Additive models

The models that we have considered in earlier sections have been **additive models**, and there has been an implicit assumption that the different components affected the time series additively.

$$\text{Data} = \text{Seasonal effect} + \text{Trend} + \text{Cyclical} + \text{Residual}$$

For monthly data, an additive model assumes that the difference between the January and July values is approximately the same each year. In other words, the **amplitude** of the seasonal effect is the same each year.

The model similarly assumes that the residuals are roughly the same size throughout the series -- they are a random component that adds on to the other components in the same way at all parts of the series.

Multiplicative models

In many time series involving **quantities** (e.g. money, wheat production, ...), the absolute differences in the values are of less interest and importance than the percentage changes.

For example, in seasonal data, it might be more useful to model that the July value is the same **proportion** higher than the January value in each year, rather than assuming that their difference is constant. Assuming that the seasonal and other effects act proportionally on the series is equivalent to a **multiplicative model**,

$$\text{Data} = (\text{Seasonal effect}) \times \text{Trend} \times \text{Cyclical} \times \text{Residual}$$

Fortunately, multiplicative models are equally easy to fit to data as additive models! The trick to fitting a multiplicative model is to take logarithms of both sides of the model,

$$\begin{aligned}\log(\text{Data}) &= \log(\text{Seasonal effect} \times \text{Trend} \times \text{Cyclical} \times \text{Residual}) \\ &= \log(\text{Seasonal effect}) + \log(\text{Trend}) \\ &\quad + \log(\text{Cyclical}) + \log(\text{Residual})\end{aligned}$$

After taking logarithms (either natural logarithms or to base 10), the four components of the time series again act additively.

To fit a multiplicative model, take logarithms of the data, then analyse the log data as before.

It is important to recognise when multiplicative models are appropriate. However fitting the models is no harder than fitting additive models.