

MTE 203 – Advanced Calculus

Homework 7

Directional Derivatives

Problem 1: [12.8, Prob. 11]

Find the rate of change of the function with respect to distance travelled along the curve $y = x^2 - 1$, $z = -2x$ in the direction of increasing x

$$f(x, y, z) = xy + z^2 \text{ at } (1, 0, -2)$$

Problem 2: [12.8, Prob. 15]

Find the direction in which the function increases most rapidly at the point. What is the rate of change in that direction?

$$f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}} \text{ at } (1, -3, 2)$$

Problem 3: [12.8, Prob. 25]

Find points on the curve $C: x = t, y = 1 - 2t, z = t$ at which the rate of change of $f(x, y, z) = x^2 + xyz$ with respect to the distance travelled along the curve vanishes.

Problem 4: [12.8, Prob. 31]

Rates of change of a function $f(x, y, z)$ at a point (x_0, y_0, z_0) in directions $\hat{i} + \hat{j}$, $2\hat{i} + \hat{k}$, and $\hat{i} - \hat{j} + \hat{k}$ are 1, 2 and -3 , respectively. What is its partial derivative with respect to z at the point?

Tangent Lines and Tangent Planes

Problem 5: [12.9, Prob. 19]

Find equations for the tangent line to the curve at the point $(1, 1, \sqrt{2})$

$$x = t^2, y = t, z = \sqrt{t + t^4}$$

Problem 6: [12.9, Prob. 25]

Find an equation for the tangent plane to the surface at the point $(-1, -1, 1)$

$$x = y \sin\left(\frac{\pi z}{2}\right)$$

Problem 7: [12.9, Prob. 31]

Find the derivative for the function $f(x, y, z) = xyz + xy + xz + yz$ at $(1, -2, 5)$ perpendicular to the surface $z = x^2 + y^2$

Problem 8: [12.9, Prob. 39]

Show that the sum of the intercepts on the x -, y -, and z -axes of the tangent plane to the surface $\sqrt{x} + \sqrt{y} + \sqrt{z} = \sqrt{a}$ at any point is a .

Solutions to the following problems can be found at the back of your textbook.

Warm-Up Problems

1. S. 12.8, Probs. 2, 4, 12, 14, 18
2. S. 12.9, Probs. 2, 6, 10, 16, 22

Extra Practice Problems

1. S. 12.8, Probs. 20, 26, 30
2. S. 12.9, Probs. 28, 34, 36

Extra Challenging Problems

1. S. 12.8, Probs. 34
2. S. 12.9, Probs. 38