Group Assignment1-MLM

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1

The multilevel analysis(modeling) is used statistical analysis of data have a hierarchical or clustered structure. Such data are educational, for instance students are nested within schools. It is worth highlighting that data within same cluster(e.g school) are more similar than data across different clusters hence, the independence assumption is violated, yet Standard statistical tests are not at all robust against violation of the independence assumption. But also,the degree of similarity is indicated by the intraclass correlation ρ.

Specifically, the data set we have at hand contains 198 students that are nested in 65 schools, London reading test(LRT) i.e the reading ability and exam score are separately measured per student, the two variables can identified as individual-level variables. The average reading score variable is an aggregated variable for students for every school, this school level variable(cluster). The different levels of data identified, leads to its hierarchical structure which implies multilevel regression analysis not the traditional regression analysis(multiple).

By ignoring the structure of the data and disaggregate the school-level(second-level) we can perform the conventional regression analysis, violates the independent assumption among observations. This is due to the fact that students from the same school are more likely to have similar scores than students from different schools, in other words they are dependent to each other, if the school they come from is the same. The problem of violating the assumption, is underestimation of the standard errors and this leads to inflated type I error rate. By not deploying multilevel analysis, may result in violation of other assumptions like linearity, presence of outlier and others.

On another hand, if we choose to aggregate the lower level(students) to second- level(school-level) will lead to a substantial decrease in the effective sample size hence lowering the statistical power[@hox2017multilevel]. So, performing, mulmtilevel analysis allows a proper assessment of effects of LRT-score on the outcome variable (Examscore) accross schools(clustrs), avarage score can be used to asses if partly affect the Examscore between schools.

To sum up, the available information from the dataset, we theoretically think it is appropriate to use multilevel analysis in this case.

2

Discriptive Statistics

We can view some descriptive statistics with $summary()$. where the minimum and maximum are respectively the first and last values in the output below for each variable. Alternatively,we can compute them by using with the $min()$ and $max()$ functions. Also, the function $summary()$ gives other four quantities(mean, median, first and third quartiles).
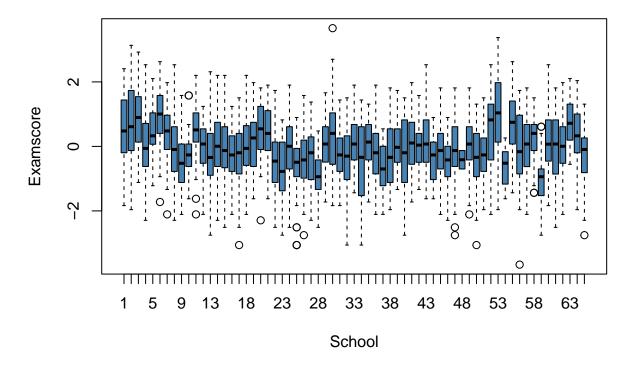
```
summary(exam)
```

```
##    ï..School        Student        Examscore          LRTscore
##  Min.   : 1.00   Min.   :  1.0   Min.   :-3.666100   Min.   :-2.93500
##  1st Qu.:14.00   1st Qu.: 16.0   1st Qu.:-0.699510   1st Qu.:-0.62071
##  Median :29.00   Median : 33.0   Median : 0.004322   Median : 0.04050
##  Mean   :31.01   Mean   : 38.7   Mean   :-0.000118   Mean   : 0.00181
##  3rd Qu.:47.00   3rd Qu.: 54.0   3rd Qu.: 0.678760   3rd Qu.: 0.61906
##  Max.   :65.00   Max.   :198.0   Max.   : 3.666100   Max.   : 3.01600
##      AvsLRT
##  Min.   :-0.755960
##  1st Qu.:-0.149340
##  Median :-0.020198
##  Mean   : 0.001811
##  3rd Qu.: 0.210520
##  Max.   : 0.637660
```

Outlier checking

Observations considered as potential outliers by the IQR criterion are displayed as points in the box-plot.Based on this criterion, there are 17 potwential outliers(2 above and 15 points below) the vertical line.

```
#create boxplot that displays examscore distribution for each school in the dataset
boxplot(Examscore~ï..School,
data= exam,
main="Exam Distribution per school",
xlab="School",
ylab="Examscore",
col="steelblue",
border="black"
)
```

Exam Distribution per school

3

a)

Yes, because we use multilevel regression analysis to accommodate nested data and the dataset we are given is nested, the dataset is of hierarchical structure.

b)

H0: $u_{0j} = 0$
vs
HA: $u_{0j} > 0$

Or by means of diviance

H0: Deviance of the null/baseline model is equal to the deviance of intercept model;

$$Deviance_{m0} = Deviance_{m1}$$

Vs

H1: Deviance of the null/baseline model is larger than the deviance of intercept model;

$$Deviance_{m0} > Deviance_{m1}$$

c)

Level-1 equation

$$examscore_{ij} = b_{0j} + e_{ij}$$

Level-2 equation

$$b_{0j} = \gamma_{00} + u_{0j}$$

Mixed Level equation

$$examscore_{ij} = \gamma_{00} + u_{0j} + e_{ij}$$

where i represents individual(student) and j represents school

d)

Model_0 and Model_1

The two models, model_0 that ignores the multilevel structure, this also known as fixed-intercept model, because it consists of an overall intercept and residual parts, thus the hierarchical structure is not considered in this model.

The second model, model_1 is a random intercept model, is fitted by adding variance component for the intercept at school level, that is to say the variance varies across schools(homogeneity is not achieved between schools).

By means of $ anova()$ function model_0 has $Deviance=$ 11509 and $AIC=$ 11513, whilst model_1 has $Deviance=$ 11011(smaller) and $AIC=$ 11017(smaller), they suggest that a model_1 is a better model. Furthermore, the Chi-square difference test indicates that the later model(model_1) fits the data significantly better than model_0(

$$\chi^2_{(1)} = 498.72, p < 0.0001$$

), this implies that we reject the null hypothesis that the variance of the intercept between schools is zero.

e) ICC

The interclass correlation is 0.17(17%), meaning that 17% of the variance is explained by second level(school level), in other words, if we choose more than one student from the same school, the expected correlation between their exam score is 0.17.

4

Model_2 and Model_3

There are two predictors i.e LRT-score(student level) and average LRT-score(school-level) in the dataset that can influence the outcome variable(Examscore), to check this we fit two models squentially, model_2, this is known as a fixed effect model with fixed effect of the LRT-score at student level.

The $Deviance_{(}4) = 11010.6, p < 0.001$ and $ AIC= 9365.2$ chi square difference test$(\chi_1^2 = 1653.4, p < 0.001)$ and Wald test$(LRT - score_{coef} = 0.56, p < 0.001)$, confirm that there is a significant fixed effect of LRT-score of Examscore. Therefore, on average, a unit increase in the LRT-test leads to a 0.56 point increase in Examscore. The interclass correlation is $icc = 0.14$, while for first level $Pseudo - R^2 = 0.32$ for the fixed effect, meaning that between students within the same school variance in the Examscore 32% is explained by student's score at first level. For the second level $Pseudo - R^2 = 0.42$, meaning 42% of the variance in Examscore is accross schools is explained by student scores(LRT score).

The model_3 is one that we add fixed effect of average LRT-score on second level.

The $Deviance_{(}5) = 9347.6, p < 0.01$ and $AIC = 9357.6$ chi square difference test$(\chi_1^2 = 9.6227, p < 0.01)$ and Wald test$(LRT - score_{coef} = 0.56, p < 0.001$ and $AvsLRT_{coef} = 0.36, p < 0.01)$, confirm that there is a significant fixed effect of LRT-score of Examscore. Therefore, on average, a unit increase in the LRT-test leads to a 0.56 point increase in Examscore. The interclass correlation is $icc = 0.12$, while for first level $Pseudo - R^2 = 0.36$ for the fixed effect, meaning that between students within the same school variance in the Examscore 32% is explained by student's score at first level. For the second level $Pseudo - R^2 = 0.44$, meaning 44% of the variance in Examscore is accross schools is explained by student scores(LRT score).

5

a)

H0: $u_{1j} = 0$
vs
HA: $u_{1j} > 0$

b)

By allowing the slope of LRT to vary between schools ($Deviance(7) = 9308.56, p < 0.001$) and $AIC = 9324.40$.A random effect model is fitted. The chi-square difference test confirm that the variance of the slope of LRT score between schools(clusters) is significant from zero ($\chi_2^2 = 37.191, p < 0.001$), this is not one or two sided, because we add variance of the slope between schools and covariance of the random intercept with random slope.

c)

We conclude that the effect of LRT score on Examscores varies between schools.Therefore, we keep the random slope in the model.This means that the fixed effect of LRT-score that only gives a limited contribution of the effect of LRT-score.

6

a)

LRT-score confidence interval at 0.95

$$[0.51, 0.59]$$

b)

```
predict(model_4)
```

```
##             1             2             3             4             5
##   0.7820965720  0.5028362126 -0.5583612623  0.5028362126  0.6145376534
##             6             7             8             9            10
##   1.8432535020 -0.3907753137 -0.3349583808  0.0001729716 -0.6141781952
##            11            12            13            14            15
##   2.0108394506  1.7874365691  0.3911340960  1.1730448573  2.0666563835
##            16            17            18            19            20
##   1.1172279244  1.0613434165  0.8937980128 -0.3907753137 -0.4466598216
##            21            22            23            24            25
##   0.1118744123 -0.5583612623 -0.2790873879 -1.2285699070  0.3352826999
##            26            27            28            29            30
##   0.6145376534  1.1172279244  0.3911340960  0.7820965720  1.7315520612
##            31            32            33            34            35
## -0.0556777488 -0.3907753137  0.5586869330  0.2794333310  0.6145376534
##            36            37            38            39            40
##   1.0613434165  0.2794333310  0.4469854922  0.7820965720  0.5586869330
##            41            42            43            44            45
##   0.1677251327  0.8937980128  0.1677251327 -0.3349583808 -0.2790873879
##            46            47            48            49            50
## -1.2285699070  0.8937980128  0.1118744123  0.8937980128 -0.1115284692
##            51            52            53            54            55
##   0.3911340960  0.1677251327  1.4523322467  0.6145376534  1.4523322467
##            56            57            58            59            60
##   0.6145376534  0.3352826999  0.1677251327 -0.0556777488  1.4523322467
##            61            62            63            64            65
##   1.1730448573 -0.0556777488  0.9496487332  0.7262458516  0.6703951312
##            66            67            68            69            70
##   1.2289293651 -0.5024767545  0.2794333310  0.5028362126  1.5081491796
##            71            72            73            74            75
## -0.2232366675  0.9496487332  0.6145376534  0.5553684858  0.9662752248
##            76            77            78            79            80
##   0.6727683820  0.9662752248  0.0270618504  1.3772032706  0.9075752767
##            81            82            83            84            85
##   1.1423821714  0.1444617467  1.3184678113  1.2010679151 -0.4425306323
##            86            87            88            89            90
##   0.0857617985  1.4358677076  1.2010679151  1.3772032706  0.7314683301
##            91            92            93            94            95
##   0.1444617467  0.4966678274  0.2618616429  0.0270618504  1.0249751730
##            96            97            98            99           100
```

```
## -0.0903451481  1.8468028556 -0.1490450962  1.8468028556  0.2031616948
##           101           102           103           104           105
##  0.5553684858  1.9054672926  1.8468028556  1.6706675001  0.1444617467
##           106           107           108           109           110
##  1.4358677076 -1.4992007209  0.6727683820  0.1444617467  0.3792686414
##           111           112           113           114           115
##  0.7314683301 -0.7360658841  0.0270618504  0.4966678274  2.1990025443
##           116           117           118           119           120
##  1.9054672926  0.6140684339  1.3772032706  2.0228671888  1.6120030631
##           121           122           123           124           125
##  0.0857617985  0.9662752248  2.0228671888  1.3772032706  0.1444617467
##           126           127           128           129           130
##  0.4379671691 -0.3251307361 -0.3251307361  0.4667263162  1.3251970225
##           131           132           133           134           135
##  1.3251970225  1.3757257769  0.4162293332  0.1637321985  0.9212174927
##           136           137           138           139           140
##  0.4667263162 -0.6442635205  0.5677239481  1.9817042364  1.2747293669
##           141           142           143           144           145
##  0.1132339935  1.2242006126  0.6182221530  0.3657250183  0.7192185629
##           146           147           148           149           150
##  0.3657250183  1.2242006126  0.7192185629 -0.1897491264  0.6687203580
##           151           152           153           154           155
##  0.9212174927  0.8707192877  0.6687203580  1.2747293669  1.9817042364
##           156           157           158           159           160
##  1.6281862523  1.1232042026  1.0727182174  1.9817042364  0.9212174927
##           161           162           163           164           165
##  1.0727182174  1.1232042026  0.7192185629  1.2242006126  0.1132339935
##           166           167           168           169           170
##  1.0727182174 -0.0887649362  0.5172251321  0.5172251321  0.8707192877
##           171           172           173           174           175
##  0.6182221530  1.0727182174  0.3152268134 -0.0382667313  1.2747293669
##           176           177           178           179           180
##  0.8202210827  0.6687203580  0.8707192877  0.8202210827  0.4667263162
##           181           182           183           184           185
##  0.2782501358 -0.1191100377  1.3000157785  0.1079558620 -0.8570450227
##           186           187           188           189           190
## -0.3461690695 -0.2326395536 -0.2894043115  0.6188455515 -0.5732349692
##           191           192           193           194           195
## -0.1758747957 -0.6867850894  1.3568148769 -0.4596985853 -0.0055750274
##           196           197           198           199           200
## -0.1191100377 -0.2894043115  0.6188455515  0.2782501358 -1.5950212162
##           201           202           203           204           205
##  0.2214853779 -0.0623384117  0.1647206200  0.4485512777  0.6756171775
##           206           207           208           209           210
## -0.5732349692 -0.5732349692 -0.8003146052 -1.1976335703  0.4485512777
##           211           212           213           214           215
## -0.0055750274  0.3917865198 -0.4596985853  0.6756171775  0.2782501358
##           216           217           218           219           220
##  0.5620807935  0.2782501358 -1.2544326686 -1.2544326686 -0.3461690695
##           221           222           223           224           225
##  0.4485512777  0.0511904173  0.0511904173 -0.6299997272  0.1079558620
##           226           227           228           229           230
## -0.1191100377  1.2432853610  0.0511904173 -0.3461690695  0.0511904173
##           231           232           233           234           235
```

```
##   1.5838739086   0.8458977151   1.6406043260   1.0729567468   0.5053160356
##            236            237            238            239            240
##   1.4703443927   0.6188455515  -0.4596985853  -0.4029338274   0.6188455515
##            241            242            243            244            245
##   0.1647206200   0.1079558620  -0.5164702113  -1.1976335703  -0.5164702113
##            246            247            248            249            250
##   0.4485512777   1.1864862626  -0.4596985853  -0.0055750274   0.3350217618
##            251            252            253            254            255
##  -0.6299997272   0.0511904173   0.5620807935  -0.1758747957   0.5620807935
##            256            257            258            259            260
##   1.3000157785   1.0162263293  -0.3461690695   0.1079558620   0.0131402393
##            261            262            263            264            265
##   0.3687988627   0.4196063593   0.2671832548   0.8260786265   0.6736499895
##            266            267            268            269            270
##   0.9784888216   0.0131402393   0.0131402393   0.5720349963   0.1655688764
##            271            272            273            274            275
##   1.3341720342   0.5212274997   0.0131402393   0.1147552325   1.4865637874
##            276            277            278            279            280
##  -0.1392822504   0.1147552325   0.4196063593   0.6228424929   0.5720349963
##            281            282            283            284            285
##  -0.4949593156   0.0131402393   0.6736499895   0.9784888216   0.3179913661
##            286            287            288            289            290
##  -0.1900897470   0.0639477359   0.5212274997   0.5720349963  -0.2409033908
##            291            292            293            294            295
##   0.0131402393   0.7752649826   0.5720349963   0.0639477359   0.2693445459
##            296            297            298            299            300
##   0.8020051845   0.8504278137   1.0441183305   1.4314935052   1.0441183305
##            301            302            303            304            305
##   1.6736359450   1.4799454282   1.0441183305   0.8988504429   0.6567314382
##            306            307            308            309            310
##   0.6567314382   1.8188745388   0.8988504429  -0.0696372935   1.8673264617
##            311            312            313            314            315
##   0.8504278137   0.8504278137   1.2378029885   0.9956957013   1.6251840220
##            316            317            318            319            320
##   0.6083088090   0.5598855939   0.4630409214   0.3177671751  -0.0696372935
##            321            322            323            324            325
##   1.2862549115   1.0441183305  -0.0211970880   1.0441183305   1.9157197971
##            326            327            328            329            330
##   0.3177671751   1.8188745388   1.2862549115   1.7220292804   0.7535766966
##            331            332            333            334            335
##   1.0925468184   1.0441183305   0.7051540674   1.3831001698   0.6083088090
##            336            337            338            339            340
##  -0.3117211456   0.8988504429   1.2378029885   1.0925468184   1.2378029885
##            341            342            343            344            345
##   0.7535766966   1.8673264617   1.6251840220   0.3177671751   0.4146124335
##            346            347            348            349            350
##   0.4630409214   1.7220292804   0.7051540674   1.3346482469   0.5598855939
##            351            352            353            354            355
##   1.5283387636   0.8504278137   0.6567314382   1.2378029885   0.7051540674
##            356            357            358            359            360
##   0.7535766966   0.8988504429   0.5114623789   0.3661898043   0.5598855939
##            361            362            363            364            365
##   0.4146124335   0.1724992875   1.0441183305   0.6083088090   1.1409577301
##            366            367            368            369            370
```

```
##   1.1894096531   0.6083088090   0.3177671751   0.8020051845   0.4630409214
##            371            372            373            374            375
##   1.4799454282   0.8504278137   1.1894096531   0.7051540674   0.4648717950
##            376            377            378            379            380
##  -0.5594968810   0.0807369304   0.2087805943  -0.0793137763   0.4328608790
##            381            382            383            384            385
##   0.4008499631   0.1767696783   0.3688386598   0.7529700386  -0.0473222257
##            386            387            388            389            390
##   0.3048172152   0.3688386598   0.7209668688   0.2728024262   0.2087805943
##            391            392            393            394            395
##   0.5288975000   0.5609084159   0.1447587623   0.2728024262   0.2407915102
##            396            397            398            399            400
##   0.3368273565   0.1447587623   1.0090573662   0.3048172152   0.2407915102
##            401            402            403            404            405
##   0.0807369304   0.2728024262   1.2971749752   1.0730791982   0.0167112254
##            406            407            408            409            410
##   0.0807369304   0.3368273565   1.1051094795   0.1447587623   0.2728024262
##            411            412            413            414            415
##   1.2331531433   0.3688386598   0.4008499631   0.3048172152   0.8810137024
##            416            417            418            419            420
##   0.1767696783   0.1767696783   0.2087805943  -0.1433356082   0.2407915102
##            421            422            423            424            425
##   0.0487221414   0.2728024262   0.3048172152   0.3688386598   0.4008499631
##            426            427            428            429            430
##   0.5929193319   0.3688386598  -0.3034095533   0.0167112254   0.1767696783
##            431            432            433            434            435
##   0.4648717950   0.6569411638   0.3048172152  -0.0152996905   0.8169918705
##            436            437            438            439            440
##   0.7529700386   0.8810137024   0.5929193319   0.1127478464   0.0487221414
##            441            442            443            444            445
##  -0.3994229359   0.2728024262   0.1767696783   0.4328608790   0.8810137024
##            446            447            448            449            450
##   0.0487221414   1.1371010301  -0.0152996905   0.4648717950   0.4968827109
##            451            452            453            454            455
##   0.2728024262   0.0807369304   0.0807369304   0.2407915102   0.5288975000
##            456            457            458            459            460
##   0.4648717950   0.5609084159   0.2087805943   0.3368273565   0.4328608790
##            461            462            463            464            465
##  -0.0473222257   0.1767696783  -0.4197403633   0.8833151507   0.5110063360
##            466            467            468            469            470
##   1.3951883922  -0.1870579114  -1.3039280501   0.5110063360   0.6506045460
##            471            472            473            474            475
##  -0.6524171847  -0.4662824843   0.8833151507  -0.0009068824   0.6971691892
##            476            477            478            479            480
##   0.0456301710  -1.2574197124   0.3248547439  -0.5593554650  -1.2574197124
##            481            482            483            484            485
##  -0.7920548086  -0.0474439358  -0.2335944018  -0.6989818278   1.0694611122
##            486            487            488            489            490
##   0.5110063360   0.4644642150  -0.3732038730   0.1387031518   0.2783182535
##            491            492            493            494            495
##  -0.2801308922  -0.2801308922  -0.3732038730  -0.6059088471  -0.3266673826
##            496            497            498            499            500
##   0.0921666614  -0.0474439358   0.5575315653  -0.2801308922   0.1387031518
##            501            502            503            504            505
```

```
## -0.0009068824 -0.4197403633  0.5110063360  0.4179277247  0.4179277247
##            506           507           508           509           510
## -0.4197403633 -0.7920548086 -0.6989818278  0.3248547439 -0.8385631462
##            511           512           513           514           515
## -0.4197403633 -0.0474439358 -0.4662824843  1.5813343537 -0.3732038730
##            516           517           518           519           520
## -0.4197403633  0.5110063360  0.1387031518  0.3248547439  0.0456301710
##            521           522           523           524           525
## -0.7454901655  0.0921666614  0.0456301710 -0.2335944018 -0.2335944018
##            526           527           528           529           530
## -0.1405214210  1.5813343537 -0.1870579114 -0.3732038730  0.4644642150
##            531           532           533           534           535
## -1.1177820885  1.6744636400 -0.5128189747  0.1387031518 -0.2801308922
##            536           537           538           539           540
## -0.1405214210  0.1852396422  0.0456301710  1.1159694499 -0.0474439358
##            541           542           543           544           545
## -0.1405214210  0.0456301710 -0.3266673826 -1.1643467317  0.6971691892
##            546           547           548           549           550
## -0.1870579114  0.6971691892 -0.3266673826 -1.2574197124 -0.6059088471
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##            556           557           558           559           560
## -1.3970010308 -0.3732038730 -0.0009068824 -0.2801308922 -0.5128189747
##            561           562           563           564           565
## -0.4662824843  0.0921666614  0.0921666614  0.4179277247 -0.1089518351
##            566           567           568           569           570
## -0.1089518351 -1.5867534149 -0.5367275879 -0.9644939303 -0.5367275879
##            571           572           573           574           575
## -1.2367353955  0.0854942554 -0.9644939303 -1.0422704844  0.2799309355
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## -0.1089518351  0.0854942554 -0.5367275879 -0.5756205701  0.0466059783
##            581           582           583           584           585
## -0.3422862026  0.2410520688 -1.0422704844 -0.8478526250 -0.1478401121
##            586           587           588           589           590
## -0.2645049433 -0.2256176073 -1.4700650579 -1.5867534149 -0.0311752810
##            591           592           593           594           595
##  0.2021590865  0.2021590865 -0.4978393108 -0.4978393108 -0.4200627567
##            596           597           598           599           600
##  0.3577074896 -0.5756205701 -0.1478401121  0.1508530526 -0.5256043189
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## -0.8976382769 -0.4579610370 -0.0182715206 -0.4579610370 -0.1873838174
##            606           607           608           609           610
## -0.3903177551 -0.1873838174 -0.3226711996 -0.0859148025  0.1170109509
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## -0.0182715206  0.1508530526 -0.3226711996 -0.2888491494 -0.2212054583
##            616           617           618           619           620
## -0.2550270993 -0.2888491494 -0.2550270993 -0.0520931616 -0.4917826779
##            621           622           623           624           625
## -0.0859148025  0.1170109509 -0.3903177551 -0.0520931616 -0.6270733338
##            626           627           628           629           630
## -0.0859148025 -0.2888491494  0.4552273602 -0.0859148025 -0.6608949747
##            631           632           633           634           635
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##            636           637           638           639           640
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##  0.0493717612  0.2522975147 -0.1197364434 -0.4917826779 -0.4241393961
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## -0.6947166157 -0.6270733338 -0.3226711996  0.4545263409 -0.0887108030
##           651           652           653           654           655
##  0.8306015380 -0.0051374134  0.0366492813  0.3709478954  1.0813217067
##           656           657           658           659           660
##  1.2067070704  0.5380997304  0.5798864252  0.9141749276  0.2873745059
##           661           662           663           664           665
## -0.4648112795  0.7888401226  1.2067070704  0.1620144215  0.8724135121
##           666           667           668           669           670
##  0.2455873055  1.4574272390  0.2455873055  0.2873745059 -0.0469241082
##           671           672           673           674           675
##  0.4963130357  0.5380997304  0.3291612007  0.4127345902  0.4963130357
##           676           677           678           679           680
##  0.5380997304  0.9559869017 -0.3394259158 -0.0469241082  0.8724135121
##           681           682           683           684           685
## -0.0469241082  0.0784359761  1.3738538495  1.6663354336  0.2455873055
##           686           687           688           689           690
## -0.2558626380 -0.2558626380  0.3709478954  0.6216731200  0.7888401226
##           691           692           693           694           695
##  0.7888401226  1.2484684858  0.3709478954  0.7470281485  0.4963130357
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##  0.9141749276  1.0395602912  1.1648950963 -0.5065726949  1.0813217067
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##           711           712           713           714           715
## -0.5522710309  0.2553617386 -0.6372730967  0.5104193652  0.2978679144
##           716           717           718           719           720
##  0.0003195407  0.2978679144 -0.4247473605 -0.2972288330 -0.0846938396
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##           741           742           743           744           745
## -0.5097648551 -1.0623348550  0.7229245297  0.3403740902  0.1278380682
##           746           747           748           749           750
## -0.1697103055  0.3403740902 -0.0421871494 -0.3822411847 -0.6798049871
##           751           752           753           754           755
## -0.4247473605 -0.2547226572  0.5954317168  0.2553617386 -0.0421871494
##           756           757           758           759           760
## -0.1271989868  0.1703493869 -0.8498073039 -0.6052036208 -1.1921980092
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##            806           807           808           809           810
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##  0.5353717552 -0.1386006321  0.2465325207 -0.7163018174  0.0539633232
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12

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##   0.1983916775   0.5835417218   0.4390900688   0.3428142071   0.1502508343
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##           1031           1032           1033           1034           1035
##  -0.1546134489  -0.0502890107  -0.3111048394  -0.2067759835  -0.0502890107
##           1036           1037           1038           1039           1040
##  -0.6762444750   0.5235234836  -0.8327491191  -0.7284063785   0.5235234836
##           1041           1042           1043           1044           1045
```

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##  -0.5197524534  0.5235234836 -0.3111048394 -0.3111048394  0.1583649144
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##   0.2626887214  0.0540347962  0.0018728927  0.0540347962  0.0540347962
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##   0.0018728927 -0.2067759835  0.4191807431 -0.1546134489  0.1583649144
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##   0.0470778928 -0.1644012953 -0.9045677926 -0.1291554747  0.1175823274
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##  -0.0939096541  0.0118278077  0.2585656098  0.4700405335  0.2232984669
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##          1156          1157          1158          1159          1160
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##          1166          1167          1168          1169          1170
##  -0.0939096541 -0.3406350893  0.2585656098 -0.2701430216 -0.0586638335
##          1171          1172          1173          1174          1175
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##          1176          1177          1178          1179          1180
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##          1186          1187          1188          1189          1190
## -0.3053888423 -0.7635930391 -0.0939096541 -0.6226054922  0.4700405335
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##          1206          1207          1208          1209          1210
## -0.2416653571 -0.0745503155 -0.7848145169 -0.4505642141  0.5103523301
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##          1226          1227          1228          1229          1230
##  0.3014686379 -0.4505642141 -0.7430104820  0.1343485414  0.9699439691
##          1231          1232          1233          1234          1235
## -0.2834491724 -0.3252279328 -0.2416653571 -1.2443556068  0.1343485414
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##          1256          1257          1258          1259          1260
##  0.1343485414 -0.7848145169  0.6774673718 -0.3670066933 -0.1998876077
##          1261          1262          1263          1264          1265
##  0.0507910206 -0.5341217349 -1.1607980860 -0.6176843106 -0.4087854537
##          1266          1267          1268          1269          1270
## -0.9101255236 -0.5759055502 -0.1163290759  0.3850413234  0.3014686379
##          1271          1272          1273          1274          1275
## -0.0327715551  0.1761273018 -0.2416653571 -0.3252279328 -0.4087854537
##          1276          1277          1278          1279          1280
## -0.9101255236  0.0925697810  0.0925697810 -0.1998876077  0.1343485414
##          1281          1282          1283          1284          1285
## -0.2834491724 -0.1998876077 -0.1998876077  0.0507910206 -0.9101255236
##          1286          1287          1288          1289          1290
## -0.4923429745 -0.4923429745  0.6357138859  0.6357138859 -0.0745503155
##          1291          1292          1293          1294          1295
## -0.3252279328  0.1761273018  0.3850413234 -0.3670066933 -0.1163290759
##          1296          1297          1298          1299          1300
## -0.1581083418 -0.4087854537 -0.1581083418  1.3459780873 -0.7430104820
##          1301          1302          1303          1304          1305
## -0.6594630710 -0.0327715551  0.0090072053 -0.6176843106 -0.4087854537
##          1306          1307          1308          1309          1310
## -0.6594630710 -0.1163290759 -0.3252279328 -0.0745503155 -0.0327715551
##          1311          1312          1313          1314          1315
```

```
##   0.2179060622 -1.0354870793 -0.2834491724  0.0507910206  0.5103523301
##           1316          1317          1318          1319          1320
## -0.1581083418 -0.1581083418  0.0925697810 -0.6176843106 -0.3252279328
##           1321          1322          1323          1324          1325
## -0.4923429745 -1.6621937599 -0.1163290759 -1.1190446001 -0.0500325207
##           1326          1327          1328          1329          1330
##   0.6406913405 -0.3408581338 -0.3408581338  0.1680885584 -0.2681527201
##           1331          1332          1333          1334          1335
## -0.1227374945 -0.0500325207  0.3498564908 -0.0863856674 -0.0863856674
##           1336          1337          1338          1339          1340
##   0.0226733327  0.0590260396 -0.5226392614  0.2044412652  0.2771466788
##           1341          1342          1343          1344          1345
##   0.3498564908  0.0590260396 -0.0500325207  0.3134993856 -0.0500325207
##           1346          1347          1348          1349          1350
##   0.1317358516  0.1680885584 -0.0863856674 -0.2318000133  0.1680885584
##           1351          1352          1353          1354          1355
## -0.0136793741 -0.1954473065 -0.4499206526  0.1680885584 -0.0136793741
##           1356          1357          1358          1359          1360
## -0.0500325207 -0.7043808035 -0.1590945997 -0.3772108406 -0.3408581338
##           1361          1362          1363          1364          1365
##   0.0953787464  0.0590260396 -0.3045054270 -0.3408581338  0.3134993856
##           1366          1367          1368          1369          1370
## -0.0863856674  0.1317358516  0.1680885584  0.1680885584  0.2044412652
##           1371          1372          1373          1374          1375
## -0.0500325207 -0.0136793741  0.2407939720  0.0953787464 -0.4862733594
##           1376          1377          1378          1379          1380
## -0.3408581338 -0.1590945997 -0.4135679458  0.1317358516  0.0226733327
##           1381          1382          1383          1384          1385
## -0.3045054270  0.0953787464 -0.0136793741  0.0953787464 -0.7043808035
##           1386          1387          1388          1389          1390
## -0.3408581338  0.6406913405  0.2044412652 -0.2318000133  0.7861021678
##           1391          1392          1393          1394          1395
##   0.4589058145  0.2044412652 -0.1227374945 -0.2681527201 -0.0863856674
##           1396          1397          1398          1399          1400
##   0.4952805133  0.4225750996 -0.5226392614  0.3498564908  0.0953787464
##           1401          1402          1403          1404          1405
## -0.3408581338 -0.2318000133  0.2044412652  0.1680885584  0.4952805133
##           1406          1407          1408          1409          1410
## -0.4135679458 -0.1227374945  0.2407939720 -0.0863856674 -0.7407555023
##           1411          1412          1413          1414          1415
## -0.2318000133 -0.0863856674 -0.3045054270 -0.3408581338  0.0953787464
##           1416          1417          1418          1419          1420
##   0.0953787464 -0.1590945997  0.1317358516  0.0226733327  0.1317358516
##           1421          1422          1423          1424          1425
##   0.0590260396  0.1317358516 -0.4135679458 -0.2318000133  0.1680885584
##           1426          1427          1428          1429          1430
##   0.3134993856 -0.0136793741 -0.0136793741  0.0953787464  0.4589058145
##           1431          1432          1433          1434          1435
##   0.2407939720 -0.1227374945 -0.3772108406  0.0953787464  0.2771466788
##           1436          1437          1438          1439          1440
##   0.2044412652 -0.1954473065 -0.1590945997  0.4225750996  0.2771466788
##           1441          1442          1443          1444          1445
##   0.0590260396 -0.0136793741  0.1680885584  0.2407939720 -0.8943274592
##           1446          1447          1448          1449          1450
```

```
##   0.3263819939  0.1737879275  0.3263819939  0.6824450720  0.4281072688
##            1451          1452          1453          1454          1455
##   1.0384527643  0.1737879275  0.1229252900  0.5298325437  0.5298325437
##            1456          1457          1458          1459          1460
##   0.1737879275 -0.2331193262 -0.1822566887  0.6315516646  0.2755193564
##            1461          1462          1463          1464          1465
## -0.0296638531  0.1229252900  0.1737879275  0.7332769396  0.3772446313
##            1466          1467          1468          1469          1470
##   0.0720626526  0.5298325437 -0.0296638531  0.1229252900 -0.4874386674
##            1471          1472          1473          1474          1475
##   0.6824450720  0.4789699062  0.1229252900 -0.0805252598 -0.0296638531
##            1476          1477          1478          1479          1480
##   0.0720626526  0.6315516646 -0.2839819636  0.6824450720  0.3263819939
##            1481          1482          1483          1484          1485
##   0.5807013351  0.3772446313  0.1229252900  0.7332769396 -0.0805252598
##            1486          1487          1488          1489          1490
##   0.6315516646 -0.2331193262 -0.7417703167  0.4281072688 -0.5383013049
##            1491          1492          1493          1494          1495
##   0.6824450720  0.5298325437  0.5298325437 -0.1313940512  0.6824450720
##            1496          1497          1498          1499          1500
##   0.1737879275  0.8350022145  0.1737879275 -0.2331193262  0.3915958040
##            1501          1502          1503          1504          1505
##   1.5750272934  0.6191789945 -0.2911482606  0.1640181205  0.2095325558
##            1506          1507          1508          1509          1510
##   0.2095325558  0.7557311117 -0.2456327239 -0.3366803184  0.2550486432
##            1511          1512          1513          1514          1515
## -0.5187424652  0.2550486432  0.8467787061 -0.0635650701 -0.2911482606
##            1516          1517          1518          1519          1520
## -0.1090806068  1.0743288545  0.6191789945 -0.1090806068  0.5281479211
##            1521          1522          1523          1524          1525
##   1.3019340732  0.2095325558  0.6191789945  0.3915958040  1.5294842216
##            1526          1527          1528          1529          1530
##   1.1198719264  1.0288408530  0.2550486432  1.1198719264  1.5294842216
##            1531          1532          1533          1534          1535
##   0.6646945312  0.5281479211  0.5736634578  0.7102100679  1.1198719264
##            1536          1537          1538          1539          1540
##   0.2550486432 -0.2911482606  0.3460802673 -0.3442877531  0.4095550554
##            1541          1542          1543          1544          1545
## -0.2029304695  0.2682143029  0.1268689900  0.0326422017  0.6451277266
##            1546          1547          1548          1549          1550
##   0.8807174989  0.2682143029  0.7393545150  0.1268689900 -0.1086979808
##            1551          1552          1553          1554          1555
##   0.6922411208  0.8807174989 -0.1086979808  0.5037875441  1.3518514406
##            1556          1557          1558          1559          1560
##   0.1268689900  0.7864736095  0.4095550554  1.0220291797  0.8335756030
##            1561          1562          1563          1564          1565
##   0.4095550554  0.6451277266  1.1162559680  0.2211003387  0.3153282671
##            1566          1567          1568          1569          1570
##   1.5873899098  0.3153282671 -0.2029304695 -0.2029304695  0.0797555959
##            1571          1572          1573          1574          1575
##   0.4566684496  0.3624416613  1.3989363331  0.4566684496  0.5509009383
##            1576          1577          1578          1579          1580
## -0.4855994338 -1.0981020598  0.4566684496  0.5037875441  0.5509009383
##            1581          1582          1583          1584          1585
```

```
## -0.6740530105  0.5980143325  0.7864736095  0.7393545150 -0.6740530105
##          1586          1587          1588          1589          1590
##  0.9278023913 -0.2500438637 -0.8154216948  0.1268689900  0.5509009383
##          1591          1592          1593          1594          1595
##  0.5509009383  0.0326422017 -0.1086979808  0.6451277266  0.2211003387
##          1596          1597          1598          1599          1600
## -0.2029304695  0.7864736095 -0.4385145414  0.7393545150  0.2211003387
##          1601          1602          1603          1604          1605
##  0.5037875441  0.4095550554  0.5037875441  0.2211003387 -0.0144711925
##          1606          1607          1608          1609          1610
##  0.6922411208  0.8335756030  1.0220291797  0.4566684496  0.9278023913
##          1611          1612          1613          1614          1615
##  0.1739880846 -0.8764309976  1.1212846090 -0.2813748712  0.3136966830
##          1616          1617          1618          1619          1620
##  0.4837097805 -0.1538599055 -0.4938912431  0.2286901342 -0.7489160319
##          1621          1622          1623          1624          1625
##  0.2286901342 -0.6214062088 -0.5789029344 -0.1963631798 -0.1963631798
##          1626          1627          1628          1629          1630
## -0.7914193063 -0.2813748712  0.0586616090 -0.3238781455 -1.0464595228
##          1631          1632          1633          1634          1635
##  0.2711676958 -0.2813748712 -1.5989763770 -1.4714922666 -0.4513889972
##          1636          1637          1638          1639          1640
## -0.3238781455 -0.1963631798 -1.0039305356 -0.5363996600 -0.5789029344
##          1641          1642          1643          1644          1645
## -0.3663814199 -0.4938912431 -0.7064127575 -0.1113566311 -0.7064127575
##          1646          1647          1648          1649          1650
## -0.2388715968 -0.0263500823 -1.1314660716 -1.3864857179  0.2286901342
##          1651          1652          1653          1654          1655
## -0.8339277232 -0.8764309976 -0.3238781455 -0.7064127575 -0.0688533567
##          1656          1657          1658          1659          1660
## -1.1314660716 -0.4088852086  0.0161531921 -0.7914193063 -0.7489160319
##          1661          1662          1663          1664          1665
##  0.3987032317 -0.4088852086 -0.7064127575  0.3987032317 -0.1113566311
##          1666          1667          1668          1669          1670
## -0.6214062088  0.6537228780  0.0161531921 -0.9614529741 -0.8339277232
##          1671          1672          1673          1674          1675
## -0.6639094831 -0.4088852086 -1.5989763770 -1.2164726203 -0.7914193063
##          1676          1677          1678          1679          1680
##  0.0161531921 -1.0039305356 -0.0688533567 -0.4513889972 -1.3014791691
##          1681          1682          1683          1684          1685
## -1.1739436332 -0.7914193063 -0.8764309976 -0.3238781455 -0.5363996600
##          1686          1687          1688          1689          1690
## -0.9614529741  0.4837097805 -0.1963631798 -0.3663814199  0.0161531921
##          1691          1692          1693          1694          1695
## -0.9189342720 -1.0889370844 -0.5789029344 -1.3864857179 -0.8764309976
##          1696          1697          1698          1699          1700
## -0.5363996600 -0.4088852086  0.5261873421 -0.4513889972 -0.6214062088
##          1701          1702          1703          1704          1705
##  0.3136966830 -0.6068698558 -0.8318513607 -0.3443868967  0.2555637829
##          1706          1707          1708          1709          1710
## -1.0943252462 -1.3193067510 -0.0068919553 -1.0193314112 -0.4568776491
##          1711          1712          1713          1714          1715
## -0.3068854424 -0.6068698558 -0.2693885249 -0.0818994007 -1.0193314112
##          1716          1717          1718          1719          1720
```

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```
## -1.5818078575  0.1430957146 -0.4943750203 -0.6443667733 -1.2443129161
##          1721          1722          1723          1724          1725
## -0.3818838142 -0.1568977725 -0.4943750203 -0.2318916074 -0.8318513607
##          1726          1727          1728          1729          1730
## -1.2443129161 -0.9818571779 -0.4193807316 -1.3193067510  0.8655109867
##          1731          1732          1733          1734          1735
##  0.4950102305 -0.1224677870 -0.1636323241  0.0833598790  0.2068534903
##          1736          1737          1738          1739          1740
## -1.3162841880 -0.5752727145 -0.2871309160  0.0833598790  0.1245234200
##          1741          1742          1743          1744          1745
##  0.7419924725  0.2891825645  0.2891825645 -0.9869180854  0.0421903613
##          1746          1747          1748          1749          1750
## -0.3282954531  0.7831819125  0.4538456934 -0.6987912287 -0.1224677870
##          1751          1752          1753          1754          1755
##  0.2068534903  0.3715166193  0.1245234200 -0.3282954531 -0.0813032499
##          1756          1757          1758          1759          1760
## -0.0401387128 -0.4929436403 -0.5341330803 -0.4929436403  0.1245234200
##          1761          1762          1763          1764          1765
##  0.1656884551 -0.6164621545 -0.2459663789 -0.2047968612  0.1245234200
##          1766          1767          1768          1769          1770
## -0.2047968612 -0.1027300638 -1.1249049412 -0.4582745288  0.0750448573
##          1771          1772          1773          1774          1775
## -0.8138351253 -0.8582506254 -1.2137897132 -0.6804918358  0.2972514100
##          1776          1777          1778          1779          1780
## -0.8138351253 -1.1249049412  0.4750209539 -0.3249419936 -0.1471724497
##          1781          1782          1783          1784          1785
## -0.6804918358 -1.3915592571  0.5194902258 -1.2137897132 -1.5249133009
##          1786          1787          1788          1789          1790
## -1.3026744852 -0.0582876778 -1.3915592571 -0.2805006831 -1.6582135730
##          1791          1792          1793          1794          1795
## -1.0360201693 -0.5027169147 -0.6804918358  0.3861361820 -0.4582745288
##          1796          1797          1798          1799          1800
##  0.0306024714 -0.1027300638 -1.1693742131 -1.3026744852 -0.5916016867
##          1801          1802          1803          1804          1805
## -0.2805006831 -0.2360577594 -1.3915592571 -0.8582506254 -0.5471593007
##          1806          1807          1808          1809          1810
## -0.2360577594 -0.5916016867 -0.3693897568 -0.1916148357 -0.2805006831
##          1811          1812          1813          1814          1815
##  0.3861361820 -0.6360440727 -0.8138351253 -0.2360577594  0.1194872433
##          1816          1817          1818          1819          1820
##  0.0306024714 -0.6804918358 -0.5471593007 -1.3026744852 -0.3693897568
##          1821          1822          1823          1824          1825
## -0.9471353973  0.6972597697 -1.0360201693 -1.0804894412 -0.9027198973
##          1826          1827          1828          1829          1830
## -1.8360368887 -0.4138321428 -0.9027198973 -0.6360440727 -0.5471593007
##          1831          1832          1833          1834          1835
## -0.6804918358 -1.2137897132 -1.0804894412 -0.1471724497 -0.5471593007
##          1836          1837          1838          1839          1840
## -0.9027198973 -0.9916046692 -1.3471437570  0.4750209539 -1.2074997596
##          1841          1842          1843          1844          1845
##  0.0182692880 -0.6627135162 -0.7081398343  0.0182692880 -0.4357247410
##          1846          1847          1848          1849          1850
##  0.0636681416 -0.4811290875  0.1998701954 -0.4811290875 -0.4357247410
##          1851          1852          1853          1854          1855
```

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```
## -0.9805329560 -0.7535112234 -0.6627135162  0.2906679026 -0.0271301149
##          1856          1857          1858          1859          1860
## -0.3903258874 -0.8443089307 -0.0271301149 -0.3903258874 -0.4357247410
##          1861          1862          1863          1864          1865
## -1.5253191994 -0.6627135162  0.0636681416  0.2452690490 -0.1179272728
##          1866          1867          1868          1869          1870
##  0.4722688100  0.4722688100  0.0182692880  0.1998701954  0.1090669952
##          1871          1872          1873          1874          1875
## -0.5719267948 -0.5265279412 -0.3903258874 -1.0259043451 -0.2541293266
##          1876          1877          1878          1879          1880
## -0.3449270338  0.2452690490 -0.8897352488 -0.9805329560 -0.0271301149
##          1881          1882          1883          1884          1885
## -1.0259043451 -0.3903258874  0.0636681416 -0.7081398343 -1.1167020523
##          1886          1887          1888          1889          1890
## -0.6627135162  0.1998701954 -1.4345214922 -1.2529260777 -0.5719267948
##          1891          1892          1893          1894          1895
## -0.0725295178  0.7900497995 -0.5719267948 -0.1633316193 -0.1179272728
##          1896          1897          1898          1899          1900
## -1.3890951740 -0.2087304730 -0.1179272728 -0.5719267948  0.3360667562
##          1901          1902          1903          1904          1905
## -0.4357247410  0.4722688100 -0.9805329560 -0.9351066379 -0.1179272728
##          1906          1907          1908          1909          1910
## -0.1633316193 -0.5719267948 -0.3903258874 -1.0713306632 -1.5707455175
##          1911          1912          1913          1914          1915
## -0.1179272728 -0.4357247410 -0.5719267948 -0.9805329560  0.3912490214
##          1916          1917          1918          1919          1920
## -1.6700247394 -0.5456822612  0.0633233559 -1.3420764020 -0.1709108818
##          1921          1922          1923          1924          1925
## -0.3582937375 -1.1078194926  0.0633233559 -0.1240651679  0.1570147837
##          1926          1927          1928          1929          1930
##  0.0164770751 -0.0303692056  0.5786262092 -0.3582937375  0.3912490214
##          1931          1932          1933          1934          1935
## -0.7799278348 -1.4357678299 -0.5925449791 -0.8736192627 -0.5456822612
##          1936          1937          1938          1939          1940
##  0.4380947353  0.2975575935 -0.7799278348 -0.4051394515 -0.5925449791
##          1941          1942          1943          1944          1945
## -0.1709108818 -0.4519908334 -1.5294592578  0.2975575935 -0.3582937375
##          1946          1947          1948          1949          1950
## -0.4519908334 -0.4519908334 -0.7330537811 -0.1240651679 -0.8267452089
##          1951          1952          1953          1954          1955
## -0.0303692056 -0.2646023097  0.1101690698 -0.5307956687 -0.3410201069
##          1956          1957          1958          1959          1960
## -0.9419715881 -0.7205674037 -0.6256815362 -1.2898826088 -0.7205674037
##          1961          1962          1963          1964          1965
## -0.3726487294 -1.4480448554 -0.8470857206 -0.0563471971 -0.9103429656
##          1966          1967          1968          1969          1970
## -0.7205674037 -1.0052326599 -1.0052326599 -0.5940529137 -0.5307956687
##          1971          1972          1973          1974          1975
## -0.7838284756 -0.5624242912 -0.7521998531 -0.3726487294 -0.6256815362
##          1976          1977          1978          1979          1980
## -1.1001108738 -1.0052326599 -0.6256815362 -0.7838284756 -0.4991632194
##          1981          1982          1983          1984          1985
## -1.1950158754 -1.1950158754 -0.3726487294 -0.4675345969 -0.5307956687
##          1986          1987          1988          1989          1990
```

```
## -1.1317586304 -0.4359059744 -1.1001108738 -1.0052326599 -1.3215303654
##          1991          1992          1993          1994          1995
## -0.5307956687 -0.7521998531 -0.5624242912 -0.8470857206 -0.9419715881
##          1996          1997          1998          1999          2000
## -1.5113021004 -0.6889395466 -0.1828616871 -1.7959788371 -0.5307956687
##          2001          2002          2003          2004          2005
## -1.2582731204 -1.2898826088 -0.8154570981 -0.6573105414 -0.5624242912
##          2006          2007          2008          2009          2010
## -0.9103429656 -0.9419715881 -1.0052326599 -0.7838284756 -0.9736040374
##          2011          2012          2013          2014          2015
##  0.6358733582 -0.2973405657  0.5581035710 -0.5306358139 -0.0251439583
##          2016          2017          2018          2019          2020
##  0.4414559469  0.2859210770  0.5581035710  0.0137385830 -0.2195613696
##          2021          2022          2023          2024          2025
## -0.0251439583 -0.4917297501  0.7136525544 -0.0251439583 -0.9583202464
##          2026          2027          2028          2029          2030
##  0.2081555239 -0.1806788282  0.3248036184 -0.2195613696  0.3248036184
##          2031          2032          2033          2034          2035
## -0.7250249983 -0.0251439583  0.2470385357 -0.2973405657 -0.2973405657
##          2036          2037          2038          2039          2040
## -0.1806788282  0.0915036658  0.3248036184  0.4025734056  0.1692725121
##          2041          2042          2043          2044          2045
## -0.7250249983 -0.2195613696  0.3636861597  0.3248036184 -0.1806788282
##          2046          2047          2048          2049          2050
##  0.0137385830  0.6747464906 -0.5306358139 -0.3361995847  0.2470385357
##          2051          2052          2053          2054          2055
## -0.0640264997 -0.1417915824 -0.2195613696 -0.3751056485 -0.2584439109
##          2056          2057          2058          2059          2060
##  0.4803384883  0.7914176371 -0.3751056485  0.0915036658  0.0137385830
##          2061          2062          2063          2064          2065
## -0.0251439583  0.5581035710  0.3636861597  0.0137385830  0.1303909116
##          2066          2067          2068          2069          2070
## -0.8805551637 -0.3751056485 -0.3751056485 -0.3361995847 -0.2973405657
##          2071          2072          2073          2074          2075
##  0.4803384883  0.4803384883 -0.0251439583 -0.1029090410  1.5301624004
##          2076          2077          2078          2079          2080
##  0.1692725121  0.4414559469  0.4414559469 -0.7250249983 -0.3361995847
##          2081          2082          2083          2084          2085
## -0.7250249983  0.4025734056  0.1303909116  0.3248036184 -0.0251439583
##          2086          2087          2088          2089          2090
## -0.1029090410  0.5581035710 -0.1806788282 -0.0251439583 -0.0456001364
##          2091          2092          2093          2094          2095
##  0.0698351317  0.2429775594  0.1852627185 -1.2576250620  0.5892693977
##          2096          2097          2098          2099          2100
## -0.2187425641  1.4549745532 -1.3730533472 -0.3918919747 -0.1033142789
##          2101          2102          2103          2104          2105
## -0.0456001364  0.4161199871 -0.5073202598  1.2818670403 -0.1610284215
##          2106          2107          2108          2109          2110
##  0.7046976829  0.3006917020  1.1664387552  0.5892693977  0.0121140062
##          2111          2112          2113          2114          2115
##  0.7046976829  0.2429775594  0.7046976829 -0.3918919747  1.6281518957
##          2116          2117          2118          2119          2120
##  1.0510104701  0.5315552552  0.9355821849  0.0698351317 -0.5650553513
##          2121          2122          2123          2124          2125
```

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```
## -1.0267684918  0.6469835403  0.8201329509 -0.1610284215 -0.1033142789
##          2126          2127          2128          2129          2130
##  2.1475442640 -0.2187425641  2.1475442640  1.4549745532  0.9355821849
##          2131          2132          2133          2134          2135
##  0.0121140062  0.0845543424 -0.4984260335 -0.3318580526 -0.0820086003
##          2136          2137          2138          2139          2140
## -0.3318580526 -0.0820086003  0.0845543424  0.0012723672 -0.2069358455
##          2141          2142          2143          2144          2145
## -0.4984260335 -0.0820086003  0.0845543424 -0.7482603711  0.2094815877
##          2146          2147          2148          2149          2150
##  0.0845543424 -0.6233633551 -0.2902173169 -0.0820086003 -1.2063336546
##          2151          2152          2153          2154          2155
## -1.0397707119 -0.1652951098  0.2094815877 -0.2069358455 -0.9564892405
##          2156          2157          2158          2159          2160
## -0.3734987882 -0.0820086003 -0.1652951098 -0.0403688722  0.0429136067
##          2161          2162          2163          2164          2165
##  0.2927630590  0.0845543424 -0.4567852978 -0.7066448265 -0.3318580526
##          2166          2167          2168          2169          2170
## -0.2485765812 -0.5816974284 -0.1236543742 -0.4984260335  0.2511223233
##          2171          2172          2173          2174          2175
## -0.0403688722 -0.6233633551 -0.5400818838 -0.5816974284 -0.3318580526
##          2176          2177          2178          2179          2180
## -0.0403688722 -0.1236543742 -0.9148233139 -0.1652951098 -0.0820086003
##          2181          2182          2183          2184          2185
## -1.0482427861 -1.1507870333 -0.1253197475 -0.4329524890  0.6950714509
##          2186          2187          2188          2189          2190
## -1.6122051279 -1.8173556575  0.3361417718 -0.1253197475  0.6437683097
##          2191          2192          2193          2194          2195
##  0.2848696482  0.2335913211 -0.7918511506 -1.0994838921 -1.0994838921
##          2196          2197          2198          2199          2200
##  0.3361417718 -0.6893069034 -1.6122051279 -0.3816803655 -0.3304082419
##          2201          2202          2203          2204          2205
## -0.3304082419  1.0539452984 -0.5867750633 -1.0482427861  0.0285022064
##          2206          2207          2208          2209          2210
##  0.1310470739 -0.3304082419  0.2848696482 -0.0740414204 -0.8431542918
##          2211          2212          2213          2214          2215
##  0.0285022064  0.3361417718 -0.0740414204 -0.6380657975 -0.9456985390
##          2216          2217          2218          2219          2220
## -1.1507870333 -0.7406100446  0.6437683097 -0.3304082419 -0.9969396449
##          2221          2222          2223          2224          2225
## -0.0227705376 -1.4071166336  0.5342259137  0.6214310053  0.0546738642
##          2226          2227          2228          2229          2230
##  0.0546738642 -0.0325148761 -0.8172182861 -0.6428397505 -0.4684717642
##          2231          2232          2233          2234          2235
##  0.0546738642  0.7521885339 -0.5992714897  0.3162474696  0.1418636595
##          2236          2237          2238          2239          2240
## -0.2068986862 -0.1197094185 -0.2504933201  0.6649992661  0.7521885339
##          2241          2242          2243          2244          2245
##  0.3598421035 -0.1197094185  0.6649992661  1.1445666119  0.4470313713
##          2246          2247          2248          2249          2250
##  0.0110787029  0.0546738642 -0.0325148761 -0.0325148761 -0.2940879540
##          2251          2252          2253          2254          2255
##  0.3162474696 -0.2068986862  0.4470313713  0.0546738642 -0.9480285608
##          2256          2257          2258          2259          2260
```

```
##  -0.5556504827   0.1854582934   0.2290529272  -0.2068986862   0.0546738642
##           2261           2262           2263           2264           2265
##  -0.0761147846  -0.1197094185   0.5342259137   0.1854582934  -1.4711641674
##           2266           2267           2268           2269           2270
##   0.0546738642   0.8829988086   1.2753241406  -0.0761147846   0.2290529272
##           2271           2272           2273           2274           2275
##   0.1418636595   0.0110787029  -0.2068986862   0.0982690256  -0.2504933201
##           2276           2277           2278           2279           2280
##   0.1418636595   0.1418636595   0.2290529272   0.6649992661  -0.0761147846
##           2281           2282           2283           2284           2285
##  -0.2068986862  -0.5556504827  -0.0761147846   0.1854582934  -0.2504933201
##           2286           2287           2288           2289           2290
##  -0.0325148761   0.1418636595   0.4906260052   0.0546738642  -0.1197094185
##           2291           2292           2293           2294           2295
##   0.0982690256   0.6214310053   0.4906260052   0.1418636595  -0.1197094185
##           2296           2297           2298           2299           2300
##  -0.0761147846   0.2290529272   0.0546738642  -0.1197094185   1.0936281669
##           2301           2302           2303           2304           2305
##  -0.4584041481   0.0767836512  -1.3147201676  -1.7963509829  -0.0302526136
##           2306           2307           2308           2309           2310
##  -0.9400608644  -0.0302526136   0.4514170533  -0.2443316185  -0.6724831530
##           2311           2312           2313           2314           2315
##   0.1838263913  -0.8865751084  -1.5822784533   0.5049416610   0.4514170533
##           2316           2317           2318           2319           2320
##   0.7725193725  -1.9569377565  -0.4584041481   1.4147369613  -0.6189650206
##           2321           2322           2323           2324           2325
##  -1.5822784533   0.0232655188  -1.0470971292   0.1303017836  -0.4048860157
##           2326           2327           2328           2329           2330
##  -0.1705046883  -0.2129318460  -0.0007960575   0.3386314707  -0.0007960575
##           2331           2332           2333           2334           2335
##   0.1689177066   0.3810586284   0.5507672592  -0.0007960575  -0.0432232152
##           2336           2337           2338           2339           2340
##   0.7629235810  -0.1705046883  -0.0432232152  -0.4250881678  -0.0432232152
##           2341           2342           2343           2344           2345
##   0.0416362335  -0.1280775306   0.1264900356   0.9326322118  -0.4250881678
##           2346           2347           2348           2349           2350
##  -0.0432232152  -0.5099424832  -0.5947967986   0.3810586284  -0.0856503729
##           2351           2352           2353           2354           2355
##   0.1264900356  -0.2553590036   0.7629235810  -0.1280775306   0.0840623646
##           2356           2357           2358           2359           2360
##  -0.0856503729  -0.2129318460   0.9326322118  -0.2977912947   0.5931995502
##           2361           2362           2363           2364           2365
##   0.1264900356  -0.6796511139  -0.0856503729  -0.9441402551  -0.7516906791
##           2366           2367           2368           2369           2370
##   0.6726317998  -0.8286798246  -0.5976984156  -0.6361929883  -0.0587650822
##           2371           2372           2373           2374           2375
##  -1.0596472610  -0.0587650822  -0.9056689701  -0.4437154671  -0.2512379459
##           2376           2377           2378           2379           2380
##   0.1337124390  -0.8286798246  -0.4437154671  -0.8671511096  -0.9056689701
##           2381           2382           2383           2384           2385
##  -0.2512379459   0.2876860724   0.2876860724  -0.0972596549  -0.5207046125
##           2386           2387           2388           2389           2390
##   0.4416643634  -0.3667263216   0.2107015845  -0.3667263216  -0.5976984156
##           2391           2392           2393           2394           2395
```

```
##   0.0952178663   0.0952178663  -0.0202705094  -0.3667263216   0.0182287208
##           2396           2397           2398           2399           2400
## -0.4437154671  -0.9826581155  -1.4060751279   0.1722070118   0.5186535089
##           2401           2402           2403           2404           2405
##   0.0952178663   0.2876860724  -0.2512379459   0.4031930784   0.4031930784
##           2406           2407           2408           2409           2410
##   0.0567232936  -0.1357542276   0.2876860724  -0.2897371761  -0.1357542276
##           2411           2412           2413           2414           2415
## -1.4445929884  -0.0202705094  -0.2512379459   0.5956426543   0.5956426543
##           2416           2417           2418           2419           2420
## -0.6361929883  -0.7516906791   1.1730845331  -0.3282317489   0.5956426543
##           2421           2422           2423           2424           2425
## -0.7516906791  -0.2512379459  -1.2520968370  -0.7516906791  -0.7516906791
##           2426           2427           2428           2429           2430
## -0.7516906791  -0.2512379459   0.7496209453  -0.4052208943   0.0182287208
##           2431           2432           2433           2434           2435
##   0.4416643634  -0.7516906791  -0.1357542276   0.3388199775  -1.1989248706
##           2436           2437           2438           2439           2440
## -0.6047956027  -0.6397433882  -0.5698478172  -0.2553135188  -1.2338515141
##           2441           2442           2443           2444           2445
## -1.2338515141  -0.8494470151  -1.2688204417  -0.8494470151   0.0941643366
##           2446           2447           2448           2449           2450
## -0.6047956027  -1.2688204417  -1.3736426563  -0.1155181483  -0.4650044605
##           2451           2452           2453           2454           2455
## -0.3252095128  -0.5698478172  -0.3252095128  -0.8144780876  -0.4300566750
##           2456           2457           2458           2459           2460
## -0.3155358766  -1.4361743428   0.2691445496  -0.0231956635  -0.2668134903
##           2461           2462           2463           2464           2465
## -0.7053208624   0.3665893223  -0.7053208624   0.1716997769  -0.1206410257
##           2466           2467           2468           2469           2470
##   0.9512520635  -0.6565984761   0.1716997769  -0.6565984761   0.0742491092
##           2471           2472           2473           2474           2475
## -0.5591478084  -0.4129806493  -0.5104254220  -0.7540609339   1.0486968362
##           2476           2477           2478           2479           2480
##   0.5614729727  -1.2899777086   0.1716997769   0.6589177454  -0.8515057066
##           2481           2482           2483           2484           2485
## -0.8515057066  -1.6310638882   0.1229773906   0.0742491092  -0.2180852089
##           2486           2487           2488           2489           2490
## -1.1438400247  -1.3387295701   0.0255267228   0.5127800615  -0.3155358766
##           2491           2492           2493           2494           2495
## -0.9976433905   0.8051143796  -0.7540609339   0.1716997769   0.1716997769
##           2496           2497           2498           2499           2500
##   0.2204221633  -0.6078760897   0.7076696069  -0.4129806493  -0.0231956635
##           2501           2502           2503           2504           2505
## -0.3642582630  -0.4129806493  -0.2180852089   0.0742491092  -0.0719180499
##           2506           2507           2508           2509           2510
##   0.0742491092  -1.3874224813  -1.0950881632  -0.8515057066  -0.4189952756
##           2511           2512           2513           2514           2515
##   0.0381229426   0.3705593010  -0.2527594995   0.7445646588   0.7860934366
##           2516           2517           2518           2519           2520
## -1.3747604899   0.4536671336  -0.3774262762  -0.2943184435  -0.9176422717
##           2521           2522           2523           2524           2525
## -0.4189952756  -0.0034360015  -0.3774262762  -0.0034360015  -0.2112055832
##           2526           2527           2528           2529           2530
```

```
## -0.0449899178  0.2874464406  1.0769959895  0.1212302725  0.8692012693
##          2531          2532          2533          2534          2535
## -0.9176422717  0.9523091020 -0.5852109410  0.7860934366 -0.1280977505
##          2536          2537          2538          2539          2540
##  0.6614568261 -0.2943184435 -0.7929553842  0.0796758534 -0.1280977505
##          2541          2542          2543          2544          2545
## -0.2112055832  0.8276724915 -0.1696516668  0.0796758534 -0.8345344390
##          2546          2547          2548          2549          2550
##  0.4121132173  0.3705593010 -0.4189952756 -0.2527594995  1.0354169347
##          2551          2552          2553          2554          2555
## -0.8760632169  0.0381229426  0.2458925242 -0.0865438342  0.0381229426
##          2556          2557          2558          2559          2560
## -0.1280977505  0.3290053846 -0.1888593479 -1.2656635599  1.3186681124
##          2561          2562          2563          2564          2565
##  0.2957100064 -0.2965391828 -0.4580664265 -0.2965391828 -0.1888593479
##          2566          2567          2568          2569          2570
## -0.1350184533 -0.7811143998  0.1341892768 -1.0503025872  0.4572242216
##          2571          2572          2573          2574          2575
## -0.5119066697  0.2957100064  0.6187775224 -0.7272741566 -0.6195871561
##          2576          2577          2578          2579          2580
## -1.6425778333  0.1880295200 -0.8888144289  0.0265022762  0.0265022762
##          2581          2582          2583          2584          2585
## -0.7811143998  0.1341892768 -1.2656635599  0.7264580087  0.0803490337
##          2586          2587          2588          2589          2590
##  0.7264580087  0.4033970069 -0.2427002425 -0.8888144289  0.0803490337
##          2591          2592          2593          2594          2595
## -1.7502583196 -0.9964949152 -0.1888593479 -0.4042261834 -0.1350184533
##          2596          2597          2598          2599          2600
## -0.7811143998  0.8341384951 -0.5119066697 -0.0273379669 -0.6195871561
##          2601          2602          2603          2604          2605
## -0.0273379669 -1.0503025872  0.1880295200  0.2418697632 -0.9964949152
##          2606          2607          2608          2609          2610
## -0.2965391828  0.0265022762  0.3495502495 -1.6425778333 -0.7811143998
##          2611          2612          2613          2614          2615
##  0.6725851943 -0.5119066697  0.4033970069 -1.4810245326 -0.6195871561
##          2616          2617          2618          2619          2620
## -0.2427002425  0.7264580087  1.4263485987  0.7264580087 -0.6734273992
##          2621          2622          2623          2624          2625
## -0.7272741566  0.6187775224 -0.5119066697 -0.0273379669 -0.5119066697
##          2626          2627          2628          2629          2630
##  0.1880295200 -0.6195871561  0.5649047080  0.5533372151  0.2917479552
##          2631          2632          2633          2634          2635
## -0.2314305646  0.3789426169 -1.1469903368 -1.3650033656 -0.3622383819
##          2636          2637          2638          2639          2640
## -0.4494330435  0.3789426169 -0.1442306280  0.4661425535 -0.8417826463
##          2641          2642          2643          2644          2645
## -0.0134386355 -0.2750278954 -0.3622383819 -0.3186252262 -0.6673933230
##          2646          2647          2648          2649          2650
##  0.3353452861 -0.2750278954 -0.0134386355  0.0301586953  0.5533372151
##          2651          2652          2653          2654          2655
##  0.4225452227 -0.0134386355  0.8585396307  0.5969345460  0.2917479552
##          2656          2657          2658          2659          2660
## -0.5366277052 -1.0597956751  0.1173586319 -0.7110170285  0.5969345460
##          2661          2662          2663          2664          2665
```

```
##  0.2917479552  0.5533372151  0.2917479552  0.5969345460  0.0301586953
##          2666          2667          2668          2669          2670
##  0.2917479552  0.6405318768  0.2045527661 -0.4930039997  0.5533372151
##          2671          2672          2673          2674          2675
##  0.4661425535  0.2045527661  0.2917479552  0.1173586319 -0.4494330435
##          2676          2677          2678          2679          2680
##  0.1609549078 -0.1006332971 -0.5801986613 -0.1006332971  0.2045527661
##          2681          2682          2683          2684          2685
##  0.2481506244  0.3789426169  0.6405318768 -0.5801986613  0.6405318768
##          2686          2687          2688          2689          2690
##  0.5969345460 -0.2750278954 -0.0570359663  0.2127041285 -0.1137695575
##          2691          2692          2693          2694          2695
##  0.0494687668  0.0902778541  0.2535127221 -0.5218752433  0.3351348468
##          2696          2697          2698          2699          2700
##  0.0902778541  0.0494687668 -0.5626591493 -0.5626591493 -0.1545781511
##          2701          2702          2703          2704          2705
##  0.4983692210  0.0902778541 -0.1137695575 -0.1137695575  0.0494687668
##          2706          2707          2708          2709          2710
##  0.0086611607 -0.2770039318 -0.4402580562  0.3759434403 -0.5218752433
##          2711          2712          2713          2714          2715
##  0.1718955350  0.1310869414 -1.0523622721  0.3351348468  0.2535127221
##          2716          2717          2718          2719          2720
##  0.5799814707 -0.1545781511  0.3351348468 -0.6442763364 -0.3586260565
##          2721          2722          2723          2724          2725
##  0.8656663132  0.4167520339  0.2127041285  0.1718955350  0.6615986578
##          2726          2727          2728          2729          2730
##  0.7840491261 -0.2770039318 -0.6442763364 -0.1953867447  0.1310869414
##          2731          2732          2733          2734          2735
## -0.3586260565 -0.0321523704 -0.1545781511 -0.5218752433  0.5799814707
##          2736          2737          2738          2739          2740
## -0.2361953382  0.0902778541  0.0494687668  0.3759434403 -0.3586260565
##          2741          2742          2743          2744          2745
## -0.6851096176 -0.2770039318  0.9064502192 -0.3994346500  0.2943262532
##          2746          2747          2748          2749          2750
##  0.1718955350 -0.0042760705 -0.4031140778  0.0455779264  0.1452859202
##          2751          2752          2753          2754          2755
## -0.3034060839  0.6937100460 -0.5028220716  0.4942759625 -0.2535520870
##          2756          2757          2758          2759          2760
## -0.6025360974 -0.0042760705 -0.0042760705 -0.1039852707 -0.0042760705
##          2761          2762          2763          2764          2765
## -0.4031140778  0.1951459491  0.3447079398  0.3447079398  0.8432418771
##          2766          2767          2768          2769          2770
##  0.6438258894  0.5940020522  0.1951459491  0.2449999460  0.2449999460
##          2771          2772          2773          2774          2775
##  0.7435338832  0.2948539429 -0.0541306706 -0.9016781747  0.6438258894
##          2776          2777          2778          2779          2780
##  1.0426578647  0.2449999460  1.3916660029 -0.1039852707  0.7435338832
##          2781          2782          2783          2784          2785
##  0.3447079398  0.4444159337  0.4444159337 -0.0042760705  0.2948539429
##          2786          2787          2788          2789          2790
## -0.0541306706  0.4444159337 -0.3532600809  0.2948539429  0.3945619368
##          2791          2792          2793          2794          2795
##  0.1951459491 -0.2036980901 -0.3034060839  0.4942759625  0.7435338832
##          2796          2797          2798          2799          2800
```

```
##   0.8931260337   0.4444159337  -0.1538380613  -0.0541306706  -0.4529680747
##           2801           2802           2803           2804           2805
##   1.1922500152   0.1452859202   0.0954319233  -0.0042760705   0.4444159337
##           2806           2807           2808           2809           2810
##  -0.9515020119  -0.1039852707  -0.2651709124  -0.6437846000  -0.5680590224
##           2811           2812           2813           2814           2815
##  -0.3030318689   0.6056439132  -0.1515889591  -0.4544775272  -0.7952174320
##           2816           2817           2818           2819           2820
##  -0.5301985240  -0.3030318689  -0.1137284607  -0.4923380256  -1.1359848218
##           2821           2822           2823           2824           2825
##  -0.5301985240  -0.0001423846  -0.3030318689  -0.4166170288   0.4542019196
##           2826           2827           2828           2829           2830
##  -0.0380028830  -1.3631478123  -0.1894494575  -1.5524732084  -0.6816450984
##           2831           2832           2833           2834           2835
##  -0.3408914511  -0.0380028830  -1.2117058186  -0.1515889591  -0.0380028830
##           2836           2837           2838           2839           2840
##   0.1134391106   0.0945879340   0.2324964810   0.2324964810   0.0026526105
##           2841           2842           2843           2844           2845
##   0.1405611575   0.6002600218   0.4623292280   0.6461998750  -1.1925065948
##           2846           2847           2848           2849           2850
##  -0.0433150512  -0.9167006243  -0.5029972304   0.0026526105  -0.5489704538
##           2851           2852           2853           2854           2855
##  -0.5029972304  -1.4683125653   0.2324964810  -0.0433150512  -0.6409057773
##           2856           2857           2858           2859           2860
##  -0.1352514870  -1.0546314182  -0.6409057773  -0.5949381156  -0.2731589217
##           2861           2862           2863           2864           2865
##  -0.2731589217  -0.5489704538  -0.9167006243   0.0486202723  -0.5489704538
##           2866           2867           2868           2869           2870
##  -0.3191265834   0.0945879340  -0.1812180364  -0.2271912599  -1.0546314182
##           2871           2872           2873           2874           2875
##   0.4163726897  -0.7328299774   0.9220058454  -0.2731589217  -0.2271912599
##           2876           2877           2878           2879           2880
##  -0.4110619069  -0.5029972304  -0.0892832691   0.2784641427  -0.5949381156
##           2881           2882           2883           2884           2885
##   0.0026526105   0.1865288192  -1.1465667417  -0.0433150512   0.7841306688
##           2886           2887           2888           2889           2890
##   0.4163726897   0.0026526105   0.3703994662  -0.9626960947  -0.4611623339
##           2891           2892           2893           2894           2895
##  -0.5009055091   0.1349901028  -0.5009055091   0.2144764532  -0.1829552988
##           2896           2897           2898           2899           2900
##  -0.8983324525  -0.1829552988  -0.0637209646  -0.1432073150   0.0555085610
##           2901           2902           2903           2904           2905
##  -0.5406486843   0.0157653858   0.0952565448  -1.1765587220  -0.3419284805
##           2906           2907           2908           2909           2910
##  -0.7393693689  -0.8586133203  -1.0573051533  -0.0637209646   0.1349901028
##           2911           2912           2913           2914           2915
##  -0.1829552988  -0.6201350347  -0.4611623339  -0.1432073150  -0.8983324525
##           2916           2917           2918           2919           2920
##  -0.5803918595  -0.6996213851  -0.8188557193  -0.8983324525  -0.8188557193
##           2921           2922           2923           2924           2925
##  -1.1367915037  -0.5406486843  -0.1034641398  -0.1432073150   0.6516754235
##           2926           2927           2928           2929           2930
##   0.1349901028   0.4132163723  -0.8188557193  -0.2624416492   0.5324218548
##           2931           2932           2933           2934           2935
```

```
## -0.0637209646 -0.3021848244 -0.4611623339 -0.5406486843 -0.2624416492
##          2936          2937          2938          2939          2940
## -0.3816721366  0.0952565448 -0.4214143501 -0.5009055091 -0.1829552988
##          2941          2942          2943          2944          2945
## -0.2226984740 -0.1034641398 -0.1432073150 -0.5803918595 -1.3355314228
##          2946          2947          2948          2949          2950
## -0.4214143501  0.0157653858 -0.6598782099 -0.3419284805 -0.2226984740
##          2951          2952          2953          2954          2955
## -0.2624416492 -0.3816721366 -0.5406486843 -0.4214143501 -0.3021848244
##          2956          2957          2958          2959          2960
##  0.0952565448 -0.8188557193 -0.8586133203 -1.3355314228 -0.2226984740
##          2961          2962          2963          2964          2965
## -0.5406486843 -0.6598782099 -1.3355314228 -1.1367915037 -0.6201350347
##          2966          2967          2968          2969          2970
## -0.8983324525 -1.3355314228 -0.0637209646 -0.4214143501 -0.8586133203
##          2971          2972          2973          2974          2975
## -0.4214143501 -0.8983324525 -0.8239497241  0.5053244726 -0.2615732899
##          2976          2977          2978          2979          2980
##  0.3519473944  0.0963127450  0.0963127450 -1.0284524950  0.3008217017
##          2981          2982          2983          2984          2985
## -0.3126989827 -0.0059392591  0.1985641304 -0.4149503681  0.5053244726
##          2986          2987          2988          2989          2990
##  0.0963127450 -0.5172079394 -0.7216983387 -0.3126989827 -1.0284524950
##          2991          2992          2993          2994          2995
##  0.3519473944 -1.3352066514 -0.5172079394 -0.6706035750  0.0963127450
##          2996          2997          2998          2999          3000
## -0.6706035750 -1.3352066514 -0.3638246754 -0.6194593248  0.0451870522
##          3001          3002          3003          3004          3005
## -1.2841118877  0.5053244726  0.4030730871  0.8120724431 -0.5683336321
##          3006          3007          3008          3009          3010
##  0.6587262940  0.9654804503  0.2496960089 -0.4149503681 -0.3126989827
##          3011          3012          3013          3014          3015
##  0.1985641304  0.3008217017 -0.4149503681  0.5564563511 -0.1593219045
##          3016          3017          3018          3019          3020
## -0.3126989827  0.6075696722  0.1985641304 -0.7728549604 -0.0570655704
##          3021          3022          3023          3024          3025
##  0.3519473944 -0.4149503681  0.6075696722 -0.6194593248 -0.0059392591
##          3026          3027          3028          3029          3030
##  0.2496960089 -0.5683336321 -0.4660760609  1.1699832213 -1.0796091168
##          3031          3032          3033          3034          3035
##  0.2496960089 -0.3638246754 -0.0059392591  0.0451870522 -0.0059392591
##          3036          3037          3038          3039          3040
## -0.0570655704 -0.3126989827 -0.4660760609 -0.6706035750  0.0963127450
##          3041          3042          3043          3044          3045
## -0.3638246754 -0.0570655704 -0.3638246754  0.8632290649 -0.3126989827
##          3046          3047          3048          3049          3050
## -0.1081900260  0.2496960089  0.4541987798  0.0451870522  0.0963127450
##          3051          3052          3053          3054          3055
## -0.4149503681 -0.0570655704 -0.0570655704 -0.2104475972 -0.3415967705
##          3056          3057          3058          3059          3060
## -0.3870991474  0.3146798415 -0.6116696799  0.1041419227  0.4409995361
##          3061          3062          3063          3064          3065
##  0.3988929712  0.5252177604  0.4409995361 -0.1063959962  0.2304616172
##          3066          3067          3068          3069          3070
```

```
##   0.3567864064   0.3567864064   0.1041419227   0.3567864064  -0.0221777719
##           3071           3072           3073           3074           3075
##   0.4409995361  -0.5695885878   0.1462484875   0.2304616172   0.1041419227
##           3076           3077           3078           3079           3080
##   0.4831061010   0.0620348484   0.1041419227  -0.6538017176   0.0199277741
##           3081           3082           3083           3084           3085
## -0.4011470447   0.2304616172   0.1041419227   0.1041419227   0.0199277741
##           3086           3087           3088           3089           3090
## -0.1906091259  -0.0642894313  -0.2327156907   0.6515272658  -0.3590404799
##           3091           3092           3093           3094           3095
##   0.1883550524   0.5252177604   0.2304616172   0.3988929712  -0.2748222556
##           3096           3097           3098           3099           3100
##   0.1462484875  -0.9064411067   0.0620348484   0.7778724332  -0.4432536096
##           3101           3102           3103           3104           3105
##   0.2304616172  -0.4432536096  -0.8643090690   0.3146798415   0.1883550524
##           3106           3107           3108           3109           3110
## -0.9064411067  -0.5695885878   0.6936593035   0.3988929712   0.9041666550
##           3111           3112           3113           3114           3115
## -0.3169288205   0.5252177604   0.2725732767   0.7357403955  -0.2327156907
##           3116           3117           3118           3119           3120
##   0.9041666550   0.0199277741   0.3146798415  -0.2748222556   0.3146798415
##           3121           3122           3123           3124           3125
##   0.1041419227  -0.8222279770  -0.1485025610  -0.7800959393   0.2304616172
##           3126           3127           3128           3129           3130
## -0.4011470447   0.7357403955   0.6094461738   0.0199277741   0.2304616172
##           3131           3132           3133           3134           3135
##   0.0620348484   0.2304616172   0.2725732767   0.4409995361   0.4409995361
##           3136           3137           3138           3139           3140
## -0.5274565502  -0.3169288205   0.3567864064  -0.3590404799  -0.1485025610
##           3141           3142           3143           3144           3145
## -0.4432536096  -1.2011615879   0.9883797847  -0.4432536096  -0.3169288205
##           3146           3147           3148           3149           3150
##   0.8199535252  -0.2748222556  -0.1485025610   0.5252177604  -0.4432536096
##           3151           3152           3153           3154           3155
## -0.5274565502   0.5252177604   0.3988929712   0.3146798415  -0.6538017176
##           3156           3157           3158           3159           3160
## -0.1906091259  -0.1906091259   0.1883550524  -0.3169288205   0.1462484875
##           3161           3162           3163           3164           3165
## -0.4011470447   0.6094461738   0.0199277741   0.6515272658  -0.0221777719
##           3166           3167           3168           3169           3170
## -0.2327156907  -0.0221777719  -0.6116696799  -0.4011470447  -0.8155969885
##           3171           3172           3173           3174           3175
## -0.4628158762   0.4443259736   0.4947528921  -0.1100347639   0.6963386148
##           3176           3177           3178           3179           3180
##   0.3939600308   0.0915570564   0.4947528921  -1.4707383922  -1.6723241149
##           3181           3182           3183           3184           3185
## -0.5636087376   0.1923499178   0.1923499178   0.1923499178  -1.4203724494
##           3186           3187           3188           3189           3190
## -0.0092358050  -0.8155969885  -0.4628158762   0.3435331122   0.3939600308
##           3191           3192           3193           3194           3195
##   0.6459116963  -0.0596383332  -0.1604311946  -0.6140051682   0.3435331122
##           3196           3197           3198           3199           3200
## -0.7147980296  -0.5636087376   0.4443259736  -0.6140051682   0.7971314762
##           3201           3202           3203           3204           3205
```

```
## -1.5211653108 -0.8155969885  0.4443259736 -0.1604311946 -0.7147980296
##          3206          3207          3208          3209          3210
## -0.5636087376 -0.8155969885 -0.4628158762 -0.8659934192 -0.9667740854
##          3211          3212          3213          3214          3215
## -0.0596383332  0.6963386148  0.2427463484 -0.4124194455 -0.4124194455
##          3216          3217          3218          3219          3220
## -0.3116217061  0.2931488767 -0.4628158762 -0.2612246657 -0.7652005579
##          3221          3222          3223          3224          3225
## -0.2108276252 -1.4707383922 -0.1604311946  0.2931488767 -0.3116217061
##          3226          3227          3228          3229          3230
## -1.1683598081 -0.3620169173 -1.0172010040  0.4443259736  0.2427463484
##          3231          3232          3233          3234          3235
## -0.6140051682 -0.5636087376  0.4443259736 -1.3699455309 -0.8659934192
##          3236          3237          3238          3239          3240
##  0.3435331122 -0.1604311946 -0.1100347639 -0.5132123069 -0.3116217061
##          3241          3242          3243          3244          3245
## -0.5636087376 -0.3116217061  0.0261321274 -0.3869555425  0.0949835432
##          3246          3247          3248          3249          3250
##  0.1982544195  0.3015211307 -1.2475628378 -0.8000348825 -0.6279375804
##          3251          3252          3253          3254          3255
## -0.0771387488 -1.0754238858 -1.1787155870  0.3703683815  0.0949835432
##          3256          3257          3258          3259          3260
## -0.8344793328 -0.3181082917  0.0605599178 -0.3181082917 -0.1459868326
##          3261          3262          3263          3264          3265
## -0.4902305837 -0.3525319171 -0.1459868326 -1.0754238858 -0.5590903295
##          3266          3267          3268          3269          3270
## -0.4902305837 -0.4213791679 -0.6967848312 -0.0082914980 -0.4213791679
##          3271          3272          3273          3274          3275
##  0.1294071686 -0.0771387488 -1.1098683361 -1.1442711366 -0.5246542092
##          3276          3277          3278          3279          3280
## -0.3525319171 -0.3525319171 -0.2836846663  0.6457573848 -0.1804096251
##          3281          3282          3283          3284          3285
## -0.9721738345 -0.3525319171 -0.4902305837  0.4736600827  0.1638307940
##          3286          3287          3288          3289          3290
## -0.2492610409 -0.0427151234 -0.0771387488 -0.2836846663  0.6113545844
##          3291          3292          3293          3294          3295
##  0.2326780449 -0.6623403808 -0.5934931300 -0.2836846663 -0.8688821333
##          3296          3297          3298          3299          3300
##  0.2326780449 -0.3181082917  0.3359655811 -0.3869555425  0.2671058353
##          3301          3302          3303          3304          3305
##  1.4796159415 -0.9115557859  0.6256161986  0.3978914293  1.4226504804
##          3306          3307          3308          3309          3310
##  1.0241471159  0.3409597203  0.2840300778 -0.0575629311 -0.6838317054
##          3311          3312          3313          3314          3315
##  0.3978914293  0.0562991091  0.5117541583  0.6256161986  1.0241471159
##          3316          3317          3318          3319          3320
##  1.6503745609  1.7073400220 -0.5699696651  0.5686851785  0.3409597203
##          3321          3322          3323          3324          3325
##  0.1132301292  0.2840300778  1.2518918610  0.6256161986 -0.3991421637
##          3326          3327          3328          3329          3330
##  0.7394851271 -0.6838317054  0.0562991091  0.8533471673 -0.3991421637
##          3331          3332          3333          3334          3335
## -0.0006319110  1.5365125206  0.6256161986  1.0241471159  0.7964161472
##          3336          3337          3338          3339          3340
```

```
##    1.3657539013  -0.2852938999  -0.1144939513   0.2270921695   0.9672092076
##            3341           3342           3343           3344           3345
##    0.2840300778   0.7964161472   0.7964161472   0.1132301292   0.8533471673
##            3346           3347           3348           3349           3350
##   -0.3422455846   0.1132301292   0.9102781874  -0.9684523650   0.6256161986
##            3351           3352           3353           3354           3355
##   -0.0575629311   0.7394851271   1.4796159415   0.8533471673   0.3978914293
##            3356           3357           3358           3359           3360
##    1.9919606816   0.5686851785  -0.9684523650   0.6256161986   1.4226504804
##            3361           3362           3363           3364           3365
##    1.7642366011   0.6689472542   1.1906415351  -0.6725259183   1.1906415351
##            3366           3367           3368           3369           3370
##    1.7868883032   0.5944200150   0.7434744933   1.6378356283   1.2651678726
##            3371           3372           3373           3374           3375
##   -0.1508595903   0.5944200150   0.5944200150   1.1906415351   1.1906415351
##            3376           3377           3378           3379           3380
##    1.9359409781   2.3830990028   1.0415888602  -0.0017978983   0.9670535057
##            3381           3382           3383           3384           3385
##    0.0727284392  -0.7470973412   0.7434744933   0.3708337890   2.2340463279
##            3386           3387           3388           3389           3390
##    2.4575802547   0.9670535057   1.7123168802   1.5632642053   0.5198954810
##            3391           3392           3393           3394           3395
##    1.1161151977   2.3085275798   1.0415888602   0.4453601264   1.8613695551
##            3396           3397           3398           3399           3400
##    1.9359409781  -0.0763332528   1.1906415351   1.5632642053   0.9670535057
##            3401           3402           3403           3404           3405
##    1.3396942100   0.7434744933   0.2217811141   0.5198954810   0.9670535057
##            3406           3407           3408           3409           3410
##   -1.5668419677  -0.5980446663  -0.0763332528   1.0415888602   0.2963074515
##            3411           3412           3413           3414           3415
##    1.4887559020   1.4887559020   1.4142205475   0.6689472542   2.5321516777
##            3416           3417           3418           3419           3420
##   -0.9706312681   2.1594749049   1.1161151977   1.0415888602   0.2963074515
##            3421           3422           3423           3424           3425
##    0.6689472542   1.7123168802   1.4887559020   1.4887559020  -0.2999393165
##            3426           3427           3428           3429           3430
##    1.4142205475   2.0104222300   1.1906415351   1.3396942100   1.4142205475
##            3431           3432           3433           3434           3435
##    1.3396942100  -0.2857621714  -0.3175755924   0.0005586177  -0.0312471049
##            3436           3437           3438           3439           3440
##    0.0641854598  -0.2539487503  -0.6039002306   0.0641854598   1.4038437178
##            3441           3442           3443           3444           3445
##    1.0378672218   1.5606483175   0.5673522177   0.6196309598   0.8810309956
##            3446           3447           3448           3449           3450
##    0.6719097019  -0.0600053381   0.8810309956   1.1947287494   1.5606483175
##            3451           3452           3453           3454           3455
##    1.2469758649   1.8220736545   0.8810309956  -0.1122840802   0.8287522535
##            3456           3457           3458           3459           3460
##    1.1424183807   0.7241884440   0.8810309956   0.0445584714   1.1424183807
##            3461           3462           3463           3464           3465
##    0.7764735114  -1.1055991559   0.8810309956   0.5673522177   0.6196309598
##            3466           3467           3468           3469           3470
##    0.2536734398   0.5673522177   0.2013946977   0.1491159556  -1.0010416717
##            3471           3472           3473           3474           3475
```

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##   0.5150728431   0.5150728431  -0.0077265960   1.7697632859   0.4105159914
##           3476           3477           3478           3479           3480
##   0.8287522535   0.0445584714   1.4560908333   0.5673522177   1.5084012020
##           3481           3482           3483           3484           3485
##   1.6652058017   0.0968372135   1.2992862336  -0.3213863979   0.2536734398
##           3486           3487           3488           3489           3490
##   0.3582309240  -0.0077265960   0.8810309956   0.7764735114   0.1491159556
##           3491           3492           3493           3494           3495
##   0.8587343763  -0.2197903876   0.3478643462   0.2910933785   0.6316848452
##           3496           3497           3498           3499           3500
## -0.4468467867  -0.1062621880   0.4613925458  -0.6739306579   0.0640362925
##           3501           3502           3503           3504           3505
##   0.4046284460  -0.5036177545  -0.2197903876   0.2343292787   0.8587343763
##           3506           3507           3508           3509           3510
## -0.1062621880   0.2343292787  -0.4468467867   0.4613925458  -1.1280434562
##           3511           3512           3513           3514           3515
## -0.4468467867  -0.2765544874   0.4613925458   0.4613925458  -0.6739306579
##           3516           3517           3518           3519           3520
## -1.2415716558  -0.8441886172  -0.0494912203  -0.1630262878   0.1208010791
##           3521           3522           3523           3524           3525
##   0.5749207454   0.5181566456  -1.0145152566  -0.7874588575   0.8587343763
##           3526           3527           3528           3529           3530
## -0.3900826870   1.2561174148  -0.2197903876   0.2548677411  -0.0378434819
##           3531           3532           3533           3534           3535
##   0.4012292552  -0.5257073255  -0.2329901999   0.8891049041  -0.0866345884
##           3536           3537           3538           3539           3540
## -0.8672214602   1.0842457194   0.3524440514   1.2793865348  -0.0378434819
##           3541           3542           3543           3544           3545
##   0.2548677411  -0.4769221216  -0.7208363356   0.0597263355   0.2548677411
##           3546           3547           3548           3549           3550
##   0.6939640887   0.0597263355   0.3036588476  -0.2817754037  -0.7696510525
##           3551           3552           3553           3554           3555
## -1.0135475586   0.1085121296  -0.3793458114  -1.0623622755   0.0597263355
##           3556           3557           3558           3559           3560
##   0.2548677411   0.2060825373  -0.3305606075   0.1085121296  -0.5257073255
##           3561           3562           3563           3564           3565
## -0.5745102372  -0.6232659279   0.1572973334   0.4987996629  -1.1111179662
##           3566           3567           3568           3569           3570
##   0.4987996629  -1.4526439062   0.6451493718  -0.1842049960   0.2060825373
##           3571           3572           3573           3574           3575
##   0.7427197795  -0.1354197922   0.8402901871   0.4987996629  -0.0378434819
##           3576           3577           3578           3579           3580
##   0.1572973334   0.1085121296  -0.7696510525   0.1572973334  -0.0866345884
##           3581           3582           3583           3584           3585
## -0.6232659279  -0.0378434819   0.2548677411   0.8402901871  -0.4281369178
##           3586           3587           3588           3589           3590
##   0.3036588476   0.7427197795   0.1085121296   0.4500144591  -0.3305606075
##           3591           3592           3593           3594           3595
## -0.2329901999   0.0853234442   0.7322968336   0.9749118546   0.3683791945
##           3596           3597           3598           3599           3600
##   0.4088150313   1.2579871746   0.0853234442   0.8131685073  -0.2381730353
##           3601           3602           3603           3604           3605
##   0.4088150313  -0.1572964692  -0.2786088722   0.1257583026   0.6514251599
##           3606           3607           3608           3609           3610
```

```
##   0.6918854589  0.4896867050 -0.2786088722  0.0448827149  0.0853234442
##           3611          3612          3613          3614          3615
##   0.4896867050  0.9749118546  0.0448827149  0.4088150313 -0.2786088722
##           3616          3617          3618          3619          3620
##  -0.3594952231  0.3279384652  0.3279384652  0.5705583787  0.0853234442
##           3621          3622          3623          3624          3625
##   0.0853234442 -0.5616499452  0.0853234442 -0.1572964692  0.4492508682
##           3626          3627          3628          3629          3630
##  -0.1572964692  0.6918854589 -0.0764247956 -1.1405129272 -0.5993914753
##           3631          3632          3633          3634          3635
##  -0.8158409290 -1.7176911019 -1.7176911019 -0.7436890743 -0.8879892920
##           3636          3637          3638          3639          3640
##  -0.6715398383 -1.1044387457 -1.0683645642 -0.0582787528 -1.0683645642
##           3641          3642          3643          3644          3645
##  -0.4550903846 -0.8879892920 -1.0322860180 -1.3208969288 -0.8158409290
##           3646          3647          3648          3649          3650
##  -1.2487485658 -0.4911645661 -0.4550903846 -0.5633172938 -1.0683645642
##           3651          3652          3653          3654          3655
##  -0.9601376550 -1.0322860180 -0.8879892920 -0.6715398383 -1.8619878279
##           3656          3657          3658          3659          3660
##  -1.1405129272 -0.7436890743 -0.5633172938 -1.2126525608 -1.1044387457
##           3661          3662          3663          3664          3665
##  -0.7436890743 -1.5733943758 -1.2487485658 -1.3930452918 -1.3208969288
##           3666          3667          3668          3669          3670
##  -1.6816387438 -0.7076144563 -0.8158409290 -0.0221827478 -0.7436890743
##           3671          3672          3673          3674          3675
##  -0.5633172938 -1.2487485658 -0.7797623828 -0.6715398383 -1.6455427388
##           3676          3677          3678          3679          3680
##  -0.0021854545 -0.4629363677 -0.3605582879 -0.7189435097  1.0729206572
##           3681          3682          3683          3684          3685
##   0.0490097797 -0.3605582879  0.4073826130  0.7657554464  0.8169568748
##           3686          3687          3688          3689          3690
##   0.1002050138  0.6121697439 -1.0261149147  1.6360992041  0.4073826130
##           3691          3692          3693          3694          3695
##  -0.1557711570 -0.0533806887  0.4585778472  0.1002050138 -0.1045759228
##           3696          3697          3698          3699          3700
##   0.3049921447  0.5097792756  0.6121697439 -0.2581678195 -0.7701077727
##           3701          3702          3703          3704          3705
##   0.6121697439  0.6633649781 -0.7189435097  0.4585778472  0.1002050138
##           3706          3707          3708          3709          3710
##  -0.7701077727 -0.4117721047  0.6121697439  1.1241468624  0.3049921447
##           3711          3712          3713          3714          3715
##  -0.0533806887  1.2265373308  0.5097792756  0.3049921447 -0.1557711570
##           3716          3717          3718          3719          3720
##   0.2537962911  0.3049921447 -0.3093630537  0.8169568748  0.7145602122
##           3721          3722          3723          3724          3725
##  -0.3093630537  0.3561873789 -0.2581678195  0.7657554464 -0.3605582879
##           3726          3727          3728          3729          3730
##  -0.9748887094  0.2026004375  0.4073826130  0.3561873789  0.8681397205
##           3731          3732          3733          3734          3735
##   0.6121697439  0.4073826130 -0.5141625730 -0.0533806887 -0.2069663911
##           3736          3737          3738          3739          3740
##   0.1514064422  0.2537962911  0.4073826130  0.2026004375  1.0217563941
##           3741          3742          3743          3744          3745
```

```
##   0.1514064422   1.0217563941   0.8681397205  -0.6677173044  -0.4629363677
##           3746           3747           3748           3749           3750
## -0.4629363677  -0.2069663911  -0.7701077727   0.0490097797   0.9193659258
##           3751           3752           3753           3754           3755
## -0.0533806887   0.3049921447   0.3049921447   0.7145602122  -0.3093630537
##           3756           3757           3758           3759           3760
## -1.1971431996   0.2407359682  -0.7013248655  -0.6517730272  -0.5030095299
##           3761           3762           3763           3764           3765
## -0.7509366940   0.1911481358   0.1911481358  -0.3542580307   0.0424026356
##           3766           3767           3768           3769           3770
## -0.9988158659  -0.3046761973   0.6374146316   0.0424026356   0.6374146316
##           3771           3772           3773           3774           3775
## -0.1063428646  -0.1063428646  -0.0567622310  -0.8996521991  -0.1063428646
##           3776           3777           3778           3779           3780
##   0.7365782984  -0.0071797977  -0.2550943639   0.1415663024  -0.1559306970
##           3781           3782           3783           3784           3785
##   0.4886451353  -1.0484276944   0.5878028031  -0.1063428646   0.1415663024
##           3786           3787           3788           3789           3790
##   0.2407359682   0.3894814685  -0.7509366940   0.2903178017  -0.1063428646
##           3791           3792           3793           3794           3795
## -0.5525913633   0.1415663024   0.2407359682   0.3398996351  -0.5030095299
##           3796           3797           3798           3799           3800
##   0.1911481358   0.7861301367   1.3811121376  -0.2550943639  -0.7013248655
##           3801           3802           3803           3804           3805
##   0.1911481358   0.0424026356  -1.2467550280   0.0424026356   1.6786031380
##           3806           3807           3808           3809           3810
## -0.0071797977  -0.0567622310  -1.1475913612   0.0919844690  -0.4534216975
##           3811           3812           3813           3814           3815
##   1.2323966324   0.5382329677   0.3398996351  -0.6021731967  -0.3542580307
##           3816           3817           3818           3819           3820
##   0.4886451353   0.2407359682  -0.7509366940   0.0424026356  -0.2438577157
##           3821           3822           3823           3824           3825
## -0.3777314467   0.5594170650   0.0685197223   0.2916480063   0.3808971603
##           3826           3827           3828           3829           3830
##   0.0685197223  -0.0207299716   0.8271645269  -0.0653550885   0.6932637998
##           3831           3832           3833           3834           3835
## -0.4223560236   0.2470234294   0.2916480063   0.5594170650  -0.2438577157
##           3836           3837           3838           3839           3840
## -0.0653550885  -0.1992331388  -0.1992331388   0.5594170650  -0.3331068697
##           3841           3842           3843           3844           3845
## -0.3331068697   0.2023988524  -0.2438577157   0.0238951453  -0.0207299716
##           3846           3847           3848           3849           3850
##   0.0238951453   0.3808971603  -0.0653550885   0.4255217372  -0.1546085618
##           3851           3852           3853           3854           3855
## -0.1099785856  -0.6008759284   0.2916480063  -0.2438577157   0.0685197223
##           3856           3857           3858           3859           3860
##   0.5594170650  -1.2256200060   0.3808971603  -0.2884822927  -0.5562351538
##           3861           3862           3863           3864           3865
##   0.1131442992   0.2023988524   0.3808971603   0.1577688762  -0.1992331388
##           3866           3867           3868           3869           3870
## -0.1992331388  -0.5562351538  -0.4669859998   0.1131442992  -0.1546085618
##           3871           3872           3873           3874           3875
##   0.4255217372   0.2916480063   0.3362725833   0.4701517134   0.2470234294
##           3876           3877           3878           3879           3880
```

```
##   0.5147654920 -0.5116105768  0.6040146459  0.9610112617 -0.0207299716
##           3881         3882         3883         3884         3885
##   0.9164136808 -0.3331068697  0.4701517134 -0.8686233902  0.3362725833
##           3886         3887         3888         3889         3890
##   0.2470234294 -0.0653550885  0.3362725833 -0.5562351538 -0.6901250823
##           3891         3892         3893         3894         3895
##   0.7677405841  0.9062921283  0.2597070796  0.5368151971  0.6753673001
##           3896         3897         3898         3899         3900
##   0.6291834520  1.3219691124  0.3982586238  0.4444480597  0.3058909277
##           3901         3902         3903         3904         3905
##   0.7677405841  0.4444480597  0.8139244322  0.3982586238  1.8299635017
##           3906         3907         3908         3909         3910
##   0.2597070796  0.1211555354  1.1372337202  0.1673393835  0.3058909277
##           3911         3912         3913         3914         3915
##   0.9062921283  0.7215511481  0.9062921283  0.4444480597  0.7215511481
##           3916         3917         3918         3919         3920
##   0.0287822514  0.7215511481 -0.0636022084  1.1833896288 -0.0174015967
##           3921         3922         3923         3924         3925
##   0.7358223125  0.9438971114 -0.8247386790  1.1519719102  1.2560093097
##           3926         3927         3928         3929         3930
## -0.0444644771  0.6317912069  0.9959472803 -0.7207012796  0.3196727148
##           3931         3932         3933         3934         3935
##   0.2676540150 -0.2525392760  0.8398597119  0.4757351078  0.6317912069
##           3936         3937         3938         3939         3940
##   0.4757351078  0.3196727148  0.9959472803  1.2040220792  0.2676540150
##           3941         3942         3943         3944         3945
##   1.3080594786 -0.2525392760 -0.6166638801  0.2676540150 -0.4606203687
##           3946         3947         3948         3949         3950
##   0.3716977083  0.6838162005  0.6317912069 -1.1368508773  0.1636166156
##           3951         3952         3953         3954         3955
##   0.5277538075  0.0075605165  0.2676540150  0.2676540150  0.4757351078
##           3956         3957         3958         3959         3960
## -0.3565766754  0.9959472803 -1.1368508773  0.3716977083  0.6838162005
##           3961         3962         3963         3964         3965
##   1.0999846798 -0.2525392760  1.4120968781 -0.3045579757  0.5797725072
##           3966         3967         3968         3969         3970
## -0.1485018765  0.6317912069  0.8398597119  0.0595779574  0.3716977083
##           3971         3972         3973         3974         3975
##   0.3196727148  0.2156353153  0.7358223125  1.6201716769  0.6317912069
##           3976         3977         3978         3979         3980
##   0.3716977083  0.0075605165  0.2156353153  0.0595779574  0.3648423105
##           3981         3982         3983         3984         3985
## -0.9752642023 -0.0048503422 -0.2821170361 -0.9752642023  0.3648423105
##           3986         3987         3988         3989         3990
## -0.0510605260 -0.2359012613 -1.4836041797  0.2724163518 -0.5131679552
##           3991         3992         3993         3994         3995
##   0.5034616798  0.5958820475 -0.6980142816 -0.6980142816 -0.6055939139
##           3996         3997         3998         3999         4000
##   0.0413598417 -0.9752642023 -0.8828438347 -0.3283272200 -0.3745374038
##           4001         4002         4003         4004         4005
##   0.2262061680  0.4110413122 -0.0048503422 -1.2525253052 -0.1896921956
##           4006         4007         4008         4009         4010
##   0.5034616798 -0.3745374038 -0.2359012613 -1.0676845700 -0.2821170361
##           4011         4012         4013         4014         4015
```

```
## -0.3745374038 -0.1434814527 -0.5131679552 -0.8366616062 -0.8366616062
##          4016          4017          4018          4019          4020
##  0.7345405543 -1.0215023415 -0.0510605260 -0.5131679552 -0.6518040977
##          4021          4022          4023          4024          4025
## -0.7442412386  0.5034616798 -0.2359012613  0.3186265356 -0.0048503422
##          4026          4027          4028          4029          4030
##  0.1337858004 -0.8366616062 -0.7442412386  0.7345405543  0.4572794514
##          4031          4032          4033          4034          4035
## -0.2359012613  0.3648423105 -0.5131679552 -0.6518040977 -1.7608652826
##          4036          4037          4038          4039          4040
## -0.0972707098 -0.3283272200 -0.9752642023 -0.9752642023 -0.3283272200
##          4041          4042          4043          4044          4045
## -0.6980142816 -0.8366616062  0.6883024151  0.4572794514 -0.2821170361
##          4046          4047          4048          4049          4050
##  0.4572794514 -0.5131679552 -0.7904234671 -0.3745374038  0.3648423105
##          4051          4052          4053          4054          4055
##  0.4572794514 -0.1896921956 -0.8828438347 -1.0215023415  0.8731431504
##          4056          4057          4058          4059
## -0.7442412386  0.0875756165 -0.6055939139 -0.9290819739
```

LRT-score prediction interval at 0.95

$$[0.31, 0.79]$$

c)

By repeating the study infinitely many times, the 95% of the constructed confidence intervals would contain the true slope of LRT-score. The predictive interval indicates that 95% of the regression coefficients of LRT-score predicted(for a new future) to lie in this interval. The difference between the two intervals is thus that the Predictive Interval refers to the distribution of the predicted slope coefficients within this particular model, whereas the Confidence Interval makes an assertion about a hypothetical sampling distribution of the slopes in the population. It also worthy noting that the Predictive interval is expected to be wider than the confidence interval.

7

To respond to this, we fit a full multilevel regression model(model_5), from the previous model(model_4) we include the cross-level interaction term between the predictors(LRT-score and average LRT-score). The $Deviance(8) = 9302.9, p < 0.01$ and $AIC = 9318.9$, there is no centering of predictors in this model, because the scale appear to include zero, hence, the main effects are still meaningful after adding the cross-level interaction, the LRT-score were centered before.

The chi-square difference test between model_4 and model_5 ($\chi_1^2 = 7.52, p < 0.01$) confirms that LRT-score:ALRT cross-level interaction($b = 0.16$). To sum up, the average reading score explain part of the differences in the relationship between reading score and Examscore in different schools. Thus, the higher the average reading, the stronger the positive relationship between the reading score and Examscore on average. The Psuedo $R^2 = 0.36$ of fixed effects indicates that 36% of variance in the effect of reading on the Examscore between schools is explained by the schools' average reading scores.

8

We chose model_5 as the final model. This is because it only includes significant parameters and has the best model fit (Deviance(8) = 9302.9). Furthermore, model_5 has the lowest AIC (9318.9).Hence, the most appropriate model to fit the dataset.

Level-1 equation

$$examscore_{ij} = b_{0j} + b_{1j} * LRTscore_j + e_{ij}$$

Level-2 equation

$$b_{0j} = \gamma_{00} + \gamma_{01} * AvsLRT_j + u_{0j}$$
$$b_{1j} = \gamma_{10} + \gamma_{11} * AvsLRT_j + u_{1j}$$

Mixed Level equation

$$examscore_{ij} = \gamma_{00} + \gamma_{01} * AvsLRT_j + \gamma_{10} * LRTscore_j + \gamma_{11} * LRTscore_j * AvsLRT_j + u_{1j} * LRTscore_j + u_{0j} + e_{ij}$$

where $_i$ represents individual(student) and $_j$ represents school

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