MTE 203 – Advanced Calculus
Homework 1 - Solutions

Vector Operations Review

As mentioned during our first lecture, the shortest distance between two points (or the length of the line segment joining two points) $P_1\left(x_1, y_1, z_1\right)$ and $P_2\left(x_2, y_2, z_2\right)$ in 3-D space is given by,

$$\left\|P_1P_2\right\| = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2}$$

Using your previous knowledge in vector operations (MTE 119) and the concept of distance between points and lines in the 2-D space (MATH 118), solve the following problems in the 3-D space.

Problem 1 [S. 11.1, Prob. 5]:

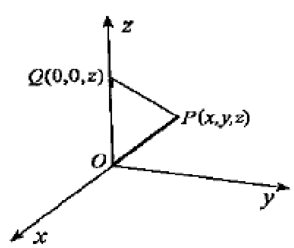
Show that the (unidirected, prependicular) distances from a point (x, y, z) to the $x-$, $y-$, and $z-$ axes are, respectively, $\sqrt{y^2+z^2}$, $\sqrt{x^2+z^2}$, $\sqrt{y^2+x^2}$.

Solution:

If we draw a line from $P(x, y, z)$ perpendicular to the z-axis, the coordinates of Q are $(0, 0, z)$. The length of the perpendicular is

$$\begin{aligned} \|PQ\| &= \sqrt{\|OP\|^2 - \|OQ\|^2} \\ &= \sqrt{x^2 + y^2 + z^2 - z^2} \\ &= \sqrt{x^2 + y^2}. \end{aligned}$$

Similar derivations give distances to the x- and y-axes.



Problem 2 [S. 11.1, Prob. 13]:

a. If $\left(\sqrt{3}-3, 2+2\sqrt{3}, 2\sqrt{3}-1\right)$ and $\left(2\sqrt{3}, 4, \sqrt{3}-2\right)$ are two vertices of an equilateral triangle, and if the third vertex lies on the z –axis, find the third vertex coordinates.
b. Can you find a third vertex on the x –axis?

Solution:

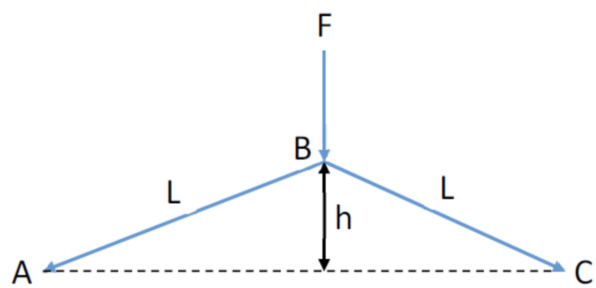
(a) If the third vertex is on the z-axis, its coordinates must be $P(0, 0, z)$. Because this point is equidistant from $Q(\sqrt{3}-3, 2+2\sqrt{3}, 2\sqrt{3}-1)$ and $R(2\sqrt{3}, 4, \sqrt{3}-2)$, we can write that $(\sqrt{3}-3)^2 + (2+2\sqrt{3})^2 + (2\sqrt{3}-1-z)^2 = (2\sqrt{3})^2 + (4)^2 + (\sqrt{3}-2-z)^2$. The solution of this equation is $z = \sqrt{3}$. Since $\|PQ\| = \|QR\| = 4\sqrt{2}$, the triangle is equilateral.

(b) If the third vertex is on the x-axis, its coordinates must be $P(x, 0, 0)$. For P to be equidistant from Q and R, we can write that $(\sqrt{3}-3-x)^2+(2+2\sqrt{3})^2+(2\sqrt{3}-1)^2 = (2\sqrt{3}-x)^2+(4)^2+(\sqrt{3}-2)^2$. The solution of this equation is $x = -1$. Since $\|PQ\| \neq \|QR\|$, the triangle is isosceles but not equilateral.

Problem 3 [11.3, Prob. 41] – Application Problem:

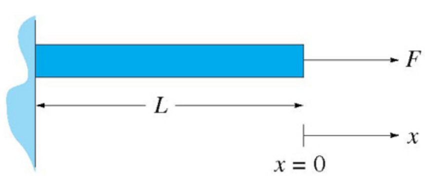
Two bars, ***AB*** and ***BC***, are pinned at ***B*** as well as at each of the ends ***A*** and ***C*** (see figure). Initially each bar is of length ***L***; and point ***B*** is at a distance ***h*** above the line ***AC***. The bars are identical, each having cross sectional area ***A*** and Young's modulus **E**. A vertical force with magnitude **F** is applied at ***B***. Show that the displacement **y** of ***B*** is related to **F** by the equation:

$$F = \frac{2\,A\,E}{L}\,(h-y)\left[\frac{L}{\sqrt{y^2 - 2\,hy + L^2}} - 1\right]$$



Hint: To solve this problem you will need to use the following concept that you learned from strength of materials:

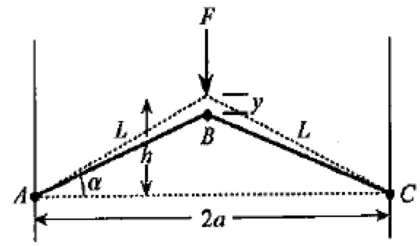
FIGURE 7.74 Stretch in rod when force is applied to one end



$$x = \frac{FL}{AE}$$

Solution:

If we let P be the magnitude of the force in each bar when B is deflected downward by an amount y, then vertical components of forces acting at B give $2P\sin\alpha - F = 0 \Longrightarrow F = 2P\sin\alpha$. From the strength of materials equation , $P = (AE/L)[L - \sqrt{(h-y)^2 + a^2}]$, and therefore



$$F = \frac{2AE}{L}[L - \sqrt{(h-y)^2 + a^2}]\frac{h-y}{\sqrt{(h-y)^2 + a^2}} = \frac{2AE}{L}(h-y)\left[\frac{L}{\sqrt{(h-y)^2 + a^2}} - 1\right]$$

$$= \frac{2AE}{L}(h-y)\left[\frac{L}{\sqrt{y^2 - 2hy + L^2}} - 1\right].$$

Extra Practice Problems

Solutions to these problems are in the Trim's Student Solution Manual

1. S. 11.1, Probs. 4, 14, 20
2. 2. 11.3, Probs. 8, 20, 30