# MTE 203 – Advanced Calculus
# Homework 10

**<u>Triple Iterated Integrals in Cylindrical Coordinates</u>**

**Problem 1: [13.11, Prob. 23]**

Evaluate the triple integral $\int_0^4 \int_0^{\sqrt{4y-y^2}} \int_0^{x^2+y} dz\ dx\ dy$

**Problem 2: [S.13.11, Prob. 25]**

Evaluate the triple integral $\int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{x^2+y^2} y^2 dz\ dx\ dy$

**Problem 3: [13.11, Prob. 20] Application problem for Moment of Inertia - Cylindrical Coordinates**

Find the moment of inertia of a uniform sphere of radius $R$ about any line through its center.

**<u>Triple Iterated Integrals in Spherical Coordinates</u>**

**Problem 4: [13.12, Prob. 13] Application problem for Centre of Mass- Spherical Coordinates**

Find the center of mass of a uniform hemispherical solid.

**Hint:** $M = \left(\frac{2}{3}\right)\pi\rho R^3$

**Problem 5: [13.12, Prob. 23]**

Find the volume bounded by the surface $(x^2 + y^2 + z^2)^2 = 2z(x^2 + y^2)$.

**Problem 6: [13.12, Prob. 25]**

A sphere of constant density $\rho$ and radius $R$ is located at the origin (figure below). If a mass $m$ is situated at a point $P$ on the z-axis (distance $d > R$ from the center of the sphere) and $dV$ is small element of the volume of the sphere, then according to Newton's universal law of gravitation, the z-component of the force on $m$ due to the mass in $dV$ is given by

$$-\frac{Gm\rho dV cos\psi}{s^2}$$

Where $G$ is a constant and $s$ is distance between $P$ and $dV$.

(a) Show that in spherical coordinates total force on $m$ due to the entire sphere has z-component

$$F_z = -\frac{Gm\rho}{2d}\int_{-\pi}^{\pi}\int_{0}^{\pi}\int_{0}^{R}\left(\frac{s^2+d^2-\mathfrak{K}^2}{s^3}\right)\mathfrak{K}^2\sin\varphi\, d\mathfrak{K}d\varphi d\theta$$

(b) Use the transformation in Exercise 24(b) to write $F_z$ in the form

$$F_z = -\frac{Gm\rho}{2d^2}\int_{-\pi}^{\pi}\int_{0}^{R}\int_{d-\mathfrak{K}}^{d+\mathfrak{K}}\mathfrak{K}\left(\frac{s^2+d^2-\mathfrak{K}^2}{s^2}\right)dsd\mathfrak{K}d\theta$$

And show that $F_z = -GmM/d^2$, where $M$ is the total mass of the sphere.

## Vector Fields:

**Problem 7: [S. 14.1, Prob. 45]**

Find all functions $f(x,y)$ such that $\vec{\nabla}f = \vec{F}$ and $\vec{F} = 2xy\hat{\imath} + x^2\hat{\jmath}$.

## Plotting Vector Fields:

**Problem 8:**

Use Matlab to plot the gradient of the function $z = xe^{-x^2-y^2}$. Then on the same plot, draw the contours of $z(x,y)$.

## Line Integrals:

**Problem 9: [S. 14.2, Prob. 13]**

Evaluate the line integral $\int_C xyds$, where C is the curve $x = 1-y^2, z = 0$ from (1,0,0) to (0,1,0).

**Problem 10: [S. 14.2, Prob. 25]**

The average value of function $f(x, y, z)$ along a curve C is defined as the value of the line integral of the function along the curve divided by the length of the curve.

Find the average value of the function along the curve

$f(x, y, z) = x^2 + y^2 + z^2$ along $x = cos\ t, y = sin\ t. z = t, 0 \leq t \leq \pi.$

**Problem 11: [S. 14.2, Prob. 31]**

In polar coordinates, small lengths along a curve can be expressed in the form $ds = \sqrt{r^2 + (dr/d\theta)^2 d\theta}$. Use this fact to evaluate the following line integral $\oint_C (x^2 + y^2) ds$ where C is the cardioid $r = 1 + \cos\theta$.

**Warm-Up Problems**

Solutions to these problems can be found at the back of your textbook

1. S. 13.11, Probs. 12, 16
2. S. 13.12, Probs. 8
3. S. 14.1, Probs. 22, 26, 46, 50
4. S. 14.2, Probs. 2, 4, 6

**Extra Practice Problems**

Solutions to these problems can be found at the back of your textbook

1. S. 13.11, Probs. 14, 24, 32
2. S. 13.12, Probs. 12, 24
3. S. 14.1, Probs. 54
4. S. 14.2, Prob. 16, 26, 32

**Extra Challenging Problems**

Solutions to these problems can be found at the back of your textbook

1. S. 13.11, Probs. 40
2. S. 13.12, Prob. 26
3. S. 14.1, Prob. 58, 60, 64
4. S. 14.2, Probs. 36