Multiple-Section Trapezoidal Rule- Derivation of General Formula

Dividing $[a,b]$ into n equal sections and applying the trapezoidal rule over each section, the sum of the results obtained for each section is the approximate value of the integral. Divide $(b-a)$ into n equal sections. Then the width of each section is

$$h=\frac{b-a}{n}$$

The integral I can be broken into multiple integrals as

$$I=\int_a^b f(x)dx = \int_a^{a+h} f(x)dx+\int_{a+h}^{a+2h} f(x)dx+\ldots+\int_{a+(n-2)h}^{a+(n-1)h} f(x)dx+\int_{a+(n-1)h}^{b} f(x)dx$$

Applying trapezoidal rule Equation (27) on each section gives

$$\begin{aligned}\int_a^b f(x)dx &= [(a+h)-a]\left[\frac{f(a)+f(a+h)}{2}\right] \\ &\quad +[(a+2h)-(a+h)]\left[\frac{f(a+h)+f(a+2h)}{2}\right] \\ &\quad +\ldots\ldots\ldots+[(a+(n-1)h)-(a+(n-2)h)]\left[\frac{f(a+(n-2)h)+f(a+(n-1)h)}{2}\right] \\ &\quad +[b-(a+(n-1)h)]\left[\frac{f(a+(n-1)h)+f(b)}{2}\right]\end{aligned}$$

$$\begin{aligned}&=h\left[\frac{f(a)+f(a+h)}{2}\right]+h\left[\frac{f(a+h)+f(a+2h)}{2}\right]+\ldots\ldots\ldots\ldots\ldots \\ &\quad +h\left[\frac{f(a+(n-2)h)+f(a+(n-1)h)}{2}\right]+h\left[\frac{f(a+(n-1)h)+f(b)}{2}\right] \\ &=h\left[\frac{f(a)+2f(a+h)+2f(a+2h)+\ldots+2f(a+(n-1)h)+f(b)}{2}\right] \\ &=\frac{h}{2}\left[f(a)+2\left\{\sum_{i=1}^{n-1} f(a+ih)\right\}+f(b)\right] \\ &=\frac{b-a}{2n}\left[f(a)+2\left\{\sum_{i=1}^{n-1} f(a+ih)\right\}+f(b)\right]\end{aligned}$$