MTE 203 – Advanced Calculus
Homework 3 - Questions

Differentiation and Integration of Vectors

Problem 1 [11.9, Prob. 7, 19]

If $f(t) = t^2 + 3$, $g(t) = 2t^3 + 3t$, $\boldsymbol{u}(t) = t\hat{\boldsymbol{i}} - t^2\hat{\boldsymbol{j}} + 2t\hat{\boldsymbol{k}}$, and $\boldsymbol{v}(t) = \hat{\boldsymbol{i}} - 2t\hat{\boldsymbol{j}} + 3t^2\hat{\boldsymbol{k}}$, find the scalar or the components of the vector in the following exercises:

a. $\frac{d}{dt}[f(t)\boldsymbol{v}(t)]$
b. $\int[f(t)\boldsymbol{u}\cdot\boldsymbol{v}]dt$

Problem 2 [11.9, Prob. 25]:

Prove that if a differentiable function $\mathbf{v}(t)$ has constant length, then at any point at which $\frac{d\mathbf{v}}{dt} \neq 0$, the vector $\frac{d\mathbf{v}}{dt}$ is perpendicular to $\mathbf{v}$

Arc Length and Length of Curves

Problem 3 [S. 11.11, Prob. 11, 13]:

Find the length of the following curves. Draw the curves.

a. $x = 2\cos t,\ y = 2\sin t,\ z = 3t,\ 0 \le t \le 2\pi$
b. $x = t^3,\ y = t^2,\ z = t^3,\ 0 \le t \le 1$

Displacement, Velocity and Acceleration

Problem 4 [11.13, Prob. 13]

A particle moves along the curve $x(t) = t,\quad y(t) = t^3 - 3t^2 + 2t,\quad 0 \le t \le 5$ in the xy plane (where t is the time). Is there any point at which its velocity is parallel to its displacement?

Problem 5 [11.13, Prob. 17]

A particle travels around the circle $x^2 + y^2 = 4$ counterclockwise at constant speed, making 2 revolutions each second. If x and y are measured in meters, what is the velocity of the particle when it is at the point $\left(1, -\sqrt{3}\right)$

Problem 6 [11.13, Prob. 21]

A plane flies with speed 600 km/h in still air. The plane is to fly in a straight line from city A to city B, where B is 1000 km northwest of A. What should be its bearing if the wind is blowing from the west at 50 km/h? How long will the trip take?

Extra Practice Problems

Solutions to these problems can be found at the back of your textbook

1. S. 11.9, Probs. 10, 18, 24
2. S. 11.11, Probs. 12,14
3. S. 11.13, Probs. 4, 14, 38