University of Houston

Homework 1 Solutions

COSC 3340
Introduction to Automata & Computability

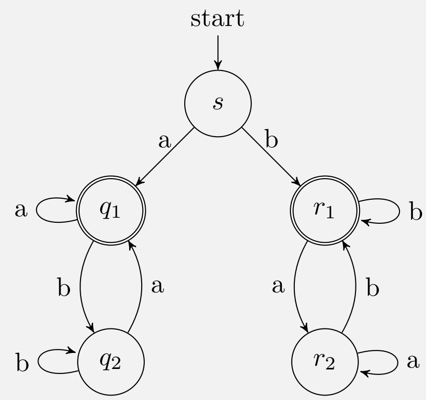
Instructor: Rathish Das
TA: Aayush Gupta

Instructions:

- This assignment comprises of 10 questions. Each question is worth 10 marks, making the total score for this assignment 100 marks.
- Handwritten Answers:
  1. Ensure your answers are clear and legible.
  2. Once completed, scan all the pages of your assignment.
  3. Save the scanned pages as a single PDF file.
- LaTeX Format:
  1. Type your answers using LaTeX in any editor of your choice.
  2. Ensure to format your answers clearly, making use of LaTeX's mathematical symbols and structures where necessary.
  3. Save your document as a PDF file.
- Submission:
  1. Only PDF files will be accepted.
  2. Name your file as YourName_PeopleSoftID.pdf (e.g., JohnDoe_12345.pdf).
  3. Submit the PDF file before the deadline.
- Scoring:
  1. Each question will be graded out of 10.
  2. Partial credit may be awarded for partially correct answers.
- Honesty Policy:
  1. Ensure all work is your own. Plagiarism will result in a score of zero for the entire assignment and further disciplinary action may be taken.
- Queries:
  1. If you have any questions or face any issues, please contact the instructor or teaching assistant

Exercise 1

The following figure shows a five.-state machine M_4. What are the accept strings?

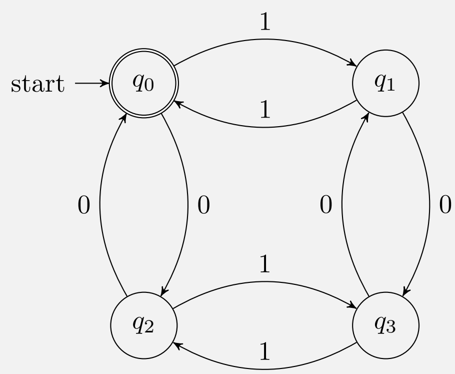


Solution a, b, aa, bb and bab.

Exercise 2

Design a DFA to accept the language

$$L = \{w|w \text{ has both an even number of 0's and an even number of 1's}\}$$



Exercise 3

Exhibit the language $L(a^*.(a+b))$ in set notation.

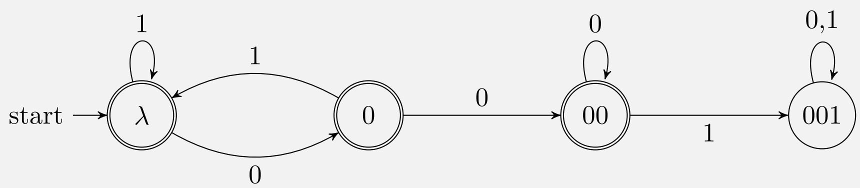
Solution

$$\begin{aligned} L(a^*.(a+b)) &= L(a^*)L(a+b) \\ &= (L(a))^*(L(a) \cup L(b)) \\ &= \{\lambda, a, aa, aaa, \dots\}\{a, b\} \\ &= \{a, aa, aaa, \dots, b, ab, aab, \dots\} \end{aligned}$$

Exercise 4

Find a dfa that accepts all the strings on $\{0, 1\}$, except those containing the substring 001.

Solution:



Exercise 5

For $\Sigma = \{0, 1\}$, give a regular expression r such that

$$L(r) = \{w \in \Sigma^* : w \text{ has at least one pair of consecutive zeros}\}$$

Solution

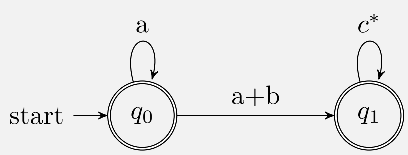
$$r = (0 + 1)^*00(0 + 1)^*$$

Exercise 6

Find an nfa that accepts

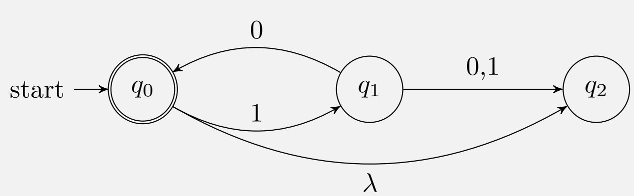
$$L(a^* + a^*(a + b)c^*)$$

Solution:



Exercise 7

What is the language accepted by the below automaton?

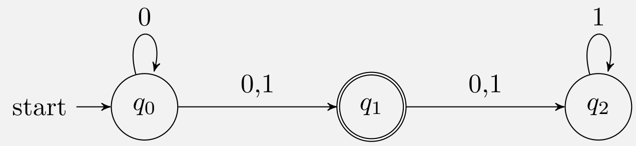


Solution:

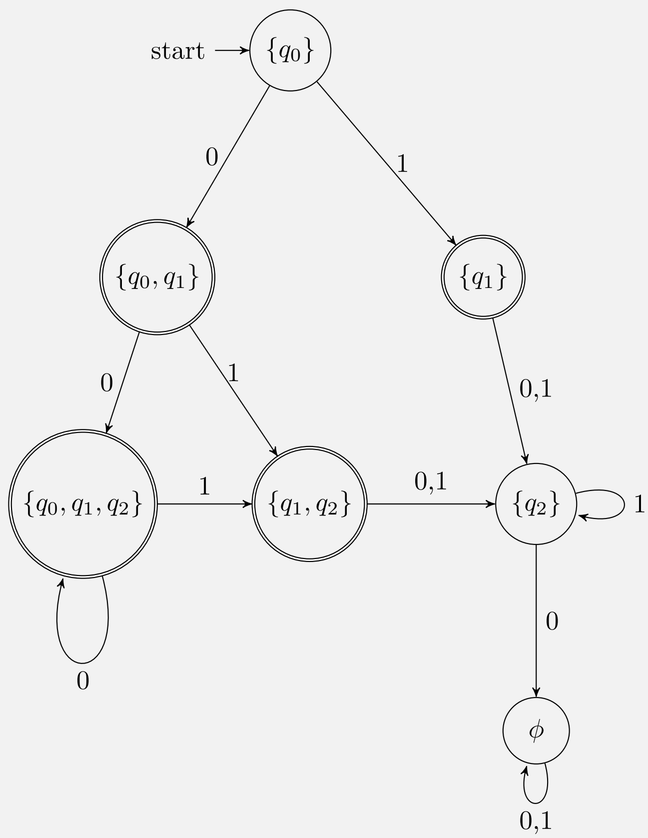
$$L = \{(10)^n : n \geq 0\}$$

Exercise 8

Convert the nfa into an equivalent deterministic machine.



Solution:



Exercise 9

Show that the language

$$L = \{a^n b^l : n \neq l\}$$

is not regular.

Solution: We need a bit of ingenuity to apply the pumping lemma directly. Choosing a string with $n = l + 1$ or $n = l + 2$ will not do, since our opponent can always choose a decomposition that will make it impossible to pump the string out of the language (that is, pump it so that it has an equal number of a's and b's).
let us take $n = m!$ and $l = (m + 1)!$. If opponent now chooses a y (by necessity consisting of all a's) of length $k < m$, we pump 4i times to generate a string with $m! + (i - 1)k$ a's. We can get a contradiction of the pumping lemma if we can pick i such that

$$m! + (i - 1)k = (m + 1)!$$

and $k \leq m$. The right side is therefore an integer, and we have succeeded in violating the conditions of the pumping lemma.
However, there is a much more elegant way of solving this problem. Suppose L were regular. Then, the language

$$L_1 = L \cap L(a^*b^*)$$

would also be regular. But $L_1 - \{a^n b^n : n \geq 0\}$, which we have already classified as nonregular. Consequently, L cannot be regular.

Exercise 10

Show that

$$L = \{a^n : n \text{ is a perfect square}\}$$

is not regular.

Solution: Given the opponent's choice of m, we pick

$$w = a^{m^2}$$

If $w = xyz$ is the decomposition, then clearly,

$$y = a^k$$

with $1 \leq k \leq m$. In that case,

$$w_0 = a^{m^2-k}$$

But $m^2 - k > (m-1)^2$, so that w_0 cannot be in L. Therefore, the language is not regular.