MTE 203 – Advanced Calculus
Homework 5

Drawing and setting up multivariable functions

Problem 1: [12.1, Prob. 5]

Find and illustrate geometrically the largest possible domain for the function:

$$f\left(x,y\right)=\sin^{-1}\left(x^2y+1\right)$$

Problem 2: [12.1, Prob.17, 19, 21]

Draw the surface defined by the following functions:

a. $f\left(x,y\right)=y-x^2$
b. $f\left(x,y\right)=\left|x-y\right|$
c. $f\left(x,y\right)=\sqrt{1+x^2-y^2}$

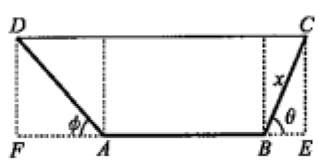
Problem 3: [12.1, Prob. 25]

Draw the level curves $f\left(x,y\right)=C$ corresponding to the values $C=-2,-1,0,1,2$ for the curve below:

$$f\left(x,y\right)=x^2-y^2$$

Problem 4: [S.12.1, Prob. 31] – Application Problem

A long piece of metal 1 m wide is bent in two places A and B (figure below) to form a channel with three straight sides. Find a formula for the cross-sectional area of the channel in terms of x, θ, and φ.



Partial Derivatives

Problem 5: [S.12.3, Probs. 21,23]

Evaluate the partial derivatives as indicated

1. $\frac{\partial f}{\partial x}$ $if\ f(x,y,z) = xyze^{x^2+y^2}$
2. $\frac{\partial f}{\partial y}$ $at\ (1,1,0)\ if\ f(x,y,z) = xy(x^2+y^2+z^2)^{\frac{1}{3}}$

Problem 6: [12.3, Prob. 39] – Application Problem

The equation of continuity for three-dimensional unsteady flow of a compressible fluid is

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

Where $\rho\left(x,y,z,t\right)$ is the density of the fluid, and

$$u\hat{\imath} + v\hat{\jmath} + w\hat{k}$$

If the velocity of the fluid at position $\left(x,y,z\right)$ and time t. Determine whether the continuity equation is satisfied if,

a. $\rho = constant, u = (2x^2 - xy + z^2)t, v = (x^2 - 4xy + y^2)t, w = (-2xy - yz + y^2)t$
b. $\rho = xy + zt, u = x^2y + t, v = y^2z - 2t^2, w = 5x + 2z$

Higher Order Partial Derivatives

Problem 7: [12.5, Prob. 21]

If $z = x^2 + xy + y^2 \sin\left(\frac{x}{y}\right)$, show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z = x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\,\partial y} + y^2\frac{\partial^2 z}{\partial y^2}$$

Problem 8: [12.5, Prob. 27] – Application Problem

A function is said to be a harmonic function in a region R if it satisfies the Laplace's equation in R and has continuous second partial derivatives in R. The Laplace's equation for a function $f\left(x,y,z\right)$ of three variables is

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$$

Find a region (if possible) in which the function,

$$f\left(x, y, z\right) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$

is harmonic.

Problem 9: [12.5, Prob. 31] – Challenging Application Problem

The figure below shows a plate bounded by the lines $x = 0$, $y = 0$, $x = 1$, and $y = 1$. Temperature along the first three sides is kept at 0°C, while that along $y = 1$ varies according to

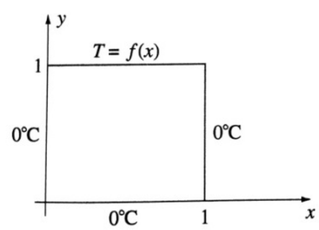
$$f(x) = \sin(3\pi x) - 2\sin(4\pi x),\ 0 \leq x \leq 1.$$

The temperature at any point interior to the plate is then

$$T(x, y) = C(e^{3\pi y} - e^{-3\pi y})\sin(3\pi x) + D(e^{4\pi y} - e^{-4\pi y})\sin(4\pi x)$$

Where $C = (e^{3\pi} - e^{-3\pi})^{-1}$ and $D = (e^{4\pi} - e^{-4\pi})^{-1}$.

Show that $T(x, y)$ is harmonic in the region $0 < x < 1,\ \ 0 < y < 1$, and that it also satisfies the boundary conditions $T(0, y) = 0$, $T(1, y) = 0$, $T(x, 0) = 0$, and $T(x, 1) = f(x)$.



Extra Problems

Solutions to these extra problems can be found at the back of your textbook (contains review from chapter 12):

Warm-Up Problems

1. S.11, Review Exercises, Probs. 42
2. S. 12.1, Probs. 2, 8, 10
3. S. 12.3, Probs. 2, 10, 14, 18, 22
4. S. 12.5, Probs. 2, 6, 12, 16

Extra Practice Problems

1. S.11, Review Exercises, Probs. 44
2. S. 12.1, Probs. 18, 20, 22, 26, 32
3. S. 12.3, Prob. 32, 36, 42
4. S. 12.5, Probs. 20, 22, 34

Extra Challenging Problems

1. Review Exercises, Probs. 46, 48
2. S. 12.1, Probs. 30
3. S. 12.3, Prob. 40
4. S. 12.5, Probs. 36, 44, 46