TIME SERIES ANALYSIS-SEASONAL DECOMPOSITION

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Introduction I

Seasonal decomposition is a common technique used in time series analysis to separate a time series into its different components. Time series data often contains various components such as trend, seasonality, and noise, and by decomposing the time series into its components, we can better understand the underlying patterns and trends in the data.

The basic idea behind seasonal decomposition is to model the time series as a combination of a seasonal component, a trend component, and a residual component. The seasonal component represents the repetitive patterns that occur at fixed intervals (e.g., daily, weekly, monthly), the trend component represents the long-term changes in the data, and the residual component represents the random fluctuations that cannot be explained by the seasonal and trend components.

There are several methods for decomposing a time series, including the classical decomposition method, the X-11 method, and the STL (seasonal and trend decomposition using Loess) method. Each method has its own strengths and weaknesses and is suited for different types of time series.

Introduction II

data. Once the time series has been decomposed, we can analyze each component separately and make predictions based on the patterns and trends observed in each component. This can be particularly useful for forecasting future values of the time series.

Overall, seasonal decomposition is an important tool in time series analysis that can help us better understand the underlying patterns and trends in the data, and make more accurate predictions for the future.

Definition I

Seasonal decomposition is a common method used in time series analysis to separate a time series into its seasonal, trend, and residual components. The decomposition process involves breaking down the time series data into its different components, with the goal of identifying patterns and trends that are present in the data.

Time series Decomposition I

Seasonal decomposition is a method used in time series analysis to decompose a time series into its trend, seasonal, and residual components. This method is used to understand the different patterns present in a time series and to separate them out for further analysis. The decomposition of a time series involves breaking it down into three components:

- Trend: The trend component is the long-term movement or pattern in the time series. It represents the overall direction of the data over time and is typically a smooth, slowly changing function.

- Seasonal: The seasonal component is the periodic pattern that repeats over a fixed interval, such as a day, week, month, or year. It represents the regular fluctuations that occur within a time period, such as seasonal changes in demand for a product or the effect of the weather on sales.

Time series Decomposition II

- Residual: The residual component is the part of the time series that cannot be explained by the trend or seasonal components. It represents the random fluctuations or noise in the data.

There are several methods for performing seasonal decomposition, including:

- Classical decomposition: This method involves using moving averages to estimate the trend and seasonal components of the time series.
- X-11 decomposition: This method is a more advanced version of classical decomposition that uses a sophisticated statistical algorithm to estimate the trend and seasonal components.
- Seasonal ARIMA models involve fitting an ARIMA model to time series data
- ARIMA model used to estimate trend, seasonal, and residual components

Time series Decomposition III

- Each component can be analyzed separately to gain insights into underlying patterns and relationships in the data
- Trend component can be used to forecast future values of time series

Time series components I

If we assume an additive decomposition, then we can write

$$y_t = S_t + T_t + R_t$$

where y_t is the data,S_t is the seasonal component, T_t is the trend-cycle component, and $R_t$is the remainder component, all at period t Alternatively, a multiplicative decomposition would be written as

$$y_t = S_t \times T_t \times R_t$$

- Additive decomposition is suitable when the seasonal fluctuations or variation around the trend-cycle remains constant with the time series level.
- Multiplicative decomposition is appropriate when the variation in the seasonal pattern or variation around the trend-cycle is proportional to the time series level.
- Economic time series often use multiplicative decompositions.

Time series components II

- Data transformation can be used to stabilize the variation over time before using additive decomposition.
- Log transformation produces results equivalent to multiplicative decomposition. $y_t = S_t \times T_t \times R_t$ is equivalent to $log\, y_t = log S_t + log T_t + log R_t$

Electrical equipment manufacturing

- Analyzing time series component of electrical equipment manufacturing data provides valuable insights into patterns and trends
- Insights can impact future demand and production levels, useful for forecasting and planning purposes
- Seasonal decomposition and regression analysis are techniques that can separate the components of the time series data
- These techniques can help identify underlying patterns and trends

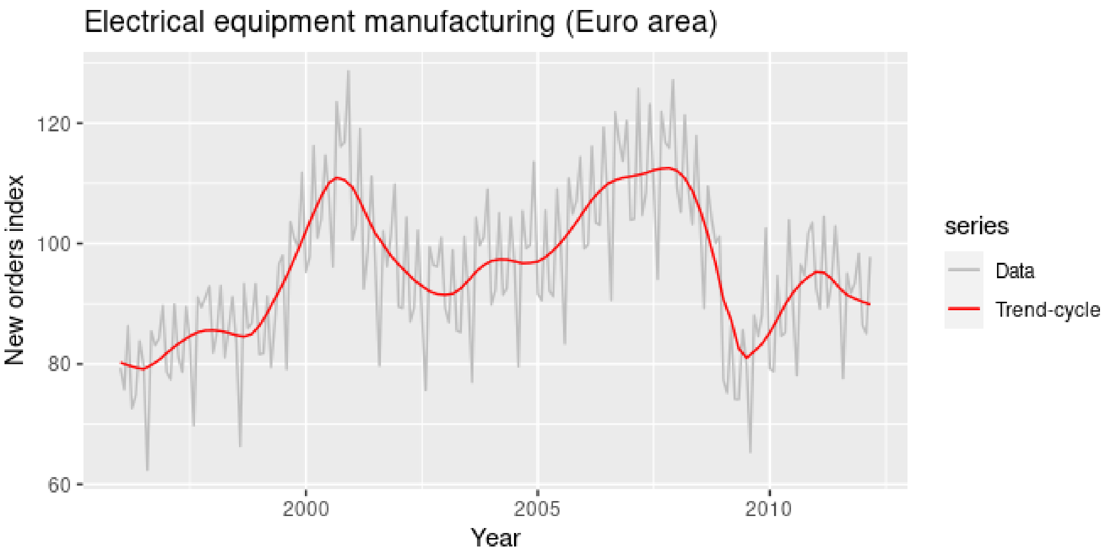
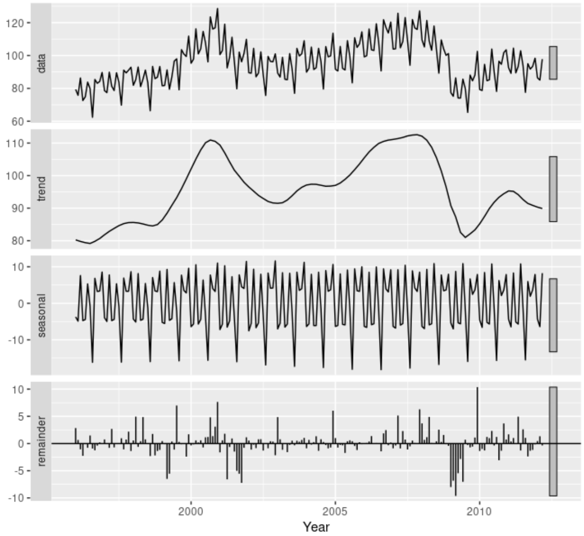

Electrical equipment manufacturing (Euro area)
New orders index
120
100
80
60
2000
2005
2010
Year
series
Data
Trend-cycle

Figure shows the trend-cycle component, T_t, in red and the original data, y_t, in grey. The trend-cycle shows the overall movement in the series, ignoring the seasonality and any small random fluctuations.

Seasonally adjusted data

- Seasonal fluctuations are regular patterns that occur in a time series data at fixed intervals, such as months, quarters, or years
- Seasonal adjustment is the process of removing the seasonal component from the data to reveal the underlying trend and irregular components
- Seasonally adjusted data is useful for identifying long-term trends and irregularities in the data, such as unexpected changes in demand or production levels
- Seasonally adjusted data is also helpful for making accurate forecasts and conducting meaningful comparisons between different time periods
- Seasonal adjustment can be performed using various techniques, such as moving averages, seasonal decomposition, and regression analysis



data
trend
seasonal
remainder
120
100
80
60
110
90
10
0
-10
5
-5
2000
2005
2010
Year

Figure shows an additive decomposition of these data. The method used for estimating components in this example is STL

Moving averages I

The classical method of time series decomposition originated in the 1920s and was widely used until the 1950s. It still forms the basis of many time series decomposition methods, so it is important to understand how it works. The first step in a classical decomposition is to use a moving average method to estimate the trend-cycle, so we begin by discussing moving averages.

Moving averages II

Moving average smoothing

A moving average of order m can be written as

$$\hat{T}_t = \frac{1}{m} \sum_{j=-k}^{k} y_t + j$$

- Trend-cycle estimation involves averaging values of the time series within k periods of time.
- The value of m is determined by the equation m=2k+1.
- Nearby observations are likely to be close in value, so averaging eliminates some of the randomness in the data.
- The result is a smooth trend-cycle component.
- This method is called an m-MA, which means a moving average of order m.

Classical decomposition I

- Classical decomposition method originated in the 1920s and is a simple procedure for time series decomposition.
- Classical decomposition has two forms: additive decomposition and multiplicative decomposition.
- Seasonal period m is used in classical decomposition, with examples including m=4 for quarterly data, m=12 for monthly data, and m=7 for daily data with a weekly pattern.
- The seasonal component is assumed to be constant from year to year in classical decomposition.
- For multiplicative seasonality, the m values that form the seasonal component are sometimes called "seasonal indices".

Classical decomposition II

Additive decomposition

- Step-1 If m is an even number, compute the trend-cycle component $\hat{T}_t$ using a $2\times m - MA.If misanoddnumber, computethetrend - cyclecomponent \hat{T}_t$ using an m-MA.
- Step-2 Calculate the detrended series:
- Step-3
  To estimate the seasonal component for each season, simply average the detrended values for that season.

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Classical decomposition III

Multiplicative decomposition

- A classical multiplicative decomposition is similar, except that the subtractions are replaced by divisions.
- Calculate the detrended series: $\frac{y_t}{\hat{T}_t}$
- The remainder component is calculated by dividing out the estimated seasonal and trend-cycle components: $\hat{R}_t = \frac{y_t}{\hat{T}_t \hat{S}_t}$

X11 decomposition

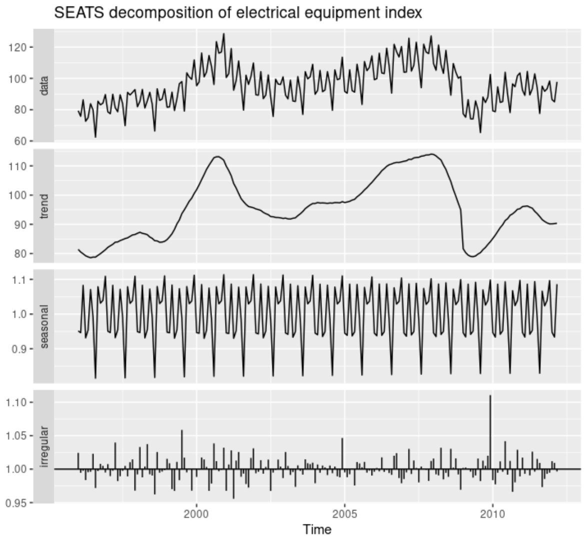
- It is based on classical decomposition but includes extra steps and features to overcome drawbacks of classical decomposition.
- Trend-cycle estimates are available for all observations, including end points, and the seasonal component can vary slowly over time.
- X11 can handle trading day variation, holiday effects, and known predictors, and supports both additive and multiplicative decomposition.
- The X11 process is automatic and is highly robust to outliers and level shifts in the time series.
- The X11 method details are described in Dagum Bianconcini (2016).
- The seas() function in the seasonal package for R is used to implement the X11 method.

SEATS decomposition I

- SEATS: Seasonal Extraction in ARIMA Time Series
- Developed at the Bank of Spain
- Widely used by government agencies worldwide
- Works with quarterly and monthly data only
- Alternative approach needed for other types of seasonality, such as daily, hourly, or weekly data
- Complete discussion available in Dagum Bianconcini (2016) Procedure can be used via the seasonal package

SEATS decomposition of electrical equipment index I

SEATS decomposition of electrical equipment index II



The result is quite similar to the X11 decomposition shown in Figure

As with the X11 method, we can use the seasonal(), trendcycle() and remainder() functions to extract the individual components, and seasadj() to compute the seasonally adjusted time series.

STL Decomposition I

- Split the time series into its three components: trend, seasonality, and remainder (or residuals).
- Use a moving average filter to estimate the trend component.
- Decompose the seasonality component by fitting a periodic function (e.g., Fourier series) to the data.
- Calculate the remainder component as the difference between the original time series and the sum of the trend and seasonality components.

Measuring strength of trend and seasonality I

- Time series decomposition can measure strength of trend and seasonality
- decomposition is written as $y_t = T_t + S_t + R_t$ where T_t is the smoothed trend component, S_t is the seasonal component and R_t is a remainder component.
- For strongly trended data, seasonally adjusted data should have more variation than remainder component
- Define strength of trend as $F_T = Max(0, 1 - \frac{Var(R_t)}{Var(T_t + R_t)})$ giving a measure between 0 and 1
- Strength of seasonality defined similarly with respect to detrended data: $F_S = Max(0, 1 - \frac{Var(R_t)}{Var(S_t + R_t)})$
- Series with seasonal strength F_s close to 0 exhibits almost no seasonality
- Series with strong seasonality will have F_s close to 1 because Var((R_t) will be much smaller than Var$(R_t + R_t)$

Forecasting with decomposition I

While decomposition is primarily useful for studying time series data, and exploring historical changes over time, it can also be used in forecasting.

- Decomposition can also be used for forecasting time series data
- Additive decomposition: $y_t = \hat{S}_t + \hat{A}_t$ where $\hat{A}_t = \hat{T}_t + \hat{R}_t$ (seasonally adjusted component)
- Multiplicative decomposition: $y_t = \hat{S}_t \hat{A}_t$ where $\hat{A}_t = \hat{T}_t \hat{R}_t$
- Forecasting involves separately forecasting the seasonal component (usually assumed to be unchanging or changing slowly) and the seasonally adjusted component
- Seasonal naïve method used for forecasting seasonal component
- Any non-seasonal forecasting method (e.g., random walk with drift model or Holt's method) may be used for forecasting the seasonally adjusted component

Reference

Reference

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Thank You!