SYDE252 - lecture notes

09/01/18

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Systems Design Engineering
note: some material (figures) borrowed from various sources

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5. Laplace+, Bode

09/11/18

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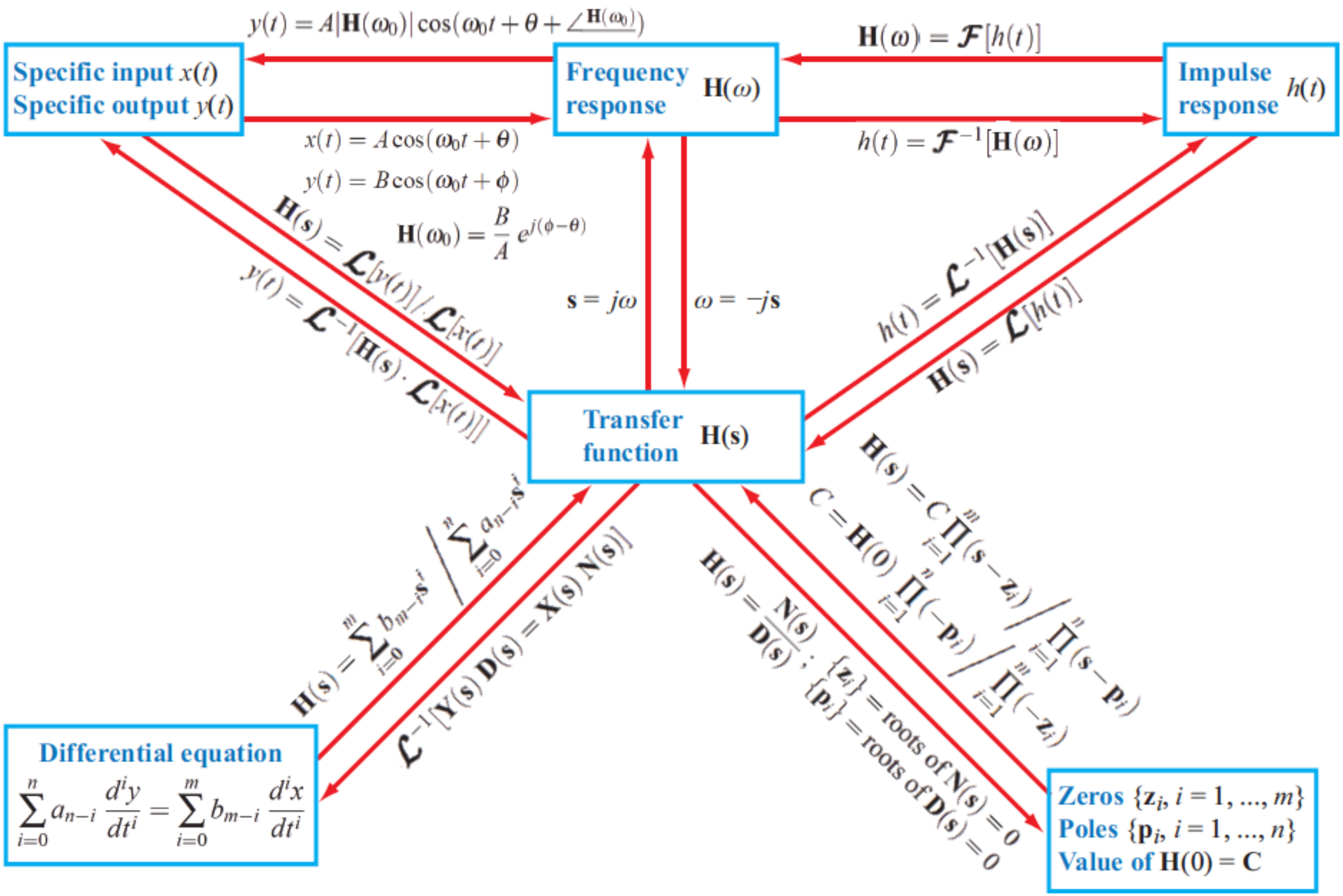
inspiration

- Laplace's Demon
- In the introduction to his 1814 Essai philosophique sur les probabilités, Pierre-Simon Laplace extended an idea of Gottfried Leibniz which became famous as Laplace's Demon, the locus classicus definition of strict physical determinism, with its one possible future.
- Laplace said,
- "We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes."
- Nous devons donc envisager l'état présent de l'universe comme l'effet de son état antérieur, et comme la cause de celui qui va suivre. Une intelligence qui pour un instant donné connaîtrait toutes les forces dont la nature est animée et la situation respective des êtres qui la composent, si d'ailleurs elle était assez vaste pour soumettre ces données à l'analyse, embrasserait dans la même formule les mouvements des plus grands corps de l'universe et ceux du plus léger atome; rien ne serait incertain pour elle, et l'avenir comme le passé serait présent a ses yeux.
- (Essai philosophique sur les probabilités, 1814)

System Description relationships



Transfer Function

- $x(t) \longleftrightarrow \mathbf{X}(\mathbf{s})$
  $h(t) \longleftrightarrow \mathbf{H}(\mathbf{s})$
  $y(t) \longleftrightarrow \mathbf{Y}(\mathbf{s})$

the system output $y(t)$ is convolution of $x(t)$ with $h(t)$,

$$y(t) = x(t) * h(t) = \int_{0^-}^{\infty} x(\tau)\, h(t-\tau)\, d\tau,$$

Because τ in the inner integral represents nothing more than a constant time shift, we can introduce the dummy variable $\mu = t - \tau$ and replace dt with $d\mu$,

$$\mathcal{L}[x(t) * h(t)] = \int_{0^-}^{\infty} x(\tau)\, e^{-\mathbf{s}\tau} \left[\int_{-\tau}^{\infty} h(\mu)\, e^{-\mathbf{s}\mu}\, d\mu \right] d\tau$$

$$= \int_{0^-}^{\infty} x(\tau)\, e^{-\mathbf{s}\tau}\, d\tau \int_{0^-}^{\infty} h(\mu)\, e^{-\mathbf{s}\mu}\, d\mu$$

$$= \mathbf{X}(\mathbf{s})\, \mathbf{H}(\mathbf{s}).$$

$$\boxed{y(t) = x(t) * h(t) \longleftrightarrow \mathbf{Y}(\mathbf{s}) = \mathbf{X}(\mathbf{s})\, \mathbf{H}(\mathbf{s}).}$$

▶ Convolution in the time domain corresponds to multiplication in the **s**-domain. ◀

In symbolic form:

	Excitation	LTI system	Response
Time domain	$x(t)$ →	$\boxed{h(t)}$ →	$y(t) = x(t) * h(t)$
		$\updownarrow$	
s-domain	$\mathbf{X}(\mathbf{s})$ →	$\boxed{\mathbf{H}(\mathbf{s})}$ →	$\mathbf{Y}(\mathbf{s}) = \mathbf{X}(\mathbf{s})\, \mathbf{H}(\mathbf{s})$

Example

The output response of a system excited by a unit step function at $t = 0$ is given by

$$y(t) = [2 + 12e^{-3t} - 6\cos 2t]\,u(t).$$

Determine: (a) the transfer function of the system and (b) its impulse response.

$X(s) = \frac{1}{s}$ $\quad x(t) = u(t)$

$Y(s) = \frac{2}{s} + \frac{12}{s+3} - \frac{6s}{s^2+4}$

$H(s) = \frac{Y(s)}{X(s)} = 2 + \frac{12s}{(s+3)} - \frac{6s^2}{s^2+4}$

$= 2 + \left(12 - \frac{36}{s+3}\right) + \left(-6 + \frac{24}{s^2+4}\right)$

$\frac{12s}{s+3} = a + \frac{b}{s+3}$ $\quad$ $as = 12$, $3a + b = 0$

$h(t) = 8\delta(t) - 36e^{-3t}u(t) + 12\sin(2t)u(t)$

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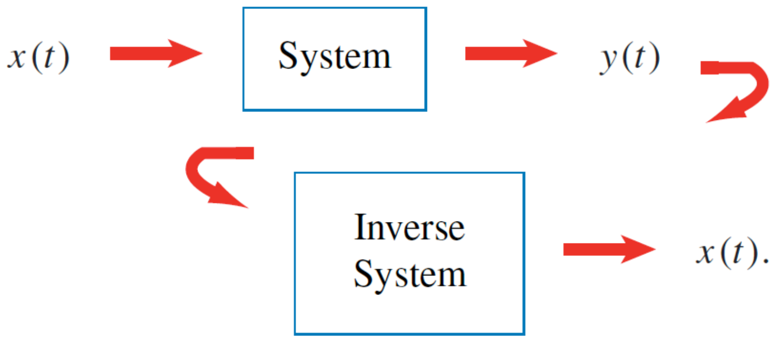
Invertible System



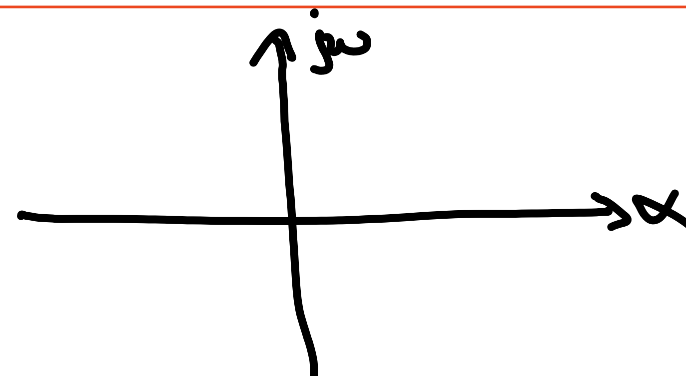
Inverse system is not stable because it is an improper rational function

$$\mathbf{H(s)} = \frac{\mathbf{Y(s)}}{\mathbf{X(s)}} = \frac{b_1\mathbf{s} + b_2}{\mathbf{s}^2 + a_1\mathbf{s} + a_2}$$

▶ A BIBO stable and causal LTI system has a BIBO stable and causal inverse system if and only if all of its poles and zeros are in the open left half-plane, and they are equal in number (its transfer function is proper). Such a system is called a ***minimum phase*** system. ◀

Transfer function of Inverse system:

$$\mathbf{G(s)} = \frac{1}{\mathbf{H(s)}} = \frac{\mathbf{s}^2 + a_1\mathbf{s} + a_2}{b_1\mathbf{s} + b_2}$$



$s = \alpha \pm j\omega$

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Transfer function simplifications

$$x(t) = e^{st}$$

$$y(t) = h(t) * e^{st}$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) e^{s(t-\tau)} d\tau$$

$$y(t) = e^{st} \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau$$

$$H(s) = \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau$$

$$\therefore y(t) = e^{st} H(s)$$

$$y(t) = x(t) * h(t)$$

$$\therefore Y(s) = X(s)H(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\mathscr{L}(y_{zs}(t))}{\mathscr{L}(x(t))}$$

If we have a particular signal $x(t) = e^{s_0 t}$ then

$$y(t) = e^{s_0 t} H(s_0)$$

$$e^{jwt} \rightarrow H(jw) e^{jwt}$$

$$\cos(wt) = R(e^{jwt}) \rightarrow R[H(jw) e^{jwt}]$$

$$H(jw) = |H(jw)| e^{j\angle H(jw)}$$

Transfer function examples

Find frequency response of $H(s) = \dfrac{s + 0.1}{s + 5}$

for inputs a) $x_1(t) = \cos(2t)$ b) $x_2(t) = \cos(10t - 50°)$

let $s = j\omega$ $H(j\omega) = \dfrac{j\omega + 0.1}{j\omega + 5}$

$|H(j\omega)| = \dfrac{\sqrt{\omega^2 + .01}}{\sqrt{\omega^2 + 25}}$

a) $|H(j2)| = \dfrac{\sqrt{2^2 + .01}}{\sqrt{2^2 + 25}} = .372$

$\phi(j2) = \tan^{-1}\dfrac{2}{.1} - \tan^{-1}\left(\dfrac{2}{5}\right) = 65.3°$

$\therefore y(t) = .372\cos(2t + 65.3°)$

$|H(j10)| = .894$

$\angle(j10) = 26°$

$y(t) = .894\cos(10t - 24°)$

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Ideal Differentiator

complex #

$= |c| \angle c$

IDEAL DIFFERENTIATOR

$H(s) = s \qquad H(j\omega) = j\omega = \omega e^{j\pi/2}$

$\therefore |H(j\omega)| = \omega \qquad \angle H(j\omega) = \frac{\pi}{2}$

$\frac{d}{dt} \cos \omega t = -\omega \sin \omega t = \omega \cos\left(\omega t + \frac{\pi}{2}\right)$

amplifies high freq ω (noise)

phase shift

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Ideal Integrator

IDEAL INTEGRATOR

$$H(s) = \frac{1}{s} \qquad H(j\omega) = \frac{1}{j\omega} = \frac{-j}{\omega} = \frac{1}{\omega} e^{-j\pi/2}$$

$$|H(j\omega)| = \frac{1}{\omega} \qquad \angle H(j\omega) = -\frac{\pi}{2}$$

$$\int \cos\omega t \, dt = \frac{1}{\omega} \sin\omega t = \frac{1}{\omega} \cos\left(\omega t - \frac{\pi}{2}\right)$$

suppresses high freq^{n} - averages (low pass)

Bode Plots

A Bode plot is a standard format for plotting frequency response of LTI systems. Becoming familiar with this format is useful because:

1. It is a standard format, so using that format facilitates communication between engineers.

2. Many common system behaviors produce simple shapes (e.g. straight lines) on a Bode plot, so it is easy to either look at a plot and recognize the system behavior, or to sketch a plot from what you know about the system behavior. endenumerate The format is a log frequency scale on the horizontal axis and, on the vertical axis, phase in degrees and magnitude in decibels.

Bode Plots

Decibels

Decibels Definition: for voltages or other physical variables (current, velocity, pressure, etc.).

decibels: $dB = 20\log_{10}\frac{V_{out}}{V_{in}}$
Since power is propotional to voltage squared (or current, velocity, pressure, etc., squared) the definition can be rewritten in terms of power as:

$$dB = 20\log_{10}\frac{V_{out}}{V_{in}} = 10\log_{10}()\frac{V_{out}}{V_{in}})^2 = 10\log_{10}\frac{P_{out}}{P_{in}}$$

Common values:

$$10\log_{10}2 = 20\log_{10}\sqrt{2} = 3db$$

$$10\log_{10}\frac{1}{2} = 20\log_{10}\frac{1}{\sqrt{2}} = -3db$$

$$10\log_{10}10 = 20\log_{10}\sqrt{10} = 10db$$

$$10\log_{10}100 = 20\log_{10}10 = 20db$$

note that 10 db for every factor of 10 in power.

Bode Plots

$K \quad \left(\frac{s}{a} - z_1\right)$

We are interested in the frequency response of an LTI system. The transfer function can be written like this:

$$H(s) = K\frac{(s - z_1)(s - z_2)...}{(s - p_1)(s - p_2)...} \qquad (6.1)$$

when we substitute $s = jw$ we get

$$H(j\omega) = K\frac{(j\omega - z_1)(j\omega - z_2)...}{(j\omega - p_1)(j\omega - p_2)...} \qquad (6.2)$$

Bode Plots

The magnitude is the product of the magnitude of all the terms. Summing terms is easy to do graphically; products are harder. However, on a log scale (e.g., dB), the product turns into a sum. Thus, if we plot the behavior of each term, we can then simply add the plots to find the total behavior. $H(s)$ is a product (or quotient) of a bunch of complex numbers. Using polar form, we can say that the angle of the product (quotient) is the sum of the angles of each term (except for division we subtract, so it's the sum of the angles for the top terms, minus the sum of the angles for the terms in the denominator).

Bode Plots

For approximating the Bode Plot, the steps to perform are:

1. Establish where the x axis crosses the y-axis. If there is a constant term, make this the value of that crossing.
2. For each pole and zero draw the approximate plot.
3. Add all approximately.
4. Add corrections if necessary (or if accuracy is needed).

Bode Plots - single pole

$H(s) = \frac{a}{s+a}$

Magnitude response:

- Low-frequency asymptote ($\omega \to 0$), flat
- Breakpoint at $\omega = a$
- High frequency asymptote, ˘20 dB/decade
- Actual curve is ˘3 dB below breakpoint

Phase response:

- Low frequency asymptote = 0 degrees
- ˘45 degrees at breakpoint ($\omega = a$)
- High frequency assymptote at -90 degrees
- Central slope crosses 0 degrees at $\omega = \frac{a}{5}$, -90 degrees at $\omega = 5a$

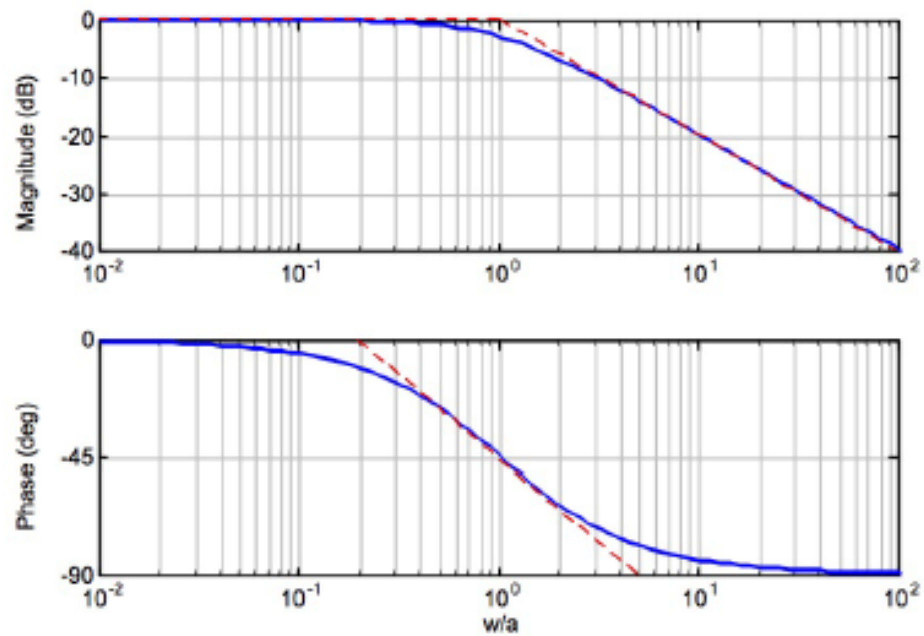

Figure 6.1: *Single Pole: Magnitude and Phase plot*

Bode Plots - single zero

Single zero
$H(s) = \frac{s+b}{s}$

Magnitude response:

- Low-frequency asymptote ($\omega \rightarrow 0$), flat
- Breakpoint at $\omega = b$
- High frequency asymptote, +20 dB/decade
- Actual curve is = 3 dB above breakpoint

Phase response:

- Low frequency asymptote = 0 degrees
- +90 degrees at breakpoint ($\omega = a$)
- High frequency assymptote at +180 degrees
- Central slope crosses 0 degrees at $\omega = \frac{a}{5}$, -180 degrees at $\omega = 5a$

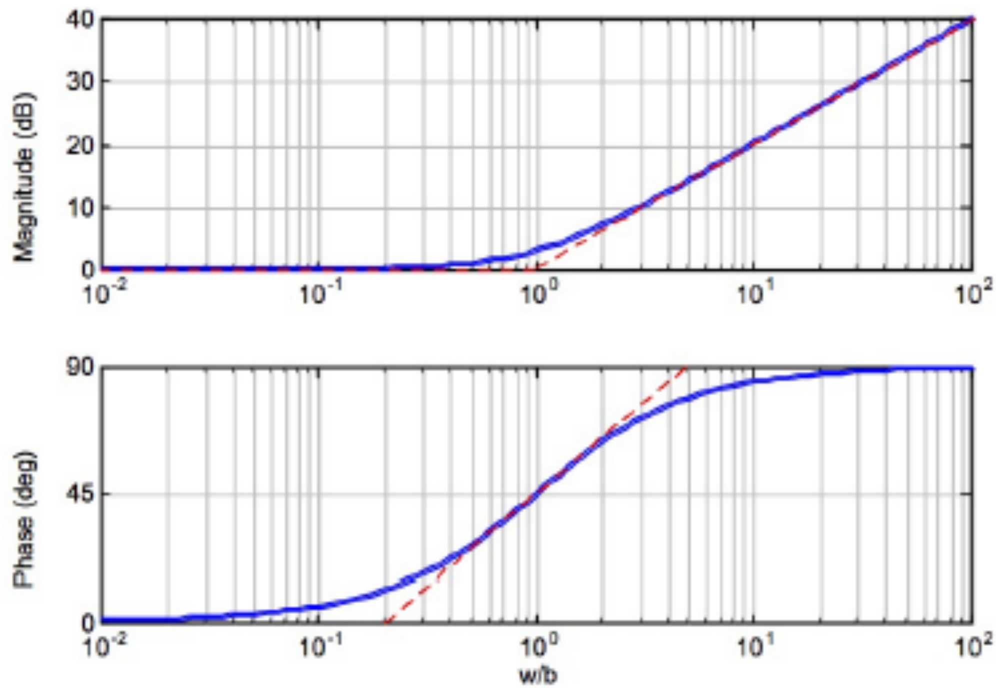

Figure 6.2: *Single Zero: Magnitude and Phase plot*

Bode Plots - double pole

$H(s) = \frac{a^2}{(s+a)^2}$

Can get those by taking 2 poles and adding them.

Magnitude response:

- Low-frequency asymptote ($\omega \to 0$), flat
- Breakpoint at $\omega = a$
- High frequency asymptote, -40 dB/decade
- Actual curve is = 6 dB above breakpoint

Phase response:

- Low frequency asymptote = 0 degrees
- $+90$ degrees at breakpoint ($\omega = a$)
- High frequency assymptote at $+180$ degrees
- Central slope crosses 0 degrees at $\omega = \frac{a}{5}$, -180 degrees at $\omega = 5a$

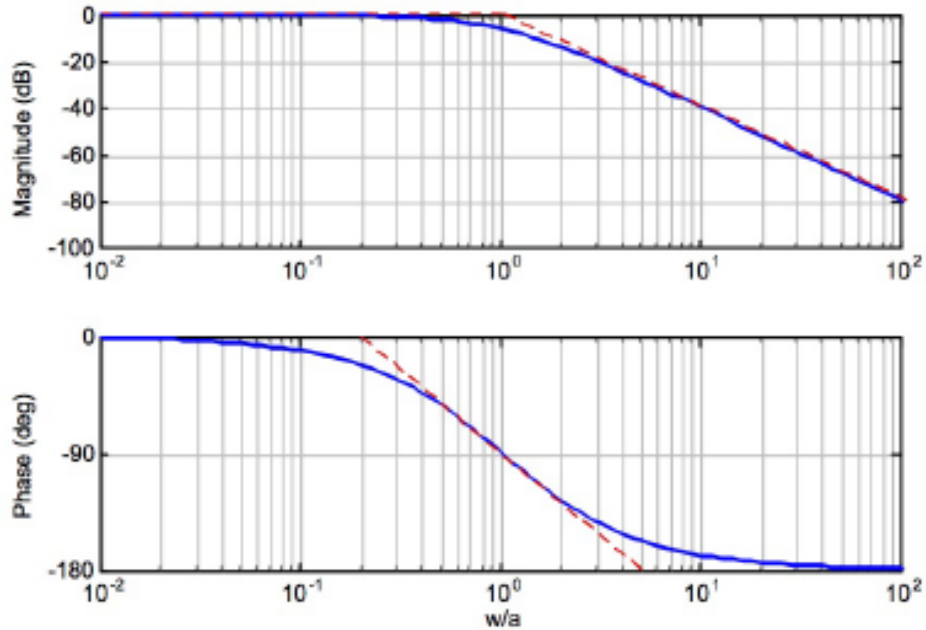

Figure 6.3: *Double Pole: Magnitude and Phase plot*

Bode Plots - 2nd order underdamped response

$H(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

Magnitude response:

- Low-frequency asymptote ($\omega \to 0$), flat
- Breakpoint at $\omega = \omega_n$
- High frequency asymptote, -40 dB/dec
- Resonant peak is at height $\frac{1}{2\zeta}$
- The actual maximum occurs at $\omega = \frac{\omega_n}{5\zeta}$

Phase response:

- Low frequency asymptote $= 0$ degrees
- $+90$ degrees at breakpoint ($\omega = \omega_n$)
- High frequency assymptote at -180 degrees
- Central slope crosses 0 degrees at $\omega = \frac{\omega}{5\zeta}$, -180 degrees at $\omega = 5\zeta\omega_n$

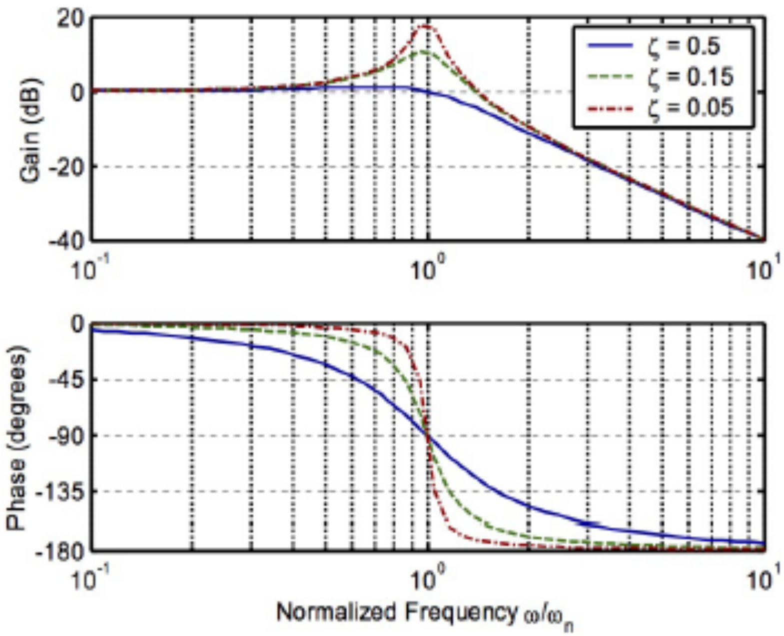

Figure 6.5: *Second order underdamped: Magnitude and Phase plot*

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Bode Plots - 2nd order underdamped response

$$H(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

	$20\log_{10}\lvert H(j\omega)\rvert$	$\angle H(j\omega)$
Exact	$-10\log_{10}\left[1-2\left(1-2\zeta^2\right)\left(\frac{\omega}{\omega_n}\right)^2+\left(\frac{\omega}{\omega_n}\right)^4\right]$	$\operatorname{atan2}\left[-2\zeta\frac{\omega}{\omega_n},\ 1-\left(\frac{\omega}{\omega_n}\right)^2\right]$
$\omega = \omega_n$	$20\log_{10}\frac{1}{2\zeta}$ dB	$-90°$
$\omega << \omega_n$	0 dB	$0°$
$\omega >> \omega_n$	-40 dB/decade	$-180°$

Figure 6.4: *Second order underdamped: Magnitude and Phase critical values*

Bode Plots - 2nd order underdamped response

Other second order responses

$H(s) = \frac{\omega_n s}{s^2+2\zeta\omega_n s+\omega_n^2}$

and

$H(s) = \frac{s^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ (bandpass and high pass respectively), the assynmptotes are different but they always cross at 0 dB and the slope changes at -40 db per decade from low to high or visa versa.

Quality factor (Q)

The empirical definition is that for a resonant peak $Q = \frac{\omega_n}{\Delta\omega}$ where $\Delta\omega$ is the distance between half power (-3 dB) points. With this, the height of a resonant peak is Q and the banbdwidth is $\frac{\omega_n}{Q}$

The general definition of Q is $2\pi \frac{energy-stored}{energy-lost-per-cycle}$ at resonance.

The relationship to the damping factor is $Q = \frac{1}{2\zeta}$

A good quick summary of making Bode plots is given in the next 2 figues taken from http://lpsa.swarthmore.edu

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Bode Plots - Bode plot rules

Term	Magnitude	Phase
Constant: K	$20 \cdot \log_{10}(\lvert K \rvert)$	$K>0$: $0°$ $K<0$: $\pm 180°$
Real Pole: $\frac{1}{\frac{s}{\omega_0}+1}$	• Low freq. asymptote at 0 dB • High freq. asymptote at -20 dB/dec • Connect asymptotic lines at ω_0,	• Low freq. asymptote at 0°. • High freq. asymptote at -90°. • Connect with straight line from $0.1 \cdot \omega_0$ to $10 \cdot \omega_0$.
Real Zero*: $\frac{s}{\omega_0}+1$	• Low freq. asymptote at 0 dB • High freq. asymptote at +20 dB/dec. • Connect asymptotic lines at ω_0.	• Low freq. asymptote at 0°. • High freq. asymptote at +90°. • Connect with line from $0.1 \cdot \omega_0$ to $10 \cdot \omega_0$.
Pole at Origin: $\frac{1}{s}$	• -20 dB/dec; through 0 dB at $\omega=1$.	• -90° for all ω.
Zero at Origin*: s	• +20 dB/dec; through 0 dB at $\omega=1$.	• +90° for all ω.
Underdamped Poles: $\frac{1}{\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1}$	• Low freq. asymptote at 0 dB. • High freq. asymptote at -40 dB/dec. • Connect asymptotic lines at ω_0. • Draw peak† at freq. ω_0, with amplitude $H(j\omega_0)=-20 \cdot \log_{10}(2\zeta)$	• Low freq. asymptote at 0°. • High freq. asymptote at -180°. • Connect with straight line from $\omega=\omega_0 \cdot 10^{-\zeta}$ to $\omega_0 \cdot 10^{\zeta}$
Underdamped Zeros*: $\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1$	• Low freq. asymptote at 0 dB. • High freq. asymptote at +40 dB/dec. • Connect asymptotic lines at ω_0. • Draw dip† at freq. ω_0, with amplitude $H(j\omega_0)=+20 \cdot \log_{10}(2\zeta)$	• Low freq. asymptote at 0°. • High freq. asymptote at +180°. • Connect with straight line from $\omega=\omega_0 \cdot 10^{-\zeta}$ to $\omega_0 \cdot 10^{\zeta}$
Time Delay: e^{-sT}	• No change in magnitude	• Phase drops linearly. Phase = $-\omega T$ radians or $-\omega T \cdot 180/\pi°$. On logarithmic plot phase appears to drop exponentially.

Notes:

ω_0 is assumed to be positive

* Rules for drawing zeros create the mirror image (around 0 dB, or 0°) of those for a pole with the same ω_0.

† We assume any peaks for $\zeta>0.5$ are too small to draw, and ignore them. However, for underdamped poles and zeros peaks exists for $0<\zeta<0.707=1/\sqrt{2}$ and peak freq. is not exactly at, ω_0 (peak is at $\omega_{peak}=\omega_0\sqrt{1-2\zeta^2}$).

For n^{th} order pole or zero make asymptotes, peaks and slopes n times higher than shown. For example, a double (i.e., repeated) pole has high frequency asymptote at -40 dB/dec, and phase goes from 0 to -180°). Don't change frequencies, only the plot values and slopes.

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Quick Reference for Making Bode Plots

If starting with a transfer function of the form (some of the coefficients b_i, a_i may be zero).

$$H(s) = C\frac{s^n + \cdots + b_1 s + b_0}{s^m + \cdots + a_1 s + a_0}$$

Factor polynomial into real factors and complex conjugate pairs (p can be positive, negative, or zero; p is zero if a_0 and b_0 are both non-zero).

$$H(s) = C \cdot s^p \frac{(s+\omega_{z1})(s+\omega_{z2})\cdots(s^2+2\zeta_{z1}\omega_{0z1}s+\omega_{0z1}^2)(s^2+2\zeta_{z2}\omega_{0z2}s+\omega_{0z2}^2)\cdots}{(s+\omega_{p1})(s+\omega_{p2})\cdots(s^2+2\zeta_{p1}\omega_{0p1}s+\omega_{0p1}^2)(s^2+2\zeta_{p2}\omega_{0p2}s+\omega_{0p2}^2)\cdots}$$

Put polynomial into standard form for Bode Plots.

$$H(s) = C\frac{\omega_{z1}\omega_{z2}\cdots\omega_{0z1}^2\omega_{0z2}^2\cdots}{\omega_{p1}\omega_{p2}\cdots\omega_{0p1}^2\omega_{0p2}^2\cdots}\cdot s^p \frac{\left(\frac{s}{\omega_{z1}}+1\right)\left(\frac{s}{\omega_{z2}}+1\right)\cdots\left(\left(\frac{s}{\omega_{0z1}}\right)^2+2\zeta_{z1}\left(\frac{s}{\omega_{0z1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0z2}}\right)^2+2\zeta_{z2}\left(\frac{s}{\omega_{0z2}}\right)+1\right)\cdots}{\left(\frac{s}{\omega_{p1}}+1\right)\left(\frac{s}{\omega_{p2}}+1\right)\cdots\left(\left(\frac{s}{\omega_{0p1}}\right)^2+2\zeta_{p1}\left(\frac{s}{\omega_{0p1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0p2}}\right)^2+2\zeta_{p2}\left(\frac{s}{\omega_{0p2}}\right)+1\right)\cdots}$$

$$= K \cdot s^p \frac{\left(\frac{s}{\omega_{z1}}+1\right)\left(\frac{s}{\omega_{z2}}+\right)\cdots\left(\left(\frac{s}{\omega_{0z1}}\right)^2+2\zeta_{z1}\left(\frac{s}{\omega_{0z1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0z2}}\right)^2+2\zeta_{z2}\left(\frac{s}{\omega_{0z2}}\right)+1\right)\cdots}{\left(\frac{s}{\omega_{p1}}+1\right)\left(\frac{s}{\omega_{p2}}+1\right)\cdots\left(\left(\frac{s}{\omega_{0p1}}\right)^2+2\zeta_{p1}\left(\frac{s}{\omega_{0p1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0p2}}\right)^2+2\zeta_{p2}\left(\frac{s}{\omega_{0p2}}\right)+1\right)\cdots}$$

Take the terms (constant, real poles and zeros, origin poles and zeros, complex poles and zeros) one by one and plot magnitude and phase according to rules on previous page. Add up resulting plots.

Bode Plots - Bode plots

$x(t) \rightarrow \boxed{H(s)} \rightarrow y(t)$

freq^n units in radians/sec (rad/s)
- also called angular freq^n

2π rad/s = 1 Hz

1 rad/s ≅ .159155 Hz ≈ 57.29 deg/sec

1 rad/s ≅ 9.5493 rpm (revolutions per minute)

magnitude plotted on a linear scale in decibels

$H_{dB}(j\omega) \triangleq 20 \log_{10} |H(j\omega)|$

phase plotted in degrees

x-axis for frequency uses log scale

Building Blocks - 1st magnitude
- 2nd Phase

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Bode Plots - Bode plots

consider a transfer fn

$$H(s) = \frac{K(s+a_1)(s+a_2)}{s(s+b_1)(s^2+b_2 s+b_3}$$

POLES - are roots of denominator

$s=0$, $s=-b_1$ & solⁿ of quadratic

ZEROS - are roots of denominator

$s=-a_1$, $s=-a_2$

Bode Plots - example 1

$$H(s) = \frac{100}{s+30}$$

$$H(s) = 100 \frac{(s+1)}{(s+10)(s+100)} = 100 \frac{(s+1)}{s^2 + 110s + 1000}$$

Bode Plots
- example 2

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$$H(s) = 10\,\frac{s+10}{s^2+3s} = 10\,\frac{10}{3}\left(\frac{1+\frac{s}{10}}{s\left(\frac{s}{3}+1\right)}\right)$$

Bode Plots - example 3

Example $H(s) = \dfrac{20s\,(s+100)}{(s+2)(s+10)}$

Bode Plots - example 4

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