MTE 203 – Advanced Calculus
Homework 12

Surface Integrals Involving Vector Fields

Problem 1: [S. 14.8, Prob.9]

Evaluate the surface integral.

$\oiint_S \left(x^2y\hat{\boldsymbol{\imath}} + xy\hat{\boldsymbol{\jmath}} + z\hat{\boldsymbol{k}}\right) \cdot \hat{\boldsymbol{n}}\, dS$, where S is the surface enclosing the volume defined by $x = 0,\ x = 2,\ z = 0,\ z = y,\ y + z = 2\ and\ \hat{\boldsymbol{n}}$ is the unit outer normal to S.

Problem 2: [S. 14.8, Prob. 13]

Evaluate the surface integral

$\oiint_S \vec{\boldsymbol{F}} \cdot \hat{\boldsymbol{n}}\, \boldsymbol{d\sigma},$ where $\hat{F} = (z^2 - x)\hat{\imath} - xy\hat{\jmath} + 3z\hat{k}$, S is the surface enclosing the volume defined by $z = 4 - y^2, x = 0, x = 3, z = 0$ and the vector $\hat{\boldsymbol{n}}$ is the unit outer normal to S.

The Divergence Theorem

Problem 3: [S. 14.9, Prob.7]

Use the divergence theorem to evaluate the surface integral:

$\oiint_S \left(z\hat{\boldsymbol{\imath}} - x\hat{\boldsymbol{\jmath}} + y\hat{\boldsymbol{k}}\right) \cdot \hat{\boldsymbol{n}}\, dS$ where S is the surface enclosing the volume defined by the surface $z = \sqrt{4 - x^2 - y^2}, z = 0$, and $\hat{\boldsymbol{n}}$ is the unit outer normal to S.

Problem 4: [S. 14.9, Prob.11]

Use the divergence theorem to evaluate the surface integral:

$\oiint_S \left(y\hat{\boldsymbol{\imath}} - xy\hat{\boldsymbol{\jmath}} + zy^2\hat{\boldsymbol{k}}\right) \cdot \hat{\boldsymbol{n}}\, dS$ where S is the surface enclosing the volume defined by $y^2 - x^2 - z^2 = 4, y = 4$, and $\hat{\boldsymbol{n}}$ is the unit inner normal to S.

Problem 5: [S. 14.9, Prob.13] - Challenging

Use the divergence theorem to evaluate the surface integral.

$\oiint_S (x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} + z\widehat{\boldsymbol{k}}) \cdot \widehat{\boldsymbol{n}}\, dS$ where S is the top half of the ellipsoid $x^2 + 4y^2 + 9z^2 = 36$, and $\widehat{\boldsymbol{n}}$ is the unit outer normal to S.

Hint 1: Since the surface S needs to enclose a volume, you need to introduce an additional surface ($\acute{S}$).

Hint 2: The volume of an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{B^2} + \frac{Z^2}{C^2} = 1$ is $V = 4\pi abc/3$ (see exercise 27 in section 13.9 for the proof).

Stoke's Theorem

Problem 6: [S. 14.10, Prob.1]

Use stoke's theorem to evaluate the line integral

$\oint_C\ x^2ydx + y^2zdy + z^2xdz$ where C is the curve $z = x^2 + y^2$, and $x^2 + y^2 = 4$, directed counterclockwise as viewed from the origin.

Problem 7: [S. 14.10, Prob. 7]

$\oint_C zy^2dx + xydy + (x^2 + z^2)dz,$ where C is the curve $x^2 + z^2 = 9,\ y = (x^2 + z^2)^{\frac{1}{2}}$ directed counterclockwise as viewed from origin.

Problem 8: [S. 14.10, Prob.13]

Use Stoke's theorem to evaluate the line integral

$\oint_C\ z(x + y)^2dx + (y - x)^2dy + z^2dz$ where C is the smooth curve of intersection of the surfaces $x^2 + z^2 = a^2$, and $y^2 + z^2 = a^2$ which has a portion in the first octant, directed so that z decreases in the first octant.

Warm-Up Problems

Solutions to these problems can be found at the back of your textbook

1. S. 14.8, Probs. 2, 4,
2. S. 14.9, Probs. 2, 4
3. S. 14.10, Probs. 2, 4

Extra Practice Problems

Solutions to these problems can be found at the back of your textbook

1. S. 14.8, Probs. 6, 10, 20
2. S. 14.9, Prob. 6,8,14
3. S. 14.10, Probs. 6,10,16

Extra Challenging Problems

Solutions to these problems can be found at the back of your textbook

1. S. 14.8, Probs. 22
2. S. 14 (Review Exercises) Prob. 24, 28