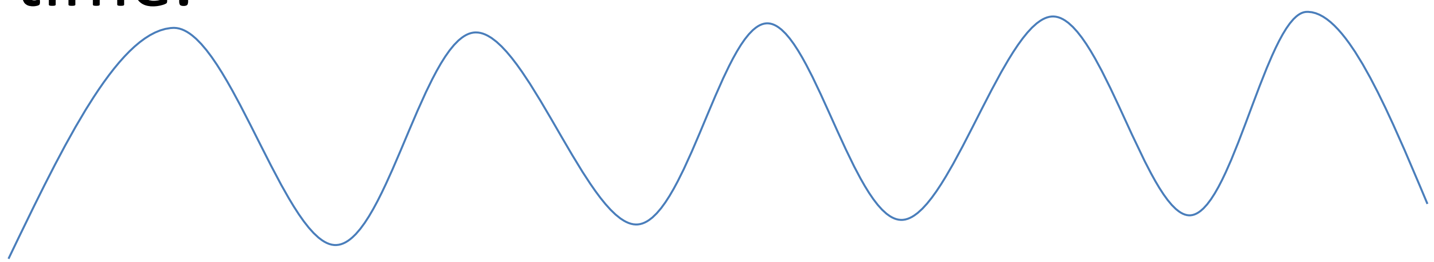
SEASONAL TIME SERIES MODELS

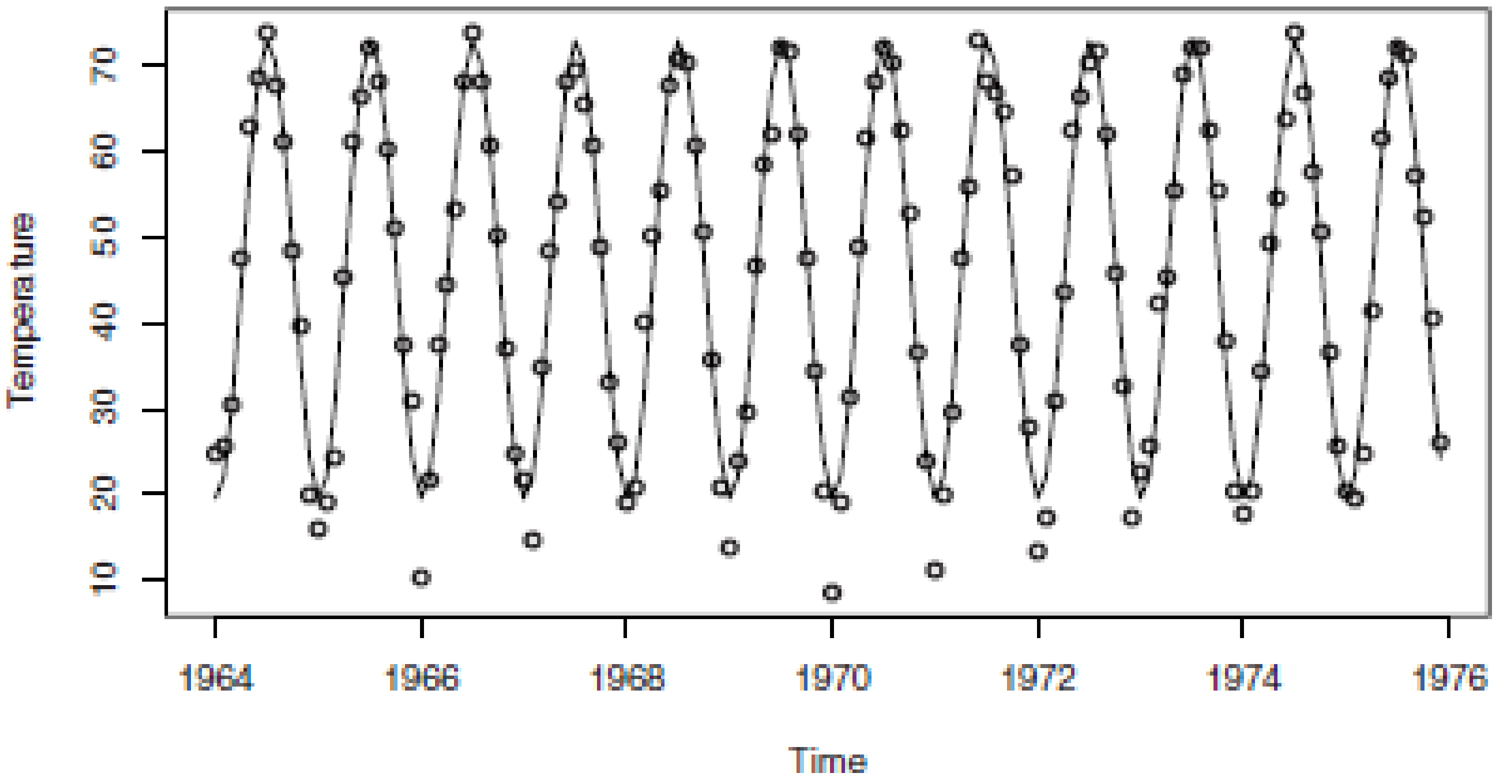
SEASONAL TIME SERIES

- A time series repeats itself after a regular period of time.



- "Business cycle" plays an important role in economics. In time series analysis, business cycle is typically represented by a seasonal (or periodic) model.
- A smallest time period for this repetitive phenomenon is called a seasonal period, s.

SEASONAL TIME SERIES



SEASONAL TIME SERIES

Seasonality

Stochastic

SARIMA

Deterministic

- **Seasonal means (dummies) + linear time trend**
- **Sums of cosine curves at various frequencies + linear time trend**

SEASONAL TIME SERIES

- For deterministic function *f(.)*, we say that *f(.)* is periodic with a periodicity s if

$$f(t) = f(t + k \times s), k = 0, \pm 1, \pm 2, \cdots$$

- A typical example of a deterministic periodic function is a trigonometric series,

e.g. $sin(\theta) = sin(\theta + 2k\pi)$ or $cos(\theta) = cos(\theta + 2k\pi)$.

- The trigonometric series are often used in econometrics to model time series with strong seasonality. [In some cases, seasonal dummy variables are used.]

SEASONAL TIME SERIES

- For stochastic process Y_t, we say that it is a seasonal (or periodic) time series with periodicity s if Y_t and Y_{t+ks} have the same distribution.
- For instance, the series of monthly sales of a department store in the U.S. tends to peak at December and to be periodic with a period 12.

OR quarterly ice cream sales is seasonal with period 4.

- In what follows, we shall use s to denote periodicity of a seasonal time series. Often $s = 4$ and 12 are used for quarterly and monthly time series, respectively.

MODELLING SEASONALITY BY SEASONAL DUMMIES

- One approach to model seasonality is **regression on seasonal dummies.** It is a simple application of dummy variables defined to reflect movement across the "seasons" of the year.
- For quarterly data, $s = 4$,
- *For* monthly data, $s = 12$,
- For weekly data, $s=52$.
- *For daily data, s=7.*

SEASONAL DUMMY

- Then we construct s *seasonal dummy* variables to indicate the season. So, if we have quarterly data and assuming the first observation we have is in the first quarter, we create:

$$D_1 = (1,0,0,0,\ 1,0,0,0,\ 1,0,0,0,\ldots)$$
$$D_2 = (0,1,0,0,\ 0,1,0,0,\ 0,1,0,0,\ldots)$$
$$D_3 = (0,0,1,0,\ 0,0,1,0,\ 0,0,1,0,\ldots)$$
$$D_4 = (0,0,0,1,\ 0,0,0,1,\ 0,0,0,1,\ldots)$$

SEASONAL DUMMY

- D_1 indicates whether *we are in the first quarter (i.e., it takes on the value 1 in the first 1* quarter and 0 otherwise),
- D_2 *indicates whether we are in the second quarter,*
- D_3 *indicates* whether we are in the third quarter,
- D_4 *indicates whether we are in the fourth quarter.*

SEASONAL DUMMY

- The pure seasonal dummy model is given by:

$$Y_t = \sum_{i=1}^{s} \gamma_i D_{it} + a_t$$

- This is simply a regression on an intercept in which we allow for a different intercept in each season.
- These different intercepts are called the seasonal factors and reflect the seasonal pattern over the year.

SEASONAL DUMMY

- If we have *s seasons, an alternative is to include just s-1 seasonal dummies and an intercept. In* this case:

  (i) the constant term is the intercept for the omitted season; and

  (ii) the coefficients on the seasonal dummies indicate the seasonal increase/decrease relative to the omitted season.

- **Never include *s seasonal dummies and an intercept.*** *T*his will cause a serious problem.

SEASONAL DUMMY AND LINEAR TIME TREND

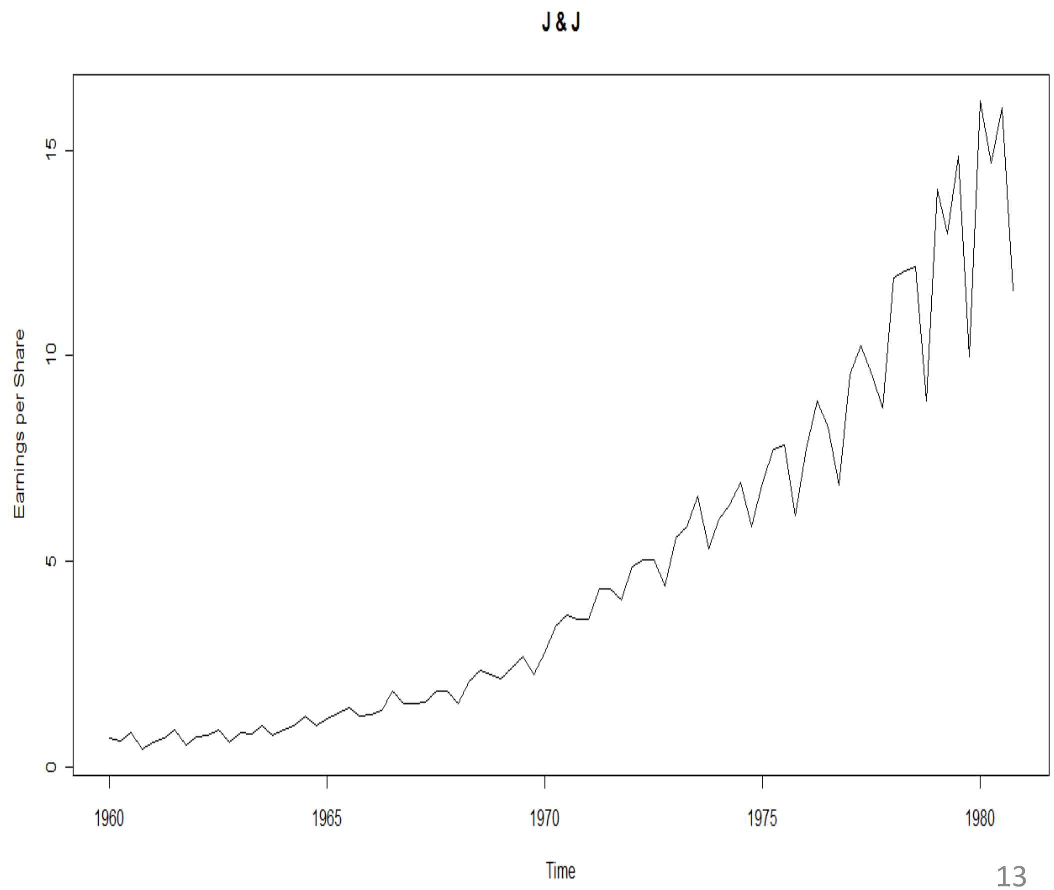
- If a variable Y *exhibits both trend and seasonality, we can combine the trend model* with the seasonal model and obtain:

$$Y_t = \beta t + \sum_{i=1}^{s} \gamma_i D_{it} + a_t$$

- Note that since we have used *s* seasonal dummies, we have dropped the intercept term from the linear trend part of the model.

EXAMPLE

```
> jj=read.table('c:/jj.dat', header=FALSE)
> jj = ts(jj, start=1960, frequency=4)
> time(jj)
        Qtr1    Qtr2    Qtr3    Qtr4
1960 1960.00 1960.25 1960.50 1960.75
1961 1961.00 1961.25 1961.50 1961.75
1962 1962.00 1962.25 1962.50 1962.75
1963 1963.00 1963.25 1963.50 1963.75
1964 1964.00 1964.25 1964.50 1964.75
1965 1965.00 1965.25 1965.50 1965.75
1966 1966.00 1966.25 1966.50 1966.75
1967 1967.00 1967.25 1967.50 1967.75
1968 1968.00 1968.25 1968.50 1968.75
1969 1969.00 1969.25 1969.50 1969.75
1970 1970.00 1970.25 1970.50 1970.75
1971 1971.00 1971.25 1971.50 1971.75
1972 1972.00 1972.25 1972.50 1972.75
1973 1973.00 1973.25 1973.50 1973.75
1974 1974.00 1974.25 1974.50 1974.75
1975 1975.00 1975.25 1975.50 1975.75
1976 1976.00 1976.25 1976.50 1976.75
1977 1977.00 1977.25 1977.50 1977.75
1978 1978.00 1978.25 1978.50 1978.75
1979 1979.00 1979.25 1979.50 1979.75
1980 1980.00 1980.25 1980.50 1980.75
```

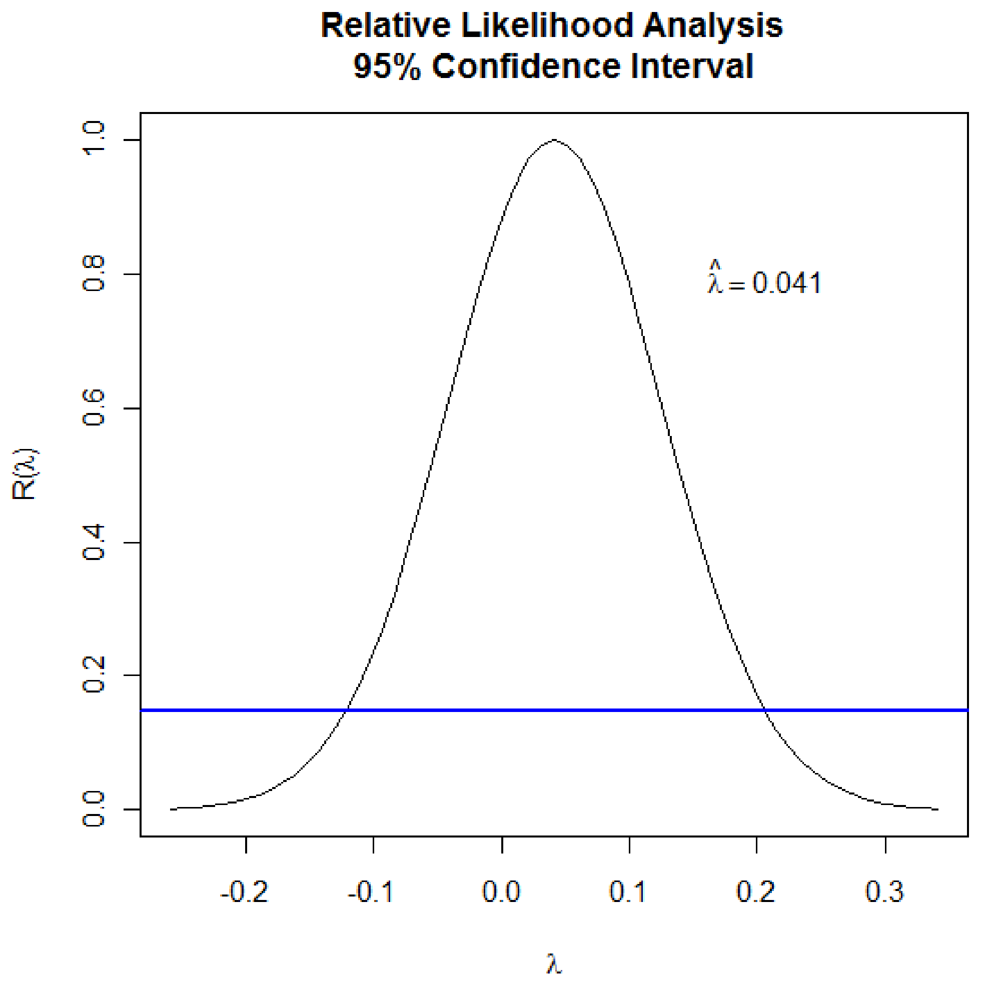


EXAMPLE

Load FitAR. Write the time series as vector. Look at the Box-Cox results

```
> jj.ts=as.vector(jj)
> BoxCox(jj.ts)
```

Use either 0.041-th power of the series or do ln transformations since the 0.041 is very close to 0.

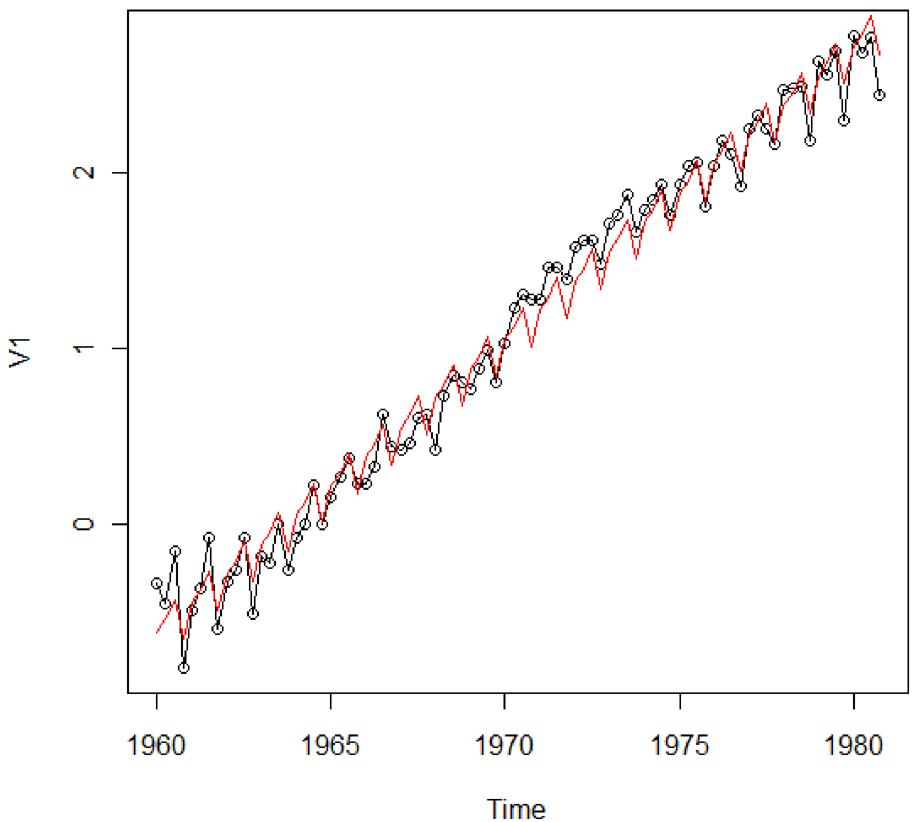


EXAMPLE

```
> Q = factor(rep(1:4,21))      # make (Q)uarter factors [that's repeat 1,2,3,4, 21 times]
> trend = time(jj)-1970   # not necessary to "center" time, but the results look nicer
> reg = lm(log(jj)~0+trend+Q, na.action=NULL)  # run the regression without an intercept
> #-- the na.action statement is to retain time series attributes
> summary(reg)

Call:
lm(formula = log(jj) ~ 0 + trend + Q, na.action = NULL)

Residuals:
     Min        1Q    Median        3Q       Max 
-0.29318 -0.09062 -0.01180  0.08460  0.27644 

Coefficients:
      Estimate Std. Error t value Pr(>|t|)    
trend 0.167172   0.002259   74.00   <2e-16 ***
Q1    1.052793   0.027359   38.48   <2e-16 ***
Q2    1.080916   0.027365   39.50   <2e-16 ***
Q3    1.151024   0.027383   42.03   <2e-16 ***
Q4    0.882266   0.027412   32.19   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1254 on 79 degrees of freedom
Multiple R-squared: 0.9935,     Adjusted R-squared: 0.9931 
F-statistic:  2407 on 5 and 79 DF,  p-value: < 2.2e-16
```

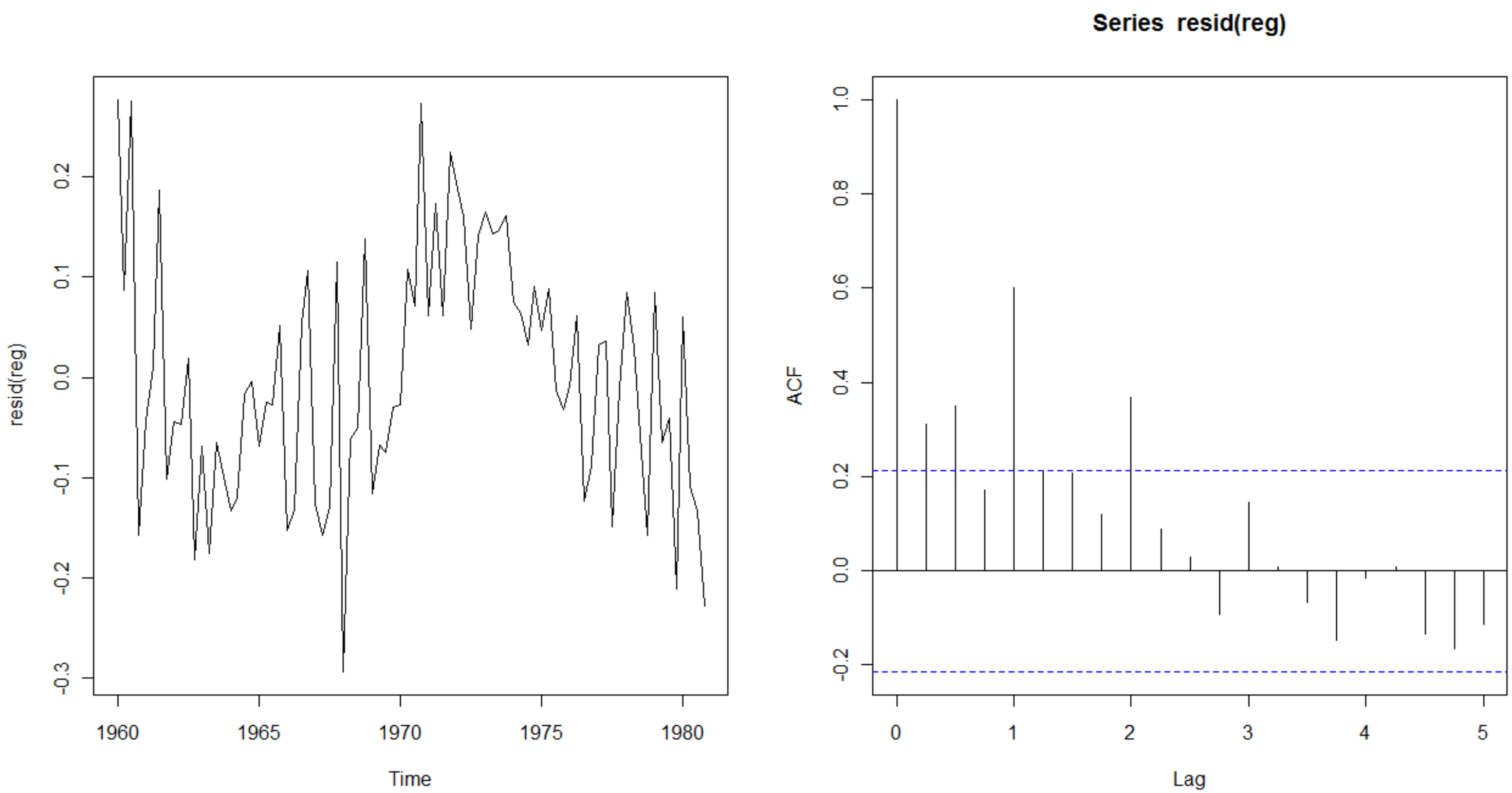
EXAMPLE

```
> plot(log(jj), type="o")    # the data in black with little dots
> lines(fitted(reg), col=2) # the fitted values in bloody red -
or use lines(reg$fitted, col=2)
```



EXAMPLE

Plot of the residuals and the ACF of the residuals:



FORECASTING SEASONAL SERIES

- The full model is given by:

$$Y_t = \beta t + \sum_{i=1}^{s} \gamma_i D_{it} + a_t$$

- So, at time $t = n+h$, *we have:*

$$Y_{n+h} = \beta(n+h) + \sum_{i=1}^{s} \gamma_i D_{i,n+h} + a_{n+h}$$

- Note that to construct this forecast we will set a_{n+h} to its unconditional expectation of zero.

FORECASTING SEASONAL SERIES

- To make this point forecast operation we replace the unknown population parameters with OLS point estimates:

$$\hat{Y}_{n+h} = \hat{\beta}(n+h) + \sum_{i=1}^{s} \hat{\gamma}_i D_{i,n+h}$$

- Finally, forecasts are formed.

SEASONALITY

- Seasonality can reflect other types **calendar effects. The "standard" seasonality model with** "seasonal dummies" is one type of calendar effect. Two other types of seasonality are **holiday variation and trading-day variation.**
- Holiday variation refers to the fact the dates of some holidays change over time. Easter Sunday is an important example, and we may want to include in a model with monthly data an "Easter dummy" which equals 1 if the month contains Easter and 0 otherwise.

SEASONALITY

- Likewise, **trading-day variation** refers to the fact that different months contain numbers of trading or business days. In a model of monthly retail sales, it would certainly seem to matter if there were, for example, 28, 29, 30, or 31 trading days in the month. To account for this we could include a trading-day variable which measures the number of trading days in the month.
- We will not focus on holiday and trading-day variation effects, even though they are important in the analysis of many time series.

PURE SEASONAL TIME SERIES

- SARIMA$(P,D,Q)_s$

$$\Phi_P\left(B^s\right)\left(1-B^s\right)^D Y_t=\theta_0+\Theta_Q\left(B^s\right)a_t$$

where θ_0 is constant,

$$\Phi_P\left(B^s\right)=1-\Phi_1 B^s-\Phi_2 B^{2s}-\cdots-\Phi_P B^{sP}$$

$$\Theta_Q\left(B^s\right)=1-\Theta_1 B^s-\Theta_2 B^{2s}-\cdots-\Theta_Q B^{sQ}$$

$SARIMA(0,0,1)_{12} = SMA(1)_{12}$

- This is a simple seasonal MA model.

$$Y_t = \theta_0 + a_t - \Theta a_{t-12}$$

- Invertibility: $|\Theta| < 1$.
- $E(Y_t) = \theta_0$.

$$Var(Y_t) = (1 + \Theta^2)\sigma_a^2$$

$$ACF : \rho_k = \begin{cases} \dfrac{-\Theta}{1+\Theta^2}, |k| = 12 \\ 0, o.w. \end{cases}$$

SARIMA$(1,0,0)_{12}$

- This is a simple seasonal AR model.

$$\left(1-\Phi B^{12}\right)Y_t = \theta_0 + a_t$$

- Stationarity: $|\Phi| < 1$.

$$E(Y_t) = \frac{\theta_0}{1-\Phi} \qquad Var(Y_t) = \frac{\sigma_a^2}{1-\Phi^2}$$

$$ACF : \rho_{12k} = \Phi^k, k = 0, \pm 1, \pm 2, \cdots$$

When $\Phi = 1$, the series is non-stationary. To test for a unit root, consider seasonal unit root tests.

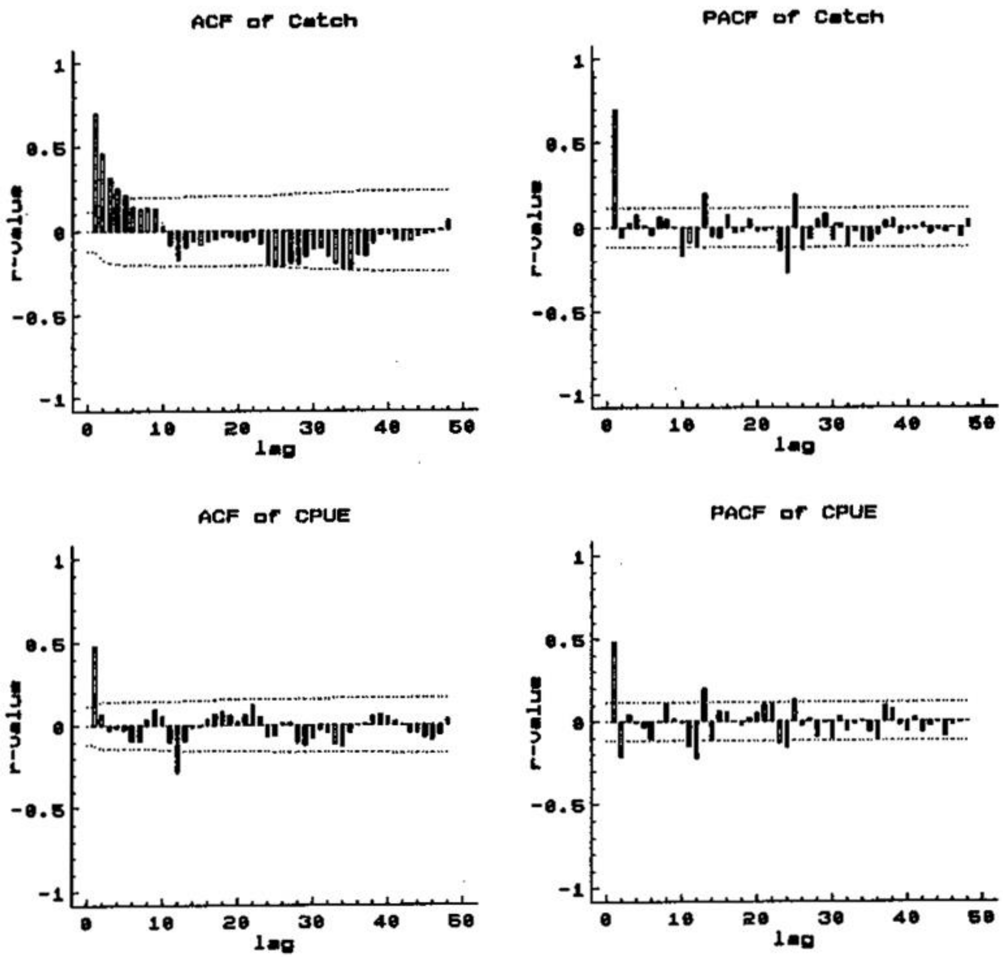
MULTIPLICATIVE SEASONAL TIME SERIES

- A special, parsimonious class of seasonal time series models that is commonly used in practice is the multiplicative seasonal model $ARIMA(p, d, q)(P,D,Q)_s$.

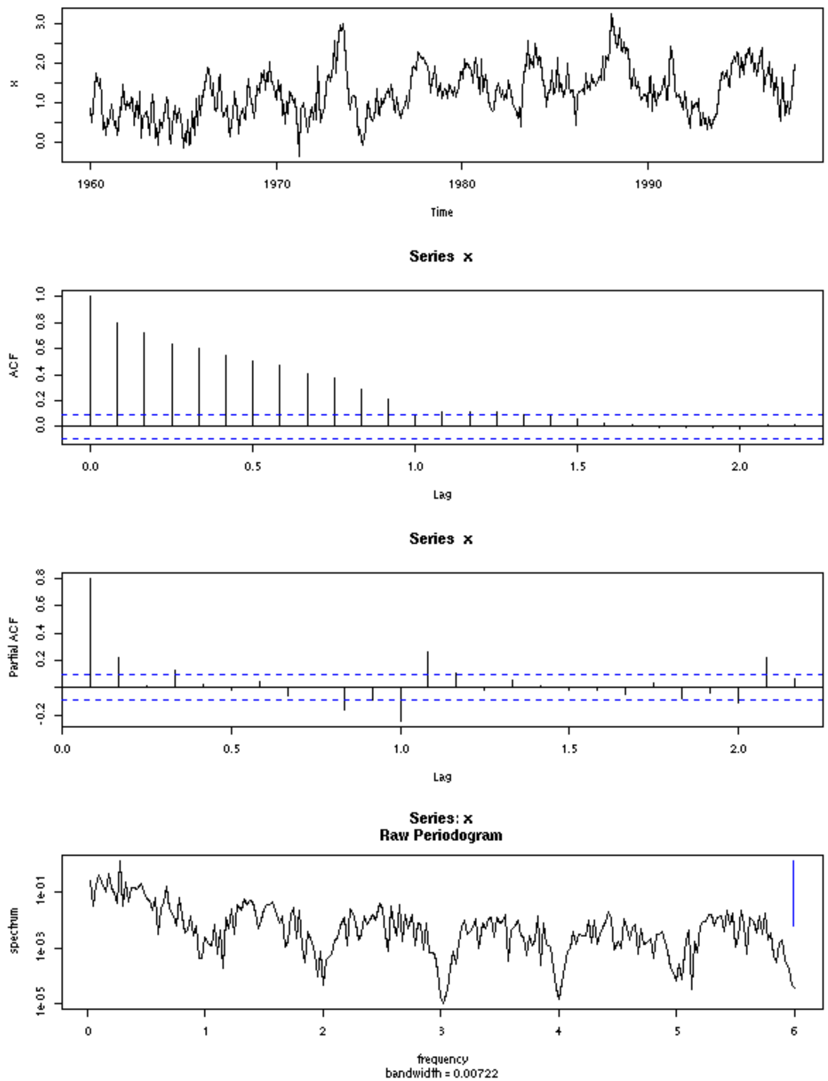
$$\phi_p(B)\Phi_P\left(B^s\right)(1-B)^d\left(1-B^s\right)^D Y_t = \theta_0 + \theta_q(B)\Theta_Q\left(B^s\right)a_t$$

where all zeros of $\phi(B)$; $\Phi(B^s)$; $\theta(B)$ and $\Theta(B^s)$ lie outside the unit circle. Of course, there are no common factors between $\phi(B)\Phi(B^s)$ and $\theta(B)\Theta(B^s)$.

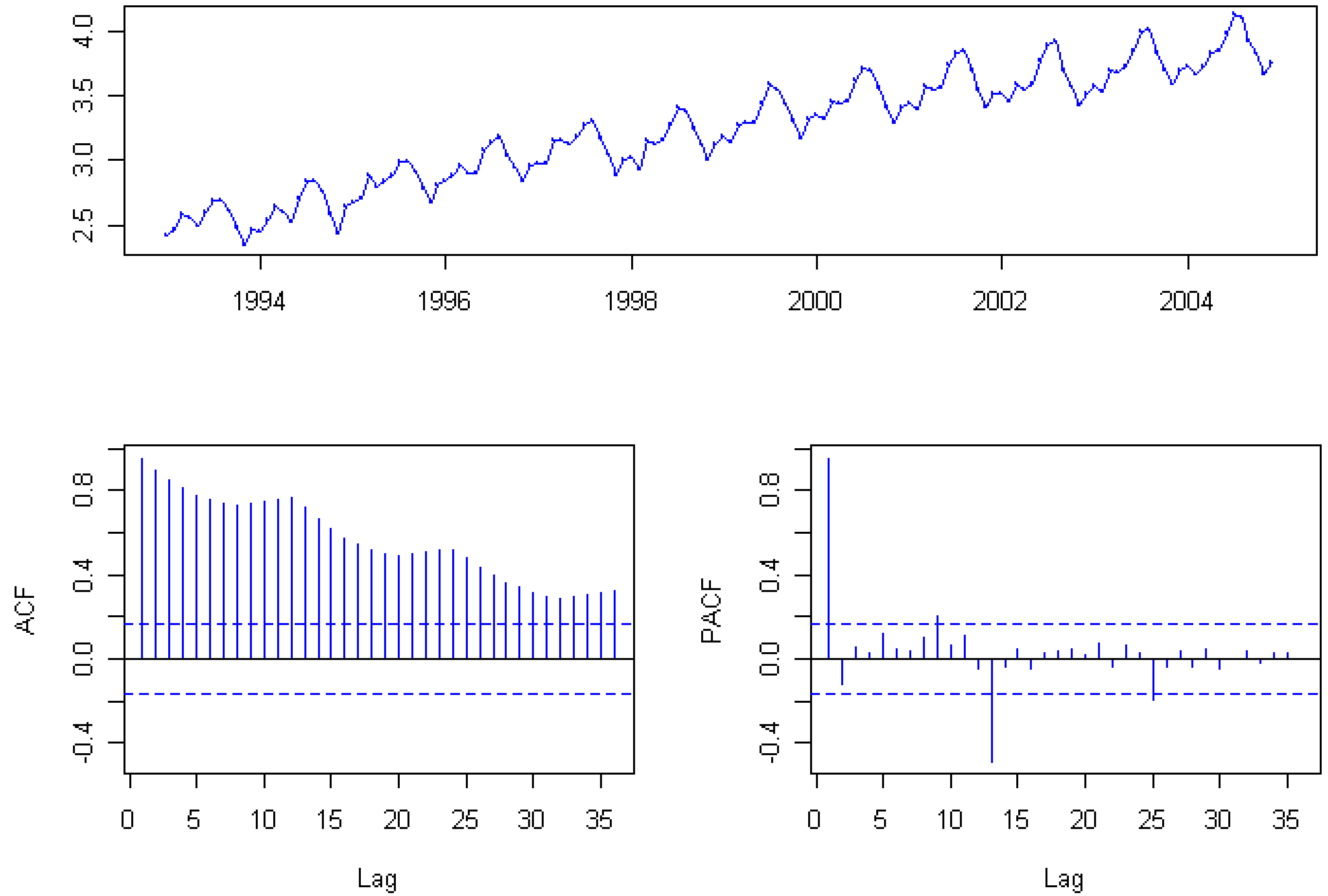
MULTIPLICATIVE SEASONAL TIME SERIES



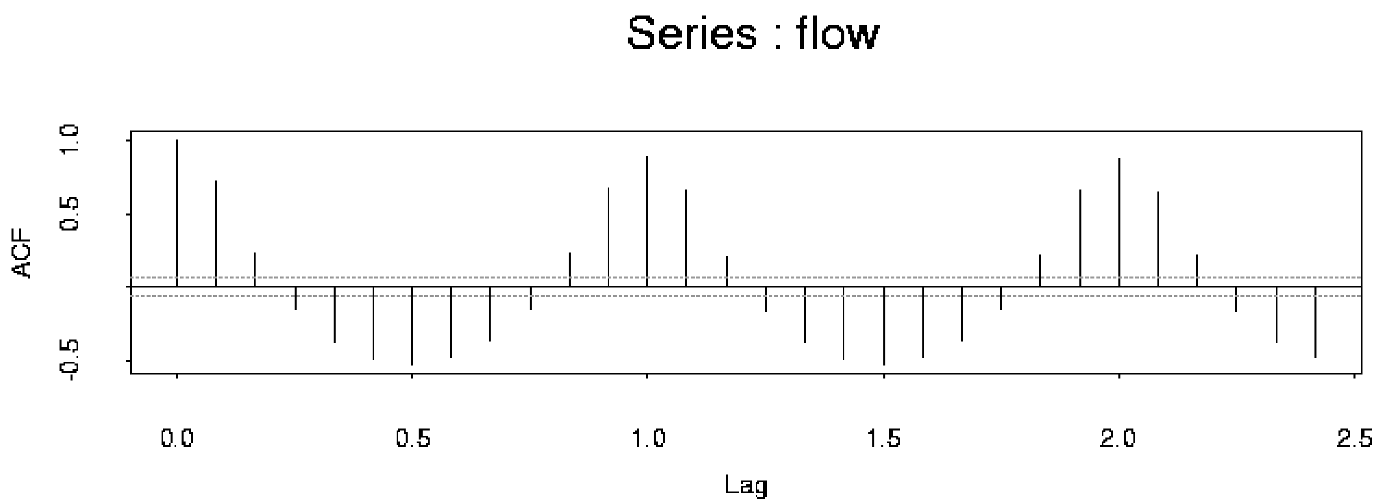
MULTIPLICATIVE SEASONAL TIME SERIES

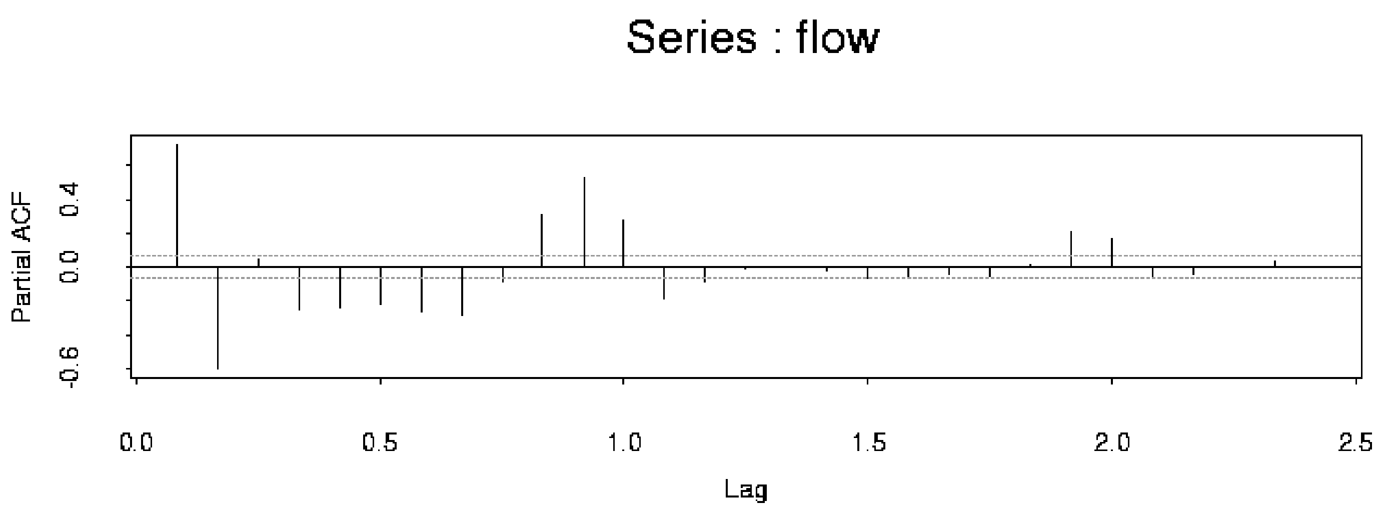


MULTIPLICATIVE SEASONAL TIME SERIES

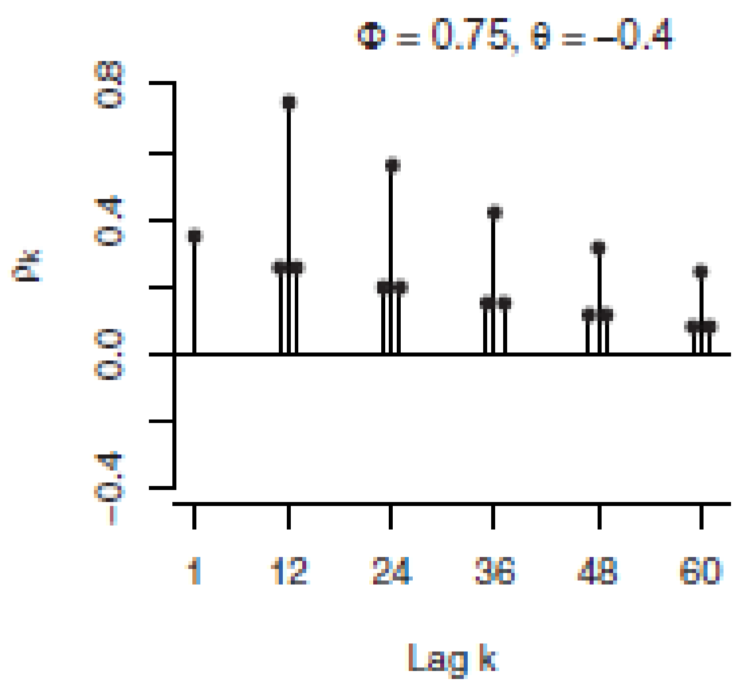


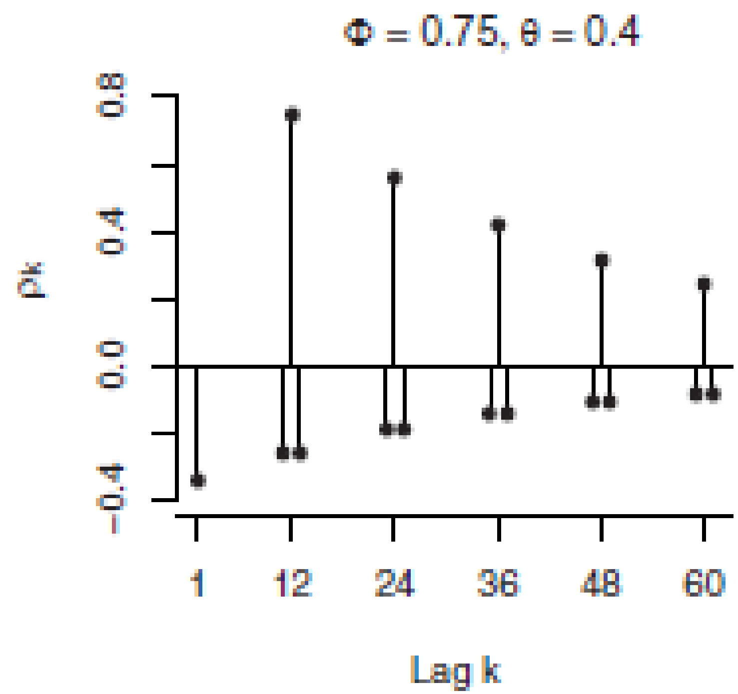
MULTIPLICATIVE SEASONAL TIME SERIES



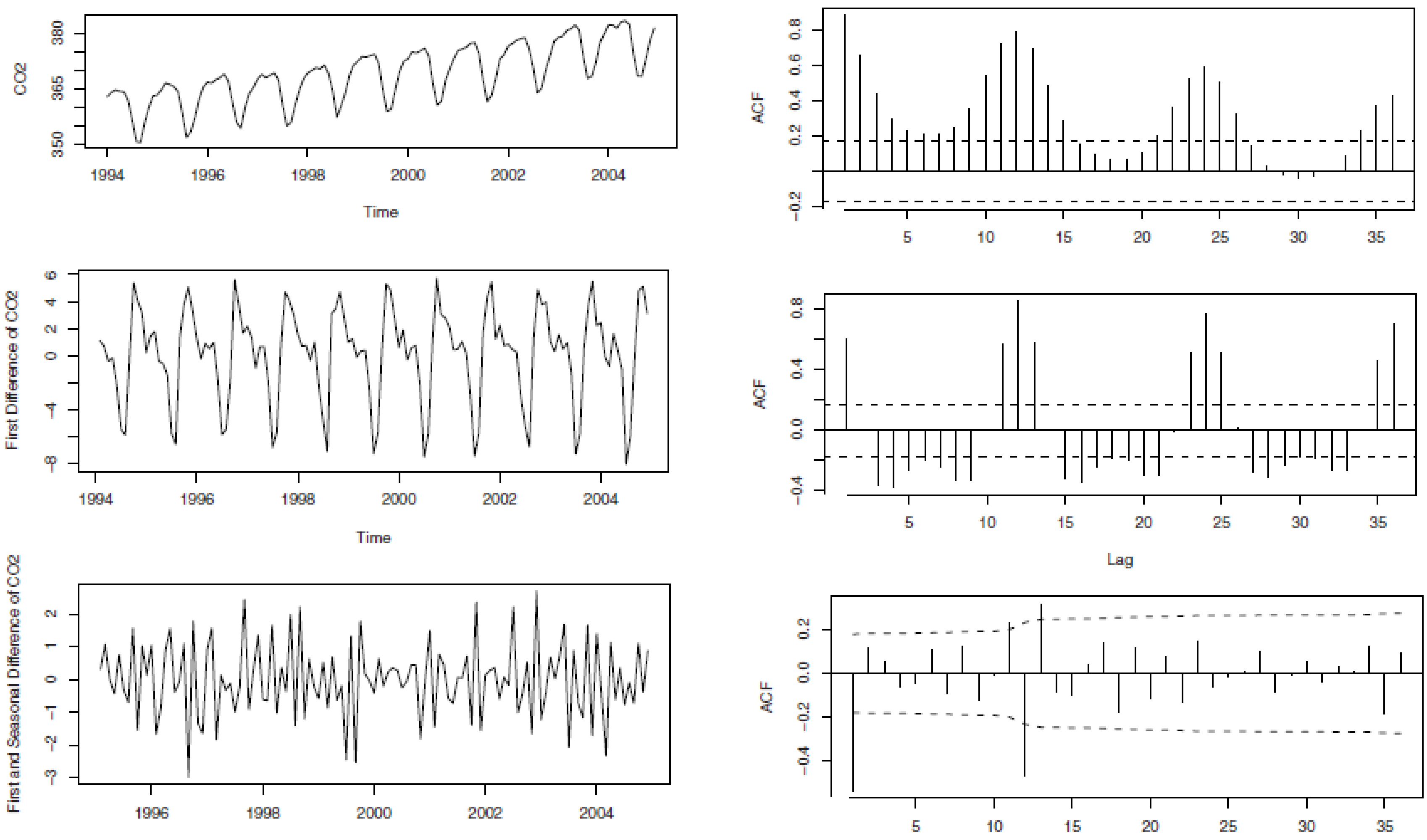


MULTIPLICATIVE SEASONAL TIME SERIES





Monthly Carbon Dioxide Levels at Alert, NWT, Canada



AIRLINE MODEL

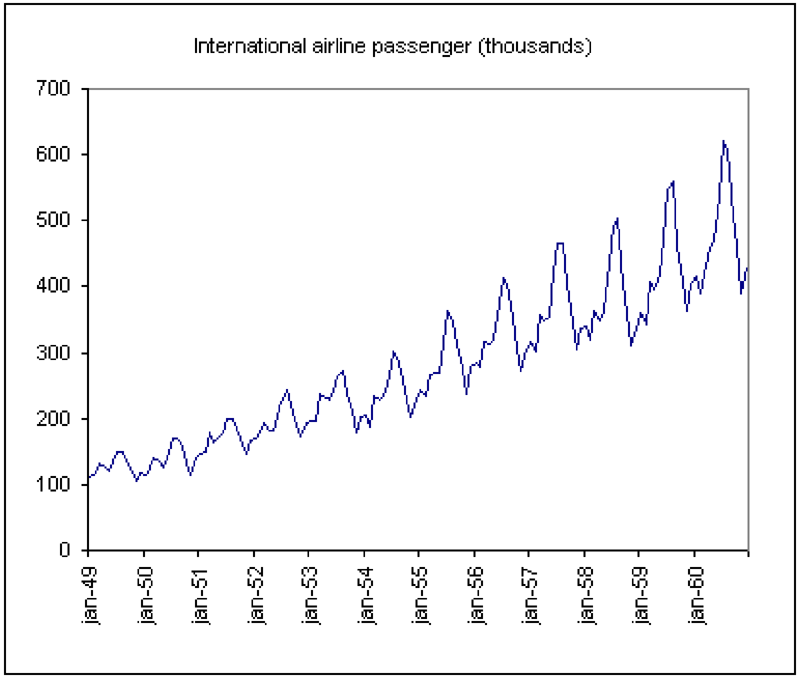
- SARIMA$(0,1,1)(0,1,1)_{12}$

$$(1-B)(1-B^{12})Y_t = (1-\theta B)(1-\Theta B^{12})a_t$$

where $|\theta|<1$ and $|\Theta|<1$.

- This model is the most used seasonal model in practice. It was proposed by Box and Jenkins (1976) for modeling the well-known monthly series of airline passengers.

AIRLINE MODEL



- Let $W_t = (1 - B)(1 - B^{12})Y_t$, where $(1 - B)$ and $(1 - B^{12})$ are usually referred to as the "regular" and "seasonal" difference, respectively.

AIRLINE MODEL

$$W_t = (1-\theta B)\left(1-\Theta B^{12}\right)a_t$$

$$W_t = a_t - \theta a_{t-1} - \Theta a_{t-12} + \theta\Theta a_{t-13}$$

$$W_t \sim I(0)$$

$$\gamma_k = \begin{cases} \left(1+\theta^2\right)\left(1+\Theta^2\right)\sigma_a^2, k=0 \\ -\theta\left(1+\Theta^2\right)\sigma_a^2, |k|=1 \\ -\Theta\left(1+\theta^2\right)\sigma_a^2, |k|=12 \\ \theta\Theta\,\sigma_a^2, |k|=11{,}13 \\ 0, o.w. \end{cases} \qquad \rho_k = \begin{cases} \dfrac{-\theta}{\left(1+\theta^2\right)}, |k|=1 \\ \dfrac{-\Theta}{\left(1+\Theta^2\right)}, |k|=12 \\ \dfrac{\theta\Theta}{\left(1+\theta^2\right)\left(1+\Theta^2\right)}, |k|=11{,}13 \\ 0, o.w. \end{cases}$$

SEASONAL UNIT ROOTS

- Seasonal unit roots and testing for seasonal integration is discussed in Charemza and Deadman (1997, 105-9) and Pfaff (2008).
- The main advantage of seasonal unit root tests is where you need to make use of data that cannot be seasonally adjusted or even as a pretest before seasonal adjustment.

SEASONAL UNIT ROOTS

- If a series has seasonal unit roots, then standard ADF test statistic do not have the same distribution as for non-seasonal series. Furthermore, seasonally adjusting series which contain seasonal unit roots can alias the seasonal roots to the zero frequency, so there is a number of reasons why economists are interested in seasonal unit roots.
- Hylleberg, S., Engle, R.F., Granger, C. W. J., and Yoo, B. S., Seasonal integration and cointegration,(1990), Journal of Econometrics, 44: pages 215{238.

THE DICKEY-HASZA-FULLER TEST

- The first test for testing for seasonal unit root is developed by Dickey, Hasza and Fuller (DHF) in 1984.
- Assuming that the process is SAR(1), the DHF test can be parameterized by

$$\Delta_s z_t = \delta_0 z_{t-1} + \sum_{i=1}^{k} \delta_i \Delta_s y_{t-i} + \varepsilon_t.$$

- In the null hypothesis, we are testing $\delta_0 = 0$ versus $\delta_0 < 0$.

DHF TEST

- After the OLS estimation, the test statistics is obtained as

$$t_{\hat{\alpha}} = \frac{\frac{1}{n}\sum_{t=1}^{n} y_{t-s} a_t}{\tilde{\sigma}\sqrt{\frac{1}{n^2}\sum_{t=1}^{n} y_{t-s}^2}}$$

- Again, the asymptotic distribution of this test statistics is a non-standard distribution. The critical values were obtained by Monte-Carlo simulation for different sample sizes and seasonal periods.

DHF TEST

- The problem of the DHF test is that, under the null hypothesis, one has exactly s unit roots. Under the alternative, one has no unit root. This is very restrictive, as some people may wish to test for specific seasonal or non-seasonal unit roots. The HEGY test by Hylleberg, Granger, Engle, Yoo can do this. Therefore, it is the most customary test.

SEASONAL UNIT ROOTS

- The **HEGY test** for seasonal integration is conducted by estimating the following regression (special case for quarterly data):

$$\Delta^4 Y_t = \alpha + \beta t + \sum_{j=2}^{4} b_j Q_{jt} + \sum_{i=1}^{4} \pi_i W_{it-1} + \sum_{\ell=1}^{k} \gamma_\ell \Delta^4 Y_{t-\ell} + a_t$$

where Q_{jt} is a seasonal dummy, and the W_{it} are given below.

$$W_{1t} = (1+B)(1+B^2)Y_t$$

$$W_{2t} = -(1-B)(1+B^2)Y_t$$

$$W_{3t} = -(1-B)(1+B)Y_t$$

$$W_{4t} = -B(1-B)(1+B)Y_t = W_{3t-1}$$

HEGY TEST

- After OLS estimation, tests are conducted for $\pi_1 = 0$, for $\pi_2 = 0$ and a joint test of the hypothesis $\pi_3 = \pi_4 = 0$.
- The HEGY test is a joint test for LR (or zero frequency) unit roots and seasonal unit roots. If none of the π_i are equal to zero, then the series is stationary (both at seasonal and nonseasonal frequencies).

HEGY TEST

- Interpretation of the different π_i is as follows:

1. If $\pi_1 < 0$, then there is no long-run (nonseasonal) unit root. π_1 is on $W_{1t} = S(B)Y_t$ which has had all of the seasonal roots removed.
2. If $\pi_2 < 0$, then there is no semi-annual unit root.
3. If π_3 and $\pi_4 < 0$, then there is no unit root in the annual cycle.

Cycles implied by the roots of the seasonal difference operator

Root $+1$ Factor $(1-L)$	Root -1 Factor $(1+L)$	Root $+i$ Factor $(1-iL)$	Root $-i$ Factor $(1+iL)$
$y_{t+1} = y_t$	$y_{t+1} = -y_t$	$y_{t+1} = iy_t$	$y_{t+1} = -iy_t$
	$y_{t+2} = -(y_{t+1}) = y_t$	$y_{t+2} = i(y_{t+1}) = -y_t$	$y_{t+2} = -i(y_{t+1}) = -y_t$
		$y_{t+3} = i(y_{t+2}) = -iy_t$	$y_{t+3} = -i(y_{t+2}) = iy_t$
		$y_{t+4} = i(y_{t+3}) = y_t$	$y_{t+4} = -i(y_{t+3}) = y_t$

HEGY TEST

- Just as in the ADF tests, it is important to ensure that the residuals from estimating the HEGY equation are white noise. Thus, in testing for seasonal unit roots, it is important to follow the sequential procedures detailed above.
- Again, begin by testing for the appropriate lag length for the dependent variable (to ensure serially uncorrelated residuals), and then test whether deterministic components belong in the model.

HEGY TEST

- The presence of seasonal unit roots at some frequency and not at other frequencies can lead to problems of interpretation. The presence of a seasonal unit root at a certain frequency implies that there is no deterministic cycle at that frequency but a stochastic cycle.
- The power of unit root tests is low, that is, it is not easy to distinguish between genuine unit roots and near-unit roots. The literature suggests that this might not be too large a problem, as erroneously imposing a unit root seems better than not imposing it when one should.

OSBORN-CHUI-SMITH-BIRCHENHALL (1988) TEST

- Osborn, Chui, Smith and Birchenhall [1988] test is the modification of Dickey, Hasza and Fuller [1984].
- In R, seasonal unit root tests are implemented in the CRAN-package **forecast** package.
- Osborn et al. [1988] suggested replacing $\Delta_s z_t$ with $\Delta_s y_t$ as the dependent variable.

OSBORN-CHUI-SMITH-BIRCHENHALL (1988) TEST

- Incidentally, if $h = 0$, this is equivalent with an ADF regression for the seasonal differences; i.e.,

- The lag orders k and h should be determined similarly to the procedures proposed for the ADF test.

OSBORN-CHUI-SMITH-BIRCHENHALL (1988) TEST

- Furthermore, it should be noted that deterministic seasonal dummy variables can also be included in the test regression. The relevant critical values are provided in Osborn et al. [1988] and are dependent on the inclusion of such deterministic dummy variables and whether the data have been demeaned at the seasonal frequency.

OSBORN-CHUI-SMITH-BIRCHENHALL (1988) TEST

- If the null hypothesis of the existence of a seasonal unit root is rejected for a large enough absolute t ratio, then one might conclude that stochastic seasonality is not present or that stochastic seasonality, which can be removed by using s-differences, does not exist. On the other hand, if the null hypothesis cannot be rejected, it is common practice to consider the order of non-seasonal differencing required to achieve stationarity instead of considering higher orders of seasonal differencing.

Number of differences required for a stationary series (ndiffs)

- ndiffs {forecast}

ndiffs uses a unit root test to determine the number of differences required for time series x to be made stationary. If test="kpss", the KPSS test is used with the null hypothesis that x has a stationary root against a unit-root alternative. Then the test returns the least number of differences required to pass the test at the level alpha. If test="adf", the Augmented Dickey-Fuller test is used and if test="pp" the Phillips-Perron test is used. In both of these cases, the null hypothesis is that x has a unit root against a stationary root alternative. Then the test returns the least number of differences required to fail the test at the level alpha.

ndiffs {forecast}

`nsdiffs` uses seasonal unit root tests to determine the number of seasonal differences required for time series `x` to be made stationary (possibly with some lag-one differencing as well). If `test="ch"`, the Canova-Hansen (1995) test is used (with null hypothesis of deterministic seasonality) and if `test="ocsb"`, the Osborn-Chui-Smith-Birchenhall (1988) test is used (with null hypothesis that a seasonal unit root exists).

```
ndiffs(x, alpha=0.05, test=c("kpss","adf", "pp"), max.d=2)
nsdiffs(x, m=frequency(x), test=c("ocsb","ch"), max.D=1)
```

```
> ndiffs(WWWusage)
[1] 1
> nsdiffs(log(AirPassengers))
[1] 1
> ndiffs(diff(log(AirPassengers),12))
[1] 1
```

Example: Austrian industrial production

- Data for log production (without taking first differences) is for 1957-2009. AIC and also BIC recommend three additional augmenting lags, and we estimate the regression:

$$\begin{aligned} \Delta_4 y_t &= \sum_{s=1}^{4} \gamma_s \delta_{st} + \pi_1 y_{t-1}^{(1)} - \pi_2 y_{t-1}^{(2)} - \pi_3 y_{t-2}^{(3)} - \pi_4 y_{t-1}^{(3)} \\ &\quad + \sum_{j=1}^{3} \psi_j \Delta_4 y_{t-j} + \varepsilon_t \end{aligned}$$

by OLS. First, we analyze the t–statistics for π_1 and π_2, and then the F–statistic for $\pi_3 = \pi_4 = 0$.

Example (Contd.)

- The statistic $t(\pi_1)$ is 2.10. Using the usual Dickey-Fuller μ, we see that this is insignificant. There is evidence on a unit root at +1, as expected.
- The statistic $t(\pi_2)$ is 2.74. According to HEGY, we revert its sign. The literature gives a critical 5% value at − 3.11 and a critical 10% value at −2.54 . Because −2.54 > −2.74 > −3.11, the unit root at −1 is rejected at 10% but not at 5%.
- The statistic $F(\pi_3, \pi_4)$ is 8.08. This is larger than the 5% significance point by HEGY of 6.57, though smaller than the 1% point of 8.79 . The unit root pair at $\pm i$ is rejected at the usual 5% level.

Example (Contd.)

- No seasonal unit root at $\pm i$ but some evidence on a unit root at -1 and convincing evidence on a unit root at $+1$. The joint F–test $F(\pi_2, \pi_3, \pi_4)$ has a 1% point of 7.63, which is surpassed by the observed value of 8.48. Thus, the joint test would tend to reject all seasonal unit roots.

HEGY for monthly series (Lecture Note of Matthieu Stigler)

- The HEGY test has been extended for monthly series (12 roots) by Franses (1990) and Beaulieu and Miron (1993).
- The roots are the same as HEGY (1,-1, i,-1) plus $\pm 1/2(1 \pm 3^{1/2}\, i)$, $\pm 1/2(3^{1/2} \pm i)$

HEGY Test for Monthly Series (Lecture Note of Matthieu Stigler)

root	freq	cycles per year
1	π	6
i	$\pi/2$	3
i	$-\pi/2$	9
$-1/2(1+\sqrt{3}i)$	$-2\pi/3$	8
$-1/2(1-\sqrt{3}i)$	$2\pi/3$	4
$1/2(1+\sqrt{3}i)$	$\pi/3$	2
$1/2(1-\sqrt{3}i)$	$-\pi/3$	10
$-1/2(\sqrt{3}+i)$	$-5\pi/6$	7
$-1/2(\sqrt{3}-i)$	$5\pi/6$	5
$1/2(\sqrt{3}+i)$	$\pi/6$	1
$1/2(\sqrt{3}-i)$	$-\pi/6$	11

HEGY tests with R (Lecture Note of Matthieu Stigler)

- The HEGY test and its extension to monthly data ara available in R in:

```
> library(uroot)
> data(AirPassengers)
> lairp <- log(AirPassengers)
> test <- HEGY.test(wts = lairp, itsd = c(1, 1, c(1:11)), regvar = 0,
+ selectlags = list(mode = "bic", Pmax = 12))
> test@stats
```

```
                Stat.   p-value
tpi_1       -2.577797 0.1000000
tpi_2       -4.396433 0.0100000
Fpi_3:4     18.519107 0.1000000
Fpi_5:6      4.823309 0.0100000
Fpi_7:8      8.656624 0.1000000
Fpi_9:10     7.119685 0.0494419
Fpi_11:12    2.854972 0.0100000
Fpi_2:12    18.373828        NA
Fpi_1:12    19.336146        NA
```

STATIONARITY TEST FOR SEASONAL SERIES

- The test developed by Canova and Hansen (1995) takes as the null hypothesis that the seasonal pattern is deterministic.
- From:

$$y_t = \alpha y_{t-1} + \sum_{i=1}^{S-1} D_{it}\beta_i + \varepsilon_t$$

The idea is (provided stationarity, i.e. $|\alpha| < 1$) to test for instability of the β_i parameters as the KPSS test does.

STATIONARITY TEST FOR SEASONAL SERIES

$$y_t = \alpha y_{t-1} + \sum_{i=1}^{S-1} D_{it}\beta_{it} + \varepsilon_t$$

$$\beta_{it} = \beta_{it-1} + u_t$$

and test that $Var(u_t)=0$.

- The null hypothesis in the Canova-Hansen test is rejected in case seasonality of a series is not constant. After seasonal adjustment the Canova-Hansen test therefore should not reject. Note that having no seasonal pattern at all also implies constant seasonality.

STATIONARITY TEST FOR SEASONAL SERIES

- Canova-Hansen suggest a Lagrange Multiplier test statistic whose distribution is known as von Mises distribution. The test is rejected for the large values of L-statistics.
- Canova and Hansen use the assumption that both the process under investigation and the explanatory variables in the null regression do not contain any non-stationary behavior at the zero frequency.

CANOVA-HANSEN TEST IN R

Description

This function computes the Canova-Hansen statistic recursively along subsamples of the original data.

Usage

CH.rectest (wts, type="moving", nsub=48, frec=NULL, f0=1, DetTr=FALSE, ltrunc=NULL, trace=list(remain=1, plot=0, elaps=1))

Arguments

wts a univariate time series object. type a character string indicating how subsamples are selected. See details. nsub the number of observations in each subsample.

frec a vector indicating the frequencies to analyse.

f0 a 0-1 (No-Yes) vector of length one indicating wether a first lag of the dependent variable is included or not in the auxiliar regression. See details.

DetTr a logical argument. If TRUE a linear trend is included in the auxiliar regression.

ltrunc lag truncation parameter for computing the residuals covariance matrix. By default, *round(s*(N/100)^0.25)*, where eqn{s} is the periodicity of the data and *N* the number of observations.

trace a list object indicating if a trace of the iteration progress should be printed. Three levels of information can be printed: remain, the percentage of the whole procedure that has been completed; plot, a plot of the computed statistics; and elaps, how much time the whole procedure has consumed.

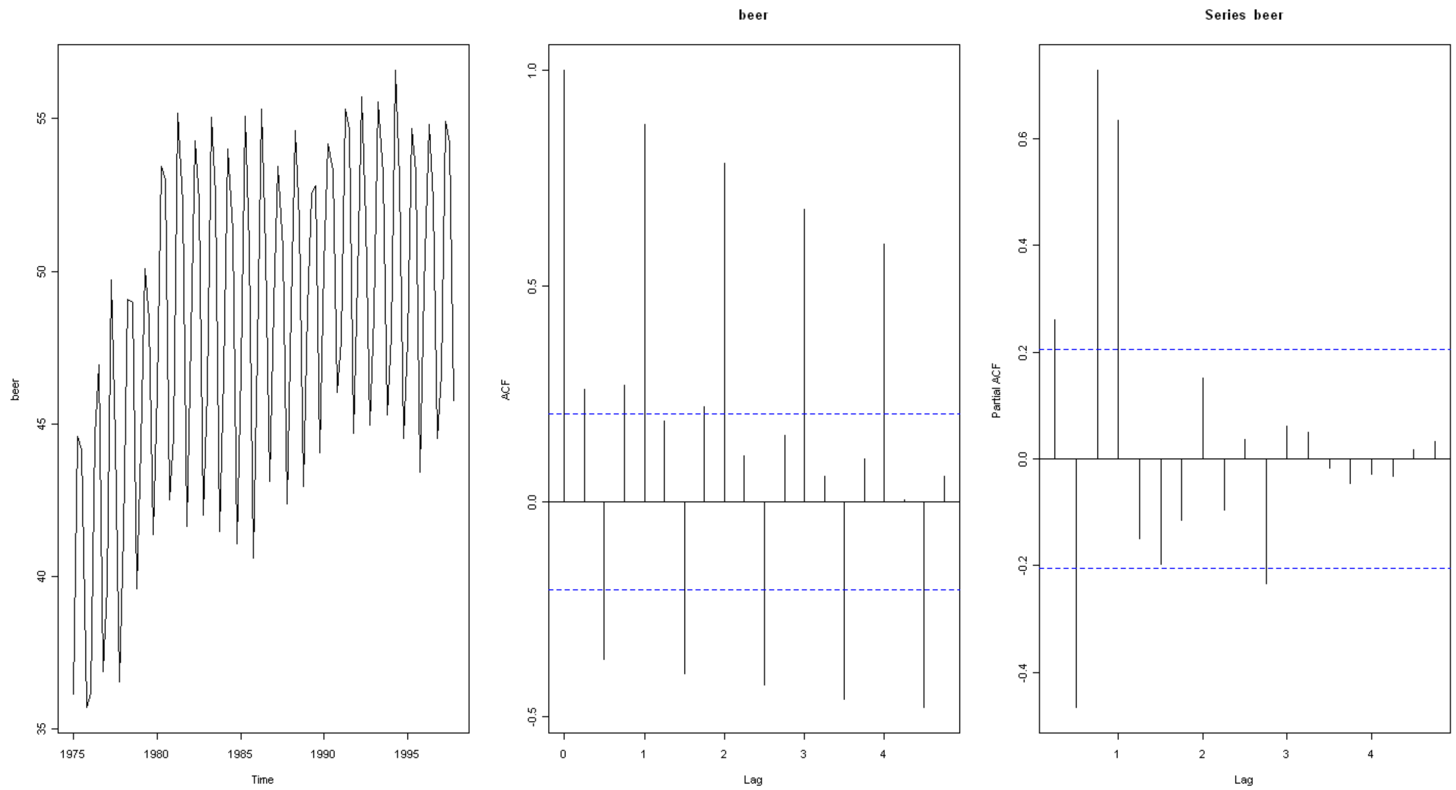
Details

Elements of frec must be set equal to 0 if the season assigned to this element is not considered and equals to 1 for the frequencies to analyse. The position of each frequency in the vector is as follows: c(pi/2, pi) for quarterly series and c(pi/6, pi/3, pi/2, 2pi/3, 5pi/6, pi) for monthly series.

Rejection of the null hypothesis implies that the analysed cycles are non-stationary.

EXAMPLE

- Quarterly US beer production data from 1975 to 1997.



EXAMPLE (contd.)

> library(uroot)

> CH.test(beer)

------ - ------ ----

Canova & Hansen test

------ - ------ ----

Null hypothesis: Stationarity.

Alternative hypothesis: Unit root.

Frequency of the tested cycles: pi/2 , pi ,

L-statistic: 0.817

Lag truncation parameter: 4

Cannot reject H_0. Seasonality pattern is deterministic. No seasonal unit root

Critical values:

0.10 0.05 0.025 0.01

0.846 **1.01** 1.16 1.35

EXAMPLE (contd.)

```
> HEGY.test(wts =beer, itsd = c(1, 1, c(1:3)), regvar = 0,selectlags = list(mode = "bic", Pmax = 12))
  ---- ----
  HEGY test
  ---- ----

  Null hypothesis: Unit root.
  Alternative hypothesis: Stationarity.

----
  HEGY statistics:

          Stat.   p-value
tpi_1   -3.339    0.085
tpi_2   -5.944    0.010
Fpi_3:4 13.238    0.010
Fpi_2:4 18.546      NA
Fpi_1:4 18.111      NA
```

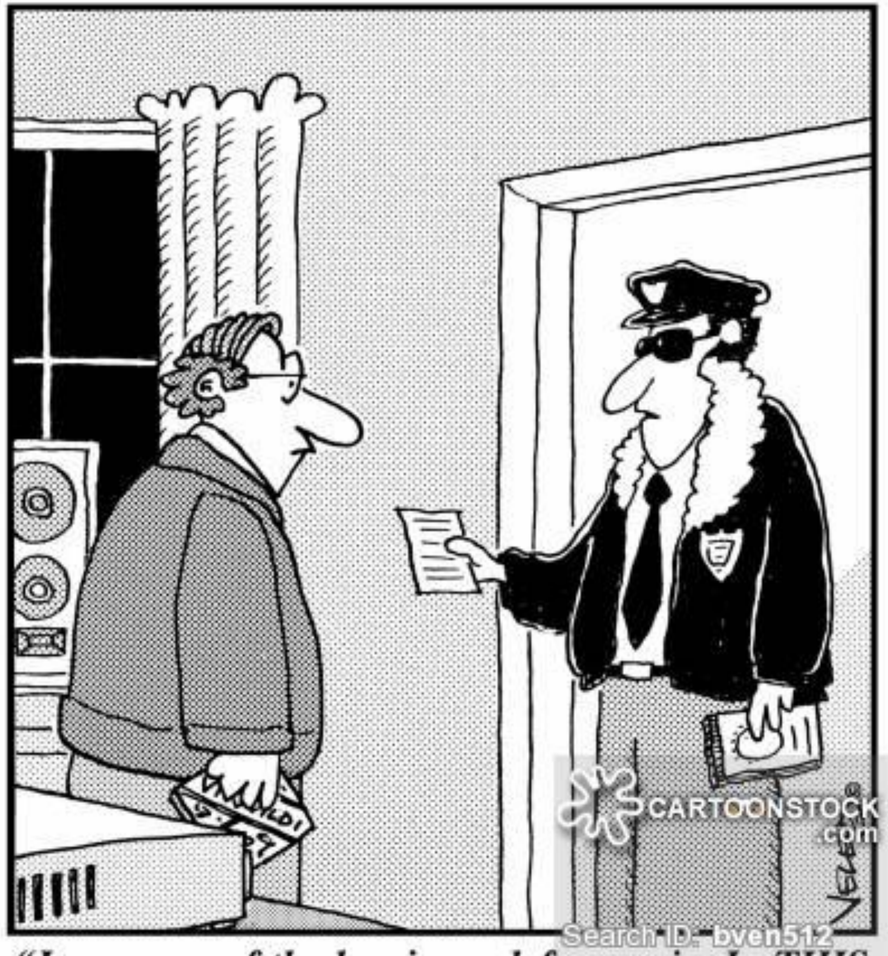
There is a unit root. The first order differencing is required.

No seasonal unit root. Nonseasonal differencing is needed.

STATIONARITY TEST IN R

```
> CH.test(wts = AirPassengers, frec = c(1, 1, 1, 1, 1, 1), f0 = 1,
+     DetTr = FALSE)
```

```
  ------ - ------ ----
  Canova & Hansen test
  ------ - ------ ----

  Null hypothesis: Stationarity.
  Alternative hypothesis: Unit root.
  Frequency of the tested cycles: pi/6 , pi/3 , pi/2 , 2pi/3 , 5pi/(

  L-statistic: 1.836
  Lag truncation parameter: 13

  Critical values:

 0.10 0.05 0.025 0.01
 2.49 2.75  2.99 3.27
```



"Ignorance of the law is no defense, sir. In THIS town, we've got ordinances against playing the 'Spring' section of Vivaldi's 'Four Seasons' on a crisp Fall evening like this!"