Rankers

Sorting Algorithms.

Insertion Sort

Take a random sequence of cards in one hand → Take a random card in other

↓

Now one by one transfer cards where they belong & tho its done!

3Blue one Brown

LINEAR ALGEBRA

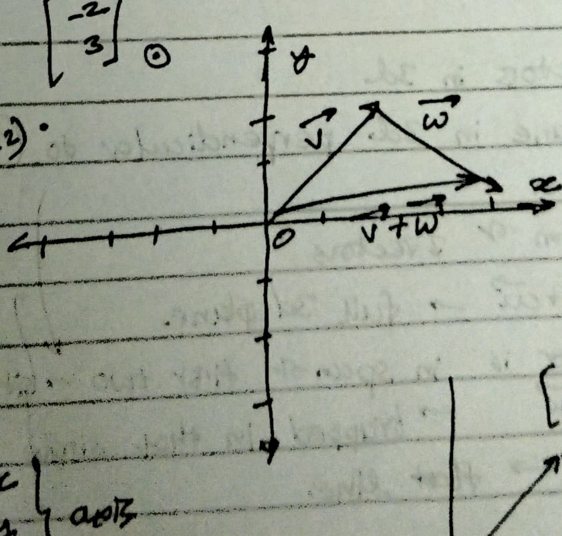
vedio1 Linear Algebra Preview

sine function examples
↳ formula (Numerical) ×
↳ Gemotrical ✓ conceptual

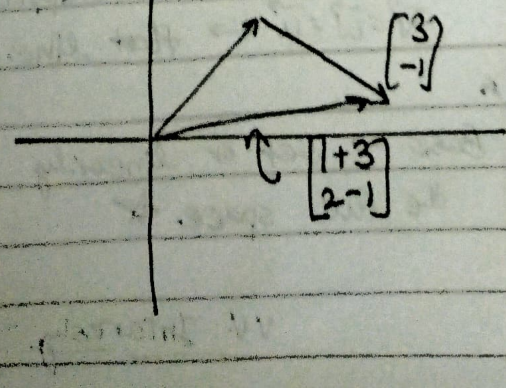
$\boxed{\text{Chapter 1 Vectors}}$

Physics Mathematical Computer Student

Adding / Subtracting → $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$ ⊙ (-4, 2)·



In 3d $\left.\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}\begin{matrix} x \\ y \\ z \end{matrix}\right\}$ axis



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \end{bmatrix}$$

· multiplying a vector → scaling

Boring tha

Video2

$\hat{i}$ & $\hat{j}$ are basic vectors
of xy coordinate system

Linear combination of vectors

$a\vec{v} + b\vec{w}$ → full 2D plane
In case $\vec{v} = \vec{w}$ → single line
· $\vec{v} = \vec{w} = 0$ → origin

} span

Span of $\vec{v}$ & $\vec{w}$ is the set of linear combinations

Think of vectors ~~and~~ as points & becomes easy

Span of two vectors in 3d
↳ a plane in 3d perpendicular to $(\vec{v} \times \vec{w})$

Linear combination of 3 vectors

$a\vec{v} + b\vec{w} + c\vec{u}$ → full 3d plane
If third vector is in span of first two → sitting in this plane
If $\vec{u} = a\vec{v} + b\vec{w}$ → trapped in that single plane
$\vec{v} = \vec{w} = \vec{u}$ → ~~flat~~ line

} SPAN IN 3D

☆ Basis → set of linearly independent vectors that span the full space ☆

VV. Interesting MT sirki yaad agyi

Video-3

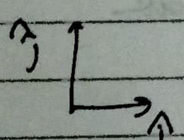
Linear transformation
- Lines remain Lines
- Origin remains fixed

$\vec{v} = -1\hat{i} + 2\hat{j}$ ↷ After transformation

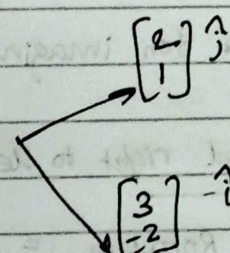
Transformed $\vec{v} = -1$ (Transformed $\hat{i}$) $+ 2$ (Transformed $\hat{j}$)

$\hat{i}$ lands $\quad$ $\hat{j}$ lands

$$\begin{bmatrix} x \\ y \end{bmatrix} = x\begin{bmatrix} 1 \\ -2 \end{bmatrix} + y\begin{bmatrix} 3 \\ 6 \end{bmatrix} - \begin{bmatrix} 1x + 3y \\ -2x + 0y \end{bmatrix}$$



Normal Plane



After Transformation

2x2 Matrix $\quad$ Initial vector

$$\cancel{\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}} \qquad \begin{bmatrix} 5 \\ 7 \end{bmatrix} \qquad \begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$$

$$\cancel{5\begin{bmatrix} 3 \\ -2 \end{bmatrix}} \qquad 5\begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix} + 7\begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$$

where the vector lands

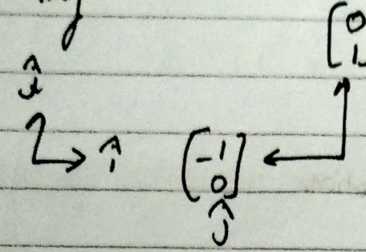
2x2 Matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = x\begin{bmatrix} a \\ c \end{bmatrix} + y\begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

New coordinates of $\hat{i}$ $\qquad$ New coordinates of $\hat{j}$

lets try

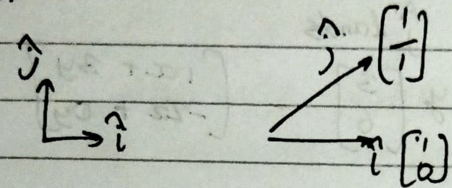


Vimp

Change in $\hat{i}$ — Change in $\hat{j}$

$$\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix} \rightarrow \text{This will rotate every vector } 90^\circ \text{ cw}$$

Sheer



$$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

Bacha hua toh imagine krlo nalayak

video 4

Read right to left $\longleftarrow$

Sheer x Rotation = Composition

$$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix} = \begin{bmatrix}1 & -1\\1 & 0\end{bmatrix}$$

Sheer

$$\underset{M_2}{\begin{bmatrix}0 & 2\\1 & 0\end{bmatrix}}\underset{M_1}{\begin{bmatrix}1 & -2\\1 & 0\end{bmatrix}} = \begin{bmatrix}? & ?\\? & ?\end{bmatrix}$$

for Refer from here

$$\begin{bmatrix}0 & 2\\1 & 0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = 1\begin{bmatrix}0\\1\end{bmatrix} + 1\begin{bmatrix}2\\0\end{bmatrix} = \begin{bmatrix}2\\1\end{bmatrix}$$

$$\begin{bmatrix}0 & 2\\1 & 0\end{bmatrix}\begin{bmatrix}-2\\0\end{bmatrix} = -2\begin{bmatrix}0\\1\end{bmatrix} + 0\begin{bmatrix}2\\0\end{bmatrix} = \begin{bmatrix}0\\-2\end{bmatrix}$$

$$\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}e & f\\g & h\end{bmatrix} = \begin{bmatrix}ae+bg & af+bh\\ce+dg & cf+dh\end{bmatrix}$$

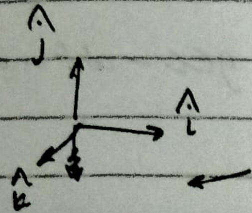
Just Imagine

$$M_1 M_2 \neq M_2 M_1$$

Video 5

3d vectors

Nahi soch sakte, marna nahi hain



$\hat{i} \times \hat{j} = \hat{k}$

$$\underbrace{\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}}_{\text{Transformation}} \overbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}^{\text{Input vector}} = x\begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix} + y\begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} + z\begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$$

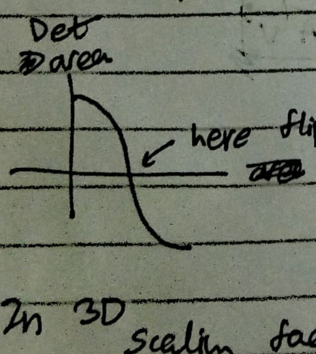
Baki everything same

Vedio 6

In 2D

☆☆ Scaling factor by Area is called $\boxed{\text{determinant}}$

Orientation or Space inverted → Value of determinant -ve



In 3D Scaling factor by volume.

if det 0
↳ columns/ rows linearly dependent

~~if i × j = k~~

{ ~~if i × j~~ if i, j, k   i × j = k
& i × j ≠ k after orientation
val is -ve }

describe scalar multiplication of $\hat{i}$ & $\hat{j}$

$$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \cancel{ab - cd} = ad - bc$$

flipping term

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = aei + bfg + dhc - gec - ahf - dbi$$

$$\det(M_1 M_2) = \det(M_1)\det(M_2)$$

Sedio Video 7

$$2x + 5y + 3z = -3$$
$$4x + 0y + 8z = 0$$
$$1x + 3y + 0z = 2$$

$$\underbrace{\begin{bmatrix} 2 & 5 & 3 \\ 4 & 0 & 8 \\ 1 & 3 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}}_{\vec{v}}$$

$$A\vec{x} = \vec{v}$$

looking for $\vec{x}$ which after transformation lands on $\vec{v}$

$$\cancel{AA^{-1}} = \cancel{AA^{-1}}$$

$$A^{-1}A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Identity (which does nothing actually)

$$\therefore \boxed{\vec{x} = A^{-1}\vec{v}}$$

$\det(A) \neq 0$

A^{-1} exists

output of transformation.

line	Rank 1
Two D plane	Rank 2
3D plane	Rank 3

set of all possible $A\vec{v}$ = "column space" of A

This set of vectors that land on origin is called "null space" or "kernel"

ex 8

$$\begin{bmatrix} 3 & 1 \\ 4 & 1 \\ 5 & 9 \end{bmatrix}$$

Mapping 2D in 3D

$$\begin{bmatrix} 3 & 1 & 4 \\ 1 & 5 & 9 \end{bmatrix}$$

Mapping 3D on 2D plane

Dot Product

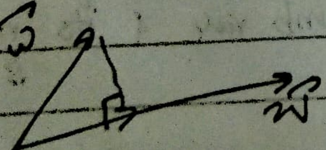
$\vec{v}\,\vec{w}$ Just projection ~~a~~ $\vec{v}$ on $\vec{w} \times \vec{w}$

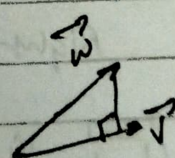
or $\vec{w}$ on $\vec{v} \times \vec{v}$

$\vec{v}\cdot\vec{w} > 0$ Angle $< \cancel{180}\ 90$

$\vec{v}\cdot\vec{w} = 0$ $\angle = 90^\circ$

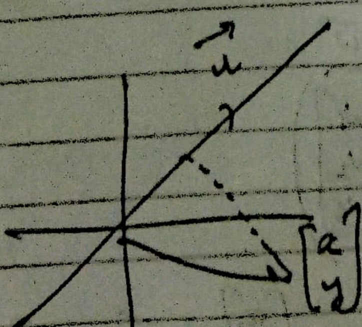
$\vec{v}\cdot\vec{w} < 0$ Angle $> \cancel{180}\ 90$





$\vec{v}.\vec{w}$

$(2\vec{v}).\vec{w} = 2(\vec{v}.\vec{w})$



$$\begin{bmatrix} u_x & u_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = u_x x + u_y y$$

$$\Downarrow$$

$$\begin{bmatrix} \cancel{u_x \ u_y} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \_\_\_$$

Matric vector product $\quad \begin{bmatrix} u_x & u_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = u_x x + u_y y$

$\Updownarrow$

Dot product $\quad \begin{bmatrix} u_x \\ u_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = u_x . x + u_y . y$

~~Vedio~~ video 11

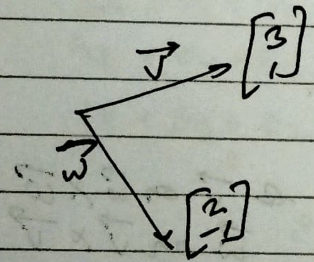
Cross product

$\vec{v} \times \vec{w} \rightarrow$ Area of parallelogram

Right hand thumb rule. $\uparrow$ +ve

$\downarrow$ -ve

How to find?



can be kept as orient as learnt

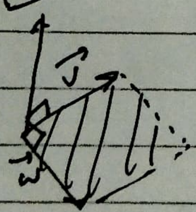
$$\vec{v} \times \vec{w} = \det\left(\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}\right)$$

$\Uparrow$ Shows us area

use orientation changing to find area

this is cross product $\boxed{\vec{w} \times \vec{v}}$

Now



vector

$\llcorner$ p. $\perp^{or}$ to the area defined by Right hand thumb rule & its magnitude is area

$$\begin{matrix} v_1 & w_1 \\ v_2 & w_2 \\ v_3 & w_3 \end{matrix} = \det\left(\begin{bmatrix} \hat{i} & v_1 & w_1 \\ \hat{j} & v_2 & w_2 \\ \hat{k} & v_3 & w_3 \end{bmatrix}\right)$$

Duality cross product

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \det\left(\begin{bmatrix} x & v_1 & w_1 \\ y & v_2 & w_2 \\ z & v_3 & w_3 \end{bmatrix}\right)$$

(Area of $\|^{m}$elogram) (component of $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ p$\perp^{or}$ to $\vec{v}$ & $\vec{w}$)

change of basis

our grid → Jennifer's grid

$$\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

our language ← Jennifer's language

Jennifer's → our grid

$$\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}^{-1}$$

our language → Jennifer's language

How to translate a a matrix

⊥ar translation

$$\underbrace{\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}^{-1}}_{\text{Inverse change of Basis matrix}} \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Multiply \& making it } \perp\text{ar}} \underbrace{\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}}_{\text{conversion to our lang}} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1/3 & -2/3 \\ 5/3 & -1/3 \end{bmatrix}$$

$\vec{v}$

Jennifer lang (vector)

$\boxed{\text{used to rotate } 90^\circ \text{ in Jennifer's language}}$

$\boxed{A^{-1} M A}$

empathy

Video 14

Eigenvectors & Eigenvalues

vector which does not rotate (Axis of rotation)
(move)

Matrix vector multiplication

$$\{ A\vec{v} = \lambda\vec{v} \}$$

scalar vector multiplication

Eigen vector

Eigenvalue

$\vec{v}$ should be non zero

$\downarrow$

$(A - \lambda I)\vec{v} = 0$

$\Downarrow$

Squishification $\det(A - \lambda I) = 0$

$(A - \lambda I)\vec{v} = \vec{0}$

* If matrix $\rightarrow$ diagonal matrix

Basis vectors $\rightarrow$ eigen vectors

entries $\rightarrow$ eigen values

To compute high powers we use it so that eigen vectors become basis vectors (changing basis)

$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ $\quad A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ $\quad A^3 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

Fa

$A^n = \begin{bmatrix} 1 & n-1 \\ n-1 & n \end{bmatrix}$

i) ~~ex~~

Video 15

1) $tr\left(\begin{bmatrix} \boxed{a} & b \\ c & \boxed{d} \end{bmatrix}\right) = a+d = \lambda_1+\lambda_2 = m$ (mean)

eigen values

mean $(a,d) =$ mean (λ_1, λ_2)

2) $\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad-bc = \lambda_1\lambda_2 = p$ (product)

m-d m+d

λ_1 m λ_2

$\begin{bmatrix} 8 & 4 \\ 2 & 6 \end{bmatrix}$ $m=7$

$p=40$

$40 = (7+d)(7-d)$

$40 = 49-d^2$

$d=3$

$d^2 = m^2 - p$

3) $\boxed{\lambda_1, \lambda_2 = m \pm \sqrt{m^2-p}}$

Video 16

2D vector

Kuch nahi tha

Relation b/w functions of linear algebra (well obvious)

vector

↳ many embodiments

↳ can't be defined actually