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TESTS FOR STOCHASTIC SEASONALITY APPLIED TO DAILY FINANCIAL TIME SERIES*

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We develop tests for seasonal unit roots for daily data by extending the methodology of Hylleberg *et al.* ('Seasonal Integration and Cointegration', *Journal of Econometrics*, Vol. 44 (1990), No. 1–2, pp. 215–238) and apply our tests to UK and US daily stock market indices. We also investigate a suggestion by Franses and Romijn ('Periodic Integration in Quarterly Macroeconomic Variables', *International Journal of Forecasting*, Vol. 9 (1993), No. 4, pp. 467–476) and Franses ('A Multivariate Approach to Modelling Univariate Seasonal Time Series', *Journal of Econometrics*, Vol. 63 (1994), No. 1, pp. 133–151) and create a price series for each day of the week and test for cointegration amongst these series. Our Monte Carlo experiments indicate that the Hylleberg *et al.* procedure is robust to autoregressive conditional heteroscedasticity type errors, while the Franses and Romijn procedure is less so. Finally, we employ Harvey's (*Time Series Models*, Hemel Hempstead, Harvester Wheatsheaf, 1993) basic structural model to test for the presence of stationary stochastic seasonality. Our results suggest that we can reject the existence of seasonal unit roots at the daily frequency in both of these markets; however, we do find evidence of stationary stochastic seasonality.

1 Introduction

Researchers in finance have documented the existence of deterministic seasonal patterns in daily stock and bond market returns. In particular, it has been found that equity markets tend to experience abnormally high returns on Fridays and abnormally low returns on Mondays (see Cross,

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1973; French, 1980). This phenomenon is at odds with the weakest form of the efficient markets hypothesis which asserts that in an informationally and operationally efficient market asset prices should not be predictable using information about past returns. A number of explanations have been advanced to explain the 'weekend effect'. Some researchers have suggested that it is due to a 'closed market effect' (see Ariel, 1990), while others have investigated the role of settlements procedures on daily stock prices (see Board and Sutcliffe, 1988); others have investigated whether returns are generated by a process which is due to calendar time, or to trading time. However, we believe that the concentration on deterministic seasonal patterns in daily asset returns may be misleading if stochastic seasonality exists in daily financial market data. According to Beaulieu and Miron (1993), an examination of seasonal patterns should begin with a study of stochastic seasonality. To this end, we extend the work of Hylleberg *et al.* (1990) (hereafter HEGY), Osborn (1990), Franses (1991a, 1991b) and Beaulieu and Miron (1993) by developing tests for seasonal unit roots for daily financial market data, generating the appropriate critical values by simulation. We also explore the logical implication of tests based upon the HEGY methodology for seasonal unit roots, namely that there exist five independent random walks within one series (in the case of daily financial market data), and extend the work of Franses (1991b) and Franses and Romijn (1993) who adapt Johansen's cointegration procedure for quarterly data to daily financial market data.

In anticipation of our results, we test for the presence of long-run and seasonal unit roots in UK and US daily equity indices over the period 31 December 1979 to 17 June 1994, using our generated critical values. We find that we can reject the hypothesis that there are unit roots at all seasonal frequencies in these series, but that we cannot reject the hypothesis that there is a long-run unit root in both markets. Therefore we find that the appropriate filter to induce stationarity in the series is $1 - L$ in both cases (where L is the lag operator: $L^i Y_t = Y_{t-i}$). We confirm this result using the Franses and Romijn procedure, finding that the cointegrating space for the reconstructed UK and US markets has dimension $v = 4$, once again rejecting the hypothesis that seasonal unit roots are present and accepting the existence of one long-run unit root in each equity market. Having rejected the existence of seasonal unit roots in our data, we found that when we employed Harvey's model using a Kalman filter the variance of the random seasonal component is larger than that of the irregular component and highly significant for both stock indices. The rest of this paper is organized as follows: in Section 2 we outline our seasonal unit root tests, the cointegration approach to the study of seasonality and Harvey's (1993) model; in Section 3 we present our results; finally, Section 4 concludes.

2 Methodology and Testing Procedures

2.1 Extending the HEGY Methodology to Daily Data

A number of authors have investigated the issue of stochastic seasonality. The existence of d_0 zero frequency (or long-run) unit roots and d_p unit roots at seasonal frequencies in a time series y_t with quarterly periodicity can be tested against a stationary alternative using tests due to Osborn (1990) and HEGY. Franses (1991a, 1991b) extends HEGY's testing procedure to the cases of bimonthly and monthly data. The asymptotic theory for the general case of arbitrary seasonal period is given, in addition to further applications, by Smith and Taylor (1998a, 1998b) and Taylor (1998). Since daily data are commonly used in empirical work in financial markets we extend HEGY's procedure in order to test for seasonal unit roots in daily financial market data. Financial economists have documented a deterministic seasonal pattern known as the 'weekend effect', where asset returns are significantly higher on Fridays and significantly lower on Mondays (see Cross, 1973; French, 1980; Gibbons and Hess, 1981). However, according to Beaulieu and Miron (1993), in any examination of seasonality we should begin with a search for seasonal unit roots, since an analysis of deterministic seasonality may be misleading if seasonal unit roots are present. See Franses *et al.* (1995) for a formal analysis of misleading inference when there are seasonal unit roots.

There is a substantial literature dedicated to testing for seasonal unit roots in quarterly data (see HEGY; Osborn, 1990; Franses and Romijn, 1993; Franses, 1994; Ghysels *et al.*, 1994; amongst many others) and, more recently, in monthly data (see Franses, 1991b; Clare *et al.,* 1995). All these tests have non-standard distributions and therefore in expanding the HEGY procedure to test for seasonal unit roots in daily data we must derive our own, appropriate critical values. HEGY develop a procedure to test for seasonal unit roots in quarterly data, which allows us to test for separate unit roots at the long-run and seasonal frequencies and for a wide range of combinations of deterministic components under the stationary alternative. The essential principle of this procedure is given by Proposition 1 in HEGY (pp. 221–222). We now adapt this procedure to the test for seasonal unit roots in daily financial market data, where the relevant polynomial for our study is $1-L^5$. This polynomial has one real root equal to 1, and two pairs of complex conjugate roots $\alpha_1, \alpha_2 = \frac{1}{4}[\surd 5 - 1 \pm \surd(-2\surd 5 - 10)] \approx 0.309 \pm 0.951i$, and, replacing $\surd 5$ by $-\surd 5$, $\alpha_3, \alpha_4 \approx -0.809 \pm 0.588i$, where $i = \surd(-1)$. From (3.3) in HEGY (p. 222), $\psi(L)$ with $p = 5$ can be written as

$$\psi(L) = \lambda_1 L\psi_1(L) + \lambda_2 L\frac{1}{\alpha_1}\left(1 - \frac{1}{\alpha_2}L\right)\psi_2(L) + \lambda_3 L\frac{1}{\alpha_2}\left(1 - \frac{1}{\alpha_1}L\right)\psi_2(L) + \lambda_4 L\frac{1}{\alpha_3}\left(1 - \frac{1}{\alpha_4}L\right)\psi_3(L) + \lambda_5 L\frac{1}{\alpha_4}\left(1 - \frac{1}{\alpha_3}L\right)\psi_3(L) + \psi^*(L)\psi_4(L) \quad (1)$$

where

$$\psi_1(L) = (1 - \zeta L + L^2)[1 + (1/\zeta)L + L^2]$$
$$\psi_2(L) = (1 - L)[1 + (1/\zeta)L + L^2]$$
$$\psi_3(L) = (1 - L)(1 - \zeta L + L^2)$$
$$\psi_4(L) = 1 - L^5$$

and $\zeta = (\surd 5 - 1)/2 \approx 0.618$, the 'golden mean', and $-\zeta$ and $1/\zeta$ are the roots of $L^2 - L - 1$. Note that α_1 and α_2 are the roots of $1 - \zeta L + L^2$, and α_3 and α_4 are the roots of $1 + (1/\zeta)L + L^2$. λ_2 and λ_3 are complex conjugates and we now define $\mu_2/2 = R(\lambda_2)$ and $\mu_3/2 = I(\lambda_2)$; similarly we define μ_4 and μ_5 as twice the real and imaginary parts of the conjugate pair λ_4, λ_5. This allows us to write (1) as

$$\psi(L) = \lambda_1 L\psi_1(L) - L[(\mu_3\gamma_2 + \mu_2\gamma_1) + \mu_2 L]L\psi_2(L) - L[(\mu_5\gamma_4 + \mu_4\gamma_3) + \mu_5 L]L\psi_3(L) + \psi^*(L)\psi_4(L) \quad (2)$$

where γ_i for $i = 1, 2, 3, 4$, is defined from $1/\alpha_1 = \alpha_2 = \gamma_1 + i\gamma_2$ and $1/\alpha_3 = \alpha_4 = \gamma_3 + i\gamma_4$. Thus $\gamma_1 = R(\alpha_1)$, $\gamma_2 = -I(\alpha_1)$, $\gamma_3 = R(\alpha_3)$ and $\gamma_4 = -I(\alpha_3)$. Finally, we define $\pi_1 = -\lambda_1$, $\pi_2 = \mu_3\gamma_2 + \mu_2\gamma_1$, $\pi_3 = \mu_2$, $\pi_4 = \mu_5\gamma_4 + \mu_4\gamma_3$ and $\pi_5 = \mu_5$, which in turn allows us to rewrite (2) as

$$\psi(L) = -\pi_1 L\psi_1(L) + (\pi_2 + \pi_3 L)L\psi_2(L) + (\pi_4 + \pi_5 L)L\psi_3(L) + \psi^*(L)\psi_4(L) \quad (3)$$

Expression (3) is comparable to expression (3.4) in HEGY (p. 223) for the quarterly case. We can now derive the testing procedure for the case of daily data. We assume that the data-generating process (DGP) is an autoregressive process given by

$$\psi(L)y_t = \varepsilon_t \quad (4)$$

where ε_t is n.i.d.(0,1). Substituting (4) into (3),

$$\psi^*(L)y_{4t} = \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{2,t-2} + \pi_4 y_{3,t-1} + \pi_5 y_{3,t-2} + \varepsilon_t \quad (5)$$

where $\psi^*(L)$ is some polynomial function of L with all the roots outside the unit circle, and

$$y_{it} = \psi_i(L)y_t \qquad i = 1, 4$$
$$= -\psi_i(L)y_t \qquad i = 2, 3$$

TABLE 1
CRITICAL VALUES OF THE t RATIOS FOR THE NULL HYPOTHESIS THAT π_1 IS ZERO

Auxiliary regressors	*0.01*	*0.025*	*0.05*	*0.10*
		$\pi_1 = 0$		
I, SD, T	−3.96	−3.66	−3.40	−3.12
I, SD	−3.42	−3.11	−2.85	−2.55

Notes: I is the constant; SD are seasonal dummies; T is the trend; $N = 800$ is the sample size.

with ψ_i given as in (1). Equation (5) can be estimated using ordinary least squares. The augmented version of the test can be obtained in the usual way through the inclusion of additional lags of $y_{4t} = \psi_4(L)y_t = (1 - L^5)y_t$ on the right-hand side of (5).

From Proposition 1 (HEGY, pp. 221–222), testing for a root at a point θ_k, where θ_k is any of the five solutions of $1 - L^5 = 0$, is equivalent to testing for $\lambda_k = 0$ in (1), or equivalent to testing for $\pi_i = 0$ in (5). If we cannot reject the null that $\pi_1 = 0$ using a one-sided t-type test (see below), then we cannot reject that there is a long-run unit root in the data, and the filter $1 - L$ is appropriate to induce stationarity at the zero frequency of the data. For the complex roots corresponding to the seasonal unit roots, the corresponding λ_k, $k = 2$ or $k = 3$, will be zero only if both corresponding π_i for $i = 2, 3$, or $i = 4, 5$, are equal to zero, which suggests the use of a joint test for each pair of πs. The corresponding appropriate filters are given by $\psi_3(L)$ and $\psi_2(L)$ as defined in (1). Finally, if using these tests we cannot reject that $\pi_i = 0$, for all i, then the filter $1 - L^5$ is the appropriate choice.

The alternative for all these tests is stationarity. For $\psi(1) = 0$, the alternative is that $\psi(1) > 0$ implying that $\pi_1 < 0$, and therefore we propose the use of a one-sided t-type test. For the complex roots, the alternative to $|\psi(\theta_k)| = 0$ is $|\psi(\theta_k)| > 0$ for $k = 2$ and 3 or $k = 4$ and 5, which suggests the use of an F-type test for the joint null. Under the alternative stationarity hypothesis in (5), if $\pi_i \neq 0$ for $i = 2, 3, 4, 5$, then this means that no seasonal unit roots are present in the data and that the use of seasonal dummies may be appropriate to model potential stationary seasonal patterns. $\pi_1 \neq 0$ means that no long-run unit root is present in the data. Therefore, if $\pi_i \neq 0$ for all i, the data are stationary.[1]

The critical values reported in Tables 1 and 2 are generated by Monte Carlo simulation using GAUSS with $M = 100{,}000$ replications. They are

[1]On the other hand, if we accept $\pi_1 = 0$ and test $\pi_i = 0$, $i = 2, \ldots, 5$, we are testing for seasonal unit roots in the first difference of the data as well as the levels, e.g. in the returns as well as in the indices.

Table 2
Critical Values for the Null Hypothesis that Sets of π Parameters are Zero

Auxiliary regressors	*0.90*	*0.95*	*0.975*	*0.99*
		$\pi_2 = \pi_3 = 0$		
I, SD, T	5.58	6.57	7.52	8.69
I, SD	5.61	6.64	7.64	8.85
		$\pi_4 = \pi_5 = 0$		
I, SD, T	5.59	6.61	7.62	8.82
I, SD	5.62	6.64	7.64	8.85
		$\pi_2 = \pi_3 = \pi_4 = \pi_5 = 0$		
I, SD, T	4.84	5.51	6.13	6.87
I, SD	4.87	5.54	6.16	6.93

Notes: I is the constant; SD are seasonal dummies; T is the trend; $N = 800$ is the sample size.

subject to Monte Carlo sampling error and further approximate the asymptotic distribution by using a sample of $N = 800$. The DGP used is given by (4), and under the alternative we included either a constant and seasonals or a constant, trend and seasonals. Table 1 gives the critical values for the t-type test for the null that H_0: $\pi_1 = 0$;[2] Table 2 gives the critical values for the F-type tests for the pairs of conjugate complex unit roots, i.e. for H_0: $\pi_i = 0$ for $i = 2, 3$, and H_0: $\pi_i = 0$ for $i = 4, 5$, and for the null hypothesis of no unit root seasonality, i.e. H_0: $\pi_i = 0$ for $i = 2, 3, 4, 5$ (or H_0: $\cap_{i=2}^{5} H_{0i}$). Kendall and Stuart (1961, pp. 371–378) give a method for calculating distribution-free confidence intervals; the 95 per cent intervals here range from (at worst) ± 0.03 for Table 1 through ± 0.04 for the 0.95 column of Table 2 to (8.74, 8.92) around the 8.85 entry in Table 2. With respect to the sample size approximation, previous studies have found $N = 400$ or $N = 800$ sufficient: in fact, independent simulations with $N = 400$ produce results almost invariably lying within the 95 per cent confidence interval for $N = 800$, which suggests that the Monte Carlo sampling variability dominates.

These simulations are conducted with n.i.d. disturbances; however, it is well documented that high frequency financial data exhibit conditional heteroscedasticity. For example, Franses and Paap (1995) find periodically

[2]As HEGY discuss, a two-step procedure can be used, where the first step is given by a t-type double-sided test for $\pi_3 = 0$ ($\pi_5 = 0$) and, if this hypothesis cannot be rejected, the second step is given by a t-type one-sided test for π_2 (π_4) against the alternative that $\pi_2 < 0$ ($\pi_4 < 0$). The drawback of this procedure is that 'potentially [it] could lack power if the first-step assumption is not warranted' (HEGY, p. 224). In Andrade *et al.* (1996) we give the critical values for these t-type tests in Tables 2–6.

integrated generalized autoregressive conditional heteroscedasticity (PAR-PIGARCH) models adequate for similar data. It is thus of interest to know how misleading these critical values might be if applied to data with such errors. Repeating the simulations for IGARCH(1,1) errors with IGARCH parameters $\alpha = 0.85$ and $\beta = 0.15$ gives the sizes and alternative critical values listed in Appendix A for the I, SD and I, SD, T cases. The 95 per cent critical values increase in absolute size, by up to 41 per cent, and the 99 per cent critical values increase by up to 116 per cent. The type I errors corresponding to the standard critical values were similarly affected. At worst, a nominal 5 per cent test can have size 13.7 per cent, and a nominal 1 per cent test a size of 7.1 per cent. How serious a distortion one considers this to be is a matter of judgement.

2.2 The Franses and Romijn Procedure

An alternative to the HEGY approach uses the fact that under the null there exist five independent random walks for daily data. For quarterly data, Franses (1991b), Franses and Romijn (1993) and Franses (1994) define a vector of quarters (VQ) and test whether the four series are cointegrated. For financial daily data, we can define a days vector (DV) as $x_T = [x_{1T}x_{2T}x_{3T}x_{4T}x_{5T}]'$, where the prime denotes transpose, and x_{1T} corresponds to the series of observations of stock prices on Mondays, x_{2T} to Tuesdays, x_{3T} to Wednesdays, x_{4T} to Thursdays and x_{5T} to Fridays. The unrestricted vector autoregression (VAR) model used in the Johansen procedure (see Johansen and Juselius, 1990, for example) can be written for DV as

$$\Delta x_T = \Gamma_1 \Delta x_{T-1} + \ldots + \Gamma_{j-1} \Delta x_{T-j+1} + \Pi x_{T-j} + \varepsilon_T \tag{6}$$

The rank of $\Pi = v$ gives the dimension of the cointegrating space and can be tested using Johansen's maximal eigenvalue and trace test statistics. If there are long-run and seasonal unit roots in y_t and $1 - L^5$ is the appropriate filter to transform the data, then the series of days are not cointegrated and $v = 0$, i.e. the five random walks are independent. However, if all the series are stationary, then $v = 5$. If $0 < v < 5$, then the series are cointegrated and Π can be factorized as $\Pi = \alpha\beta'$. The columns of β give the coefficients of the cointegrating relationships amongst the series, i.e. the linear combinations $\beta' x_T$ which are stationary.

An important case is when $v = 4$ and we can write that $\beta' \boldsymbol{i} = \mathbf{0}$, where $\boldsymbol{i}$ is a 5×1 vector of ones and $\mathbf{0}$ is the null vector. We can therefore re-standardize (change the base of the cointegration space) the cointegrating vectors in terms of one variable, say Monday, x_{1T}. We can partition β' as $\beta' = [\boldsymbol{b} \mid B]$, where β' is 4×5, $\boldsymbol{b}$ is a column vector and B is a non-singular square matrix of order 4; then, re-standardizing $B^{-1}\beta' = [B^{-1}\boldsymbol{b} \mid I]$. If

$\beta' \boldsymbol{i} = \mathbf{0}$, $B^{-1}\beta' \boldsymbol{i} = \mathbf{0} \Rightarrow B^{-1}\boldsymbol{b} = -\boldsymbol{i}$, which allows us to write β' as $\beta' = [-\boldsymbol{i} \mid I]$:

$$\beta' = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

An alternative would be to formulate

$$\beta' = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

That is, the stationary relationships are given by $x_{5T} - x_{4T}$, $x_{4T} - x_{3T}$, $x_{3T} - x_{2T}$ and $x_{2T} - x_{1T}$. The appropriate filter to stationarize y_t is $1 - L$. There is only a long-run unit root in the data and no seasonal unit roots. The same sort of derivation of relevant restrictions on the coefficients of β can be obtained for the cases when $v = 1, 2, 3$, where one may also have a periodically varying difference filter. For this testing procedure, the available critical values from Johansen are still appropriate. We investigate the sensitivity of these tests to conditional heteroscedasticity and report these results in Appendix B. We find that the critical values appropriate for this particular IGARCH process are between 34 and 61 per cent larger than the corresponding normal values. They are, of course, only indicative of sensitivity to other values of the IGARCH parameters.

2.3 Stationary Stochastic Seasonality

We can test for the existence of stationary stochastic seasonality by employing the 'basic structural model' due to Harvey (1993, Section 5.6) which, he suggests, can be fitted with stationary seasonality. This model can be adapted for daily data and augmented by fixed seasonal effects. We can define the augmented basic structural model as

$$y_t = \mu + \delta_t + \gamma_t + \varepsilon_t \tag{7}$$

which decomposes the observations (on differenced natural logarithms in our case) y_t into an overall mean μ, a fixed seasonal effect δ_t, with $\delta_t = \delta_{t-5}$ and $\sum_{i=0}^{4} \delta_{t-i} = 0$, and a random seasonal component γ_t, with $\sum_{i=0}^{4} \gamma_{t-i} = u_t$, u_t i.i.d.$(0, \sigma_u^2)$, independent of the irregular component ε_t, which is i.i.d.$(0, \sigma_\varepsilon^2)$. This model (7) can be written in state space form as follows:

$$\begin{aligned} y_t &= Z_t \alpha_t + \varepsilon_t \qquad t = 1, 2, \ldots, n \\ \alpha_t &= T\alpha_{t-1} + \eta_t \end{aligned} \tag{8}$$

where $\alpha_t' = [\mu\, \delta_1\, \delta_2\, \delta_3\, \delta_4\, \gamma_t\, \gamma_{t-1}\, \gamma_{t-2}\, \gamma_{t-3}]$ and

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{I}_5 & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{T}^* \end{bmatrix}$$

with

$$\boldsymbol{T}^* = \begin{bmatrix} -1 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$\boldsymbol{I}_p$ is a unit matrix of order p, $\boldsymbol{O}$ is a null matrix of appropriate dimension, $\eta_t' = [0\,0\,0\,0\,0\,u_t\,0\,0\,0\,0]$, and $\boldsymbol{Z}$, i.e. $[Z_1\, Z_2 \ldots Z_n]'$, is a repetition of its first five rows, which are

$$\left[\boldsymbol{i}, \begin{bmatrix} -\boldsymbol{i}' \\ \boldsymbol{I}_4 \end{bmatrix}, \boldsymbol{i}, \boldsymbol{O}\right]$$

the null matrix being 5 by 3 and the $\boldsymbol{i}$s columns of 1s of appropriate lengths. The prime denotes transpose. This model has a 'random walk' seasonal component as we will now show: if we define $\Gamma_t' = [\gamma_t\, \gamma_{t-1}\, \gamma_{t-2}\, \gamma_{t-3}]$ as a sub-vector of α_t, from (8) we can write

$$\Gamma_t = \boldsymbol{T}^* \Gamma_{t-1} + [u_t\, 0\, 0\, 0]' \tag{9}$$

As $\gamma_t = -\gamma_{t-1} - \gamma_{t-2} - \gamma_{t-3} - \gamma_{t-4} + u_t$, the five daily deviations γ_t, viewed as separate series, are cointegrated with the dimension of the cointegrating space equal to one, and thus we have four seasonal unit roots. If, instead, we want to consider a stationary random seasonal component in the model, we introduce a damping factor ρ, with $|\rho| < 1$ (Harvey, 1993, Section 6.5), replacing $\boldsymbol{T}^*$ by $\rho\boldsymbol{T}^*$ in (9) and therefore in model (8). Furthermore, either the fixed (δ_t) or the random (γ_t) seasonal components can be written in trigonometric form. For the random component, if we define $\lambda_j = 2\pi j/5$, $c_j = \cos(\lambda_j)$ and $s_j = \sin(\lambda_j)$, then

$$\boldsymbol{T}^* = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ 0 & c_2 & 0 & s_2 \\ -s_1 & 0 & c_1 & 0 \\ 0 & -s_2 & 0 & c_2 \end{bmatrix}$$

replacing the original $\boldsymbol{T}^*$ matrix in model (8). The last four columns of the $\boldsymbol{Z}$ matrix are now all columns of 1s. Again, this version is rendered stationary by the inclusion of a damping factor ρ. The model thus contains a fixed seasonal term δ_t plus a damped random component, which contributes a constant proportion of the variation in y_t.

Table 3
Daily Means, Deleting Ten Observations

	Mean × 10^3	$\lvert t\rvert$		*Deviation* × 10^3	$\lvert$*Robust t*$\rvert$[a]
UK					
Monday	−0.574	1.7	(Mean)	0.578	4.2
Tuesday	0.724	2.4	Monday	−1.152	3.8
Wednesday	1.075	3.6	Tuesday	0.146	0.5
Thursday	0.664	2.2	Wednesday	0.496	1.9
Friday	1.003	3.3	Friday	0.424	1.6
USA					
Monday	0.097	0.3	(Mean)	0.426	1.1
Tuesday	0.660	1.9	Monday	−0.328	1.0
Wednesday	0.973	2.9	Tuesday	0.235	0.8
Thursday	0.221	0.6	Wednesday	0.547	1.7
Friday	0.176	0.5	Friday	−0.249	0.7

Notes: For the UK, SD of dependent variable = 0.008220, 'regression' standard error 0.008199, $R^2 = 0.00503$, $F(4, 3636)$ for deviations = 4.60 (p value 0.001). For the USA, SD of dependent variable = 0.009169, 'regression' standard error 0.009162, $R^2 = 0.00136$, $F(4, 3601)$ for deviations = 1.23 (p value 0.298).

[a] Robust t calculated using White's (1980) standard errors.

3 Results

We now employ the methodologies outlined in Section 2 using 3775 daily observations on the UK's Financial Times All Shares Stock Index and on the USA's S&P 500 Index: from Monday, 31 December 1979, to Friday, 17 June 1994. We used the data as provided by Datastream. For public holidays we were obliged to use the previous trading day's value. We begin by examining the fixed seasonal effects in the data. One difficulty is the number of outliers. Removing even considerable numbers of outliers still leaves the data very erratic, and *ad hoc* removal becomes difficult to justify. In the results presented below (where necessary) we omit the ten observations between 19 October 1987 and 30 October 1987 to exclude 'Black Monday' and its immediate aftermath.[3]

Table 3 shows the daily effects in the natural logarithm of the UK index, detrended by taking first differences. The individual daily effects are all significant, and small. One can extract the slight overall drift in the data as a mean of 0.000578 with significant deviations for Monday (negative) and, marginally, for Wednesday (positive). The White (1980) heteroscedastic robust standard errors are provided for the deviations, but the adjustment makes very little difference. The 'errors' contain serial correlation and heteroscedasticity, and exhibit non-normality. These effects can be reduced, but not eliminated, by deleting outliers. The

[3]The choice of ten observations to omit was made as a matter of judgement after inspecting the data and the results for Tables 3 and 4.

TABLE 4
MODEL WITH ARCH ERRORS

	UK		*USA*	
	Coefficient	*|Robust t|*[a]	*Coefficient*	*|Robust t|*[a]
Mean	0.00049	4.0	0.00045	3.1
Monday	−0.00108	4.1	−0.00047	1.6
Tuesday	0.00027	1.1	0.00026	0.9
Wednesday	0.00039	1.6	0.00061	2.2
Friday	0.00045	1.8	−0.00038	1.2
y_{t-1}	0.139	7.7	n/a	–
y_{t-10}	0.02986	1.8	n/a	–
γ_0	0.00004	14.7	0.00005	11.4
γ_1	0.09734	3.0	0.05312	2.5
γ_2	0.1037	4.2	0.04801	2.0
γ_3	0.07401	3.0	0.07773	3.4
γ_4	0.1214	4.7	0.1130	3.0
γ_5	0.07007	3.0	0.09940	4.2
Log-likelihood	12469.93	–	11911.54	–
BP[b]	4.514		4.339	
	[0.4780]		[0.5017]	
BG[c]	0.972		0.9673	
	[0.4335]		[0.4365]	
ARCH(5)[d]	0.4446		0.1413	
	[0.8174]		[0.9826]	

Notes: [a] Robust t estimated using analytic formula; the robustness is to non-normality in the quasi-maximum likelihood estimates. See Gourieroux *et al.* (1984).
[b] BP, Box–Pierce test statistic for white noise residuals calculated using five lags.
[c] BG, Breusch–Godfrey test statistic (F) for white noise residuals using five lags.
[d] ARCH(5), F version of the Engle (1982) test.

significance of the deviations is robust to this process. In this static 'model' the main problem with such errors is a loss of efficiency. Here, given the large number of observations, the daily deviations are detectable, and jointly significant at 1 per cent. We re-estimated the fixed seasonal effects with ARCH(5) errors, but this had almost no effect on the associated t values (see Table 4). The ARCH coefficients, although significant, are small, ranging from 0.0622 to 0.1150. The pattern is similar in the USA (see Table 3) with returns rising from Monday to Wednesday and then falling, except for Friday, which continues the downward trend in the USA but not in the UK. None of the deviations is significant at the 5 per cent level, although Wednesday is at the 10 per cent level. However, if one fits an ARCH(5) error process (see Table 4), there are some changes in the coefficients—Wednesday becomes significant at 5 per cent, Monday marginal at 10 per cent and the deviations are jointly significant at 10 per cent using a likelihood ratio test. There is no evidence of serial correlation in the ARCH residuals. Thus in our sample there is reasonable evidence of daily effects in the UK stock market, and weaker evidence for the US market.

TABLE 5
3775 DAILY OBSERVATIONS FROM 31 DECEMBER 1979 TO 17 JUNE 1994 ON THE UK AND US STOCK MARKET INDICES

	UK				*USA*			
	Coefficient	$\|t\|$	*Coefficient*	$\|t\|$	*Coefficient*	$\|t\|$	*Coefficient*	$\|t\|$
I	0.016	2.4	0.003	1.9	0.029	3.1	0.002	1.2
D_2	0.001	3.4	0.002	3.4	0.001	2.1	0.001	2.1
D_3	0.002	4.4	0.002	4.4	0.001	2.7	0.001	2.7
D_4	0.001	3.0	0.001	3.0	0.001	1.0	0.001	1.0
D_5	0.002	4.1	0.002	4.1	0.001	1.0	0.001	1.0
T	0.000	2.0	–	–	0.000	2.9	–	–
π_1	−0.001	2.4	−0.000	1.7	−0.001	3.0	−0.000	1.0
π_2	0.295	26.7	0.296	26.8	0.286	26.1	0.287	26.2
π_3	−0.243	22.0	−0.242	22.0	−0.247	22.6	−0.245	22.4
π_4	0.83	51.0	0.830	51.0	0.748	47.4	0.748	47.3
π_5	−0.81	49.7	−0.809	50.0	−0.718	45.5	−0.718	45.4
F tests								
π_2–π_3		1199.8		1198.4		1203.8		1201.4
π_4–π_5		1522.6		1521.0		1285.3		1282.5
π_2–π_5		5305.1		5300.1		3735.0		3727.0
BP[a]		0.015		0.025		7.185		6.371
		[0.999]		[0.999]		[0.207]		[0.272]
BG[b]		0.406		0.356		1.750		1.571
		[0.845]		[0.878]		[0.120]		[0.165]

Notes: The auxiliary regression includes I, SD and T, or I and SD. No augmentation with additional lags was required to whiten the residuals.
[a] BP, Box–Pierce test statistic for white noise residuals calculated using five lags and approximately distributed as $\chi^2(5)$.
[b] BG, Breusch–Godfrey test statistic for white noise residuals using five lags and distributed as $F(5, 3754)$ or as $F(5, 3755)$.

3.1 Testing for Seasonal Unit Roots

In Table 5 we present the results[4] for the HEGY-type test procedure (regression (5)) using two combinations of deterministic components—a constant with seasonal dummies, and a constant with seasonal dummies and a trend. No augmentation using additional lags was required to whiten the residuals. However, there are problems of non-normality, which should not be surprising in financial data (see above). We included dummy variables for certain observations, in particular the period following the 1987 crash, which were sufficient to allow us to accept the hypothesis that the residuals were normally distributed. Extensive experiments were made excluding up to 50 outliers, detected as large residuals (or, for ARCH,

[4]In Andrade *et al.* (1996) we report the results of estimating (5) with all five combinations of deterministic components. The conclusions of the tests for seasonal unit roots are unchanged.

standardized residuals) in the relationship being fitted. The conclusions from the test did not change, and for brevity we report the results for the full data set. We can see from Table 5 and from the appropriate critical values given in Tables 1 and 2 that we cannot reject the hypothesis of a long-run unit root in either the UK or the US stock markets. However, we do reject the hypothesis that unit root seasonality at all other frequencies is a feature of these markets, at the 1 per cent and 5 per cent levels of significance. The main implication of these results is that the filter $1 - L$ is adequate to achieve stationarity in these stock market indices.

3.2 *The Franses and Romijn Procedure*

We now investigate the implication from the HEGY procedure that there exist five seasonal unit roots in daily financial time series and test for cointegration amongst day-of-the-week series. To this end we divide the 3775 daily observations on the two stock market indices into five series each formed by the observations corresponding to a day of the week, x_{iT}, $i = 1$ (Monday), 2 (Tuesday), 3 (Wednesday), 4 (Thursday) and 5 (Friday). We thus define a DV (days vector) and use Johansen's test procedure to estimate (6) and test for cointegration amongst these series. We paid particular attention to determining the lag order of the VAR so that there was no residual serial correlation, and concluded that a VAR(3) for the UK market and a VAR(2) for the US market were appropriate.[5] Using both of Johansen's test statistics, we find that the dimension of the cointegrating space in both cases is $v = 4$ (see Table 6) and therefore we can use the method derived in Section 2 to re-standardize β'. (This inference is unaffected if one uses the critical values for the particular IGARCH process in Appendix B.) We obtain $\beta' = [-\boldsymbol{i} \mid I]$ to four figures accuracy for the UK data and three figures for the US data, and so $x_{2T} - x_{1T}$, $x_{3T} - x_{1T}$, $x_{4T} - x_{1T}$ and $x_{5T} - x_{1T}$ are stationary. These stationary relationships can be written as $x_{5T} - x_{4T}$, $x_{4T} - x_{3T}$, $x_{3T} - x_{2T}$ and $x_{2T} - x_{1T}$ which shows immediately that the appropriate filter to induce stationarity in the original series y_t is $1 - L$ and that no seasonal unit roots are present. These results reinforce the ones obtained using our alternative methodology above. Franses (1994) shows that one can use the χ^2 test to test for the $(1, -1)$ cointegrating relations. PcFiml provides $\chi^2(4)$ p values of 0.59 (UK) and 0.12 (US) and acceptance of the null.

3.3 *Stationary Seasonality*

Having rejected the existence of non-stationary seasonal unit roots in our

[5]Detailed results are reported in Andrade *et al.* (1996). The residuals are non-normal, but the inclusion of event-specific dummies did not change our conclusions.

TABLE 6
FRANSES AND ROMIJN'S PROCEDURE

$H_0(v)$	UK Eigen-value $\hat{\lambda}_i$	UK $\lambda_{\max}$ (1)	UK λ_{trace} (2)	USA Eigen-value $\hat{\lambda}_i$	USA $\lambda_{\max}$ (1)	USA λ_{trace} (2)	Osterwald–Lenum 95% critical value (1)	Osterwald–Lenum 95% critical value (2)
$v \leq 4$	0.003	1.93	1.93	0.001	0.77	0.77	8.18	8.18
$v \leq 3$	0.175	144.73	146.66	0.268	235.32	236.09	14.90	17.95
$v \leq 2$	0.246	212.34	359.00	0.313	282.38	518.47	21.07	31.52
$v \leq 1$	0.256	221.90	580.90	0.385	366.17	884.65	27.14	48.28
$v = 0$	0.289	257.01	837.90	0.441	437.53	1322.18	33.32	70.60

Notes: λ_{max} and λ_{trace} are Johansen's test statistics $-T\ln(1-\hat{\lambda}_i)$ and $-T\sum_i \ln(1-\hat{\lambda}_i)$, respectively. 3775 daily observations become 755 observations for each of the five series corresponding to each day of the week. UK estimation based on a VAR(3) with constant and US estimation based on a VAR(2) with constant.

data we now test for the presence of stationary stochastic seasonality using Harvey's (1993) basic structural model which we adapt for daily data and augment with fixed seasonal effects (see expressions (7) and (8) above) and estimate using the Kalman filter.

TABLE 7
ESTIMATES OF THE STATIONARY RANDOM SEASONAL MODEL

	UK Coefficient $\times 10^3$	UK |t ratio|	USA Coefficient $\times 10^3$	USA |t ratio|
μ	0.5781	14.8	0.4279	3.6
δ_1	0.1002	2.1	−0.1856	0.9
δ_2	0.4296	5.8	−0.2615	2.9
δ_3	−1.1720	14.8	−0.3321	3.7
δ_4	0.1390	2.8	0.2288	0.5
σ_ε	3.5930	(0.2038)[a]	4.0740	(0.2176)[a]
σ_u	7.2660	(0.1142)[a]	8.1940	(0.09877)[a]
ρ	−0.1657	169.7	−0.05279	53.9
Log-likelihood	12356.69	–	11808.50	–
BP[b]		3.477		4.944
		[0.6269]		[0.4228]
BG[c]		0.6867		0.9687
		[0.6335]		[0.4354]
ARCH(5)[d]		191.10		116.6
		[0.000]		[0.000]

Notes: [a] Standard errors (SEs). All SEs are calculated using numerical second derivatives post estimation.
[b] BP, Box–Pierce test statistic for white noise residuals calculated using five lags.
[c] BG, Breusch–Godfrey test statistic (F) for white noise residuals using five lags.
[d] ARCH(5), F version of the Engle (1982) test.
p values in square brackets.

TABLE 8
AUTOCORRELATION FUNCTION FOR THE RANDOM SEASONAL COMPONENT IN TABLE 7

	Lag				
ρ	*1*	*2*	*3*	*4*	*5*
−0.1	0.097	−0.0	0.0	−0.0	−0.0
0.1	−0.097	−0.0	−0.0	−0.0	0.0
0.2	−0.179	−0.001	−0.0	−0.0	0.0
0.3	−0.239	−0.006	−0.002	−0.001	0.002
0.4	−0.278	−0.015	−0.006	−0.003	0.010
0.5	−0.300	−0.030	−0.016	−0.009	0.031
0.6	−0.310	−0.052	−0.033	−0.024	0.078
0.7	−0.309	−0.080	−0.062	−0.052	0.168
0.8	−0.298	−0.118	−0.105	−0.098	0.328
0.9	−0.278	−0.172	−0.167	−0.164	0.590

Note: The sign asymmetry is given by the first two rows. The autocorrelation for lag 5, ρ_5, is equal to ρ^5 and $\rho_{k+5} = \rho^5 \rho_k$.

Preliminary investigation of the stationary random seasonal models is encouraging, in that the seasonal deviations are significant: see Tables 7 and 9 for both UK and US data. The variance of the random seasonal component is larger than that of the irregular component and highly significant. The damping parameter on the random seasonal component is small but significant, ranging from −0.25 to −0.05 depending on the model

TABLE 9
ESTIMATES OF THE TRIGONOMETRIC FORM OF THE STATIONARY RANDOM SEASONAL MODEL

	UK		*USA*	
	Coefficient × 10^3	$\lvert$*t ratio*$\rvert$	*Coefficient* × 10^3	$\lvert$*t ratio*$\rvert$
μ	0.5780	27.5	0.4252	13.3
δ_1	0.08831	8.4	−0.2018	3.8
δ_2	0.4326	5.6	−0.2516	4.8
δ_3	−1.170	17.4	−0.3331	4.8
δ_4	0.1486	8.0	0.2397	4.4
σ_ε	−0.005632	$(4.97 \times 10^{-5})^a$	4.070	$(0.3095)^a$
σ_u	8.137	$(0.008972)^a$	8.197	$(0.1374)^a$
ρ	−0.2501	203.3	−0.1167	4.9
Log-likelihood	12350.96	–	11808.28	–
BP[b]		15.79		652620
		[0.0075]		[0.23450]
BG[c]		2.943		1.096
		[0.0118]		[0.3605]
ARCH(5)[d]		192.1		116.2
		[0.000]		[0.000]

Notes: As for Table 7.

TABLE 10
AUTOCORRELATION FUNCTION FOR THE RANDOM SEASONAL COMPONENT IN TABLE 9

	Lag				
ρ	*1*	*2*	*3*	*4*	*5*
−0.1	0.025	−0.003	0.0	−0.0	−0.0
0.1	−0.025	−0.003	−0.0	−0.0	0.0
0.2	−0.050	−0.010	−0.002	−0.0	0.0
0.3	−0.075	−0.023	−0.007	−0.002	0.002
0.4	−0.100	−0.040	−0.016	−0.006	0.010
0.5	−0.125	−0.063	−0.031	−0.016	0.031
0.6	−0.150	−0.090	−0.0354	−0.032	0.078
0.7	−0.175	−0.122	−0.086	−0.060	0.168
0.8	−0.200	−0.160	−0.128	−0.102	0.328
0.9	−0.225	−0.202	−0.182	−0.164	0.590

Note: As for Table 8.

TABLE 11
ESTIMATES OF THE SEASONAL AUTOREGRESSION WITH ARCH ERRORS MODEL

	UK	
	Coefficient	$|t$ *ratio*$|$
Mean	0.0004881	4.0
Monday	−0.0010820	4.1
Tuesday	0.0002654	1.1
Wednesday	0.0003903	1.6
Friday	0.0004475	1.8
y_{t-1}	0.1390	7.7
y_{t-10}	0.02986	1.8
γ_0	0.00003537	14.7
γ_1	0.09734	3.0
γ_2	0.1037	4.2
γ_3	0.07401	3.0
γ_4	0.1214	4.7
γ_5	0.07007	3.0
Log-likelihood	12469.93	–
BP[a]	4.514	
	[0.4780]	
BG[b]	0.9720	
	[0.4335]	
ARCH(5)[c]	0.4446	
	[0.8174]	

Notes: t ratios calculated from analytic standard errors.
[a] BP, Box–Pierce test statistic for white noise residuals calculated using five lags.
[b] BG, Breusch–Godfrey test statistic (F) for white noise residuals using five lags.
[c] ARCH(5), F version of the Engle (1982) test.
p values in square brackets in each case.

and data. The autocorrelation functions for the damped model are given in Tables 8 and 10 (trigonometric form) for several different values of ρ. The use of only two parameters to represent the stationary random seasonality does impose a particular shape to this function. Given the small values of ρ observed, and the result that the correlation between days a week apart is ρ^5 in either formulation, and thus less than 0.002 for our estimates, it may be that the 'damped random seasonal component' is aliasing for serial correlation in the residuals. We found significant stationary seasonal effects, but with a small estimated damping parameter and with innovations that exhibit significant non-normality and ARCH effects. Furthermore, in terms of the likelihood, the stationary random seasonal model fits the data less well than a simple fixed seasonality model with ARCH errors, as described in Section 3.1 above for the US data and in Table 11 for the UK data.

4 Conclusions

Financial economists frequently use daily data in their research and in so doing have identified deterministic seasonal patterns in daily stock and bond market returns. However, since empirical results can be misleading in the presence of stochastic seasonality which has not been identified (see Beaulieu and Miron, 1993), in this paper we developed tests which will allow researchers in finance to check for seasonal unit roots in daily data. We began by extending the HEGY procedure to the case of daily data and calculated the appropriate critical values. We then outlined the way in which the Franses and Romijn procedure can be used to detect seasonal unit roots in daily financial market data. Our Monte Carlo experiments, reported in Appendices A and B, indicate that the HEGY procedure is robust to ARCH type errors, while the Franses and Romijn procedure is less so. We suggest that it is important to test for seasonal unit roots both for modelling and for forecasting: inappropriate seasonal differencing imposes seasonal unit roots and gives rise to a noticeably different forecasting model. Using our own tests and the Franses and Romijn procedure adapted for daily financial market data, we tested for seasonal unit roots in the daily prices of the UK's Financial Times All Shares Index and the USA's S&P 500 Index. Using both procedures, we found that stochastic seasonality was not a feature of these indices over our sample, but that there was one long-run unit root in both markets. We then tested for the presence of stationary stochastic seasonality using a stationary random seasonal version of Harvey's (1993) basic structural model. We found significant stationary seasonal effects, but with a small estimated damping parameter and with innovations that exhibit significant non-normality and ARCH effects. Furthermore, in terms of the likelihood, the stationary random seasonal model fitted the data less well than a simple model with fixed seasonal effects and ARCH errors. The main statistical conclusion from our results is that the filter $1 - L$ was sufficient to induce stationarity in the stock market indices over this period.

Appendix A:

The Effect of Conditionally Heteroscedastic Disturbances

Tables 1 and 2 were obtained using independent and identically distributed standard normal deviates. Given the nature of the data, it is relevant to enquire how robust they are to heteroscedasticity, and non-normal kurtosis. The calculations were repeated for the I, SD and I, SD, T cases; Tables 1A and 2A give the critical values for $N = 800$ for IGARCH(1,1) errors, with $\alpha = 0.85$ and $\beta = 0.15$.

There are considerable differences when comparison is made with Tables 1 and 2, as might be expected, given that the GARCH process, although stationary, has infinite variance. However, it may be more relevant to enquire how the size of the tests is affected if the normal critical values are used with GARCH errors. The same simulation yielded the estimates in Tables 1B and 2B.

Thus at worst a nominal 5 per cent test could have size 13.7 per cent, and a nominal 1 per cent test a size of 7.1 per cent. We find this result reasonably encouraging.

Table 1A
Critical Values for IGARCH Errors

Auxiliary regressors	*0.01*	*0.025*	*0.05*	*0.10*
I, SD, T	−5.02	−4.34	−3.88	−3.45
I, SD	−4.34	−3.68	−3.25	−2.82

Notes: 100,000 replications giving an accuracy (distribution-free 95 per cent confidence interval) of at worst ±0.06 for Table 1A, ±0.09 for the 0.95 column of Table 2A and (15.80, 16.64) around the 16.18 entry in Table 2A. The GARCH process was run for 50 pre-sample trials.

Table 2A
Critical Values for IGARCH Errors

Auxiliary regressors	H_0	*0.90*	*0.95*	*0.975*	*0.99*
I, SD, T	$\pi_2 = \pi_3 = 0$	6.77	8.68	11.15	16.18
I, SD		6.75	8.66	11.01	15.84
I, SD, T	$\pi_4 = \pi_5 = 0$	6.77	8.68	11.02	15.64
I, SD		6.74	8.62	11.03	15.52
I, SD, T	$\pi_2 = \pi_3 =$	6.13	7.79	10.15	14.86
I, SD	$\pi_4 = \pi_5 = 0$	6.13	7.76	10.07	14.64

Notes: As for Table 1A.

TABLE 1B
TAIL AREA PROBABILITIES (%)

Auxiliary regressors	*0.01*	*0.025*	*0.05*	*0.10*
I, SD, T	4.4	7.1	10.7	16.7
I, SD	3.7	6.3	9.6	15.3

Notes: As for Tables 1A and 2A where applicable, with accuracy between ±0.25 per cent (for the largest entry, 19.5 per cent) and ±0.12 per cent (for the smallest, 3.7 per cent).

TABLE 2B
TAIL AREA PROBABILITIES (%)

Auxiliary regressors	H_0	*0.90*	*0.95*	*0.975*	*0.99*
I, SD, T	$\pi_2 = \pi_3 = 0$	16.4	10.8	7.5	5.0
I, SD		15.8	10.4	7.2	4.6
I, SD, T	$\pi_4 = \pi_5 = 0$	16.2	10.7	7.2	4.8
I, SD		15.9	10.5	7.1	4.7
I, SD, T	$\pi_2 = \pi_3 =$	19.5	13.7	10.0	7.1
I, SD	$\pi_4 = \pi_5 = 0$	18.9	13.4	9.9	7.0

Notes: As for Table 1B.

APPENDIX B: IGARCH ERRORS IN COINTEGRATION TESTING

Table 6 was obtained using independent and identically distributed standard normal deviates. As in Appendix A, it is relevant to enquire how robust they are to heteroscedasticity, and non-normal kurtosis. Johansen critical values were obtained

TABLE 6A
JOHANSEN'S TEST: SIZES, CRITICAL VALUES FOR IGARCH ERRORS

	Sizes (%)		*Critical values*		*Osterwald–Lenum 95% critical value*	
$H_0(v)$	$\lambda_{\max}$ (1)	λ_{trace} (2)	$\lambda_{\max}$ (1)	λ_{trace} (2)	(1)	(2)
$v \leq 4$	9.8	9.8	10.93 (0.1)	10.93 (0.1)	8.18	8.18
$v \leq 3$	14.2	13.8	21.46 (0.2)	24.78 (0.2)	14.90	17.95
$v \leq 2$	19.3	18.1	31.89 (0.3)	43.30 (0.3)	21.07	31.52
$v \leq 1$	23.8	23.5	42.23 (0.4)	66.43 (0.4)	27.14	48.28
$v = 0$	28.4	27.3	53.50 (0.4)	95.21 (0.5)	33.32	70.60

Notes: $\lambda_{\max}$ and λ_{trace} are Johansen's test statistics $-T\ln(1-\hat{\lambda}_i)$ and $-T\sum_i \ln(1-\hat{\lambda}_i)$, respectively. 100,000 replications give a sampling error (95 per cent confidence interval) for the size estimates of ±0.18 per cent to ±0.28 per cent. The critical values have 95 per cent distribution-free confidence intervals plus/minus at most the figure given in parentheses: thus 10.93 ± 0.1. The interval is not exactly symmetric.

by simulation. The calculations were carried out for $N = 755$ for IGARCH(1,1) errors, with $\alpha = 0.85$ and $\beta = 0.15$. $N = 755$ was chosen as the sample size used for Table 6 as simulating for $N = 100$, 200, 400, 800 showed that the critical values, while not significantly different, appear to be increasing slightly from 400 to 800, as if the asymptote has not been reached. This effect is also noticeable for normal errors.

We see that the critical values appropriate for this particular IGARCH process are between 34 and 61 per cent larger than the corresponding normal values; alternatively, inappropriate use of the standard critical values can give a size of between 9.8 and 28.4 per cent. Our empirical conclusions would still stand. Of course, the results for other GARCH and IGARCH processes require further investigation.

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