Analyzing a panel of seasonal time series: Does seasonality converge across Europe?

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Abstract

In this paper we consider deterministic seasonal variation in quarterly industrial production for several European countries, and we address the question whether this variation has become more similar across countries over time. Due to economic and institutional factors, one may expect convergence across business cycles. When these have similar characteristics as seasonal cycles, one may perhaps also find convergence in seasonality. To this aim, we propose a new method, which is based on treating the set of production series as a panel. By testing for the relevant parameter restrictions for moving window samples, we examine the hypothesis of convergence in deterministic seasonality while allowing for seasonal unit roots. We derive the estimation bias, and show that it is very small for samples of more than three years of quarterly observations. Our main empirical finding is that there is almost no evidence for convergence in seasonality.

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1 Introduction

This paper deals with the analysis of a panel of seasonal time series. Such a panel can consist of a number N units (like countries or sectors), and the observations span ST data points, where S denotes the seasonal frequency and T the number of years. In the present paper S is 4, T is 41 and N is 14.

When modelling a panel of time series, a usual step in practice is to see if the series have any properties in common. A main reason for doing so is that imposing common structures can substantially increase of degrees of freedom, but also motivations based on economic theory may be relevant. Additionally, it may be that common structures improve out of sample forecasting, and it may facilitate parameter interpretation.

As we deal with quarterly seasonal time series we are interested in examining whether these series have deterministic seasonality in common. Of course, part of the seasonal variation may not be deterministic, but stochastic, possibly even of the unit root type, and therefore we propose methods to examine common deterministic seasonality while allowing for the potential presence of seasonal unit roots.

A second feature of our paper is that we wish to examine if there is evidence of more common deterministic seasonality over the years. We consider 14 European countries, and we wonder whether increased economic coherence is leading to more common patterns over time. From an economic point of view, strong seasonality in production may be viewed as less efficient, due to inventory and stock-piling reasons, and having a very different seasonal cycle from those in other countries might be thought of as even more inefficient. Hence, one might expect to see some convergence across the seasonal patterns in the various European countries.

Our focus is in the quarterly time series on industrial production for the following fourteen European countries: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, Spain, Sweden, and the United Kingdom. In short, these are all members of the European Union as of 2003, that is, before the Eastern enlargement, with the exception of Denmark, for which country data of comparable length were not available. The time range for all series is from 1962 to 2002. Data are taken from the OECD data base. Special remarks are due for the data on Ireland and Sweden. For Ireland, only data from 1975 are available in the current OECD data base, which we chained to older data. For Sweden, only seasonally adjusted series are reported. We used this adjusted series and multiplied it with a seasonal factor that was obtained from the production in mining and manufacturing, which is reported in a seasonally adjusted and in an unadjusted version. All industrial production series are real indexes.

Tables 1–3 report the results from regressing first-order differences of the logarithmic series on quarterly dummies. In line with the findings by Miron (1996), we see that R^2 values are rather high for most countries. Of course, this high a value might also be spurious as seasonal unit roots would also deliver such values, see Franses, Hylleberg and Lee (1995), but in any case, it indicates that seasonal variation explains a sizeable part of the overall fluctuation. For Tables 2 and 3, the sample was split in two halves. The consequences are somewhat ambiguous. While the variation in R^2 across countries has decreased significantly, that is, cross-sectional standard deviation is halved for the more recent data, and while variation in regression coefficients for the third and fourth quarters did also drop, variation has increased somewhat for the first two quarters. Most countries conform to the basic pattern of high production in the second and fourth quarters, and low production in the first and third quarters.

The outline of our paper is as follows. In Section 2 we start with an analysis of each of the series. For this, we use the familiar HEGY test approach, and we observe that some series have all seasonal unit roots, some have a few, and others have no seasonal unit roots. In Section 3, we describe two models for panels of quarterly time series, one in which we allow the autoregressive parameters to be country-specific, while in the second specification we restrict these parameters as equal. We relegate analytical evaluations of the bias in the estimators to an Appendix, although we take as a result from these analytics that when T is 12 or more, this bias is negligible. This allows us to examine for converging patterns by looking at moving window samples of size 12. The common dynamics model allows us to formulate a panel version of the HEGY test. In the second part of Section 3, we discuss the asymptotic distributions of the panel HEGY tests and we give critical values. In Section 4, we consider the various panel models for our 14 series, where we consider the full sample as well as moving window samples. Basically, our main finding is that there are signs of more common deterministic seasonality over the years. In Section 5, we conclude with a discussion of possible extensions to our work.

2 Univariate analysis

This section deals with an analysis of the time series properties of the 14 series under scrutiny. For this purpose, we rely on the HEGY test procedure.

2.1 Models and tests

At first, we model the individual industrial production series y_{kt}, $k = 1, \ldots, N$, as autoregressive processes of order $p_k + 1$. In line with the predominant view in the literature, we assume that all series have exactly one unit root at zero (non-seasonal) frequency. Thus, we investigate models of order p_k for the first differences, that is,

$$\Delta y_{kt} = m_k + \sum_{j=1}^{p_k} \varphi_{kj} \Delta y_{k,t-j} + \varepsilon_{kt}, \tag{1}$$

where Δ denotes first differencing. We are interested in whether the operator $\varphi(z) = 1 - \sum_{j=1}^{p_k} \varphi_j z^j$ has seasonal unit roots at -1 or $\pm i$, that is, whether $\varphi(-1) = 0$ or $\varphi(\pm i) = 0$.

A test for these hypotheses has been developed in Hylleberg *et al.* (1990) [HEGY] who generalized the unit-root tests by Dickey and Fuller (1979). The HEGY test uses the t–statistic τ_{-1} for $b_1 = 0$ and the F–statistic $F_{\pm i}$ for $b_2 = b_3 = 0$ in the regression

$$\begin{aligned} \Delta_4 y_{kt} &= m_k + b_1 y_{k,t-1}^{(2)} + b_2 \Delta_2 y_{k,t-2} + b_3 \Delta_2 y_{k,t-1} + \sum_{j=1}^{p^*} \gamma_j \Delta_4 y_{k,t-j} + \varepsilon_{kt}, \\ & \qquad t = p^* + 5, \ldots, T, \quad k = 1, \ldots, N, \end{aligned} \tag{2}$$

where $\Delta_4 = 1 - B^4$, $\Delta_2 = 1 - B^2$, $y_{k,t}^{(2)} = (1 - B + B^2 - B^3) y_{k,t}$, and B denotes the backward shift operator. Note that the usually included extra regressor $(1 + B + B^2 + B^3) y_{k,t-1}$ is absent, as we do not test for a unit root at $+1$. Omitting this regressor increases the test power at the other frequencies. Significance points for the statistics τ_{-1} and $F_{\pm i}$ and the corresponding null hypotheses $\varphi(-1) = 0$ and $\varphi(\pm i) = 0$ are given in HEGY. The distribution of the test statistics is affected if the deterministic intercept m_k is replaced by seasonal dummy variables.

2.2 Results

We determine the lag orders of autoregressions p_k using Akaike's AIC. These values are reported in Table 4 for the full sample. They range from only five (for the Finnish and the Irish series) to a maximum of twelve for the British series, although most of the values cluster around nine or ten. For the versions with added seasonal dummies, lag orders tend to be slightly lower, while there are some exceptions. This pattern provides some evidence against purely deterministic seasonality, as lag orders should differ markedly in that case.

Additionally, Table 4 gives the HEGY test statistics τ_{-1} and $F_{\pm i}$. For most series, seasonal unit roots cannot be rejected for the specification without seasonal dummies. The root pair at $\pm i$ tends to be rejected for the specification with seasonal dummies, while adding these dummies hardly affects the test statistic for the root at -1. There are noteworthy exceptions to this general pattern. For example, the t–statistic for the root at -1 has the wrong sign for the Irish series and may point to an explosive seasonal pattern. Of course, explosive models are not acceptable as data-generating processes, due to their implausible implications for the longer run. Such aberrant estimation results can also suggest the relevance of panel time series models, where some structure on the parameters is to be imposed.

3 Panels of time series

In this section, we provide details on the representation of the panel data models we use in our empirical work below, and on the relevant estimators and tests. We first outline various relevant models, Next, in Section 3.2, we put forward a panel version of the HEGY test.

3.1 Models

The panel models we discuss differ in terms of the structure they impose on the autoregressive parameters.

3.1.1 The individual-dynamics model

If, as in (2), all countries are assumed to follow their own idiosyncratic dynamic patterns, one has the individual-dynamics (ID) model. As said, we assume one unit root at $\omega = 0$ (the zero frequency), and we allow for fixed seasonal effects, and for autoregressive individual dynamics of order $p_k$largest integer function and $\delta(i,j)$ for Kronecker's δ function. Thus, we define $D_{jt} = \delta(j, t-4[(t-1)/4])$ as the usual quarterly dummy variables. Then, the ID model reads as

$$\begin{aligned} \Delta y_{kt} &= \sum_{j=1}^{4} d_{kj} D_{jt} + \sum_{j=1}^{p_k} \varphi_{kj} \Delta y_{k,t-j} + \varepsilon_{kt}, \\ & t = p_k + 2, \ldots, T, \quad k = 1, \ldots, N. \end{aligned} \tag{3}$$

In this general form, the ID model collects N independent autoregressions of form (1) and does not exploit any panel properties. The polynomials $\varphi_k(z) = 1 - \sum_{j=1}^{p_k} \varphi_{kj}(z)$ may have seasonal unit roots at -1 or $\pm i$. In order to avoid diverging seasonal trends, it appears plausible to impose the restrictions suggested by FRANSES AND KUNST (1999). To this aim, it is convenient to use a different representation for the deterministic seasonal part, that is, to use

$$d_{k1} D_{1t} + \ldots + d_{k4} D_{4t} = g_{k0} + g_{k1} w_{1t} + g_{k2} w_{2t} + g_{k3} w_{3t}, \tag{4}$$

where we use the abbreviations $w_{1t} = \cos(\pi t)$, $w_{2t} = \cos(\pi t/2)$, $w_{3t} = \cos(\pi(t-1)/2)$ for the cycles at the seasonal frequencies. The frequency of the deterministic seasonal cycle w_{1t} corresponds to the angular frequency $\omega = \pi$ and to the unit root at -1 in the lag polynomial $\varphi_k(z)$, while w_{2t} and w_{3t} correspond to $\omega = \pi/2$ and to the unit root at $\pm i$. Representation (4) allows to separate the individual long-term growth rates g_{k0} from the quarterly growth rates. We will not attempt to test restrictions on these growth rates, as it is, for example, done in the literature on economic convergence (cf. ANDRES *et al.*, 1996, among others). Instead, we view them as freely varying and we just focus on the coefficients g_{kj}, $j = 1, 2, 3$, which describe the shape and amplitude of the deterministic seasonal cycle.

The analysis of Franses and Kunst (1999) suggests to impose $g_{k1} = 0$ if $\varphi(-1) = 0$ and $g_{k2} = g_{k3} = 0$ if $\varphi(\pm i) = 0$. In order to simplify notation, we define the switching factors

$$\zeta_{k1} = I\{\varphi_k(-1) \neq 0\}, \quad \zeta_{k2} = \zeta_{k3} = I\{\varphi_k(\pm i) \neq 0\}.$$

Hence the following model

$$\begin{aligned} \Delta y_{kt} &= g_{ko} + \sum_{j=1}^{3} g_{kj}\zeta_{kj}w_{jt} - (\varphi_k(B) - 1)\Delta y_{kt} + \varepsilon_{kt}, \\ & \quad t = p_k + 2, \ldots, T, \quad k = 1, \ldots, N, \end{aligned} \tag{5}$$

is designed to avoid seasonal cycles that expand as $T \to \infty$. The coefficients g_{kj} are not identified for $\zeta_{kj} = 0$ but they are kept in (5) for the ease of notation.

It may be interesting to see whether the deterministic seasonal cycles show similarities across the panel, even if the autoregressive dynamics and the seasonal unit-root properties are allowed to vary among countries. To this aim, one may consider the restricted model

$$\Delta y_{kt} = g_{ko} + h_k \sum_{j=1}^{3} g_j \zeta_{kj} w_{jt} - (\varphi_k(B) - 1)\Delta y_{kt} + \varepsilon_{kt}. \tag{6}$$

This model is identified if, for example, $\|(g_1, g_2, g_3)\| = 1$ is imposed for a normalization, and if $\zeta_{kj} = 1$ for some k. It should be noted that this model allows the non-seasonal dynamics φ_k, the cases of seasonal unit roots ζ_{kj}, the growth rates g_{k0}, and the amplitude of the seasonal cycle h_k to be country-specific, whereas the shape of the deterministic seasonal cycle is restricted to be common to all countries.

Unfortunately, this restricted model does not properly reflect similarities in deterministic cycles across countries. For example, if $\varphi_k(\pm i) = 0$, then $\zeta_{k2} = \zeta_{k3} = 0$ and no restriction is imposed on the deterministic seasonal sine wave $h_k w_{1t}$ in country k. On the other hand, restricting attention to the cases without seasonal unit roots may (and actually does so in our empirical application) eliminate most countries from the analysis. Therefore, we adopt a compromise solution and exclude only those countries where seasonal unit roots at *both* seasonal frequencies have been detected. In the remaining cases, seasonal unit roots are not imposed during estimation, and deterministic cycles are allowed to consist of all components w_{jt}, $j = 1, 2, 3$.

The final model reads

$$\begin{aligned} \Delta y_{kt} &= g_{ko} + \{1 - (1-\zeta_{k1})(1-\zeta_{k2})\} h_k \sum_{j=1}^{3} g_j w_{jt} - (\varphi_k(B) - 1)\Delta y_{kt} + \varepsilon_{kt}, \\ & \quad t = p_k + 2, \ldots, T, \quad k = 1, \ldots, N. \end{aligned} \tag{7}$$

It has

$$\begin{aligned} n_R &= \sum_{k=1}^{N} p_k + N + 2 + \sum_{k=1}^{N} \{1 - (1-\zeta_{k1})(1-\zeta_{k2})\} \\ &= \sum_{k=1}^{N} p_k + N + 2 + \sum_{k=1}^{N} \zeta_{k1} + \sum_{k=1}^{N} \zeta_{k2} - \sum_{k=1}^{N} \zeta_{k1}\zeta_{k2} \end{aligned} \tag{8}$$

free coefficient parameters, while the unrestricted ID model (5) has

$$n_U = \sum_{k=1}^{N} p_k + N + \sum_{k=1}^{N} \zeta_{k1} + 2\sum_{k=1}^{N} \zeta_{k2} \tag{9}$$

such coefficients. The restriction of a common shape in the deterministic seasonal cycles can be tested by a Fisher test. The Fisher statistic is defined as

$$F = \frac{ESS_R - ESS_U}{ESS_U / DF_U} \quad , \tag{10}$$

where $DF_U = N(T-1) - \sum_{k=1}^{N} p_k - \sum_{k=1}^{N} \zeta_{k1} - 2\sum_{k=1}^{N} \zeta_{k2}$ is the degrees of freedom of the unrestricted model, that is, the difference of the NT observations and the n_U coefficient parameters to be estimated. The abbreviations ESS_U and ESS_R denote the residual sums of squares in the unrestricted and the restricted model, respectively. Under the restricted model, F is asymptotically distributed as χ^2 with $\sum_{k=1}^{N} \zeta_{k2}(1+\zeta_{k1}) - 2$ degrees of freedom.

To obtain the sums of squares ESS_R and ESS_U, the parameters in (5) and (7) must be estimated. We first calculate the individual HEGY test statistics, based on lag orders p_k selected by AIC. Then, we determine the switch factors ζ_{kj} from the 10% fractiles as tabulated by HEGY.

A convenient way to estimate the restricted model is to conduct the following steps, after eliminating those $N - N^*$ countries where seasonal unit roots at both seasonal frequencies have been found:

1. Purge the deterministic seasonal cycle from each country k by regressing $\Delta y_{kt}, t = 1, \ldots, T$ on a constant and on $w_{jt}, j = 1, 2, 3$. Call the residuals Δy_{kt}^*.

2. Estimate the individual autoregressive dynamics from the purged series by N^* least-squares regressions.

3. Filter the original Δy_{kt} by the estimated $\hat{\varphi}_k$, yielding filtered values f_{kt}.

4. Run a canonical analysis between the N^*–dimensional vector of the filtered f_{kt} and the 3–dimensional vector w_{jt}. This is equivalent to a reduced-rank regression of f_{kt} on w_{jt} with the rank of the coefficient matrix fixed at one. Estimates for the loading coefficients $h_k, k = 1, \ldots, N^*$, and for the shape coefficients $g_j, j = 1, 2, 3$, are obtained.

For the technique of canonical analysis in reduced-rank regression problems, see Tso (1981). Conditional on known shape coefficients g_j, the loading coefficients can be estimated efficiently by least squares. Thus, after obtaining the shape coefficients g_j as canonical vectors, estimates for h_k are calculated by linear regression.

3.1.2 The common-dynamics model

Panel models differ mainly with respect to how much common structure is assumed across the N individuals. In the ID model, no common structure is assumed in the general model (5) although we suggested testing for a common shape of the seasonal cycle in (7). Imposing more common structure can increase the power of restriction tests.

Alternatively, the common-dynamics (CD) model assumes that the N countries are subject to the same autoregressive dynamics. In other words, the CD model assumes that $\varphi_k \equiv \varphi$. This simplifies the structure (3) to

$$\Delta y_{kt} = g_{k0} + \sum_{j=1}^{3} g_{kj} w_{jt} + \sum_{j=1}^{p} \varphi_j \Delta y_{k,t-j} + \varepsilon_{kt},$$
$$t = p+2, \ldots, T, \quad k = 1, \ldots, N. \tag{11}$$

The common polynomial $\varphi(z)$ may contain seasonal unit roots at -1 or at $\pm i$. Then, the restrictions suggested by Franses and Kunst (1999) imply that the deterministic cycle is restricted for *all* individuals. For example, if $\varphi(-1) = 0$, then $g_{k1} \equiv 0$ for $k = 1, \ldots, N$ and the regressor w_{1t} is excluded from all individual relationships. After accounting for these restrictions, tests for common deterministic seasonality can be conducted in the CD model in a similar fashion as in the ID model.

3.2 A panel HEGY test

For a decision on the presence of seasonal unit roots in the CD model, a panel variant of seasonal unit-root tests such as the test by HEGY can be applied. The panel HEGY statistics are the t–value τ_{-1} on the coefficient b_1 and the F–statistic $F_{\pm i}$ for (b_2, b_3) in the regression

$$\Delta_4 y_{kt} = g_{k0} + b_1 y^{(2)}_{k,t-1} + b_2 \Delta_2 y_{k,t-2} + b_3 \Delta_2 y_{k,t-1} + \sum_{j=1}^{p^*} \gamma_j \Delta_4 y_{k,t-j} + \varepsilon_{kt},$$
$$t = p^* + 5, \ldots, T, \quad k = 1, \ldots, N.$$

For the case $N = 1$, the panel HEGY test becomes the traditional HEGY test. For the case $N > 1$, the simulated significance points for finite T and for $T \to \infty$ given by HEGY are no longer valid.

For $N = 1$, the HEGY statistic τ_{-1} can be shown to converge, under the null hypothesis $\varphi(-1) = 0$, for $T \to \infty$, and for the homogeneous regression with $g_{k1} = 0$, to the Dickey-Fuller distribution, which is formally written as $\int BdB(\int B^2)^{-1/2}$ in stochastic integrals over the Brownian motion $B(\omega)$. For general N, this distribution changes to

$$\frac{\sum_{k=1}^{N} \int_0^1 B_k(\omega) dB_k(\omega)}{(\sum_{k=1}^{N} \int_0^1 B_k^2(\omega) d\omega)^{-1/2}}$$

and new significance points have to be calculated. If seasonal intercepts are included in the regression, the Brownian motion $B(\omega)$ is replaced by the de-meaned Brownian motion $\bar{B}(\omega) = B(\omega) - \int_0^1 B(x)dx$

The situation is equivalent to testing for a unit root at $\omega = 0$ with $g_{k0} = 0$ and $g_{k0} \neq 0$. For related problems, see BREITUNG AND MAYER (1994) and QUAH (1994). Like BREITUNG AND MAYER, who consider the problem of testing for cointegration in panels for small fixed T and $N \to \infty$, QUAH also focuses exclusively on Dickey-Fuller tests for a unit root at $\omega = 0$. QUAH (1994) derives asymptotic properties of the coefficient estimate $\hat{\varphi}$ in the homogeneous autoregression $y_{kt} = \varphi y_{k,t-1} + \varepsilon_{kt}$ for the case that $T \to \infty$ and $N = O(T)$. From QUAH's Theorem 2.2, it follows that the t–value for $\hat{\varphi} - 1$ converges to a standard normal distribution. Although QUAH focuses on the coefficient estimate only and the normal limit distribution is not valid for fixed T and $N \to \infty$, his Monte Carlo simulations demonstrate that the standard normal approximation is also relevant for large N and $T = 25$. However, for even smaller T, the approximation becomes unreliable.

In the case of an inhomogeneous autoregression $y_{kt} = g_{k0} + \varphi y_{k,t-1} + \varepsilon_{kt}$ with individually varying intercepts g_{k0} (as we have, in contrast to QUAH, 1994), the least-squares estimation procedure is known as LSDV estimation (*least squares dummy variables*) in the literature (cf. BALTAGI, 1995). Now, the asymptotics given by QUAH no longer hold. In particular, LEVIN AND LIN (1993, Theorem 4.7(c)) show that the t–statistic diverges to $-\infty$, under the condition that $N \to \infty$ but $N = o(T)$. This behavior is essentially due to the fact that the expectation of the sample covariance of Δy_{kt} and $y_{k,t-1}$ is not 0 for any fixed k and T. This bias is a special case of the so-called Nickell bias in dynamic panels (after NICKELL, 1981, see ARELLANO, 2003). LEVIN AND LIN (1993) and LEVIN *et al.* (2001) suggest to use a modified t–statistic that corrects for this bias and that has an asymptotic standard normal distribution. However, note that the divergence of the t–statistic does not preclude its potential usefulness for discriminating unit roots from stationary models, as long as the t–statistic diverges even faster under the alternative. Significance points for uncorrected Dickey-Fuller statistics that are tailored to the sample sizes of a given empirical problem have been considered by WU AND ZHANG (1997).

For the case of seasonal unit roots, the issues parallel the case of the $\omega = 0$ unit root, but with a few modifications. Least-squares estimation of models with individually varying seasonal constants, such as

$$\begin{aligned} \Delta_4 y_{kt} &= g_{k0} + g_{k1} w_{1t} + g_{k2} w_{2t} + g_{k3} w_{3t} + \\ & \quad b_1 y_{t-1}^{(2)} + b_2 \Delta_2 y_{t-2} + b_3 \Delta_2 y_{t-1} + \sum_{j=1}^{p^*} \gamma_j \Delta_4 y_{k,t-j} + \varepsilon_{kt} \quad , \end{aligned} \tag{12}$$

is equivalent to a first-stage regression of all involved variables, that is, $\Delta_4 y_t$ and its p^* lags, $y_{t-1}^{(2)}$, $\Delta_2 y_{t-1}$ and $\Delta_2 y_{t-1}$, on four individually varying seasonal dummies, and a second-stage regression of the 'purged' $\Delta_4 y_{kt}$ on all purged non-deterministic regressors. Note that the second-stage least-squares regression has a sample size of $N(T - p^* - 4)$, whereas the N first-stage regressions only use $T - p^* - 4$ observations each. In analogy with the LSDV estimator, this estimator is called the LSSD estimator (*least squares seasonal dummies*). Like the LSDV estimator, the LSSD estimator also has a finite-sample bias. The technical analysis of this bias is relegated to the Appendix, where we consider the first-order autoregressive model $y_t = g_{k0} + g_{k1} w_{1t} + g_{k2} w_{2t} + g_{k3} w_{3t} + \varphi y_{t-1} + \varepsilon_t$. We find that, firstly, it is smaller than the LSDV bias and, secondly, it is symmetric around $\varphi = 0$.

For the LSDV and the LSSD versions of the HEGY–type test, the results parallel the main results of LEVIN AND LIN (1993) and of QUAH (1992). The t–values of $b_j, j = 1, 2, 3$, diverge for LSSD estimation and converge to standard normal for LSDV without seasonal dummies $g_{kj}, j = 1, 2, 3$. We concentrate on the LSSD form (12) and therefore we calculate significance points from Monte Carlo experiments. Table 5 shows some 5% and 10% significance points for the HEGY statistics τ_{-1} and $F_{\pm i}$, for various combinations of N and T. Table 5 confirms that both statistics diverge as $N \to \infty$, while their distribution is hardly affected by T.

LEVIN AND LIN (1992, Theorem 5.2(d)) derive an asymptotic expression for the case of a panel unit root at $\omega = 0$, which we found to coincide well with the distributional properties of τ_{-1} as $N \to \infty$. In particular, the formulae

$$\begin{aligned} f_{0.9}(\tau_{-1}) &\approx 1.240 + 1.198\sqrt{N}, \\ f_{0.95}(\tau_{-1}) &\approx 1.548 + 1.199\sqrt{N} \end{aligned}$$

yield good response surfaces for the 10% and 5% significance points. These were obtained from simulating the case $T = 100$ and varying N in the range $[5, 100]$. The estimated coefficients are slightly smaller than the coefficient of $\sqrt{15/8}$, which was derived by LEVIN AND LIN. For the statistic $F_{\pm i}$, we obtained analogous Monte Carlo approximations

$$\begin{aligned} f_{0.9}\left(F_{\pm i}\right) &\approx 8.059 + 1.590N, \\ f_{0.95}\left(F_{\pm i}\right) &\approx 10.235 + 1.633N, \end{aligned}$$

which are however less exact than those for τ_{-1}.

Although the CD model can be seen as a restricted version of the ID model, we do not consider a restriction test for CD against ID. The possible presence of seasonal unit roots in some of the individual $\varphi_k(z)$ under the alternative and the existence of sample-specific features outside the autoregressive frame makes such tests rather unreliable. We prefer to view the CD and ID models as two different angles of exploring common structures in dynamic panels.

In the limits of this study, we do not focus on the panel seasonal unit root test aspect of the CD model. Rather, noting that the CD model allows for four different events—seasonal unit roots at both frequencies $\omega = \pi, \pi/2$, at one frequency only, and at none—the evidence on convergence in deterministic seasonal patterns can be summarized by moving-window Fisher test statistics for all cases excepting the first one. If there are two seasonal unit roots in the CD model, no deterministic patterns are acceptable and the test becomes meaningless. Assuming a seasonal unit root at $\omega = \pi$ in the CD model focuses interest on the deterministic cycle at $\omega = \pi/2$, and *vice versa*. For a model without seasonal unit roots, convergence in the complete deterministic seasonal cycle can be investigated.

4 Results

In this section we apply the various panel models in the previous section to the 14 industrial production series of interest. We aim to answer the question in the tite, whether there is any sign of converging seasonal deterministics.

4.1 Evidence from individual dynamics

The first step in the ID testing procedure requires a decision rule for seasonal unit roots in individual countries. Countries with two seasonal unit roots are then excluded from the main step. If the original fractiles from HEGY and a significance level of 10% are used, only three countries are excluded, that is, Finland, the Netherlands, and the United Kingdom. If the nominal significance level is reduced, more unit roots are found and less countries are included in the test. In turn, this reduction may tend to enhance the evidence on common deterministic cycles.

Generally, the outlined Fisher tests always reject their null hypothesis of common seasonal cycles. However, in this paper the main focus is on possible convergence rather than on existing differences. Figure 3 uses a moving window of length 60 (15 years), a rather wide window, and a nominal significance level of 10%. Dates on the abscissa axis indicate the middle of the estimation window. For example, the value for 1970 is obtained from the subsample with the time range from 1963 to 1977. The Fisher statistic becomes larger following the OPEC crises of the 1970s and shows some indication of convergence toward the end of the sample. However, even at the very end of the sample the statistic has just reached values close to the minimum in the early part of the sample. In other words, seasonal cycles are as different among European countries as they have been in the 1960s, though such differences were even more pronounced in the meantime.

We note that, due to the preliminary differencing and purging of data from seasonal patterns, the identified autoregressive models for first differences have a quite low order. As a consequence, even for 60 observations, random walks with added seasonal cycles were frequently identified as the optimum models by AIC. This is to be contrasted with the relatively large orders that are shown for the full sample in Table 4.

While a common deterministic seasonal cycle is not supported statistically, the test procedure still yields an estimate of such a common cycle. Its shape conforms to intuition and shows little variation over time. Economic activity peaks in the second and fourth quarters of each year. The other two quarters show a decline in production, probably due to climatic hardships in the winter quarter and the pooling of summer vacation in the third quarter.

4.2 Evidence from common dynamics

We calculate the panel HEGY test statistics from moving windows, and we compare these calculated t– and F–values with the significance points that we tabulated in Table 5. For the full sample ($T = 164$, $N = 14$), we obtained $\tau_{-1} = 3.25$ and $F_{\pm i} = 9.08$. In summary, all statistics remained markedly below the given points, hence formally seasonal unit roots in the CD panel cannot be rejected. However, in the interest of robustness of the results, we do not rely on the decision that is suggested by the panel unit-root test and rather consider all possible cases.

If no seasonal unit roots are assumed, restrictions on the complete seasonal cycle can be tested, as all seasonal patterns are deterministic by assumption. Figure 4 shows the implications of a moving window of 24 observations and a Fisher test statistic with 39 degrees of freedom. F–statistics are displayed by the logarithms of the corresponding p–values. It is obvious that the hypothesis of identical seasonal cycles is rejected convincingly for all time points. From 1975 to 1988, the graph reveals a converging tendency, as the significance of the test statistic decreases. However, this period of convergence is then followed by an episode of divergence with subsequent stabilization, such that the end of the sample shows differences among countries of a similar intensity as in the 1970s.

In Figure 5, the experiment is repeated assuming a seasonal unit root at -1 only. The similarity of Figures 4 and 5 shows that the deterministic cycles at $\omega = \pi/2$ are mainly responsible for the non-convergence of seasonality across European economies. This interpretation is confirmed by the curious appearance of the p–values (not logarithmic p–values) in Figure 6, which is based on the reverse experiment that assumes unit roots at $\pm i$ and considers deterministic cycles at $\omega = \pi$. These cycles do not differ significantly across European countries, excepting maybe two short episodes in 1969 and 1996. Again, however, a tendency toward convergence is not confirmed.

5 Conclusion

The celebrated definition of seasonality by Hylleberg (1992) states that seasonal cycles are rooted in various sources, among which the climate, cultural traditions, and technology play a key role. While the climate is unlikely to converge across Europe, regional traditions and technologies may now have come much closer to each other than forty years ago. The increasing intra-European trade, division of labor and production processes, and integration in various fields would motivate a tendency toward a convergence of seasonal patterns, particularly as the relationship of business cycles and seasonal cycles has been demonstrated on empirical as well as on theoretical grounds (see Miron, 1996, for the former, and Ghysels, 1988, for the latter). However, the empirical evidence provided in this paper does not seem to support this idea.

One might presume that the fierce catching-up process of the Irish economy in the 1990s as well as other features specific to single countries may have distorted the general result. However, the general evidence appears to be rather robust with respect to the exclusion or inclusion of single countries.

It may be argued that the CD and the ID model represent extreme assumptions with respect to the dynamic structure in the panel. In the CD model, trade-cycle fluctuations are assumed constant across all countries, whereas, in the ID model, each nation has its own dynamics. Hence, the literature occasionally has focused on intermediate assumptions that impose some joint structure and allow other parts to be idiosyncratic. In particular, Levin *et al.* (2001) assume unit-root events to be invariant across the cross-section dimension. Such an assumption could be justified on theoretical grounds by viewing the presence of unit roots as a characteristic feature of an economic variable that should be found in repeated country experiments with idiosyncratic short-run fluctuations. An extension of this idea to the seasonal case is more problematic, as many authors found the presence of seasonal unit roots to be country-specific and saw no difficulties in different unit-root events for different countries. The reason might be the relative strength of the climatic cycle in different countries but also the relative impact of this cycle to the economy due to different shares of sectors that are more vulnerable to climatic conditions, such as the agricultural sector. Therefore, we consider the case of intermediate restrictions with respect to seasonal unit roots as potentially less interesting than with respect to unit roots at the zero frequency.

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Appendix

In this section, an analytical evaluation of the bias created by the LSSD estimator in fixed-effects autoregressions with common dynamics for $N \to \infty$ is given. Such a bias has been calculated first by Nickell (1981) for the LSDV estimator. The Nickell bias is negative for first-order autoregressions at all possible coefficients φ. For analytical expressions and discussion, see Arellano (2003, p. 85). For the LSSD estimator, the situation is different, as individual variables are regressed on cyclical patterns and not only on constants. It turns out that the bias is directed toward 0, just as the Hurwicz bias.

The distinction among these three bias terms, the Hurwicz, the Nickell, and the LSSD bias is as follows. The Hurwicz bias is created by the dependence between the numerator and denominator in least-squares estimation and is amplified by estimating a constant. Existing negative and positive dependence is reduced in the estimation. The Nickell bias is only caused by estimating the constant term, which leads to artificial under-smoothing of the variables. The dependence between numerator and denominator plays no role due to the fact that repeated measurements of the variables are available. The LSSD bias is caused by regression on cycles that are potentially different across individuals, which leads to under-smoothing if the true dependence is positive and over-smoothing if it is negative. Hence, the LSSD bias is (skew) symmetric around zero.

In order to derive the counterpart to the bias formula that was presented by Nickell (1981) for the LSDV estimator in the non-seasonal fixed-effects model, we need an exact representation for the first-order autoregression with seasonal dummies. Assuming the model

$$\begin{aligned} y_0 &\equiv y_0 \\ y_t &= \varphi y_{t-1} + a_0 + a_1 \cos(\pi t) + a_2 \cos(\pi t/2) \\ &\quad + a_3 \cos(\pi (t-1)/2) + u_t \quad , \quad t > 0, \end{aligned} \tag{13}$$

yields the representation formula

$$
\begin{aligned}
y_t &= \varphi^t y_0 + \sum_{j=0}^{t-1} \varphi^j u_{t-j} + a_0 \frac{1-\varphi^t}{1-\varphi} \\
&\quad + a_1 \frac{\cos(\pi t) - \varphi}{1+\varphi} \\
&\quad + a_2 \frac{\cos(\pi t/2) + \varphi \cos(\pi(t-1)/2) - \varphi^t}{1+\varphi^2} \\
&\quad + a_3 \frac{\cos(\pi(t-1)/2) - \varphi \cos(\pi t/2) + \varphi^{t+1}}{1+\varphi^2} \qquad (14)
\end{aligned}
$$

which is easily, though somewhat tediously, proved by separating cases of $t \bmod 4$. If the variable y is part of a panel, the only change is adding an individual subscript index k.

The LSSD estimator of φ is defined by

$$\hat{\varphi} = \frac{(NT)^{-1} \sum_{k=1}^{N} \sum_{t=1}^{T} (y_{kt} - \hat{a}_k' D_t)(y_{k,t-1} - \tilde{a}_k' D_{t-1})}{(NT)^{-1} \sum_{k=1}^{N} \sum_{t=1}^{T} (y_{k,t-1} - \tilde{a}_k' D_{t-1})^2}$$

where we use the hat notation for estimates from the time range $(1, \ldots, T)$ and tilde notation for estimates from the time range $(0, \ldots, T-1)$. The defining properties of least-squares estimation yield $\sum_{t=1}^{T} \hat{u}_{kt} = 0$ for all k, if $\hat{u}$ are the least-squares residuals. If seasonal constants are used as regressors, one has similarly $\sum_{t=1}^{T} (-1)^t \hat{u}_{kt} = 0$ and $\sum_{t=1}^{T/2} (-1)^t \hat{u}_{k,2t} = \sum_{t=1}^{T/2} (-1)^t \hat{u}_{k,2t-1} = 0$. Noticing that $\sum_{t=1}^{T} (-1)^i y_{kt}$ etc. are used for calculating the estimates in $\hat{a}_k$, it is easy to establish that

$$\hat{\varphi} = \varphi + \frac{(NT)^{-1} \sum_{k=1}^{N} \sum_{t=1}^{T} (u_{kt} - \bar{u}_{kt}' D_t)(y_{k,t-1} - \tilde{a}_k' D_{t-1})}{(NT)^{-1} \sum_{k=1}^{N} \sum_{t=1}^{T} (y_{k,t-1} - \tilde{a}_k' D_{t-1})^2} \qquad (15)$$

where $\bar{u}_{kt}$ is *not* simply the arithmetic mean of the true errors u_{kt} but rather a four-periodic vector variable formed from

$$
\begin{aligned}
\bar{u}_{kt} &= (T^{-1} \sum_{t=1}^{T} u_{kt}, T^{-1} \sum_{t=1}^{T} (-1)^t u_{kt}, (T/2)^{-1} \sum_{t=1}^{T/2} (-1)^t u_{k,2t}, \\
&\quad (T/2)^{-1} \sum_{t=1}^{T/2} (-1)^t u_{k,2t-1})' = (\bar{u}_{k,t,1}, \ldots, \bar{u}_{k,t,4})'
\end{aligned}
$$

This introduces the notation $\bar{u}_{k,t,j}, j = 1, \ldots, 4$, and we will use an analogous notation for $\bar{y}_{k,t,j}$ when we need it.

The numerator of the bias ratio in (15) can be decomposed into four terms. The cross-sums $\sum u_{kt}y_{k,t-1}$ do not play a role, as u_{kt} is independent of $y_{k,t-1}$ and the classical Hurwicz bias is dominated by the other terms. Following the law of large numbers, this part converges for $N \to \infty$ to its expectation of 0. The cross-sums $\sum \bar{u}'_{kt} D_t \tilde{a}'_k D_{t-1}$ have three kinds of terms, one of which corresponds to the frequency $\omega = 0$ and to classical summing up and hence behaves as in the NICKELL calculations.

It is convenient to re-consider the numerator in matrix notation. To do so, we define the $T \times 4$–matrix $\mathbf{W}$ whose first column is a column of constants 1 and the three other columns consist of the dummy variables w_{jt} that were introduced in Section 2. The symbols $\tilde{y}$ and u denote the column vectors $(y_0, \ldots, y_{T-1})'$ and $(u_1, \ldots, u_T)'$. With this notation, we may re-write the limit for $N \to \infty$ of the ratio in (15) as

$$\frac{E[\tilde{y}'\{\mathbf{I} - \mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}'\}u]}{E[\tilde{y}'\{\mathbf{I} - \mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}'\}\tilde{y}]} \tag{16}$$

If 4 divides T, the variables w_{jt} are exactly orthogonal and one has $(\mathbf{W}'\mathbf{W})^{-1} = \text{diag}(T^{-1}, T^{-1}, 2T^{-1}, 2T^{-1})$. For general T, exact orthogonality does not hold and calculations become messy. However, if the equivalent representation with seasonal dummies (4) is chosen, one has the general expression

$$(\mathbf{W}'\mathbf{W})^{-1} = \text{diag}(\left[\frac{T+3}{4}\right]^{-1}, \left[\frac{T+2}{4}\right]^{-1}, \left[\frac{T+1}{4}\right]^{-1}, \left[\frac{T}{4}\right]^{-1}) \tag{17}$$

using the notation [.] for the integer part. Because the projection matrix $\mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}'$ is the same for both representations, we will keep $\mathbf{W}$ for the $\mathbf{e}_{[T/4]} \otimes \mathbf{I}_4$ matrix that is extended by the first $T - 4[T/4]$ rows of $\mathbf{I}_4$ without further change in notation, where $\mathbf{e}_l$ denotes a column vector of ones with dimension l.

Using the matrix $\mathbf{\Phi}$ that is defined as

$$\mathbf{\Phi} = \begin{bmatrix} 0 & 0 & 0 & 0 & \ldots & 0 \\ 1 & 0 & 0 & 0 & & 0 \\ \varphi & 1 & 0 & 0 & & 0 \\ \varphi^2 & \varphi & 1 & 0 & \ldots & 0 \\ \ldots & & & & & \\ \varphi^{T-2} & \varphi^{T-3} & \varphi^{T-4} & \ldots & & 0 \end{bmatrix} . \tag{18}$$

and the property $\tilde{y} = \mathbf{\Phi}u$, one may re-write the numerator as $E[u'\mathbf{\Phi}'\{\mathbf{I} - \mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}'\}u]$.

This is the same as the trace of $\mathbf{\Phi}'\{\mathbf{I}-\mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}'\}$, ignoring the scaling variance that cancels in the ratio. The trace of $\mathbf{\Phi}'$ is 0, and the expression simplifies to the trace of $-\mathbf{\Phi}'\mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}'$.

The projection matrix $\mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}'$ is built as follows. It repeats the basic diagonal structure of $(\mathbf{W}'\mathbf{W})^{-1}$ as often as is possible within a $T\times T$–matrix, i.e.

$$\mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}' = \begin{bmatrix} (\mathbf{W}'\mathbf{W})^{-1} & (\mathbf{W}'\mathbf{W})^{-1} & \cdots \\ (\mathbf{W}'\mathbf{W})^{-1} & (\mathbf{W}'\mathbf{W})^{-1} & \\ \vdots & & \ddots \end{bmatrix} \tag{19}$$

In the case where T is a multiple of 4, one has simply $\mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}' = (4/T)\mathbf{E}_{T/4}\otimes\mathbf{I}_4$.

The matrix $\mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}'$ is pre-multiplied with $\mathbf{\Phi}'$. It is seen immediately that the diagonal elements of the product only contain terms in φ^{4j-1} for integer j, i.e. $\varphi^3,\varphi^7,\ldots$ A more careful evaluation of this product yields

$$\mathrm{tr}\{\mathbf{\Phi}'\mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}'\} = \sum_{j=1}^{[(T+1)/4]}\sum_{l=1}^{4}\frac{[\{T-l-4(j-1)\}/4]}{[(T-l+4)/4]}\varphi^{4j-1} \quad . \tag{20}$$

If 4 divides T, this expression simplifies to

$$\frac{4}{T}\sum_{j=1}^{T/4-1}(T-4j)\varphi^{4j-1} \quad . \tag{21}$$

Now consider the denominator. The quadratic sum $\sum(y_{t-1}-\bar{y}_{t-1})^2$ may also be written as

$$\begin{aligned} \tilde{y}'(\mathbf{I}-\mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}')\tilde{y} &= \tilde{y}'\tilde{y}-\tilde{y}'\mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}'\tilde{y} \\ &= u'\mathbf{\Phi}'\mathbf{\Phi}u - u'\mathbf{\Phi}'\mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}'\mathbf{\Phi}u \end{aligned} \tag{22}$$

where we use the matrix $\mathbf{\Phi}$ defined above. Notice that this expression would have to be modified if there were non-zero starting values y_0. The term $u'\mathbf{\Phi}'\mathbf{\Phi}u$ contributes to the expectation of the denominator directly through its trace which is found to be

$$\mathrm{tr}(\mathbf{\Phi}'\mathbf{\Phi}) = (T-1)+(T-2)\varphi^2+\ldots+\varphi^{2(T-2)} \quad . \tag{23}$$

This part corresponds to the formula given by NICKELL as

$$\lim_{N\to\infty}(NT)^{-1}\sum_{k=1}^{N}\sum_{t=1}^{T}y_{k,t-1}^2 = \frac{T-1-T\varphi^2+\varphi^{2T}}{T(1-\varphi^2)^2} \quad . \tag{24}$$

Now consider the second component $w_T = E\{u'\mathbf{\Phi}'\mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}'\mathbf{\Phi}u\} = tr\mathbf{\Phi}'\mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}'\mathbf{\Phi}$. The projection matrix $\mathbf{W}(\mathbf{W}'\mathbf{W})^{-1}\mathbf{W}'$ was already analyzed. An evaluation of the diagonal elements of the product yields

$$w_T = \sum_{j=0}^{T-2}\sum_{l=1}^{4} \frac{\tau(T,j,l)}{[(T+4-l)/4]}\varphi^{2j} \quad . \tag{25}$$

Unfortunately, it evolves that, even when T is a multiple of 4, $\tau(T,j,l)$ has no simple closed from. Algebraic evaluation yields the following polynomial representations:

$$\begin{aligned}
w_3(\varphi) &= 2+\varphi^2 \\
w_4(\varphi) &= 3+2\varphi^2+\varphi^4 \\
w_5(\varphi) &= \frac{7}{2}+\frac{5}{2}\varphi^2+\frac{3}{2}\varphi^4+\frac{1}{2}\varphi^6 \\
w_6(\varphi) &= \frac{7}{2}+3\varphi^2+3\varphi^4+\varphi^6+\frac{1}{2}\varphi^8 \\
w_7(\varphi) &= \frac{7}{2}+3\varphi^2+\frac{9}{2}\varphi^4+2\varphi^6+\varphi^8+\frac{1}{2}\varphi^{10} \\
w_8(\varphi) &= \frac{7}{2}+3\varphi^2+\frac{11}{2}\varphi^4+4\varphi^6+\frac{5}{2}\varphi^8+\varphi^{10}+\frac{1}{2}\varphi^{12} \\
w_9(\varphi) &= \frac{11}{3}+\frac{19}{6}\varphi^2+\frac{19}{3}\varphi^4+\frac{29}{6}\varphi^6+\frac{7}{2}\varphi^8+2\varphi^{10}+\frac{5}{6}\varphi^{12}+\frac{1}{3}\varphi^{14} \\
w_{10}(\varphi) &= \frac{11}{3}+\frac{10}{3}\varphi^2+\frac{41}{6}\varphi^6+\frac{17}{3}\varphi^8+5\varphi^8+3\varphi^{10}+\frac{11}{6}\varphi^{12}+\frac{2}{3}\varphi^{14}+\frac{1}{3}\varphi^{16}
\end{aligned}$$

Notice that the polynomials for $T = 3, 4$ are given for completeness only, as in these cases the fit is perfect and the denominator becomes 0. Apart from the constant and the highest-order coefficient, the generating laws for the coefficient do not follow any simple pattern. The constant is important, as it describes the behavior around $\varphi = 0$. For example, for $T = 5$ the denominator becomes $1/2$ at $\varphi = 0$ and for $T = 6$ it becomes $3/2$.

Dividing (20) by the difference of (23) and (25) results in a ratio expression for the bias. If 4 divides T, the bias ratio is

$$\frac{-4\{(T-4)\varphi^3+(T-8)\varphi^7+\ldots\}}{T\{(T-1)+(T-2)\varphi^2+\ldots+\varphi^{2(T-2)}-w_T(\varphi)\}} \quad .$$

The correspondence of this formula and of the general one to Monte Carlo averages is good and reliable. Exemplary curves of the ratio expressions for the small values of $T = 6, 8, 10$ are given in Figure 1. Notice that the amount of the bias is hardly affected by increasing T from 6 to 8 and remains constant for $\varphi = \pm 1$. Increasing T to 10 results in a more noticeable bias reduction.

The skew-symmetric shape of the bias curve as a function of φ is the same if T is increased to the value of 100, while the absolute value of the bias is, of course, much smaller (see Figure 2). Because it would have been computationally demanding to calculate $w_{100}(\varphi)$ exactly, the graph in Figure 2 was obtained by Monte Carlo simulation. This simulation used only $N = 100$ and 10 experimental replications but obviously succeeded in a smooth and skew-symmetric curve with $w_{100}(0) = 0$.

Figures and tables

Table 1: Regression on seasonal dummies.

Country	d_1	d_2	d_3	d_4	R^2
Austria	-0.078	0.071	-0.075	0.121	0.950
Belgium	-0.007	0.027	-0.115	0.118	0.906
Finland	-0.010	0.022	-0.129	0.161	0.870
France	-0.006	-0.011	-0.158	0.199	0.922
Germany	-0.053	0.031	-0.052	0.097	0.771
Greece	-0.049	0.061	0.039	-0.007	0.515
Ireland	-0.007	0.030	-0.025	0.034	0.544
Italy	0.005	0.028	-0.167	0.160	0.921
Luxembourg	0.012	0.054	-0.110	0.065	0.737
Netherlands	-0.010	-0.046	-0.108	0.195	0.853
Portugal	-0.001	0.026	-0.076	0.094	0.595
Sweden	0.007	0.013	-0.112	0.133	0.907
Spain	-0.054	0.066	-0.236	0.253	0.966
United Kingdom	0.002	-0.034	-0.051	0.098	0.827

Table 2: Regression on seasonal dummies. Sample 1962–1982.

Country	d_1	d_2	d_3	d_4	R^2
Austria	-0.083	0.077	-0.073	0.120	0.949
Belgium	-0.018	0.036	-0.112	0.125	0.905
Finland	0.012	0.007	-0.154	0.185	0.946
France	0.005	-0.007	-0.190	0.225	0.945
Germany	-0.058	0.053	-0.082	0.115	0.867
Greece	-0.030	0.077	0.041	-0.015	0.448
Ireland	-0.019	0.038	-0.014	0.016	0.721
Italy	0.007	0.034	-0.151	0.148	0.881
Luxembourg	0.012	0.057	-0.097	0.041	0.651
Netherlands	-0.029	-0.006	-0.103	0.183	0.869
Portugal	-0.010	0.035	-0.048	0.084	0.406
Sweden	0.020	0.012	-0.103	0.136	0.884
Spain	-0.045	0.055	-0.251	0.266	0.979
United Kingdom	0.003	-0.023	-0.076	0.111	0.881

Table 3: Regression on seasonal dummies. Sample 1983–2002.

Country	d_1	d_2	d_3	d_4	R^2
Austria	-0.074	0.065	-0.077	0.123	0.954
Belgium	0.005	0.019	-0.117	0.111	0.926
Finland	-0.034	0.039	-0.102	0.136	0.853
France	-0.015	-0.015	-0.124	0.172	0.956
Germany	-0.046	0.008	-0.021	0.078	0.835
Greece	-0.071	0.044	0.036	0.001	0.754
Ireland	0.007	0.022	-0.037	0.053	0.681
Italy	0.004	0.023	-0.183	0.172	0.968
Luxembourg	0.013	0.051	-0.123	0.091	0.858
Netherlands	0.010	-0.089	-0.112	0.207	0.907
Portugal	0.010	0.016	-0.105	0.104	0.867
Sweden	-0.005	0.015	-0.121	0.130	0.943
Spain	-0.065	0.077	-0.221	0.239	0.960
United Kingdom	0.003	-0.046	-0.026	0.085	0.889

Table 4: Tests for seasonal unit roots in industrial production series. Time range is 1962:1-2002:4 for all series.

	with constant			with seasonal dummies		
Country	p	τ_{-1}	$F_{\pm i}$	p	τ_{-1}	$F_{\pm i}$
Austria	9	-1.017	2.086	7	-1.222	13.940
Belgium	10	-2.287	0.713	9	-1.767	8.314
Finland	5	-1.268	1.267	5	-1.261	2.184
France	9	-2.406	0.856	8	-2.939	5.677
Germany	8	-2.549	2.887	6	-2.194	15.702
Greece	9	-0.721	0.492	7	-3.886	8.705
Ireland	5	2.646	5.450	5	2.626	6.719
Italy	8	1.061	0.820	8	-3.403	8.038
Luxembourg	11	0.052	0.621	11	0.033	5.788
Netherlands	9	-2.768	0.324	9	-1.466	3.913
Portugal	10	-2.099	1.264	11	-2.714	3.371
Sweden	9	-0.766	0.905	8	-1.162	7.409
Spain	9	-3.790	1.707	9	-3.677	4.130
United Kingdom	12	-1.282	0.833	12	-1.182	4.140

The second and fifth columns give lag orders of autoregressive models for the data in logarithms, as identified by AIC. τ_{-1} denotes the t–statistic for the test for the unit root at -1. $F_{\pm i}$ is the F–statistic for the test for the unit root pair at $\pm i$.

Table 5: Simulated significance points for HEGY–type tests for seasonal unit roots in panels with N individuals and T observations.

		t_{-1}		$F_{\pm i}$	
N	T	10% point	5% point	10% point	5% point
14	24	6.29	6.61	71.19	76.90
14	40	6.18	6.49	68.98	74.01
14	60	6.21	6.48	68.26	73.11
14	150	6.15	6.42	66.82	71.50
5	40	4.36	4.67	33.27	36.92
10	40	5.54	5.83	54.01	58.55

Generating models are random walks with 10,000 replications. Testing model is $\Delta Y_t = b_1 Y_t^{(2)} + b_2 \Delta_2 Y_{t-2} + b_3 \Delta_2 Y_{t-1} + u_t$ for the series Y defined as residuals from individual regressions on seasonal dummies.

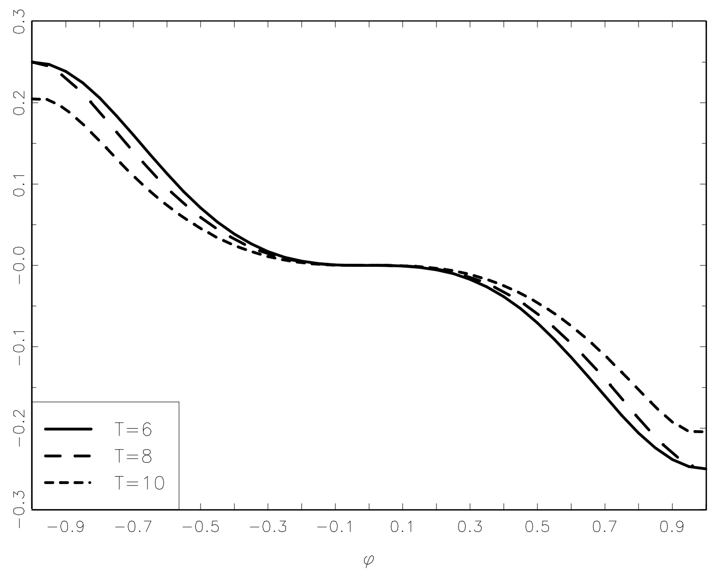

Figure 1: Exact small-sample bias of the LSSD estimate in a first-order autoregression as a function of the autoregressive coefficient φ.

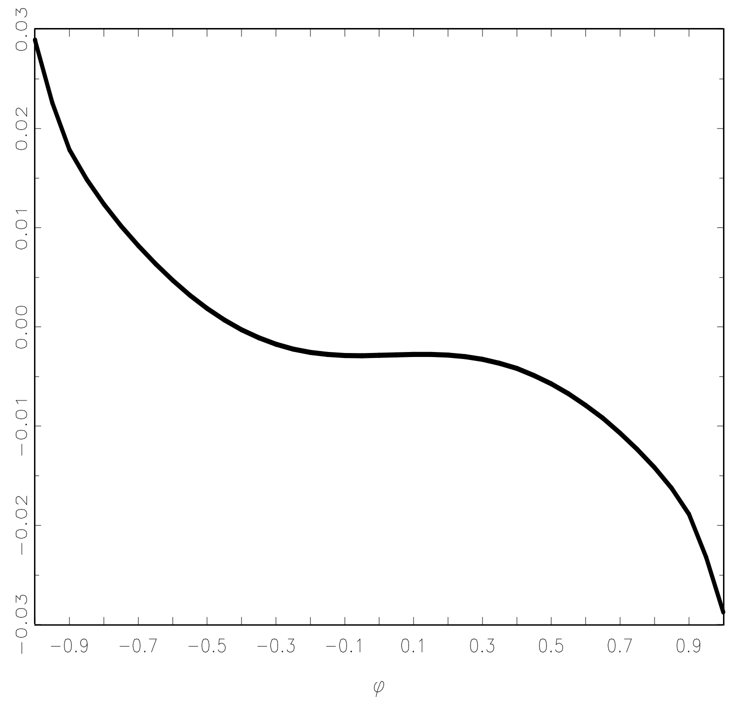

Figure 2: Simulated bias of the LSSD estimate for $T = 100$ in a first-order autoregression as a function of the autoregressive coefficient φ.

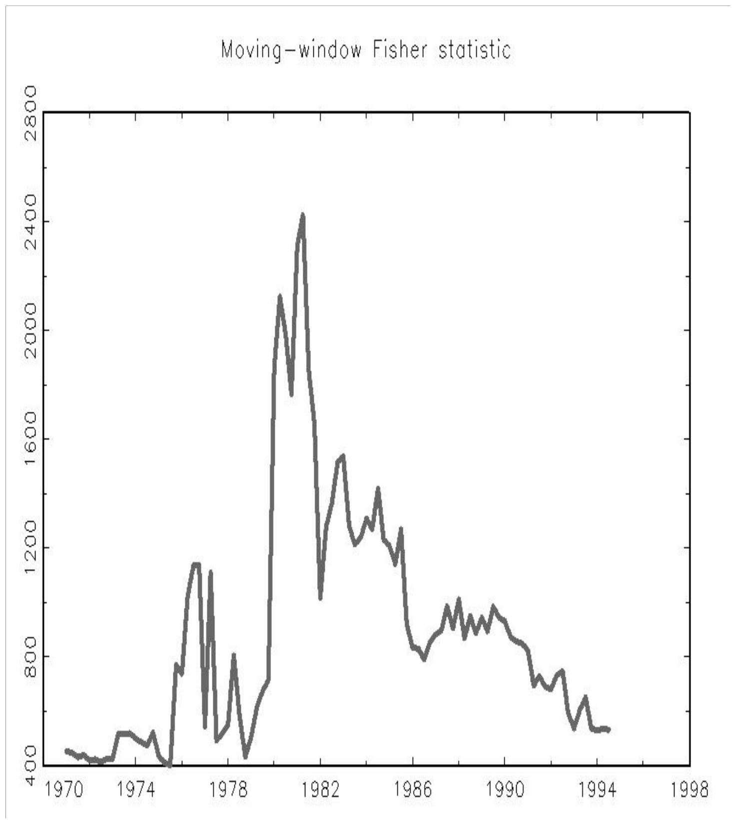

Figure 3: Fisher statistic for the null hypothesis of identical seasonal cycles across 11 European economies, as calculated from moving windows of 60 quarters. Maintained model is the ID panel model.

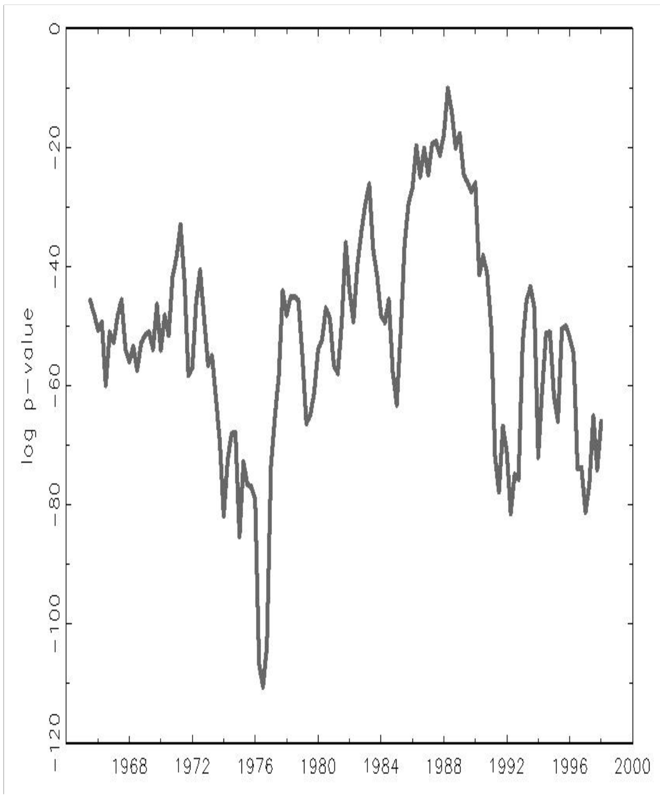

Figure 4: Logarithms of p–values of the Fisher statistic for the null hypothesis of common deterministic seasonal cycles in all 14 countries and at both seasonal frequencies. Calculation is based on a moving window of 24 observations (6 years).

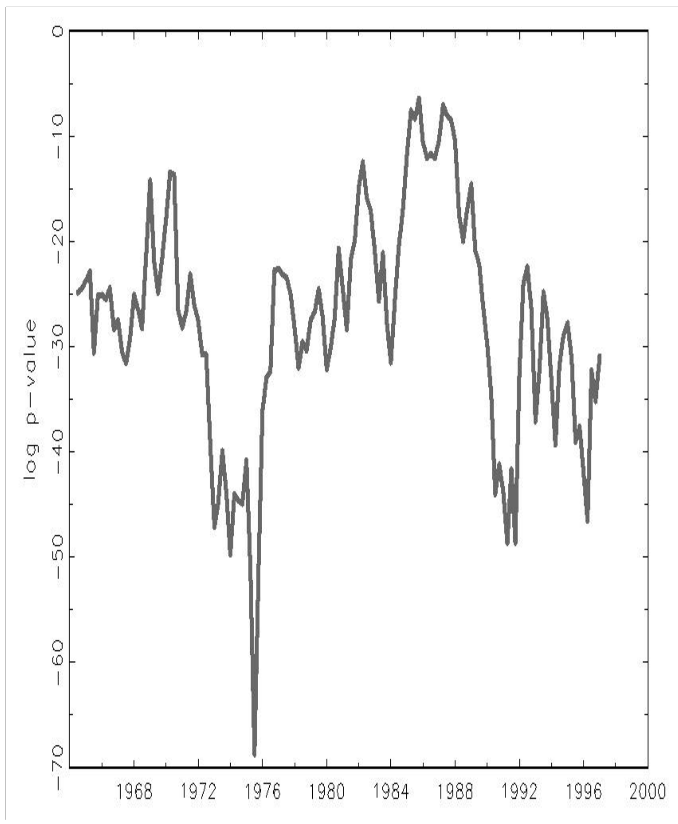

Figure 5: Logarithms of p–values of the Fisher statistic for the null hypothesis of common deterministic seasonal cycles at the seasonal frequency $\omega = \pi/2$ in all 14 countries. Calculation is based on a moving window of 24 observations (6 years).

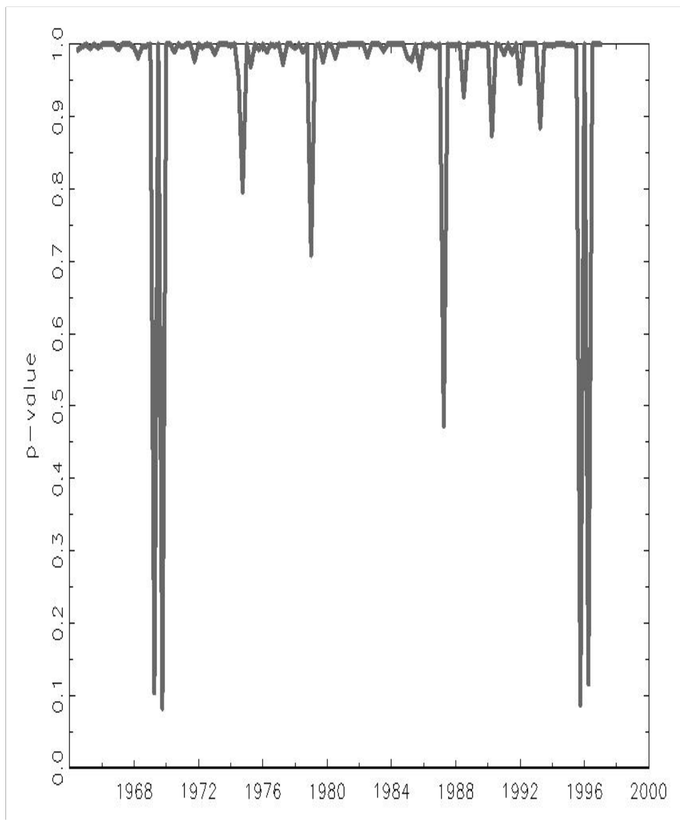

Figure 6: p–values of the Fisher statistic for the null hypothesis of common deterministic seasonal cycles at the seasonal frequency $\omega = \pi$ in all 14 countries. Calculation is based on a moving window of 24 observations (6 years).