Lecture 19- MTE204-Numerical Integration- Trapezoidal Rule-In-class Examples

Trapezoidal Rule. Example 1

The vertical distance covered by a rocket from 8 to 30 seconds is given by

$$x = \int_8^{30} \left(2000 \ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

a) Use the single segment trapezoidal rule to find the distance covered for 8 to 30 seconds.
b) Find the true error, E_t for part (a).
c) Find the absolute relative true error for part (a).

Solution---

a) $I \approx (b-a)\left[\frac{f(a) + f(b)}{2} \right]$, where $a = 8$ and $b = 30$

$$f(t) = 2000 \ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t$$

$$f(8) = 2000 \ln \left[\frac{140000}{140000 - 2100(8)} \right] - 9.8(8)$$

$$= 177.27 \text{ m/s}$$

$$f(30) = 2000 \ln \left[\frac{140000}{140000 - 2100(30)} \right] - 9.8(30)$$

$$= 901.67 \text{ m/s}$$

$$I \approx (30 - 8)\left[\frac{177.27 + 901.67}{2} \right]$$

$$= 11868 \text{ m}$$

b) The exact value of the above integral is

$$x = \int_8^{30} \left(2000 \ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

$$= 11061 \text{ m}$$

so the true error is

$E_t =$ True Value – Approximate Value $= 11061 - 11868 = -807$ m

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left| \frac{\text{True Error}}{\text{True Value}} \right| \times 100$$

$$= \left| \frac{11061 - 11868}{11061} \right| \times 100$$

$$= 7.2958\%$$

Trapezoidal Rule. Example 2

The vertical distance covered by a rocket from $t = 8$ to $t = 30$ seconds is given by

$$x = \int_8^{30} \left(2000 \ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

a) Use the <u>two-segment</u> trapezoidal rule to find the distance covered from $t = 8$ to $t = 30$ seconds.
b) Find the true error, E_t for part (a).
c) Find the absolute relative true error for part (a).

Solution--

a) The solution using 2-segment Trapezoidal rule is

$$I \approx \frac{b-a}{2n} \left[f(a) + 2 \left\{ \sum_{i=1}^{n-1} f(a + ih) \right\} + f(b) \right] \quad , \; n = 2$$

$$a = 8$$
$$b = 30$$

$$h = \frac{b-a}{n} = \frac{30-8}{2} = 11$$

$$I \approx \frac{30-8}{2(2)} \left[f(8) + 2 \left\{ \sum_{i=1}^{2-1} f(8 + 11i) \right\} + f(30) \right] = \frac{22}{4} \left[f(8) + 2f(19) + f(30) \right]$$

$$= \frac{22}{4} \left[177.27 + 2(484.75) + 901.67 \right] = 11266 \text{ m}$$

b) The exact value of the above integral is

$$x = \int_8^{30} \left(2000 \ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt = 11061 \text{ m}$$

so the true error is

$$E_t = \text{True Value} - \text{Approximate Value}$$
$$= 11061 - 11266 = -205 \text{ m}$$

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left| \frac{\text{True Error}}{\text{True Value}} \right| \times 100 = \left| \frac{11061 - 11266}{11061} \right| \times 100 = 1.8537\%$$

What do you predict in terms of error if more sections (trapezoids) are used to approximate the integral? Look at the table below that has tabulated to compare the results of integral approximation using trapezoidal rule for single trapezoid up to 8 trapezoids for this function:

$$x = \int_8^{30} \left(2000 \ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

n	Approximate Value	E_t	$\left\|\in_t\right\|\%$	$\left\|\in_a\right\|\%$
1	11868	-807	7.296	---
2	11266	-205	1.853	5.343
3	11153	-91.4	0.8265	1.019
4	11113	-51.5	0.4655	0.3594
5	11094	-33.0	0.2981	0.1669
6	11084	-22.9	0.2070	0.09082
7	11078	-16.8	0.1521	0.05482
8	11074	-12.9	0.1165	0.03560

Trapezoidal Rule. Numerical Integration. Example 3 (Group Problem Solving)

Use the multiple-segment trapezoidal rule to find the area under the curve

$f(x) = \dfrac{300x}{1+e^x}$ from $x=0$ to $x=10$.

Solution: Using two segments, we get $h = \dfrac{10-0}{2} = 5$ **,** $f(0) = \dfrac{300(0)}{1+e^0} = 0$

$$f(5) = \frac{300(5)}{1+e^5} = 10.039$$

$$f(10) = \frac{300(10)}{1+e^{10}} = 0.136$$

$$I \approx \frac{b-a}{2n}\left[f(a) + 2\left\{\sum_{i=1}^{n-1} f(a+ih)\right\} + f(b)\right] = \frac{10-0}{2(2)}\left[f(0) + 2\left\{\sum_{i=1}^{2-1} f(0+5)\right\} + f(10)\right]$$

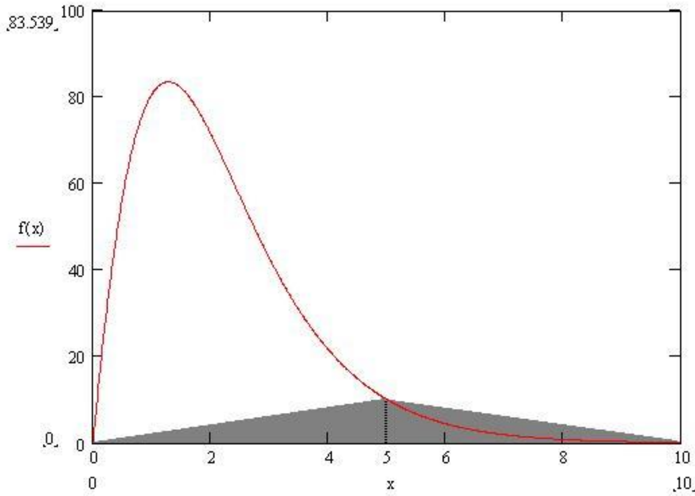
$$= \frac{10}{4}\left[f(0) + 2f(5) + f(10)\right] = \frac{10}{4}\left[0 + 2(10.039) + 0.136\right] = 50.537$$

So what is the true value of this integral?

$$\int_0^{10} \frac{300x}{1+e^x}\,dx = 246.59$$

Making the absolute relative true error of $\left|\in_t\right| = \left|\dfrac{246.59 - 50.535}{246.59}\right| \times 100 = 79.506\%$

Why is the true value so far away from the approximate values? Just take a look at Figure. As you can see, the area under the "trapezoids" covers a small portion of the area under the curve. As we add more segments, the approximated value quickly approaches the true value.



2-segment trapezoidal rule approximation.

Values obtained using multiple-segment trapezoidal rule for $\int_0^{10} \frac{300x}{1+e^x} dx$.

n	Approximate Value	E_t	$\lvert\in_t\rvert$
1	0.681	245.91	99.724%
2	50.535	196.05	79.505%
4	170.61	75.978	30.812%
8	227.04	19.546	7.927%
16	241.70	4.887	1.982%
32	245.37	1.222	0.495%
64	246.28	0.305	0.124%