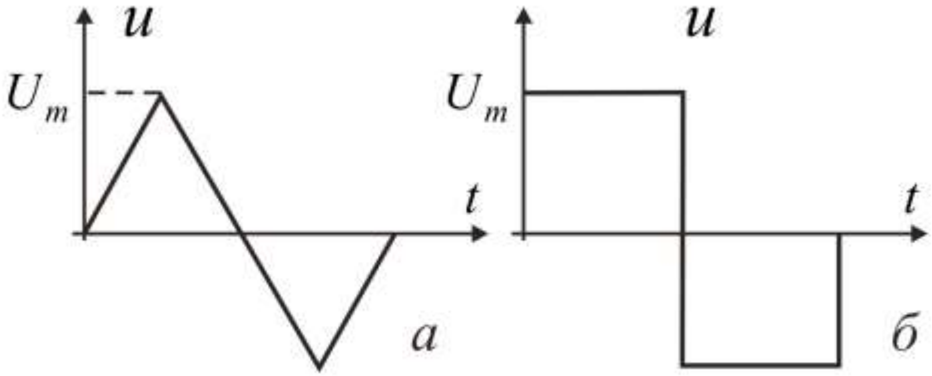
Цепи периодического несинусоидального тока



$$f(t) = f(t+T)$$

$$f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty}\left(a_k \cos k\omega t + b_k \sin k\omega t\right)$$

$$a_k = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos k\omega t\,dt,\; b_k = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin k\omega t\,dt.$$

Действующее значение периодического несинусоидального сигнала

$$i(t) = I_0 + \sum_{k=1}^{\infty} I_{mk}\cos\left(k\omega t - \psi_{ik}\right) \qquad I = \sqrt{I_0^2 + \sum_{k=1}^{\infty}\frac{I_{mk}^2}{2}} = \sqrt{I_0^2 + \sum_{k=1}^{\infty} I_k^2} = \sqrt{\sum_{k=0}^{\infty} I_k^2}$$

$$u(t) = U_0 + \sum_{k=1}^{\infty} U_{mk}\cos\left(k\omega t - \psi_{uk}\right) \qquad U = \sqrt{U_0^2 + \sum_{k=1}^{\infty}\frac{U_{mk}^2}{2}} = \sqrt{U_0^2 + \sum_{k=1}^{\infty} U_k^2} = \sqrt{\sum_{k=0}^{\infty} U_k^2}$$

Пример №1. $u(t) = 100 + 80\sin\left(\alpha t + 30^\circ\right) + 60\sin\left(3\alpha t + 20^\circ\right) + 50\sin\left(5\alpha t + 45^\circ\right)$

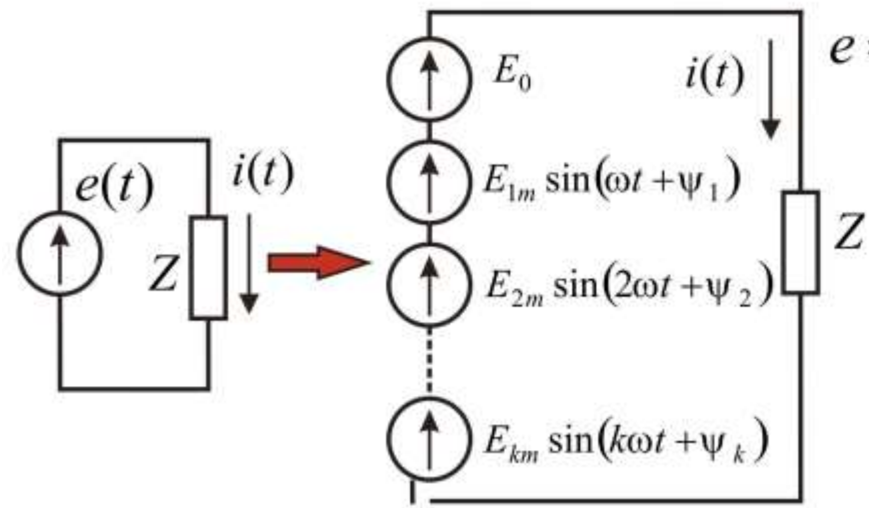
$$U = \sqrt{U_0^2 + U_1^2 + U_3^2 + U_5^2} = \sqrt{100^2 + \left(\frac{80}{\sqrt{2}}\right)^2 + \left(\frac{60}{\sqrt{2}}\right)^2 + \left(\frac{50}{\sqrt{2}}\right)^2} = 127 \text{ В.}$$

Активная мощность негармонического сигнала

$$P=\frac{1}{T}\int_{0}^{T}u(t)\,i(t)\,dt \qquad i(t)=\sum_{k=0}^{\infty}I_{mk}\cos\left(k\omega t-\psi_{ik}\right) \; u(t)=\sum_{k=0}^{\infty}U_{mk}\cos\left(k\omega t-\psi_{ik}+\varphi_{k}\right)$$

$$P=\sum_{k=0}^{\infty}U_{k}\,I_{k}\cos\varphi_{k}=\sum_{k=0}^{\infty}P_{k} \quad Q=\sum_{k=0}^{\infty}U_{k}\,I_{k}\sin\varphi_{k}=\sum_{k=0}^{\infty}Q_{k} \quad S=U\,I=\sqrt{\sum_{k=0}^{\infty}U_{k}^{2}\,\sum_{k=0}^{\infty}I_{k}^{2}}$$

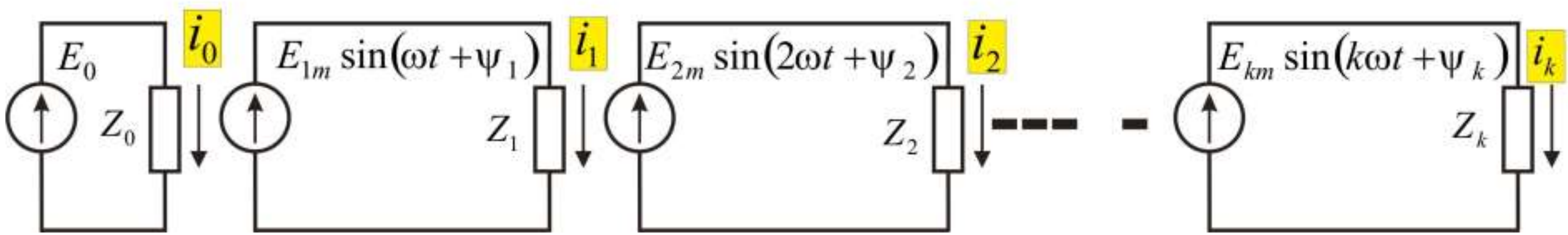
Расчет цепей при периодических негармонических воздействиях



$$e=E_0+E_{1m}\sin(\omega t+\psi_1)+E_{2m}\sin(2\omega t+\psi_2)+\ldots.$$

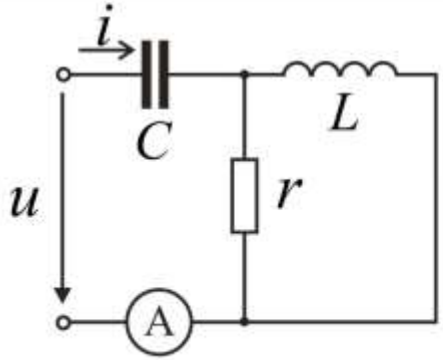
$$Z_k=R+j\,k\,\omega\,L-j\frac{1}{k\,\omega\,C}$$

$$\boxed{i=i_0+i_1+i_2+\ldots\ldots+i_k}$$



$u = 74 + 46{,}3\sin(10^3 t + 45^\circ) + 16{,}3\cdot\sqrt{2}\sin(2\cdot 10^3 t + 10{,}6^\circ);$

$r = 20;\ L = 0{,}02;\ C = 10^{-4}.\qquad i,\ \text{A},\ P = ?$



1. Нулевая гармоника

$$U_{(0)} = 74;\ Z_{(0)} = \infty \Rightarrow \boxed{I_{(0)} = 0}$$

2. Первая гармоника $\omega = 10^3$

$$\dot{U}_{(1)} = \frac{46{,}3}{\sqrt{2}}\cdot e^{j45^\circ} = 23{,}15 + 23{,}15\,j;\ x_{L(1)} = \omega L = 10^3\cdot 0{,}02 = 20;\ x_{C(1)} = \frac{1}{\omega C} = 10;$$

$$Z_{(1)} = -jx_{C(1)} + \frac{jx_{L(1)}\cdot r}{r + jx_{L(1)}} = 10;\ \dot{I}_{(1)} = \frac{\dot{U}_{(1)}}{Z_{(1)}} = 2{,}315 + 2{,}315\,j \Rightarrow \boxed{i_{(1)} = 4{,}63\sin(10^3 t + 45^\circ).}$$

3. Вторая гармоника $\omega = 2\cdot 10^3$

$$\dot{U}_{(2)} = \frac{16{,}3\sqrt{2}}{\sqrt{2}}\cdot e^{j10{,}6^\circ} = 16 + 3\,j;\ x_{L(2)} = 40;\ x_{C(2)} = 5;\ Z_{(2)} = -jx_{C(2)} + \frac{jx_{L(2)}\cdot r}{r + jx_{L(2)}} =$$

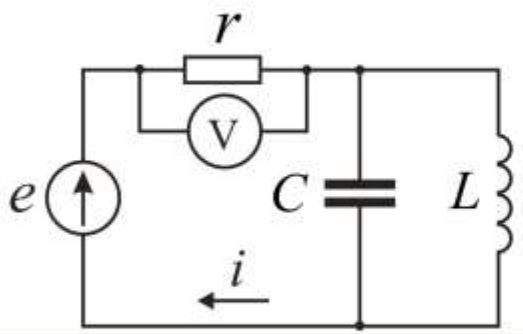
$$= 16 + 3\,j;\ \dot{I}_{(2)} = \frac{\dot{U}_{(2)}}{Z_{(2)}} = 1 \Rightarrow \boxed{i_{(2)} = \sqrt{2}\sin(2\cdot 10^3 t).}$$

$$i = I_{(0)} + i_{(1)} + i_{(2)} = \boxed{4{,}63\sin(10^3 t + 45^\circ) + \sqrt{2}\sin(2\cdot 10^3 t)}$$

$$\text{A} = \sqrt{I_{(0)}^2 + \left(\frac{I_{\text{m}(1)}}{\sqrt{2}}\right) + \left(\frac{I_{\text{m}(2)}}{\sqrt{2}}\right)^2} = \sqrt{\left(\frac{4{,}63}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{2}}{\sqrt{2}}\right)^2} = \boxed{3{,}42}$$

$$P = U_{(0)}\cdot I_{(0)} + U_{(1)}\cdot I_{(1)}\cdot\cos(\psi_{U_{(1)}} - \psi_{I_{(1)}}) + U_{(2)}\cdot I_{(2)}\cdot\cos(\psi_{U_{(2)}} - \psi_{I_{(2)}}) =$$

$$\frac{46{,}3}{\sqrt{2}}\cdot\frac{4{,}63}{\sqrt{2}}\cdot\cos(45^\circ - 45^\circ) + \frac{16{,}3\sqrt{2}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}\cdot\cos(10{,}6^\circ - 0) = \boxed{123{,}2}$$



$e = 60 + 60\sin\omega t + 30\sin 2\omega t;$

$r = x_{C_{(1)}} = 30;\ x_{L_{(1)}} = 15;\ i = ?;\ \mathrm{V} = ?$

1. Нулевая гармоника

$$E_{(0)} = 60;\ Z_{(0)} = r = 30 \Rightarrow \boxed{I_{(0)} = \frac{E_{(0)}}{Z_{(0)}} = 2}.$$

2. Первая гармоника

$$\dot{E}_{(1)} = \frac{60}{\sqrt{2}} = 30\sqrt{2};\ Z_{(1)} = r + \frac{jx_{L(1)} \cdot (-jx_{C(1)})}{j(x_{L(1)} - jx_{C(1)})} = 30 + 30j = 30\sqrt{2} \cdot e^{j45^\circ};$$

$$\dot{I}_{(1)} = \frac{\dot{E}_{(1)}}{Z_{(1)}} = \frac{30\sqrt{2}}{30\sqrt{2} \cdot e^{j45^\circ}} = e^{-j45^\circ} \Rightarrow \boxed{i_{(1)} = \sqrt{2}\sin(\omega t - 45^\circ)}.$$

3. Вторая гармоника $\quad x_{L(2)} = 30;\ x_{C(2)} = 15$

$$\dot{E}_{(2)} = \frac{30}{\sqrt{2}} = 15\sqrt{2};\ Z_{(2)} = r + \frac{jx_{L(2)} \cdot (-jx_{C(2)})}{j(x_{L(2)} - jx_{C(2)})} = 30 - 30j = 30\sqrt{2} \cdot e^{-j45^\circ};\ \dot{I}_{(2)} = \frac{\dot{E}_{(2)}}{Z_{(2)}} = \frac{15\sqrt{2}}{30\sqrt{2} \cdot e^{-j45^\circ}} = 0{,}5 \cdot e^{j45^\circ} \Rightarrow$$

$$\Rightarrow \boxed{i_{(2)} = 0{,}5\sqrt{2}\sin(2\omega t + 45^\circ)}.$$

$$\boxed{i = I_{(0)} + i_{(1)} + i_{(2)} = 2 + \sqrt{2}\sin(\omega t - 45^\circ) + 0{,}5\sqrt{2}\sin(2\omega t + 45^\circ)}$$

$$u_{\mathrm{V}} = i \cdot r = 60 + 30\sqrt{2}\sin(\omega t - 45^\circ) + 15\sqrt{2}\sin(2\omega t + 45^\circ) \Rightarrow$$

$$\Rightarrow \boxed{\mathrm{V} = \sqrt{U_{\mathrm{V}(0)}^2 + \left(\frac{U_{\mathrm{Vm}(1)}}{\sqrt{2}}\right) + \left(\frac{U_{\mathrm{Vm}(2)}}{\sqrt{2}}\right)^2} = \sqrt{60^2 + \left(\frac{30\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{15\sqrt{2}}{\sqrt{2}}\right)^2} = 68{,}7}$$