Multilevel analysis

Introduction to multilevel analysis and the basic two-level regression model

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Who

Emmeke Aarts
Course coordinator
Lecturer

Beth Grandfield
Computer labs
Exam
Feedback and grading

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Course outline - schedule

- Week 1: Mon **Lecture 1**
  Fri **DAT + Start up lab**
- Week 2: Mon **Long computer lab + Q&A**
- Week 3: Mon **Lecture 2** (hand in assignment 1 before start of Lecture 2)
  Fri **DAT + Start up lab**
- Week 4: Mon **Long computer lab + Q&A**
- Week 5: Mon **Lecture 3** (hand in assignment 2 before start of Lecture 3)
  Fri **DAT + Start up lab**
- Week 6: Mon **Long computer lab + Q&A**
- Week 8: **Exam**

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Course outline - topics

Week 1 & 2
- When/why multilevel analysis
- The multilevel regression model
- The three-level MLM
- MLM assumptions

Green: Lecture

Blue: Discussion additional topics (DAT)

Week 3 & 4
- Longitudinal model
- Contextual effects

Week 5 & 6
- Analyzing dichotomous and ordinal data
- Summary part 1

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Acknowledgements

Multilevel module content is based on course materials of past and present Utrecht colleagues, including: Cora Maas, Joop Hox, Leoniek Wijngaards-de Meij, Peter van der Heijden and Mirjam Moerbeek.

Permission to use and/or modify their course material is gratefully acknowledged.

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Today

1) Introduction: why use multilevel analysis?
2) Building the multilevel regression model
3) Analysis approach
4) Example

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Introduction

Multilevel data structures and their implications in analysis

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Multilevel regression model

Known in literature under a variety of names

- Hierarchical linear model (HLM)
  - Takes hierarchical data structure into account
- Multilevel model
  - Takes multiple levels of nesting into account
- Random coefficient model
  - Allows effects of predictors to vary across clusters
- Variance component model
  - Partitions the variance into components at individual and cluster level
- Mixed Linear Model
  - Model contains fixed and random effects

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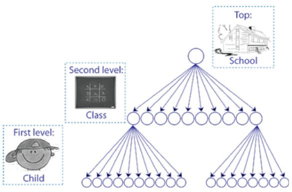
Nested data

Example: education family

level 3 (schools)
level 2 classes families
level 1 pupils members



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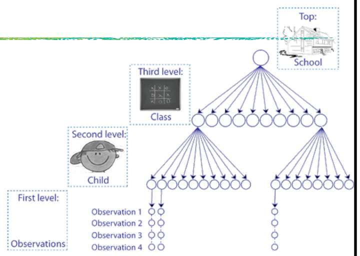
Nested data

Example: longitudinal

level 3 classes
level 2 pupils
level 1 occasions



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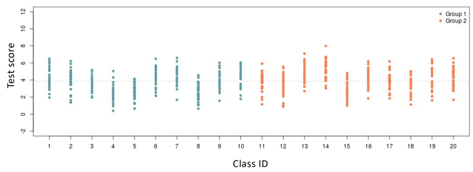
Nested data - example



When looking at this graph, do you think the collected observations can be seen as independent?

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Nested data

- Observations in the same cluster are generally not independent
  - they tend to be more similar than observations from different clusters
  - selection, shared history, mutual influence, contextual group effects
- The degree of similarity is indicated by the *intraclass correlation* ρ
- Standard statistical tests are not at all robust against violation of the independence assumption

That is why we need special multilevel techniques!

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Nested data – more challenges

Not only use proper sample size (i.e., corrected for dependency), but:
- Predict test scores using variables at all levels (student and class level)
- Relation between test score and sex can differ over the clusters (classes)

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Traditional Approaches

- Disaggregate all variables to the lowest level
  - Do standard analyses (ANOVA, multiple regression)
- Aggregate all variables to the highest level
  - Do standard analyses (ANOVA, multiple regression)
- ANCOVA with clusters as factor (i.e. use dummy variables)

- Some improvements:
  - Explanatory variables as deviations from their cluster mean
  - Have both deviation score and disaggregated cluster mean as predictor
    (separates individual and group effects)

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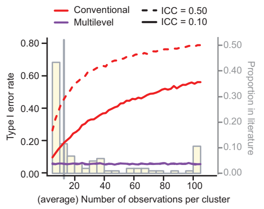
Not accommodating dependency inflates Type I error rate



- Outcomes within the same cluster are correlated
- Standard analysis does not take correlation into account
- As a result standard error of effect underestimated
- Results in inflated type I error rate

E. Aarts et al. (2014). A solution to dependency: using multilevel analysis to accommodate nested data. *Nature neuroscience*, 17(4), 491-496.

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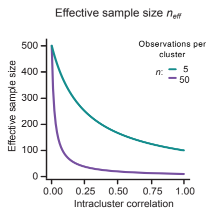
Dependency decreases the effective sample size



Given ρ (ICC) and cluster size n_j:

- Effective sample size

$$n_{eff} = n_{total}/[1 + (n_j - 1)\rho]$$

- Design effect

$$\mathrm{DE} = 1 + (n_j - 1)\rho$$

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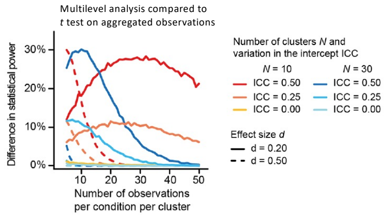
Aggregating decreases statistical power



E. Aarts et al. (2015). Multilevel analysis quantifies variation in the experimental effect while optimizing power and preventing false positives. BMC neuroscience, 16(1), 94

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Traditional Approaches

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AN(C)OVA with cluster as factor

So disaggregating and aggregating are not a good idea.
What if we use the cluster as a predictor in our model?

$$y_{ij} = \beta_0 + u_j + e_{ij}$$

Fixed effects model, with j dummy variables, one for each cluster: one way ANOVA

$$y_{ij} = \beta_0 + \beta_1 X_{ij} + u_j + e_{ij}$$

Or one-way ANCOVA (also called variance component model)

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AN(C)OVA with cluster as factor

Advantages of ANCOVA approach:
- Simple to compute
- No assumption about distribution of u_j's
- Can cope with large and extreme between cluster differences

Disadvantages of ANCOVA approach
- Large number of parameters when J is large
- Each u_j is poorly estimated if n_j, the number of level one units within each unit j, is small
- Does not make sense if J is a sample from a population and we want to make inferences about that population.
- Suboptimal use of data: some research questions cannot be answered

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ANCOVA vs. multilevel analysis

Advantages of ML over ANCOVA approach:
- Efficient estimation (only a few parameters)
- No problem if n_j's are small
- Handles explanatory variables at different levels (cannot be done in ANCOVA approach)
- Quantify variation between clusters

Disadvantages of ML over ANCOVA approach
- Assumptions about distributions of errors

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More problems with traditional approaches

- Multiple Regression assumes
  - independent observations
  - independent error terms
  - equal variances of errors for all observations (assumption of homoscedastic errors)
  - normal distribution for errors
- With hierarchical data
  - observations are not independent
  - errors are not independent
  - different observations may have errors with different variances (heteroscedastic errors)

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Why multilevel analysis?

1. Very flexible:
   - *Nested data*
   - *Longitudinal data*
   - *Logistic regression*, Poisson regression
   - Meta-analysis
   - Multivariate
   - *Contextual effects*
   - and so on

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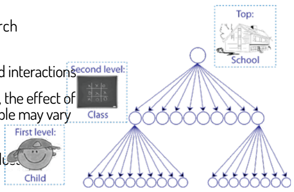
Why multilevel analysis?

2. To control for dependency
3. Enables to answer new research questions:
   - Variables at different levels and interactions
   - Variation between clusters: i.e., the effect of an individual explanatory variable may vary over clusters
   - Explaining variation between clusters
   - Contextual effects



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Coffee break

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Building the multilevel model

Formulating models at each level of the multilevel data structure

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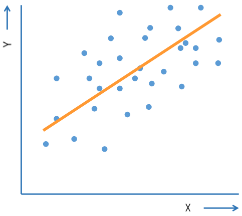
Rehearsal: traditional regression

Ordinary regression with one explanatory variable X:

$y_i = \beta_0 + \beta_1 X_i + e_i$

- y_i outcome of person i
- X_i explanatory variable
- β_0 intercept
- β_1 regression slope
- e_i residual error term



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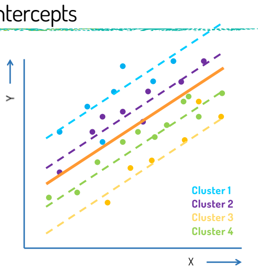
Multilevel regression – random intercepts

Multilevel regression with one explanatory variable X:

$y_{ij} = \beta_{0j} + \beta_1 X_{ij} + e_{ij}$

- subscript j denotes cluster membership
- y_{ij} outcome of person i in cluster j
- X_{ij} explanatory variable
- β_{0j} cluster dependent intercept
- β_1 regression slope
- e_{ij} residual error term, are assumed to have mean zero and variance σ_e^2



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Multilevel regression – random intercepts

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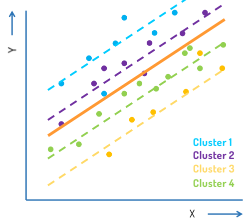

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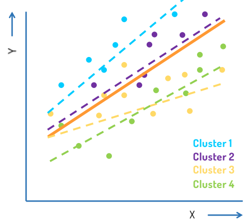
Multilevel regression – random slopes

Multilevel regression with one explanatory variable X:

$y_{ij} = \beta_{0j} + \beta_{1j} X_{ij} + e_{ij}$

- subscript j denotes cluster membership
- y_{ij} outcome of person i in cluster j
- X_{ij} explanatory variable
- β_{0j} cluster dependent intercept
- β_{1j} cluster dependent regression slope
- e_{ij} residual error term, are assumed to have mean zero and variance σ_e^2



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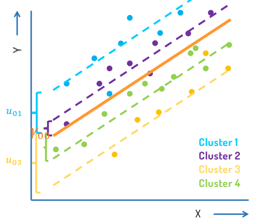
Multilevel regression – intercept variance

Multilevel regression with one explanatory variable X:

$y_{ij} = \beta_{0j} + \beta_1 X_{ij} + e_{ij}$

$\beta_{0j} = \gamma_{00} + u_{0j}$



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Multilevel regression – intercept variance

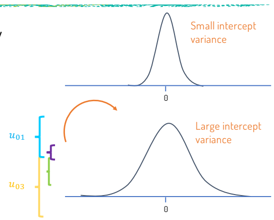
Multilevel regression with one explanatory variable X:

$y_{ij} = \beta_{0j} + \beta_1 X_{ij} + e_{ij}$

$\beta_{0j} = \gamma_{00} + u_{0j}$

$u_{0j} \sim N(0, \sigma^2_{u0})$

σ^2_{u0} = intercept variance



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Multilevel regression – slope variance

Multilevel regression with one explanatory variable X:

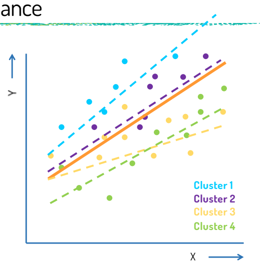
$y_{ij} = \beta_{0j} + \beta_{1j} X_{ij} + e_{ij}$

$\beta_{0j} = \gamma_{00} + u_{0j}$

$\beta_{1j} = \gamma_{10} + u_{1j}$

$u_{1j} \sim N(0, \sigma^2_{u1})$

σ^2_{u1} = slope variance



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Multilevel regression – level 1 and 2 equations

Multilevel regression with one explanatory variable X:

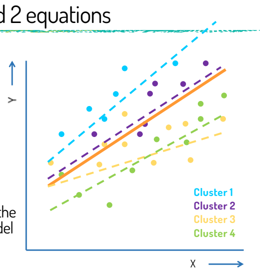
$y_{ij} = \beta_{0j} + \beta_{1j} X_{ij} + e_{ij}$ Level 1 equation (individual level)

$\beta_{0j} = \gamma_{00} + u_{0j}$

$\beta_{1j} = \gamma_{10} + u_{1j}$

Level 2 equations (cluster level)

The intercept and slope coefficients vary across the clusters, hence the term *random coefficient* model



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Multilevel regression – prediction at the second level

Multilevel regression with one explanatory variable X:

$$y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + e_{ij}$$
$$\beta_{0j} = \gamma_{00} + \gamma_{01}Z_j + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11}Z_j + u_{1j}$$

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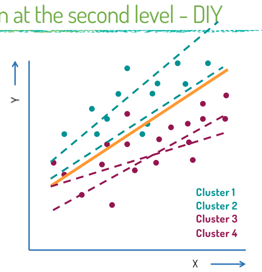
Multilevel regression – prediction at the second level – DIY

Multilevel regression with one explanatory variable X:

$$y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + e_{ij}$$
$$\beta_{0j} = \gamma_{00} + \gamma_{01}Z_j + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11}Z_j + u_{1j}$$

- What (model parameter) decreases if we add $\gamma_{01}Z_j$ to the model?
- What (model parameter) decreases if we add $\gamma_{11}Z_j$ to the model?



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Multilevel regression – prediction at the second level

Multilevel regression with one explanatory variable X:

$$y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + e_{ij}$$
$$\beta_{0j} = \gamma_{00} + \gamma_{01}Z_j + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11}Z_j + u_{1j}$$

- γ_{00} and γ_{01} are the intercept and slope to predict β_{0j} from Z_j
- γ_{10} and γ_{11} are the intercept and slope to predict β_{1j} from Z_j
- The regression coefficients γ are **fixed** across clusters
- Between cluster variation left in β's is captured by the residual error terms u_{0j} and u_{1j}

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Multilevel regression – single equation model

At the lowest (individual) level we have
$y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + e_{ij}$

and at the second (cluster) level.
$\beta_{0j} = \gamma_{00} + \gamma_{01}Z_j + u_{0j}$
$\beta_{1j} = \gamma_{10} + \gamma_{11}Z_j + u_{1j}$

Combining (substitution and rearranging terms) gives ("mixed" equation)
$y_{ij} = \gamma_{00} + \gamma_{10}X_{ij} + \gamma_{01}Z_j + \gamma_{11}Z_jX_{ij} + u_{0j} + u_{1j}X_{ij} + e_{ij}$

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Multilevel regression – single equation model

$y_{ij} = [\gamma_{00} + \gamma_{10}X_{ij} + \gamma_{01}Z_j + \gamma_{11}Z_jX_{ij}] + [u_{0j} + u_{1j}X_{ij} + e_{ij}]$

This equation has two distinct parts

- $[\gamma_{00} + \gamma_{10}X_{ij} + \gamma_{01}Z_j + \gamma_{11}Z_jX_{ij}]$ contains all the fixed coefficients, it is called the **fixed part** of the model
- $[u_{0j} + u_{1j}X_{ij} + e_{ij}]$ contains all the random error terms, it is called the **random part** of the model
- the *cross-level* interaction Z_jX_{ij} results from modeling the regression slope β_{1j} of individual level variable X_{ij} with the group level variable Z_j
- the error term u_{1j} is connected to X_{ij}. Thus the residuals are larger for large values of X_{ij}, implying *heteroscedasticity*

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Multilevel regression – interpretation

$y_{ij} = [\gamma_{00} + \gamma_{10}X_{ij} + \gamma_{01}Z_j + \gamma_{11}Z_jX_{ij}] + [u_{0j} + u_{1j}X_{ij} + e_{ij}]$

- the fixed part is an ordinary regression model
- complicated error term: $[u_{0j} + u_{1j}X_{ij} + e_{ij}]$

- Several error (co-)variances
  - σ_e^2 variance of the lowest level errors e_{ij}
  - σ_{u0}^2 variance of the highest level errors u_{0j}
  - σ_{u1}^2 variance of the highest level errors u_{1j}
  - σ_{u01} covariance of u_{0j} and u_{1j}

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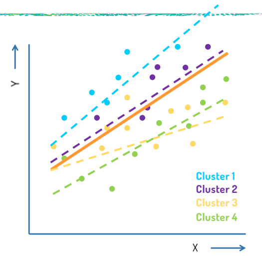
Multilevel regression - DIY

Draw three graphs that depict different scenarios of σ_{u01}, the covariance of u_{0j} and u_{1j}:

1. Positive covariance σ_{u01}
2. Zero covariance σ_{u01}
3. Negative covariance σ_{u01}

If you have time left, think of a real life scenario that would result in each of these three



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Estimation

- Maximum Likelihood (ML) estimation
- Full maximum likelihood vs Restricted Maximum likelihood (default in R!)
- Measure of model fit:
  - Model *deviance* (-2*loglikelihood)
  - AIC (deviance + 2*k*)
  - Not BIC, penalty based on sample size is ambiguous
- Estimation is iterative, check for convergence

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Analysis approach

Bottom up: from simple to complicated models

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Steps in Model Exploration

1. *Intercept-only* model
   calculate *intraclass correlation coefficient*
2. *Fixed model*, 1st level predictor variables
   test slopes for significance
   calculate proportion explained variance for intercept and residual
3. *Fixed model*, 1st and 2nd level predictor variables
   test slopes for significance
   calculate proportion explained variance for intercept
4. *Random coefficient model*
   test if any 1st level slope has a significant variance component
5. *Full Multilevel Regression Model*
   add predictors for random slopes, test for significance,
   calculate proportion explained variance for slopes

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The intercept only model

- Intercept only model (null model, baseline model)
- Contains only intercept and corresponding error terms
  - At the lowest (individual) level we have
    $y_{ij} = \beta_{0j} + e_{ij}$
  - and at the second level
    $\beta_{0j} = \gamma_{00} + u_{0j}$
  - hence
    $y_{ij} = \gamma_{00} + u_{0j} + e_{ij}$

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The intercept only model – Intraclass correlation

- Intercept only model (null model, baseline model)
  $y_{ij} = \gamma_{00} + u_{0j} + e_{ij}$
- Used to decompose the total variance and compute the *intraclass correlation* ρ
  - $\rho = \sigma_{u0}^2 / (\sigma_e^2 + \sigma_{u0}^2)$
  - ρ = cluster level variance / total variance
  - Interpretation:
  1. Expected correlation between two randomly sampled individuals in *same* clsuter
  2. Percentage of variance at the cluster level

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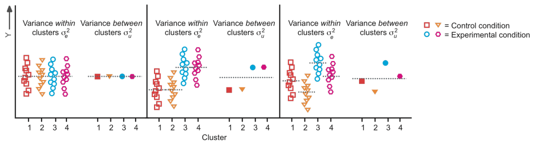
The intercept only model – DIY

The graph below displays various scenarios of the intraclass correlation (ICC; ρ) computed as $\rho = \sigma_{u0}^2/(\sigma_e^2 + \sigma_{u0}^2)$

1. What do you think the value of ICC is (approximately) in each of the three graphs?
2. What is the difference between the middle and right panel?



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The Fixed Model – level 1

Only fixed effects for (individual level) explanatory variables

Slopes are assumed not to vary across groups

- At the lowest (individual) level we have
  $y_{ij} = \beta_{0j} + \beta_1 X_{ij} + e_{ij}$
- and at the second level
  $\beta_{0j} = \gamma_{00} + u_{0j}$ and
  $\beta_{1j} = \gamma_{10}$
- hence
  $y_{ij} = \gamma_{00} + \gamma_{10} X_{ij} + u_{0j} + e_{ij}$

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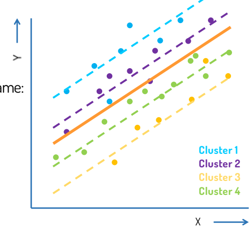
The Fixed Model – level 1

Only fixed effects for explanatory variables

Intercepts vary across clusters, slopes are the same:

$y_{ij} = \beta_{0j} + \beta_1 X_{ij} + e_{ij}$
$= (\gamma_{00} + u_{0j}) + \gamma_{10} X_{ij} + e_{ij}$



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The Fixed Model – level 1 and 2

Fixed effects for individual and cluster level explanatory variables

Slopes are assumed not to vary across groups

- At the lowest (individual) level we have
  $y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + e_{ij}$
- and at the second level
  $\beta_{0j} = \gamma_{00} + \gamma_{01}Z_j + u_{0j}$ and
  $\beta_{1j} = \gamma_{10}$
- hence
  $y_{ij} = \gamma_{00} + \gamma_{10}X_{ij} + \gamma_{01}Z_j + u_{0j} + e_{ij}$
- As we do not include (cross-level) interactions yet, only the intercept is predicted by the cluster level variable

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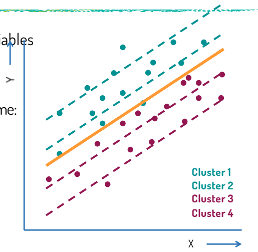
The Fixed Model – level 1 and 2

Fixed effects for level 1 and 2 explanatory variables

Intercepts vary across clusters, slopes are the same:

$$y_{ij} = \beta_{0j} + \beta_1 X_{ij} + e_{ij}$$
$$= (\gamma_{00} + \gamma_{01}Z_j + u_{0j}) + \gamma_{10}X_{ij} + e_{ij}$$



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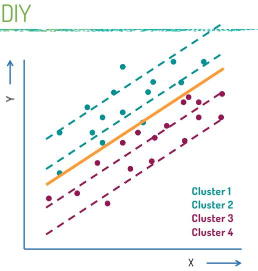
The Fixed Model – level 1 and 2 - DIY

In this model, which of the variance components will decrease, compared to the previous model? Why?

Intercepts vary across clusters, slopes are the same:

$$y_{ij} = \beta_{0j} + \beta_1 X_{ij} + e_{ij}$$
$$= (\gamma_{00} + \gamma_{01}Z_j + u_{0j}) + \gamma_{10}X_{ij} + e_{ij}$$



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The Random Coefficient Model

Assumes intercept **and slopes** vary across groups

- At the lowest level
  $y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + e_{ij}$
- and at the second level
  $\beta_{0j} = \gamma_{00} + \gamma_{01}Z_j + u_{0j}$ and
  $\beta_{1j} = \gamma_{10} + u_{1j}$
- hence
  $y_{ij} = \gamma_{00} + \gamma_{10}X_{ij} + \gamma_{01}Z_j + u_{0j} + u_{1j}X_{ij} + e_{ij}$

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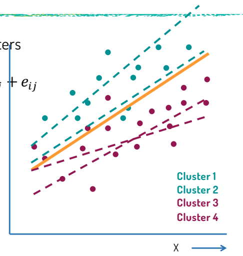
The Random Coefficient Model

Assumes intercept **and slopes** vary across clusters

$y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + e_{ij}$
$= (\gamma_{00} + u_{0j} + \gamma_{01}Z_j) + (\gamma_{10} + u_{1j})X_{ij} + e_{ij}$



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Full Multilevel Regression Model

Explanatory variables at all levels

Higher level variables predict variation of lowest level intercept and slopes

- At the lowest (individual) level we have
  $y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + e_{ij}$
- and at the second level
  $\beta_{0j} = \gamma_{00} + \gamma_{01}Z_j + u_{0j}$ and
  $\beta_{1j} = \gamma_{10} + \gamma_{11}Z_j + u_{1j}$
- hence
  $y_{ij} = \gamma_{00} + \gamma_{10}X_{ij} + \gamma_{01}Z_j + \gamma_{11}Z_jX_{ij} + u_{0j} + u_{1j}X_{ij} + e_{ij}$

- Predicting the intercept implies a direct effect
- **Predicting slopes implies cross-level interactions**

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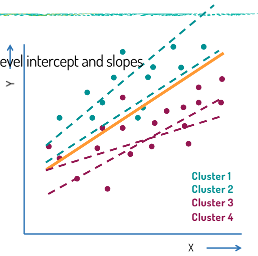
Full Multilevel Regression Model

Explanatory variables at all levels

Higher level variables predict variation of lowest level intercept and slopes

$$y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + e_{ij}$$
$$= (\gamma_{00} + u_{0j} + \gamma_{01}Z_j) +$$
$$(\gamma_{10} + u_{1j} + \gamma_{11}Z_j)X_{ij} + e_{ij}$$



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Coffee break

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Explained variance – pseudo R^2

- Intercept only model is the benchmark model for calculating pseudo R^2
- Variance is decomposed in two parts: within / residual variance and between / cluster variance
- We obtain the explained variance for both parts.
- For model with only predictors at level 1 (fixed effects):
  - Explained variance level 1: $R^2_{L1} = \frac{\sigma^2_{e|baseline} - \sigma^2_{e|model1}}{\sigma^2_{e|baseline}}$
  - Explained variance level 2: $R^2_{L2} = \frac{\sigma^2_{u0|baseline} - \sigma^2_{u0|model1}}{\sigma^2_{u0|baseline}}$

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Explained variance – pseudo R^2

- Intercept only model is the benchmark model for calculating pseudo R^2
- Variance is decomposed in two parts: within / residual variance and between / cluster variance
- We obtain the explained variance for both parts.
- When adding predictors at level 2 to the model (fixed effects):
  - ~~Explained variance level 1:~~ $\cancel{R^2_{L1} = \frac{\sigma^2_{e|baseline} - \sigma^2_{e|model2}}{\sigma^2_{e|baseline}}}$
  - Explained variance level 2: $R^2_{L2} = \frac{\sigma^2_{u0|baseline} - \sigma^2_{u0|model2}}{\sigma^2_{u0|baseline}}$
  - For subsequent models we thus look at the **added** explained variance

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Explained variance – pseudo R^2

- Can also compute total explained variance:
  - Suppose 70% of the variance is at the individual level, and 30% at the cluster level
  - We explain in total 45% of the individual variance
  - We explain in total 60% of the cluster level variance
- Total explained variance:
  - (45% of 70%) + (60% of 30%) = 50%

- When adding slope variance, we do not explain any variance
- When adding cross level interactions (i.e., full model), we only look at the explained variance of … ?

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Example 1

Popularity in Schools

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Predicting school popularity

- Simulated data set
- 100 classes, 2000 pupils
- Continuous outcome: popularity rating
- Explanatory variables pupil level
  - sex (0 = boy, 1 = girl),
  - extraversion (1-10)
- Explanatory variables class level:
  - teacher experience (in years, 2-25)

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0. Model ignoring multilevel structure

```
lm(popular ~ 1, data = popular)
```

$popularity_i = \gamma_{00} + e_i$

What does γ_{00} represent in this model?

Model:	M0: intercept only – no ML	M1: intercept only
Fixed part	Coefficients (SE)	Coefficients (SE)
Intercept γ_{00}	5.08 (0.03)	
Gender γ_{10}		
Extr γ_{20}		
T exp γ_{01}		
Extr*texp γ_{21}		
Random part		
σ_e^2	1.91	
$\sigma_{u_0}^2$		

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1. Intercept-only Model

```
lmer(popular ~ 1 + (1|class), REML = FALSE,
     data = popular)
```

$popularity_{ij} = \gamma_{00} + u_{0j} + e_{ij}$

Should we perform multilevel analysis?

$$\rho = \frac{\sigma_{u0}^2}{\sigma_e^2 + \sigma_{u0}^2} = \frac{0.69}{1.22 + 0.69} = 0.36$$

About one third of the variance is at the class level (this is unusually high)

Model:	M0: intercept only – no ML	M1: intercept only
Fixed part	Coefficients (SE)	Coefficients (SE)
Intercept γ_{00}	5.08 (0.03)	5.08 (0.09)
Gender γ_{10}		
Extr γ_{20}		
T exp γ_{01}		
Extr*texp γ_{21}		
Random part		
σ_e^2	1.91	1.22
$\sigma_{u_0}^2$		0.69

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2. Fixed Model: 1st level predictors

```
lmer(popular ~ gender + extrav + (1|class),
     REML = FALSE, data = popular)
```

$popularity_{ij} =$
$\gamma_{00} + \gamma_{10} sex_{ij} + \gamma_{20} extr_{ij} + u_{0j} + e_{ij}$

Model:	M1: intercept only	M2: level 1 predictors
Fixed part	Coefficients (SE)	Coefficients (SE)
Intercept γ_{00}	5.08 (0.09)	2.14 (0.12)
Gender γ_{10}		1.25 (0.04)
Extr γ_{20}		0.44 (0.02)
T exp γ_{01}		
Extr*texp γ_{21}		
Random part		
σ_e^2	1.22	0.59
$\sigma_{u_0}^2$	0.69	0.62
Explained var		
Level 1		0.52
Level 2		0.11

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3. Fixed Model: 1st and 2nd level predictors

```
lmer(popular ~ gender + extrav + texp +
     (1|class),
     REML = FALSE, data = popular)
```

$popularity_{ij} =$
$\gamma_{00} + \gamma_{10} sex_{ij} + \gamma_{20} extr_{ij} + \gamma_{01} texp_j + u_{0j} + e_{ij}$

Model:	M2: level 1 predictors	M3: level 1 and 2 predictors
Fixed part	Coefficients (SE)	Coefficients (SE)
Intercept γ_{00}	2.14 (0.12)	0.81 (0.17)
Gender γ_{10}	1.25 (0.04)	1.25 (0.04)
Extr γ_{20}	0.44 (0.02)	0.45 (0.02)
T exp γ_{01}		0.09 (0.01)
Extr*texp γ_{21}		
Random part		
σ_e^2	0.59	0.59
$\sigma_{u_0}^2$	0.62	0.29
Explained var		
Level 1	0.52	0.52
Level 2	0.11	0.58

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4. Random Coefficient Model

```
lmer(popular ~ 1 + extrav + gender + texp + (extrav|class),
     REML = FALSE, data = popular)
```

$popularity_{ij} =$
$\gamma_{00} + \gamma_{10} sex_{ij} + \gamma_{20} extr_{ij} + \gamma_{01} texp_j + u_{0j} + u_{1j} sex_{ij} + u_{2j} extr_{ij} + e_{ij}$

What is σ_{u02}?

Significant slope variation for *extraversion*, not for *sex*

Model:	M4: with random slope
Fixed part	Coefficients (SE)
Intercept γ_{00}	0.76 (0.20)
Gender γ_{10}	1.25 (0.04)
Extr γ_{20}	0.45 (0.03)
T exp γ_{01}	0.09 (0.01)
Extr*texp γ_{21}	
Random part	
σ_e^2	0.55
$\sigma_{u_0}^2$	1.32
$\sigma_{u_2}^2$	0.03
$\sigma_{u_{02}}^2$	-0.19

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5. Full Multilevel Regression Model

```
lmer(popular ~ 1 + extrav + gender + texp +
       extrav*texp + (extrav|class),
       REML = FALSE, data = popular)
```

$popularity_{ij} =$
$\gamma_{00} + \gamma_{10} sex_{ij} + \gamma_{20} extr_{ij} + \gamma_{01} texp_j +$
$\gamma_{21} extr_{ij} * texp_j + u_{0j} + u_{2j} extr_{ij} + e_{ij}$

Smaller slope variation for *extraversion*.

85% explained. No longer significant!

Model:	**M4: with random slope**		**M5: cross-level interaction**	
Fixed part	Coefficients (SE)		Coefficients (SE)	
Intercept γ_{00}	0.74	(0.20)	-1.21	(0.27)
Gender γ_{10}	1.25	(0.03)	1.24	(0.04)
Extr γ_{20}	0.45	(0.02)	0.80	(0.04)
T exp γ_{01}	0.09	(0.01)	0.23	(0.02)
Extr*texp γ_{21}			-0.03	(0.00)
Random part				
σ_e^2	0.55		0.55	
$\sigma_{u_0}^2$	1.30		0.45	
$\sigma_{u_2}^2$	0.03		0.00	
$\sigma_{u_{02}}^2$	-0.19		-0.03	

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Summary

- We use multilevel regression analysis to accommodate nested data
- In multilevel analysis, both the intercept and the slope (of level 1 predictors) can be random, that is, vary over clusters
- Using multilevel analysis ensures optimal use of our data: not only accommodate dependency but enables us to answer new research questions
- The amount dependency in the data is quantified by the ICC, obtained from the intercept only model
- The multilevel model is build up from simple to more complex
- The amount of explained variance is obtained both at the individual level (residual variance) and at the cluster level (intercept / slope variance)

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