MTE 203 – Advanced Calculus
Homework 9

Double Integrals

Problem 1: [13.1, Prob. 19]

Evaluate the double iterated integral $\int_0^1 \int_0^x \frac{1}{\sqrt{1-y^2}} dy\, dx$

Problem 2: [S. 13.1, Prob. 33] Application Problem for Double Integrals

In two-dimensional steady state, incompressible flow, the velocity $\mathbf{v} = u(x,y)\hat{\mathbf{i}} + v(x,y)\hat{\mathbf{j}}$, which must satisfy the *continuity equation,* $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$.

If $(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$, find all possible functions $v(x,y)$.

Problem 3: [13.1, Prob. 39] Application Problem

Stream functions $\psi(x,y)$ for two dimensional, steady state, incompressible flow satisfy

$$\frac{\partial \psi}{\partial x} = -v(x,y) \ , \qquad \frac{\partial \psi}{\partial y} = u(x,y)$$

where $\mathbf{v} = u(x,y)\hat{\mathbf{i}} + v(x,y)\hat{\mathbf{j}}$ is the velocity of the flow. Find all stream functions for the flow with

$$\mathbf{v} = -\cos x \sin y\, \hat{\mathbf{i}} + (\sin x \cos y + x)\hat{\mathbf{j}}$$

Evaluation of Double Integrals by Double Iterated Integrals

Problem 4: [13.2, Prob. 3]

Evaluate the double integral over the region

$\iint_R (x+y)\, dA$ where R is bounded by $x = y^3 + 2$ and $x = 1$ and $y = 1$

Problem 5: [13.2, Prob. 17]

Evaluate the double iterated integral by reversing the order of the integral.

$$\int_0^2 \int_0^{\frac{x^2}{2}} \frac{x}{\sqrt{1+x^2+y^2}}\, dy\, dx$$

Hint1: after revising the order use integral by substitution method ($y = \sqrt{5}\, \tan\theta$)

Hint2: $\int (\sec\theta)^3\, d\theta = \frac{1}{2}(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|)$

Double Iterated Integrals in Polar Coordinates

Problem 6: [13.7, Prob. 25]

Find the area inside the circle $x^2 + y^2 = 4x$ and outside the circle $x^2 + y^2 = 1$.

Problem 7: [13.7, Prob. 29]

Find the area of the region bounded by the curve $(x^2 + y^2)^2 = 2xy$

Triple Integrals and Triple Iterated Integrals

Problem 8: [S.13.8, Prob. 3]

Evaluate the triple integral over the region:

$\iiint_V \sin(y+z)\, dV$ Where V is bounded by $z = 0, y = 2x, y = 0, x = 1, z = x + 2y$

Problem 9: [13.8, Prob. 17]

Setup, but do not evaluate, a triple iterated integral for the triple integral.

$\iiint_V x^2y^2z^2\, dV$ where V is bounded by $x = y^2 + z^2$ and $x + 1 = (y^2 + z^2)^2$

Volumes

Problem 10: [13.9, Prob. 19]

A pyramid has a square base with side length *b* and has height *h* at its center.

(a) Find its volume by taking cross-sections parallel to the base (see section 7.9).
(b) Find its volume using triple integrals.

Problem 11: [13.9, Prob. 21] Application problem for Average - Cartesian Coordinates

Find the average value [$\bar{f} = \frac{1}{V}\iiint_V f(x,y,z)dV$] if $f(x,y,z) = x + y + z$ over the region in the first octant bounded by the surfaces $z = 9 - x^2 - y^2$, $z = 0$, and for which $0 \leq x \leq 1$, $0 \leq y \leq 1$.

Warm-Up Problems

Solutions to these problems can be found at the back of your textbook

1. S. 13.1, Probs. 2,6, 8, 10, 14
2. S. 13.2, Probs. 2, 4, 6, 14
3. S. 13.7, Probs. 2, 4, 8,
4. S. 13.8, Probs. 2,4,10
5. S. 13.9, Probs. 2,4,6

Extra Practice Problems

Solutions to these problems can be found at the back of your textbook

1. S. 13.1, Probs. 22, 24, 28, 32
2. S. 13.2, Prob. 10, 18, 36
3. S. 13.7, Probs. 12, 22, 24,30
4. S. 13.8, Probs. 12, 18, 24
5. S. 13.9, Probs. 6, 8, 16, 18,

Extra Challenging Problems

Solutions to these problems can be found at the back of your textbook

1. S. 13.1, Probs. 34, 38
2. S. 13.2, Prob. 30
3. S. 13.7, Probs. 32, 34
4. S. 13.8, Probs. 20, 22
5. S. 13.9, Probs. 26