup*

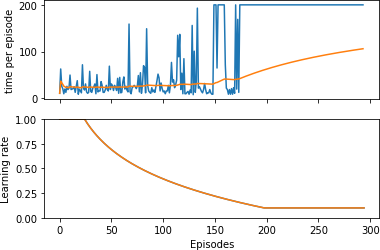
The complete implementation code for this paper is avail- able on GitHub [12]. The program is written using Python and Keras APIs [7]. It takes about a couple of hours (2- 3 hours) for running about 1000 episodes on a HP Omen laptop with a Nvidia GeForce RTX 2060 GPU card with 6 GB of video ram. It is also possible to make use of freely available GPU cloud such as Google Colab [6] [2] or Kaggle

[10] if you don’t own a GPU machine.

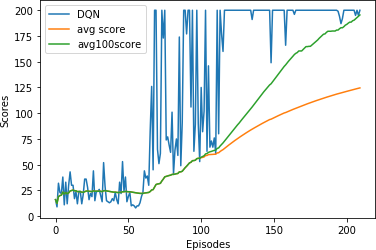
1. *Performance of various RL models*

The performance of Q-learning algorithm is shown in Fig- ure [3.](#_bookmark6) As one can see, Q-learning algorithm is able to solve the problem within 300 episodes. It also shows the learning rate that decreases monotonically with training iterations. The performance of DQN Algorithm with experience replay is shown in Figure [4.](#_bookmark6) It is clearly faster than the standard Q- learning algorithm and is found to solve the problem with

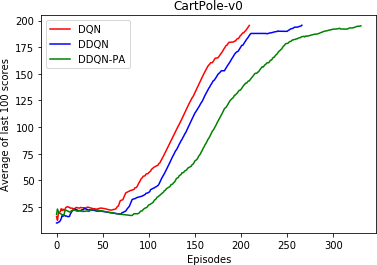
200 episodes. The performance comparison for DQN, Double DQN (DDQN) and DDQN with Polyak Averaging (PA) is shown in Figure [5.](#_bookmark6) While all of them are able to solve the problem within 300 episodes, DQN is clearly the fastest. DDQN and DDQN-PA do not provide any perceptible advantage over DQN. This could be because the problem is itself too simple and does not require these complex architectures. The replay memory size of 2000 and batch size of 24 is used for producing the result shown in [5.](#_bookmark6) Polyak Averaging (PA) tends to slow down the learning process and it is more commonly known as the soft method for updating target model. Similarly, the dueling versions of DQN or DDQN architectures fail to provide convergence within 300 episodes as shown in Figure [6.](#_bookmark6) The problem might be too simple to make use of these complex architectures. Dueling architectures with Prioritized Experience Replay (PER) has been shown to provide remarkable improvement in ATARI games. It can be seen that Dueling-DQN is faster than Dueling-DDQN as it uses less number of parameters. The performance of DQN algorithms is also affected by changing the values of parameters such as the replay memory size (MS) and batch size (BS) selected for experience replay.



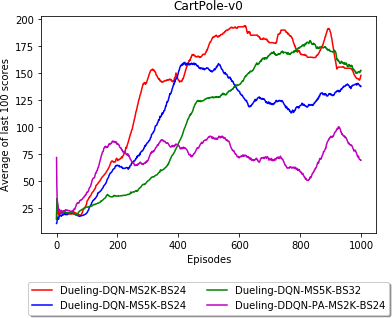
**Fig. 3:** Performance of Q-learning Algorithm. The standard Q- learning solve the problem within 300 episodes. The problem is considered solved if the average of last 100 scores is *>* = 195. Learning rate is decreased monotonically with increasing training episodes.



**Fig. 4:** Performance of DQN Algorithm. Avg100score is the average of last 100 episodes. The problem is considered to be solved when average of last 100 scores is *>* = 195 for CartPole-V0.

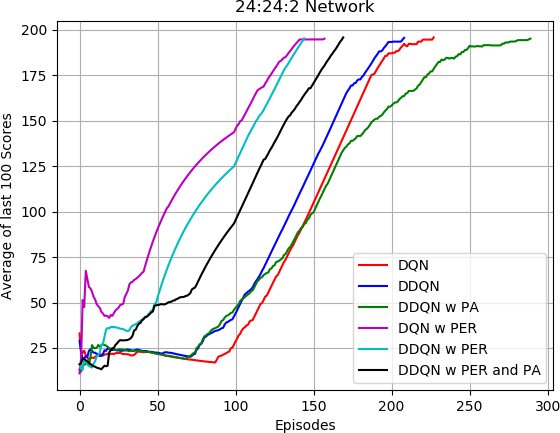


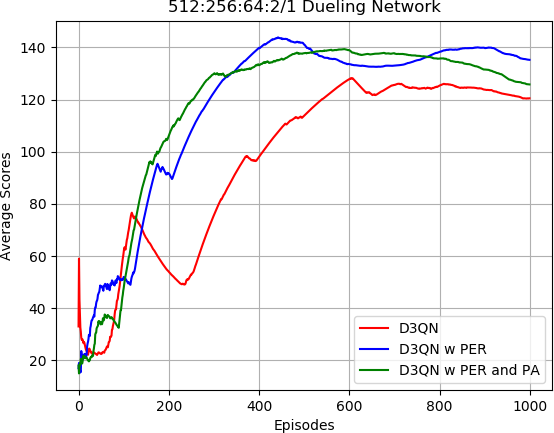
**Fig. 5:** Performance Comparison of DQN and DDQN architectures. DQN is found to solve the problem faster compared to DDQN and DDQN-PA architectures.



**Fig. 6:** Performance of Dueling DQN and DDQN architectures. Du-

eling architectures fail to solve the problem within 1000 episodes.

**Fig. 7:** Effect of Prioritized Experience Replay (PER) on DQN and DDQN network models. The program is terminated when the average of last 100 scores exceed 195.



**Fig. 8:** Effect of PER on Dueling-DDQN (D3QN) Model Archi- tecture

1. *Effect of Prioritized Experience Replay*

The effect of prioritized experience replay (PER) on DQN and DDQN architectures is shown in Figure [7.](#_bookmark6) These results are produced using a 3 layer network (24-24-2) architecture with about 770 parameters, sampling batch size of 24 and a replay memory size of 2000. The values of various hyper- parameters are as shown in Table [I.](#_bookmark5) The best performance out of 2-3 independent runs are shown in this plot. As one can see, PER provides clear improvement over the normal DQN and DDQN implementations. The same is seen in case of Dueling-DDQN (D3QN) architecture as shown in Figure

[8.](#_bookmark6) This result for this figure is produced by using a 512-256- 64-2/1 network architecture with about 150,531 parameters, a sampling batch size of 32 and a replay memory capacity of 10,000. As one can see, D3QN with PER provides higher average scores compared to that obtained using only D3QN. Soft target update using Polyak Averaging (PA) does not necessarily provide any significant advantage over PER. It has the effect of slowing down the learning process as is evident from these two figures. The best performance is obtained using DQN with PER that learns to solve the problem in about 50 episodes (average of last 100 is take as the termination criterion).

1. Conclusions

This is a tutorial paper that provides implementation details of a few reinforcement learning algorithms used for solving the Cart-Pole problem. The implementation code is written in Python and makes use of OpenAI/Gym simulation framework and Keras deep learning tools. It is observed that DQN is considerably faster compared to the standard Q- learning algorithms and allows the use of continuous state values. DDQN and Dueling architectures do not provide any significant improvement over DQN as the problem is too simple to warrant such complex architectures. Further improvement in performance is obtained by using Prioritized Experience Replay (PER). DQN with PER is shown to pro- vide the best performance so far. The codes provided could be executed on Google Colab which provides free access to a GPU cloud. We believe that these details will be of interest to students and novice practitioners and will motivate them to explore further and make novel contributions to this field.

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