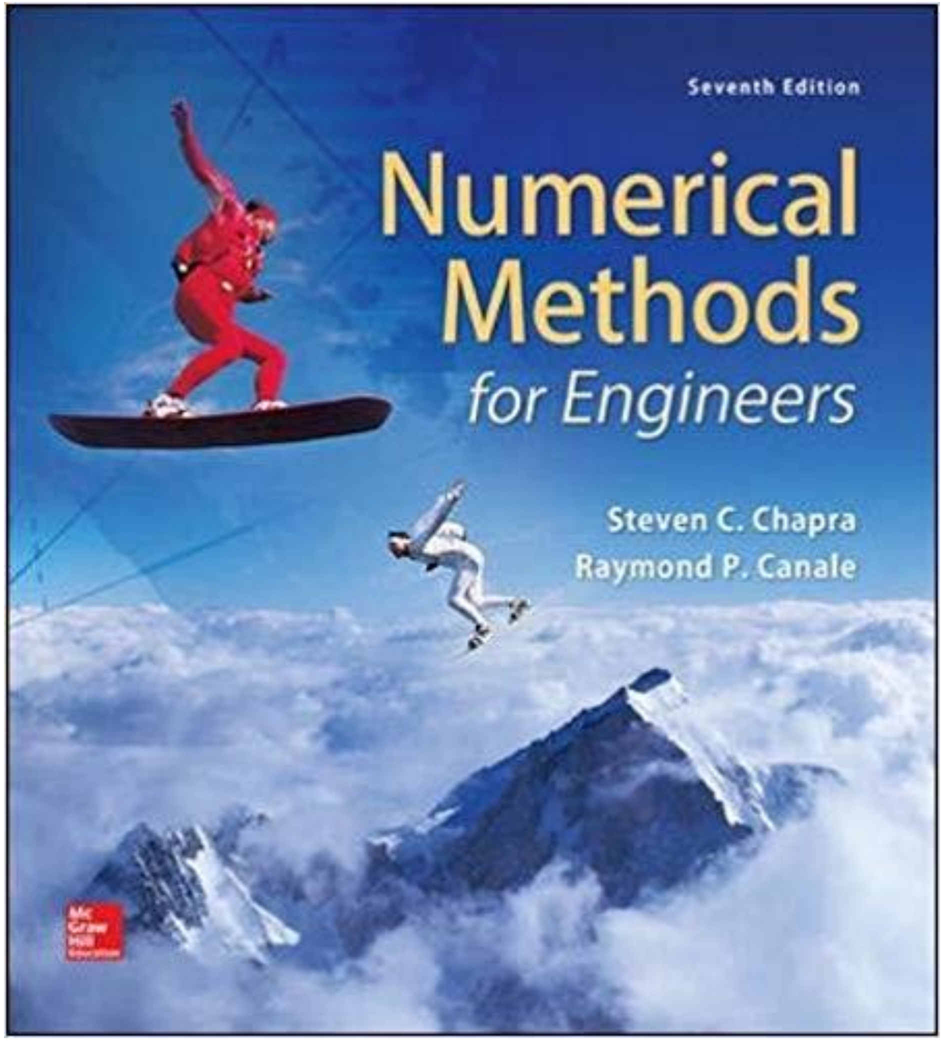
Part 1. Modeling, Computers, and Error Analysis
Chapter 3. Approximation and Round-Off Errors

Lecture 1
Introduction & Error Definitions

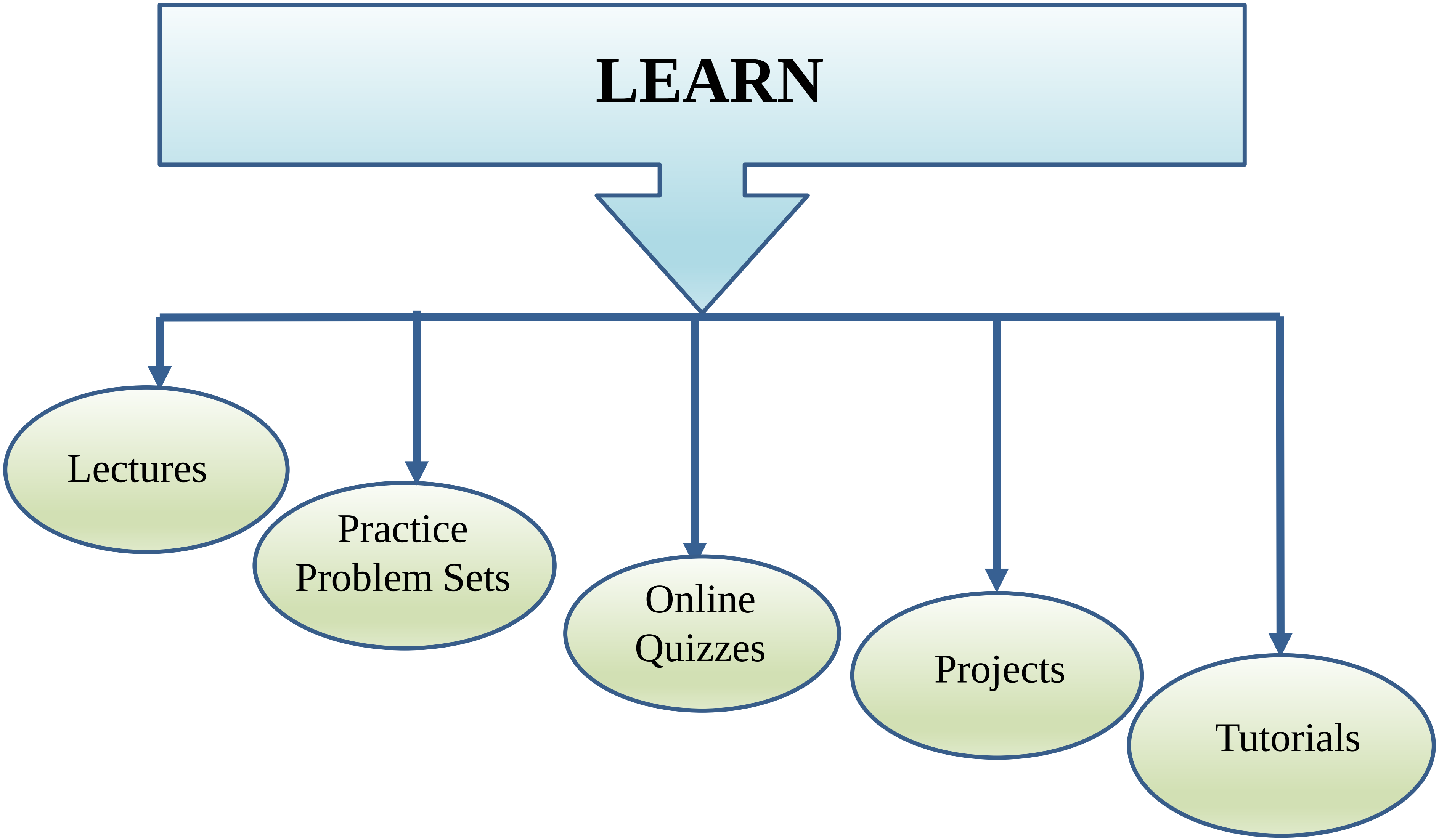
3.2, 3.3

Homeyra Pourmohammadali

Course Textbook



Course Homepage



Grading Scheme

12%	Online Quizzes (4)
10%	Projects (2)
38%	Midterm Exam (1)
40%	Final Exam (1)

Instructional Team

Homeyra Pourmohammadali

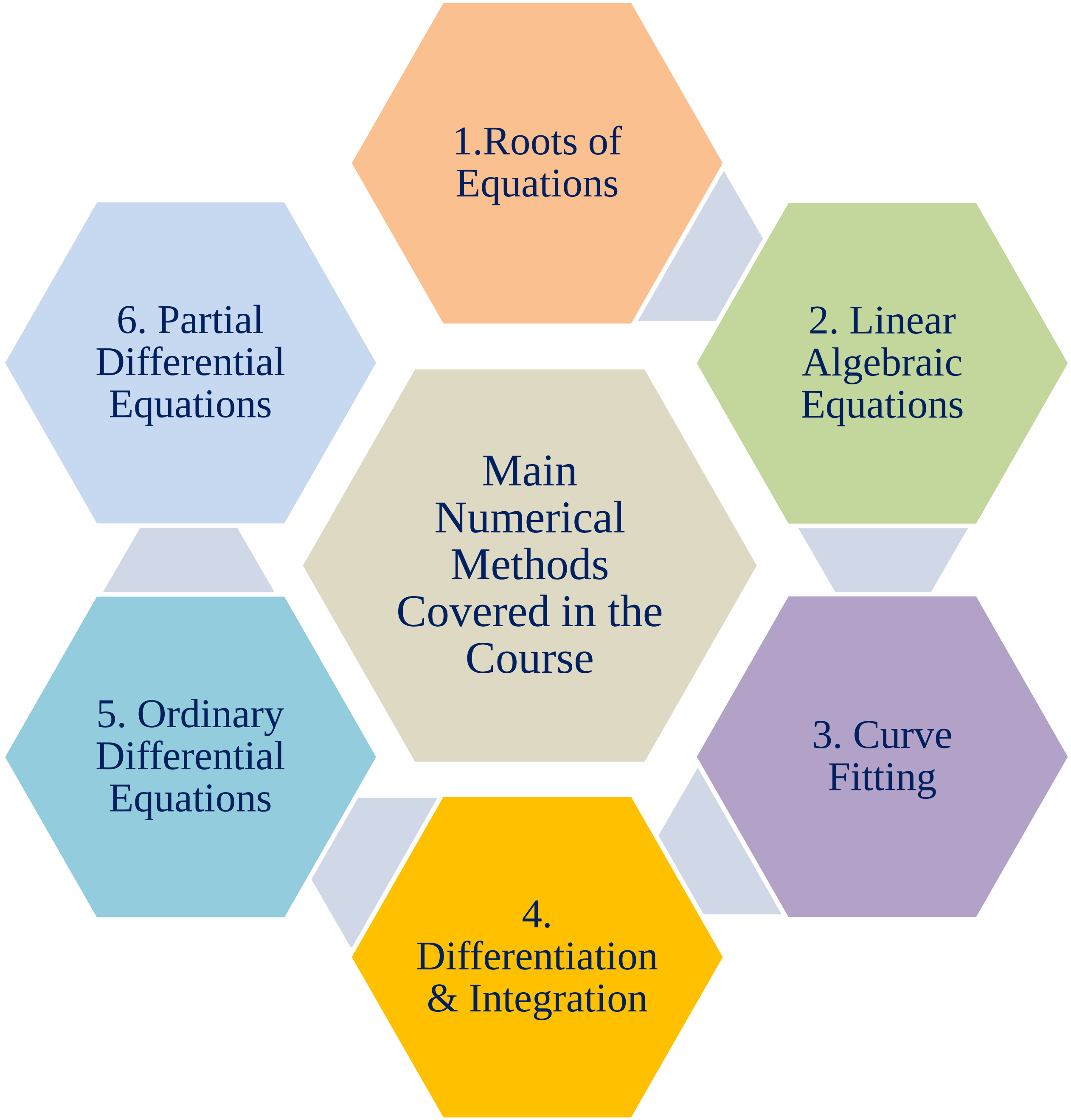
Maryam Razavi (TA)

Sepehr Ghadami (TA)

Instructional Team Availability

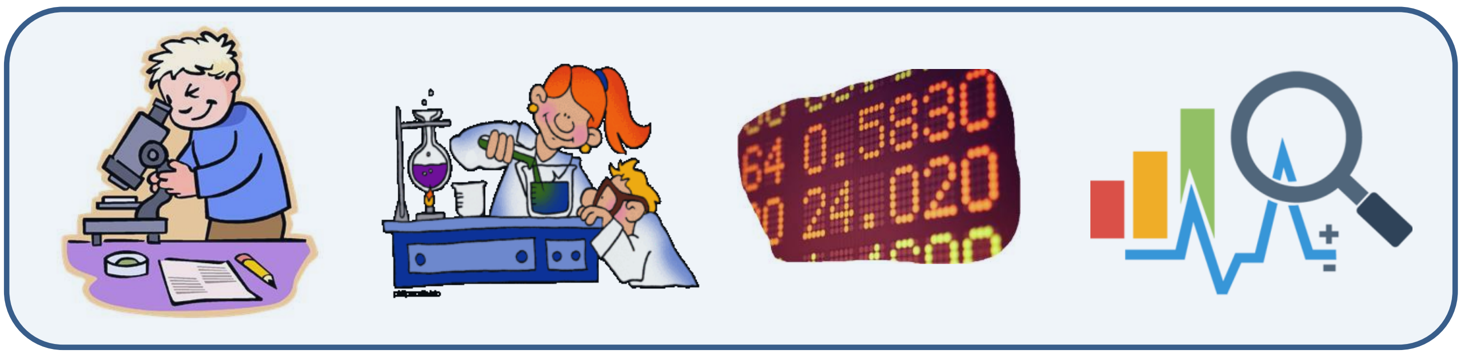
Year: 2B	Term: FALL 2018

	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
8:30 a.m.				MTE 200B Make-up Lecture E5 3101	
9:30 a.m.		MTE 204- Lecture Pourmohammadali E5 3101		MTE 204- Lecture Pourmohammadali E5 3101	
10:30 a.m.					
11:30 a.m.		MTE 200B Make-up Lecture E5 3101			
12:30 p.m.	MTE204 Homeyra's office hour (E5-3047)	MTE204 Maryam's office hour (ERC-3024)	MTE204 Sepehr's Office hour (E2-4403)		
1:30 p.m.				MTE204 Tutorial 01, MC4042 (Maryam & Sepehr)	
2:30 p.m.				MTE 204 Tutorial 02, MC4042 (Maryam & Sepehr)	
3:30 p.m.					


1.Roots of Equations
2. Linear Algebraic Equations
3. Curve Fitting
4. Differentiation & Integration
5. Ordinary Differential Equations
6. Partial Differential Equations
Main Numerical Methods Covered in the Course

Introduction:
Engineering Problem Solving

- Observations, experiments, experimental data, analysis

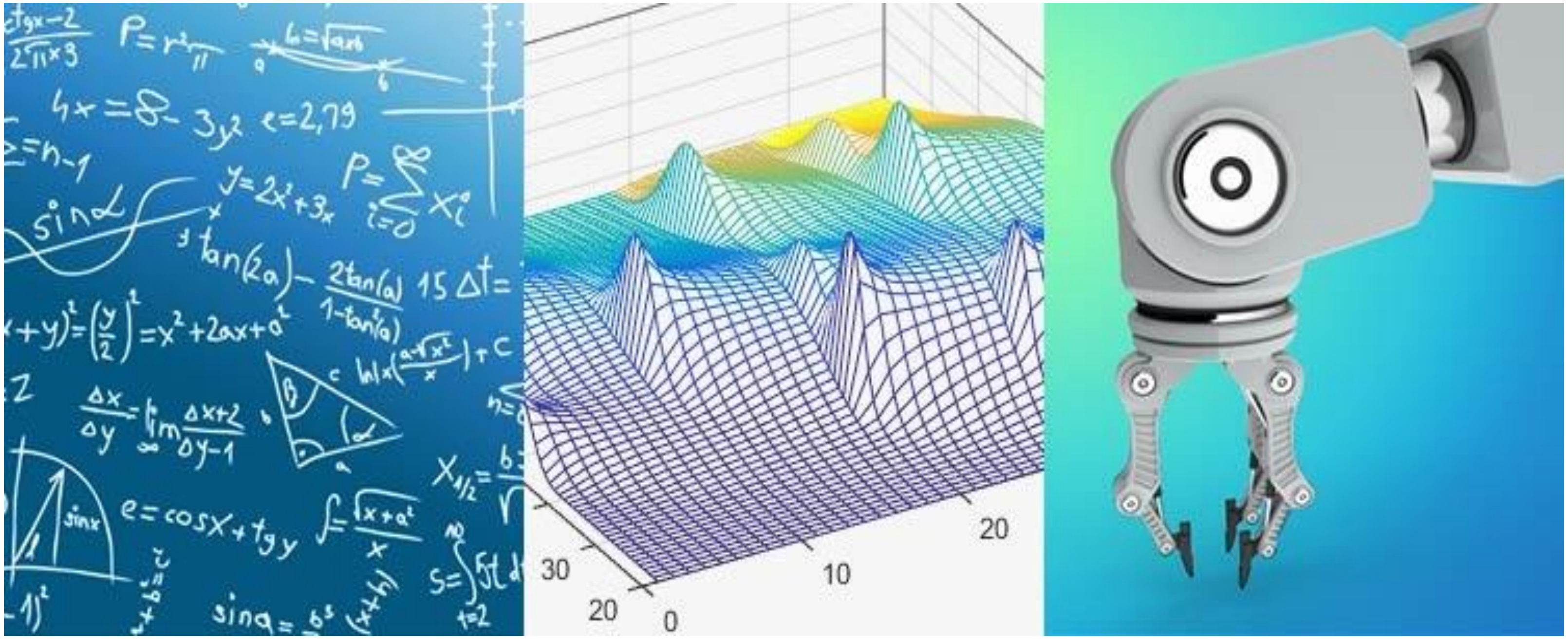


- Fundamental laws, relationships

(these are expressed in form of mathematical model)

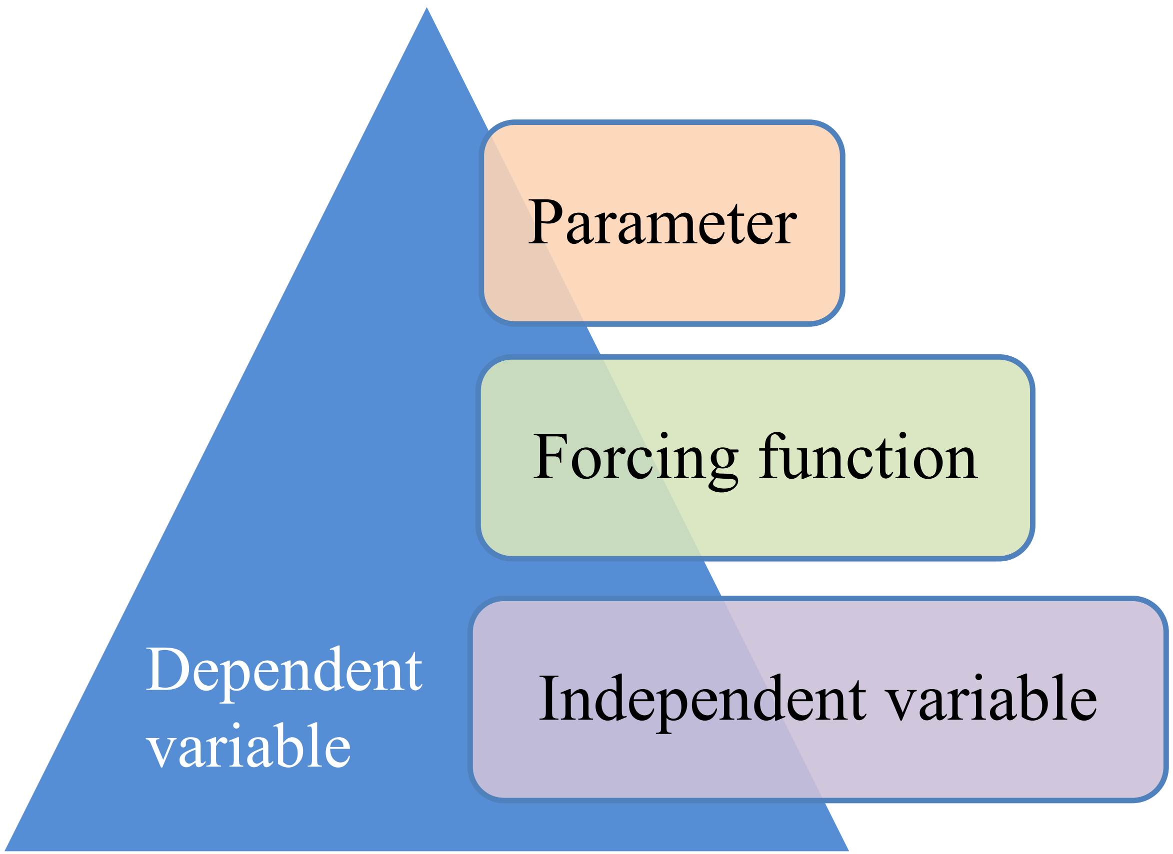
Introduction:
Mathematical Model

- Expresses essential features of physical system in mathematical terms



https://www.mathworks.com/solutions/mathematical-modeling.html

Introduction:
Mathematical Model Components & their Relation



- Dependent variable = f (independent variable, parameter, forcing functions)

Mathematical Models

Analytical Models

Solve equation

(usually explicit)

→ “exact” answers

→ can be mathematically intensive or impossible

Numerical Models

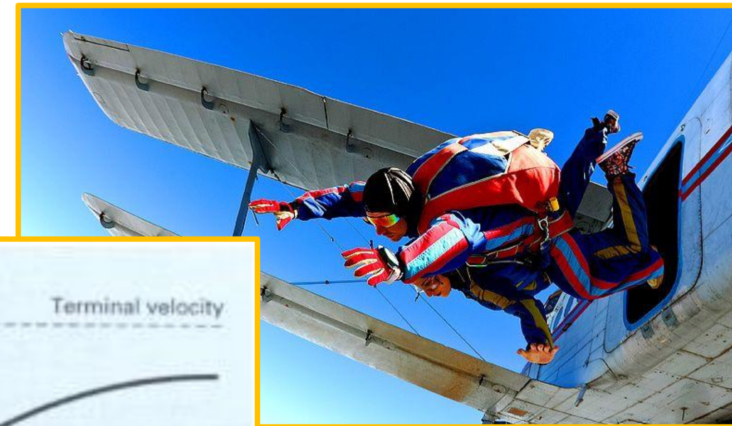
Approximation based on discretization, solving simple (often algebraic) equations in small sub intervals

→ Easier? Quicker?

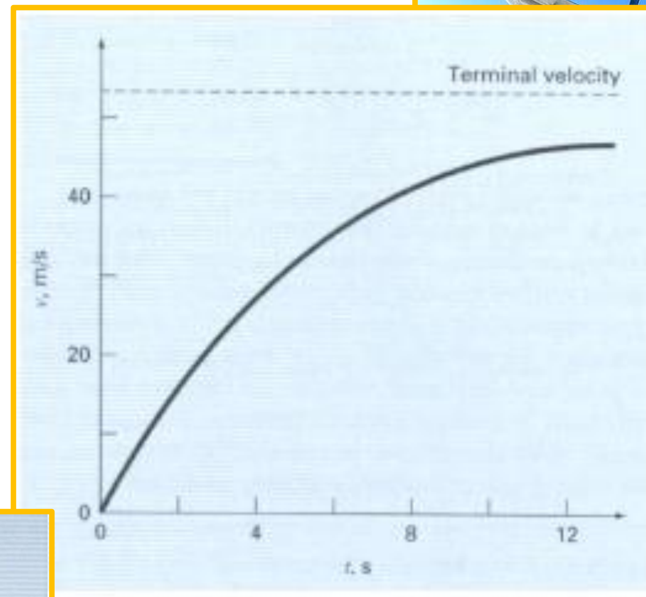
Sometimes <u>only way</u> to solve problem

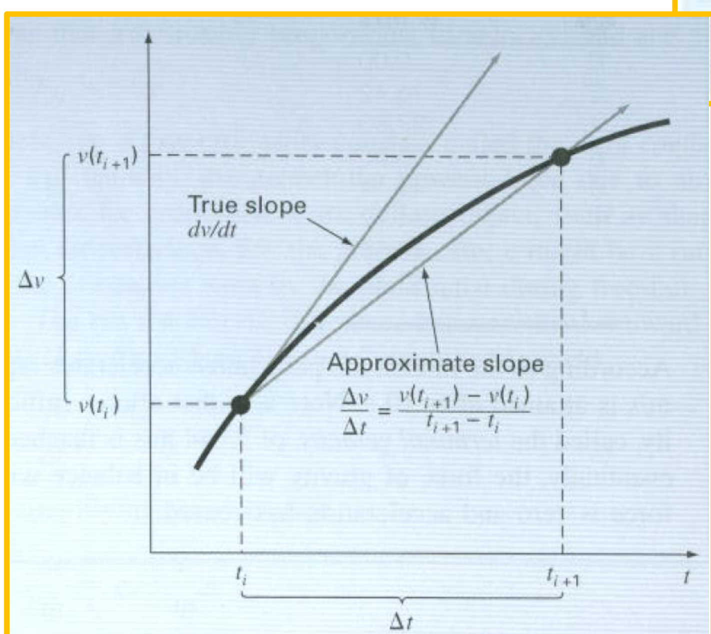
Example 1. Simple mathematical model of a falling parachutist : analytical vs numerical solution

Parachutist approaches terminal velocity



True value →





← Approximation

Approximation

- Compare true and approximate values at different times, gives true error
- At t= 6 s v = 35.68 vs 39.90 m/s

1. Does exact (analytical) solution match physical situation perfectly?

-- No, essential feature

2. What about situations where exact (analytical) solution doesn't exist, can't be calculated?

--- Settle for approximation, estimation of error

Example - Conservation of Linear Momentum

- $L = mv$
- General form:

  Change = Increase – Decrease
- $dL / dt = m (dv/dt) = ma = F$
- If $a = 0$ (body at rest, statics) → then $\sum F = 0$
- Useful for structural truss analysis

- From Newton's second law,

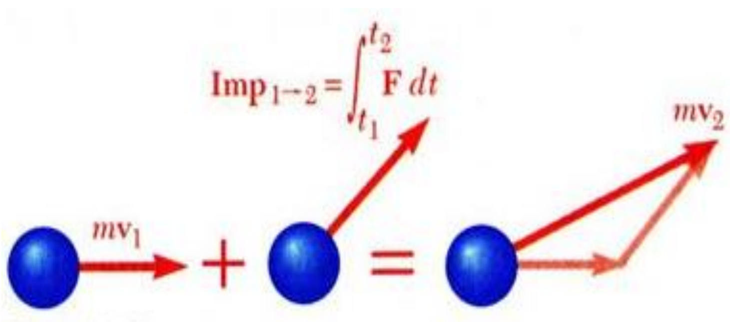
$$\bar{F} = \frac{d}{dt}(m\bar{v}) \qquad m\bar{v} = \text{linear momentum}$$

$$\bar{F}dt = d(m\bar{v})$$

$$\int_{t_1}^{t_2} \bar{F}dt = m\bar{v}_2 - m\bar{v}_1$$

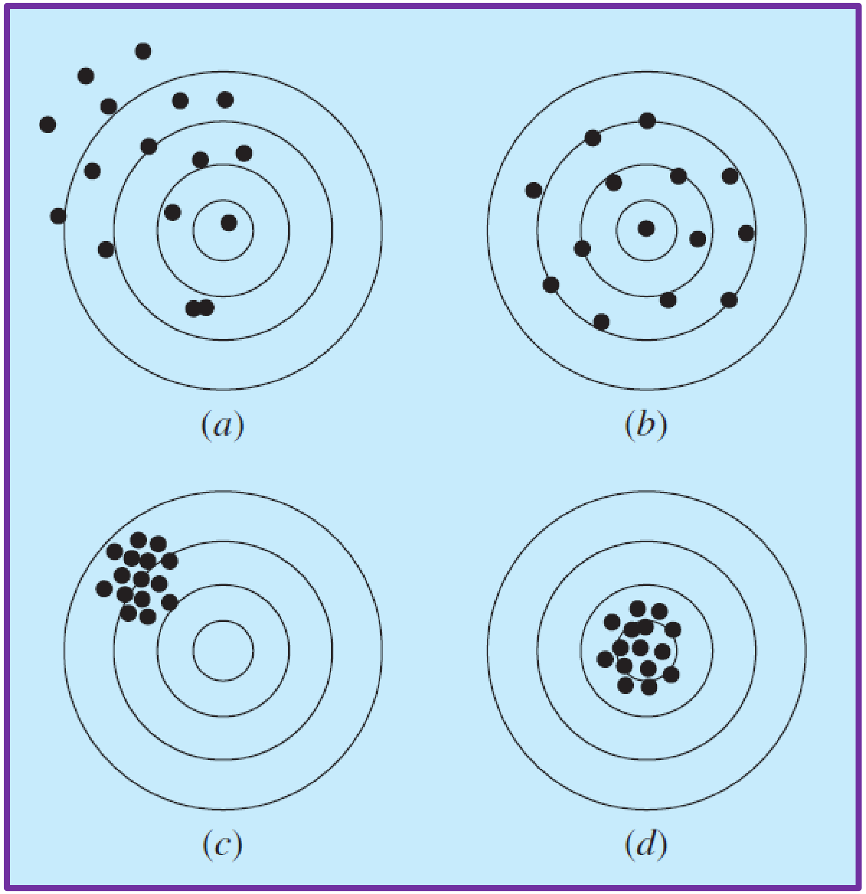
$$\int_{t_1}^{t_2} \bar{F}dt = \mathbf{Imp}_{1\to2} = \text{impulse of the force } \bar{F}$$

$$m\bar{v}_1 + \mathbf{Imp}_{1\to2} = m\bar{v}_2$$



Accuracy and Precision

- **Accuracy**: how closely a computed or measured value agrees with the true value.
- **Precision**: how closely individual computed or measured values agree with each other.
- Which one is most accurate and most precise?



Numerical Errors

- Arise from the use of approximation to represent exact mathematical operations and quantities

True value = approximation + error

E_t = " true" error, exact value of error

E_t = true value - approximation

Approximate Percent Relative Error

$$E_a = (\text{approximate error} / \text{approximation}) * 100\%$$

Approximate error is hard to find, need alternative approach

For iterative problems:

$$E_a = \frac{(\text{Current approx.} - \text{Previous approx.})}{\text{Current Approximation}} * 100\%$$

Next session you will start learning about : Two Types of Error

Round-Off Error

Truncation Error