MTE 203 – Advanced Calculus
Homework 4

Normal Vectors, Curvature, and Radius of Curvature

Problem 1 [11.12, Prob. 9] :

Find $\widehat{N}$ and $\widehat{B}$ at the point $(5,2,10)$ on the curve: $x = y^2 + 1$, $z = x + 5$, directed so that *y* increases along the curve.

Problem 2 [11.12, Probs. 15, 17]:

Find the curvature and the radius of curvature of the curve (if they exist). Draw each curve.

a. $x = t, y = t^3, z = t^2, \quad t \geq 0$
b. $x = t + 1, y = t^2 - 1, z = t + 1, \quad -\infty < t < \infty$

Problem 3: [11.12, Prob. 23]

Let *C* be the curve $x = t, y = t^2$ in the xy-plane.

a. At each point on C calculate the unit tangent $\widehat{T}$ vector and the principal normal $\widehat{N}$. What is $\widehat{B}$?
b. $\vec{F} = t^2\,\hat{\imath} + t^4\,\hat{\jmath}$ is a vector that is defined at each point P on C. Denote by F_T and F_N the components of $\vec{F}$ in the directions $\widehat{T}$ and $\widehat{N}$ respectively. Find F_T and F_N as functions of t.
c. Express F in terms of $\widehat{T}$ and $\widehat{N}$.

Tangential and Normal Components of Acceleration

Problem 4: [11.13, Prob. 25]

Calculate the normal component a_N of the acceleration of a particle using equations 11.112 and 11.113 if its position is given by $x = t^2 + 1, y = 2t^2 - 1, z = t^2 + 5t\,, t \geq 0,$ (t being time).

Note:

Equation 11.112 (Trim Book):

$$a_T = a.\hat{T} = \frac{d}{dt}|v|, a_N = a.\hat{N} = |v|\left|\frac{d\hat{T}}{dt}\right|$$

Equation11.113 (Trim Book):

$$a_N = \sqrt{|a|^2 - a_T^2}$$

Extra Practice Problems

Solutions to these problems can be found at the back of your textbook

1. S. 11.12, Probs. 18, 24, 26
2. S. 11.13, Probs. 14, 18, 24, 38