Additive models

The models that we have considered in earlier sections have been **additive models**, and there has been an implicit assumption that the different components affected the time series additively.

$$\text{Data} = \text{Seasonal effect} + \text{Trend} + \text{Cyclical} + \text{Residual}$$

For monthly data, an additive model assumes that the difference between the January and July values is approximately the same each year. In other words, the **amplitude** of the seasonal effect is the same each year.

The model similarly assumes that the residuals are roughly the same size throughout the series -- they are a random component that adds on to the other components in the same way at all parts of the series.

Multiplicative models

In many time series involving **quantities** (e.g. money, wheat production, ...), the absolute differences in the values are of less interest and importance than the percentage changes.

For example, in seasonal data, it might be more useful to model that the July value is the same **proportion** higher than the January value in each year, rather than assuming that their difference is constant. Assuming that the seasonal and other effects act proportionally on the series is equivalent to a **multiplicative model**,

$$\text{Data} = (\text{Seasonal effect}) \times \text{Trend} \times \text{Cyclical} \times \text{Residual}$$

Fortunately, multiplicative models are equally easy to fit to data as additive models! The trick to fitting a multiplicative model is to take logarithms of both sides of the model,

$$\begin{aligned}\log(\text{Data}) &= \log(\text{Seasonal effect} \times \text{Trend} \times \text{Cyclical} \times \text{Residual}) \\ &= \log(\text{Seasonal effect}) + \log(\text{Trend}) \\ &\qquad + \log(\text{Cyclical}) + \log(\text{Residual})\end{aligned}$$

After taking logarithms (either natural logarithms or to base 10), the four components of the time series again act additively.

To fit a multiplicative model, take logarithms of the data, then analyse the log data as before.

It is important to recognise when multiplicative models are appropriate. However fitting the models is no harder than fitting additive models.

Illustrative example

In the following time series, the values increase by 20% each year, except for two years -- 1983 and 1998 -- in each of which the values **decrease** by 20%.

In a plot of the raw data, 1998 appears more unusual than 1983. Indeed, it is easy to miss the fact that 1983 is an unusual year.

Since we consider 1983 and 1998 to be equally unusual because they have the same **proportional** change from the previous year, so a multiplicative model is suggested.

Drag the slider to **Log scale**. The applies a logarithmic transformation to the data. Observe that both unusual years appear the same on a log scale. Note also that the transformation linearises the trend in the series, making it easier to model.

Personal disposable income

The time series plot below shows how US personal disposable income (in billions of dollars) changed between 1959 and 2001.

In this period, personal disposable incomes have increased considerably, so a same percentage increase that would be quite noticable in 2001 would be hard to recognise on a plot of the raw data in 1959.

Drag the slider to **Log scale**. Observe that the trend is fairly linear until about 1985, implying a constant percentage increase each year. However there has been a flattening out of the time series in recent years -- the annual percentage increase in personal disposable income has become smaller.

Visitor arrivals in New Zealand

Our final example is a seasonal time series. The plot below describes the numbers of short-term visitor arrivals in New Zealand (in thousands) each quarter from 1980 to 2001.

Observe that the seasonal component affects this series multiplicatively -- in earlier years when the visitor numbers were small, the differences between the quarters were also small.

Apply a logarithmic transformation to the data, and observe that the seasonal variation now becomes more constant. A multiplicative model is therefore appropriate, and the data can be analysed by fitting an additive model to the log visitor numbers.