

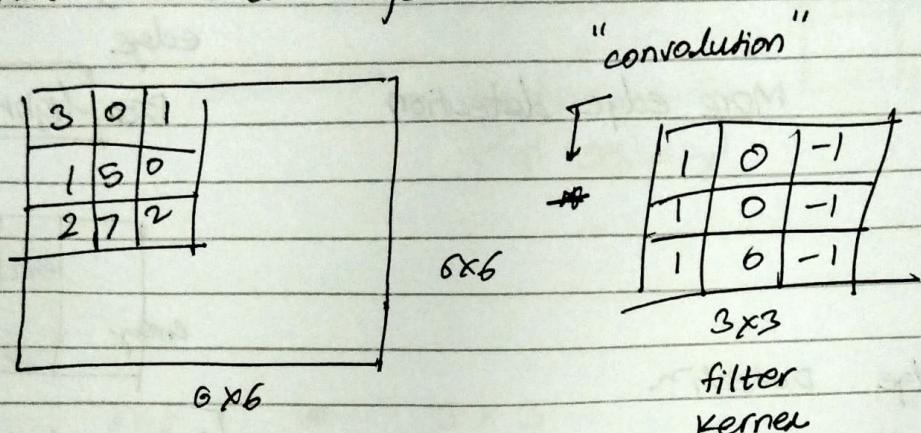
CONVOLUTIONAL NEURAL NETWORKS

Computer Vision Problems

- Image classification
- Object detection
- Neural Style transfer

Can't take huge images as input as it requires very high end computations so let's learn convolution

How to detect edges?



$$= \begin{array}{|c|c|c|c|} \hline & -5 & & \\ \hline 1 & 2 & -4 & 0 & 8 \\ \hline -10 & -2 & 2 & 3 \\ \hline 0 & -2 & -4 & -7 \\ \hline -3 & -2 & -3 & -16 \\ \hline \end{array}$$

for box

Multiply every element in per matrix and then add up these 9 numbers

python: conv-forward

Keras: conv2D

| | | | | | |
|----|----|----|---|---|---|
| 10 | 10 | 10 | 0 | 0 | 0 |
| 1 | | | | | |
| . | | | | | |
| 10 | 10 | 10 | 0 | 0 | 0 |

#

| | | |
|---|---|----|
| 1 | 0 | -1 |
| 1 | 0 | -1 |
| 1 | 0 | -1 |

| | | | |
|---|----|----|---|
| 0 | 30 | 30 | 0 |
| 0 | 30 | 30 | 0 |
| 0 | 30 | 30 | 0 |
| 0 | 30 | 30 | 0 |

white grey

white grey black

white grey black

Detects one vertical edge

More edge detection

Dark light → Dark

| | | |
|-------|--|--|
| Black | | |
| grey | | |
| grey | | |

Horizontal Edge Detection

| | | |
|----|----|----|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| -1 | -1 | -1 |

Dark → light

| | | | | | |
|------|----|----|------|---|---|
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | | | |
| 10 | 10 | 10 | grey | | |
| 10 | 10 | 10 | | | |
| grey | | | | | |

#

| | | |
|----|----|----|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| -1 | -1 | -1 |

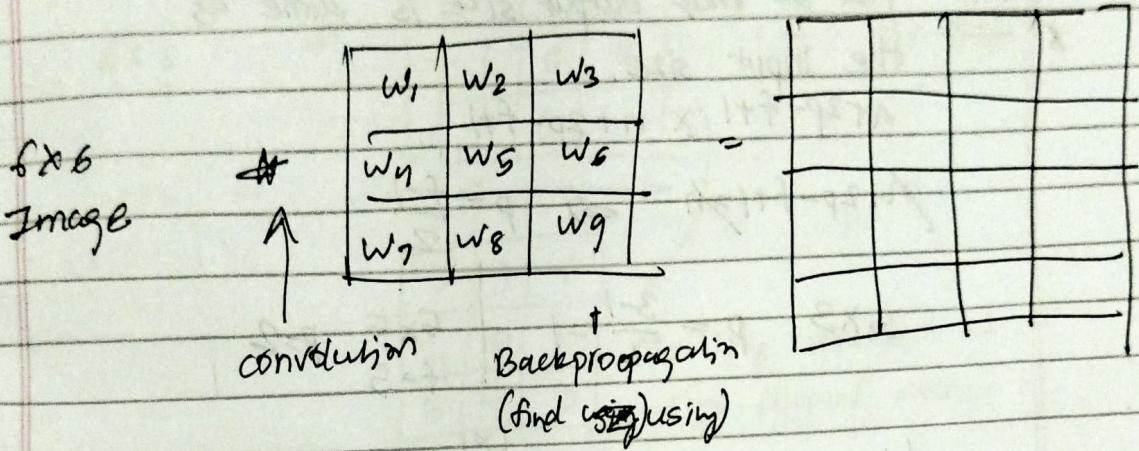
| | | | |
|----|----|-----|-----|
| 0 | 0 | 0 | 0 |
| 30 | 10 | -10 | -30 |
| 30 | 10 | -10 | -30 |
| 0 | 0 | 0 | 0 |

6x6

classmate
Date _____
Page _____

$$\begin{array}{ccc}
 \begin{matrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{matrix} &
 \boxed{\begin{matrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{matrix}} &
 \boxed{\begin{matrix} 3 & 0 & -3 \\ 10 & 0 & -10 \\ 3 & 0 & -3 \end{matrix}}
 \end{array}$$

Sobel filter Scharr filter



Padding

$$\begin{matrix}
 \begin{matrix} 6 \times 6 \\ n \times n \end{matrix} * \begin{matrix} 3 \times 3 \\ f \times f \end{matrix} = \begin{matrix} 4 \times 4 \\ n-f+1 \times n-f+1 \end{matrix}
 \end{matrix}$$

\downarrow
 6×6

→ Shrinking output

→ throwing away info from edges

$p = \text{padding} = 1$

$$n + 2p - f + 1 \Rightarrow n + 2p - f + 1$$

$$6 + 2 \cdot 3 + 1 \times \underline{\quad} = 6 \times 6$$

Valid and same convolutions

"valid": $n \times n * f \times f \rightarrow n-f+1 \times n-f+1$

$6 \times 6 * 3 \times 3 \rightarrow 4 \times 4$

"same": Pad so that output size is same as the input size.

$$n+2p-f+1 \times n+2p-f+1$$

$$n+2p-f+1 = n \Rightarrow p = \frac{f-1}{2}$$

$$3 \times 3 \quad p = \frac{3-1}{2} = 1$$

$$5 \times 5 \quad p=2 \\ f=5$$

f is usually odd

| |
|--------------|
| 1×1 |
| 3×3 |
| 5×5 |
| 7×7 |

Strided convolutions

Taking big stride skipping over one row & one column

$$7 \times 7 * 3 \times 3 = 3 \times 3$$

$n \times n$ $f \times f$
padding p stride s
 $s=2$

$$\left[\frac{n+2p-f}{s} + 1 \right] \times \left[\frac{n+2p-f}{s} + 1 \right]$$

$$\frac{7+0-3}{2} + 1 = \frac{4}{2} + 1 - 3$$

$$\lfloor 2 \rfloor = \text{floor}(2)$$

0x n-imageIgnorepadding = p

6'

cross-correlationvs convolutionwe are using
this from last
few videos

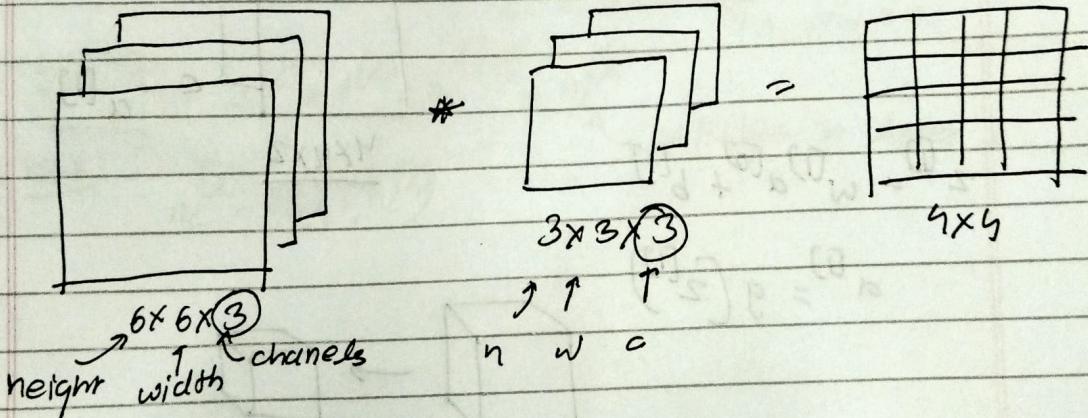
6x6

+

$$\begin{array}{|c|c|c|} \hline 3 & 4 & 5 \\ \hline 1 & 0 & 2 \\ \hline -1 & 9 & 6 \\ \hline \end{array}$$

= 4x4

$$\begin{array}{|c|c|c|} \hline 7 & 9 & -1 \\ \hline 2 & 0 & 1 \\ \hline 5 & 7 & 3 \\ \hline \end{array}$$

Multiply elements of
flipped matrixconvolutions over volume

Just multiply & add in 3d

Multiple filters

6x6x3



Vertical edge

 $3 \times 3 \times 3$ = 4×4 $4 \times 4 \times 2$ 

horizontal edge

 $3 \times 3 \times 3$ = 4×4

Summary

| | | | | |
|-------------------------|---------------|-------------------------|---------------|---------------------------------|
| $n \times n \times n_c$ | $\star \star$ | $f \times f \times n_c$ | \rightarrow | $n-f+1 \times n-f+1 \times n_c$ |
| $6 \times 6 \times 3$ | | $3 \times 3 \times 3$ | \downarrow | $4 \times 4 \times 27$ |
| | | | | # filters used |

\leftarrow One layer of conn

$$w^{(1)}_{a} z^{(1)}$$

$$z^{(1)}$$

$$\text{ReLU}(4 \times 4 + b_1) \rightarrow$$

$$4 \times 4$$

$$6 \times 6 \times 3$$

$$a^{(0)}$$

$$3 \times 3 \times 3 \rightarrow \text{ReLU}(4 \times 4 + b_2)$$

$$4 \times 4$$

$$w^{(1)}$$

$$4 \times 4 \times 2$$

$$4 \times 4$$

$$z^{(1)} = w^{(1)} a^{(0)} + b^{(1)}$$

$$a^{(1)} = g(z^{(1)})$$

$$6 \times 6 \times 3$$

$$a^{(0)}$$

$$4 \times 4 \times 2$$

$$a^{(1)}$$

Q) 10 filters that are $3 \times 3 \times 3$ in one layer of neural network, how many parameters does that layer have?

$$3 \times 3 \times 3 \\ + \text{bias}$$

$$\times 10$$

$$280$$

summary of notation

If layer l is a convolutional layer:

$$f^{(l)} = \text{filter size}$$

$$p^{(l)} = \text{padding}$$

$$s^{(l)} = \text{stride}$$

$$\text{Input: } n_H^{(l-1)} \times n_w^{(l-1)} \times n_c^{(l-1)}$$

$$\text{Output: } n_H^{(l)} \times n_w^{(l)} \times n_c^{(l)}$$

$$n_H^{(l)} = \left\lceil \frac{n_H^{(l-1)} + 2p^{(l)} - f^{(l)}}{s^{(l)}} + 1 \right\rceil$$

$$\text{Each filter: } f^{(l)} \times f^{(l)} \times n_c^{(l-1)}$$

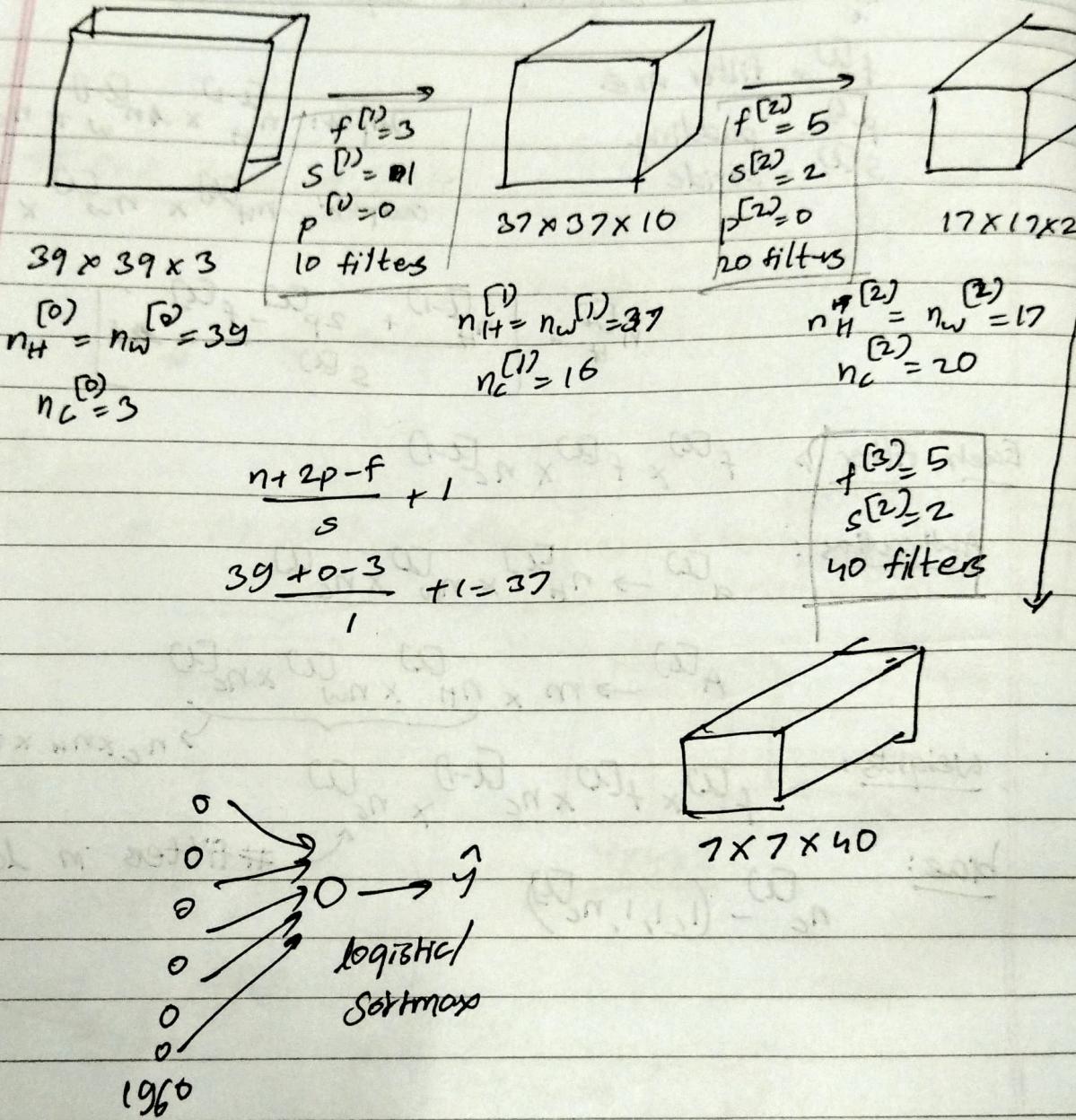
$$\text{Activations: } a^{(l)} \rightarrow n_H^{(l)} \times n_w^{(l)} \times n_c^{(l)}$$

$$A^{(l)} \rightarrow m \times \underbrace{n_H^{(l)} \times n_w^{(l)} \times n_c^{(l)}}_{n_c \times n_H \times n_w}$$

$$\text{Weights: } f^{(l)} \times f^{(l)} \times n_c^{(l-1)} \times n_c^{(l)} \rightarrow n_c \times n_H \times n_w$$

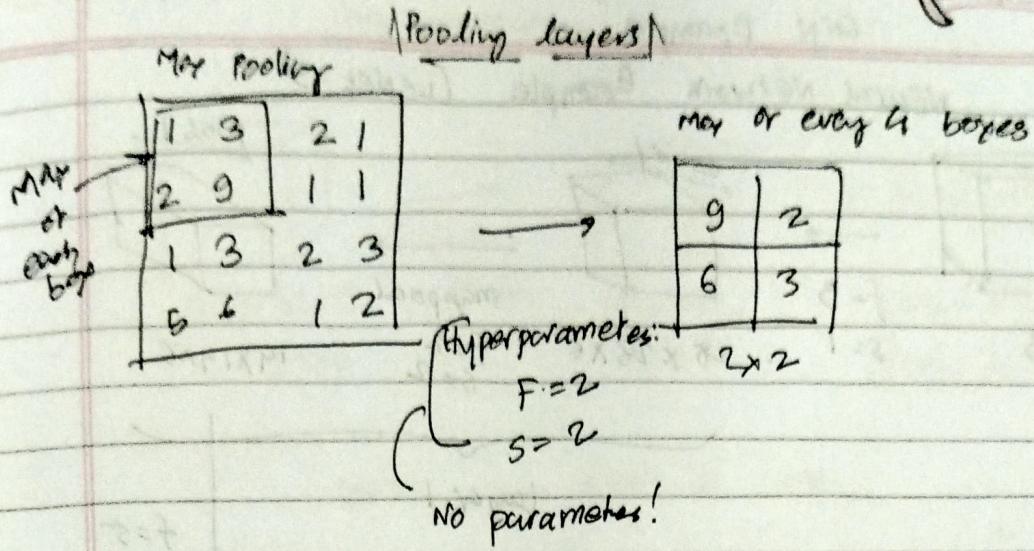
$$\text{bias: } n_c^{(l)} = (1, 1, 1, n_c^{(l)}) \quad \uparrow \# \text{filters in layer } l$$

A Simple convolutional network example



Types of layers in a convolutional network:

- convolutional (CONV)
- Pooling (POOL)
- Fully connected (FC)



$$5 \times 5 \times n_c \longrightarrow 3 \times 3 \times n_c$$

$$f=3 \\ s=1$$

Average Pooling

Instead of max, avg of all inputs is taken
for Average pooling

$$7 \times 7 \times 1000 \rightarrow 1 \times 1 \times 1000$$

To shrink, very rare

Hyperparameters:

f : filter size

s : stride

Max or average pooling

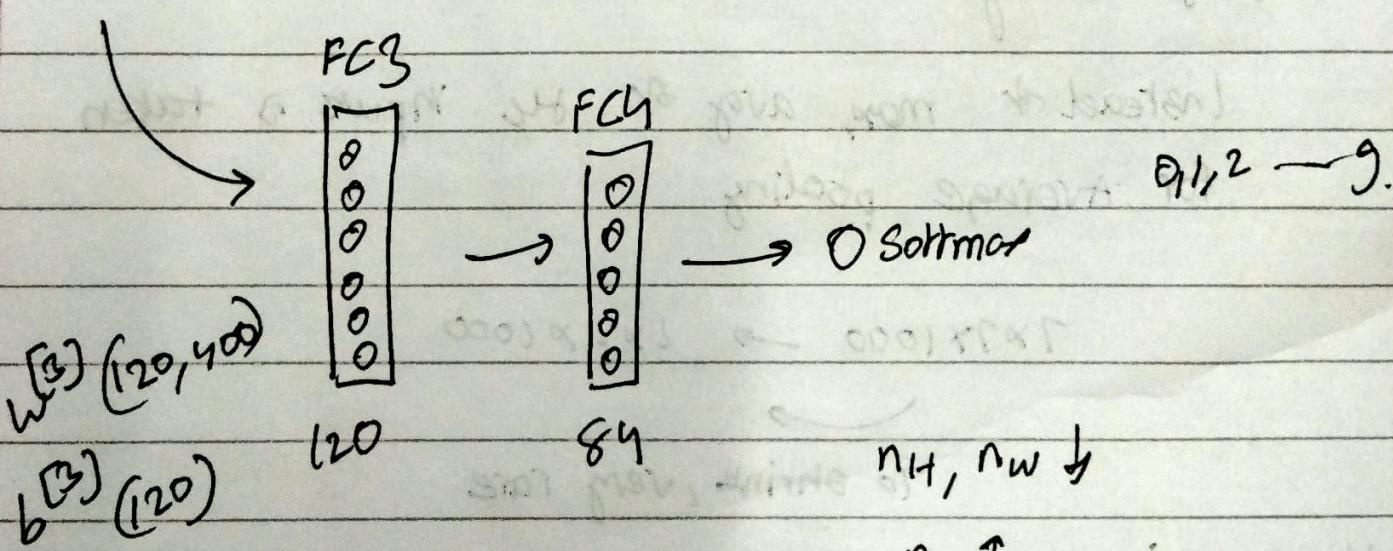
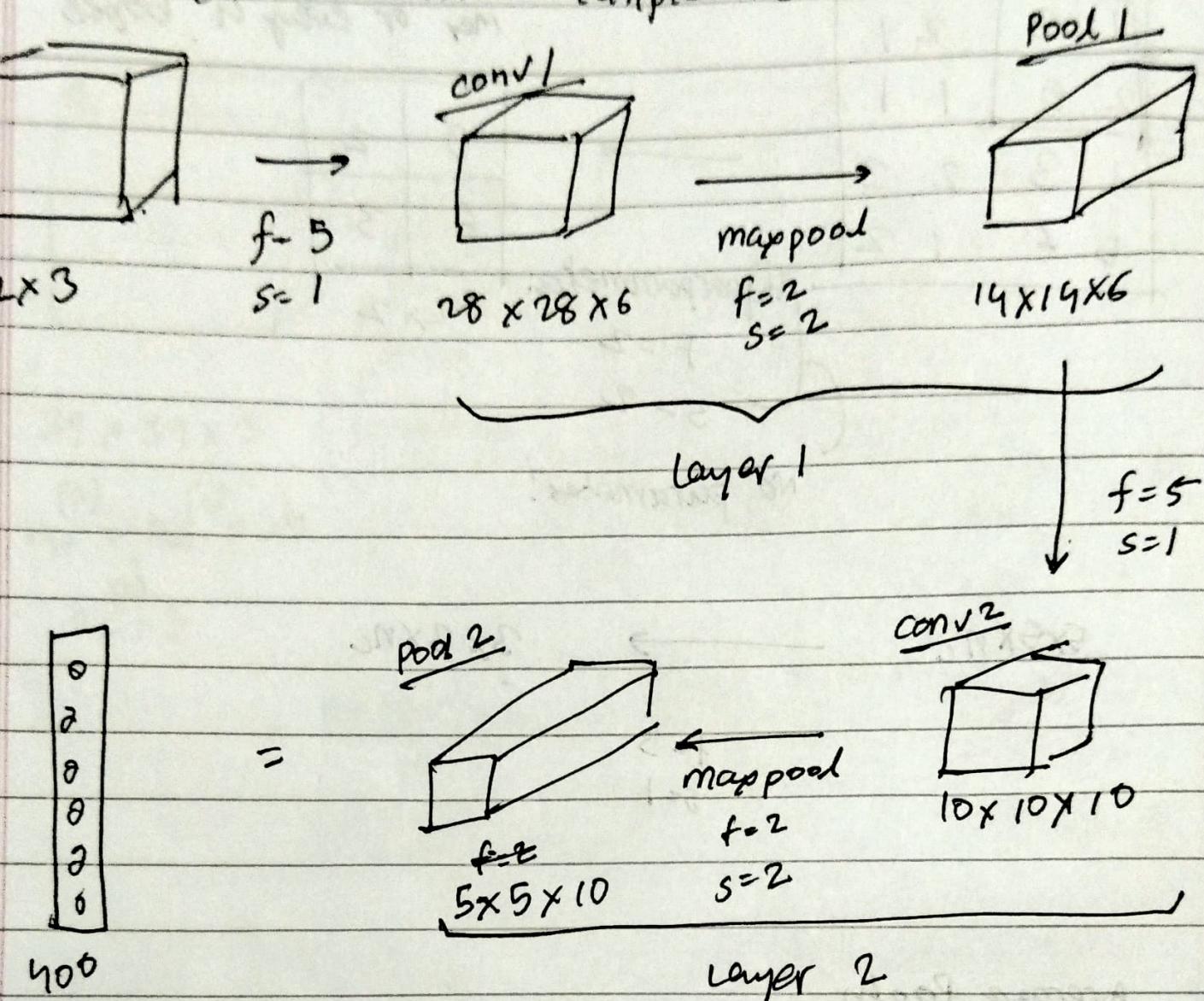
p : padding

$$n_h \times n_w \times n_c \longrightarrow \left\lfloor \frac{n_h - f + 1}{s} + 1 \right\rfloor \times \left\lfloor \frac{n_w - f + 1}{s} + 1 \right\rfloor \times n_c$$

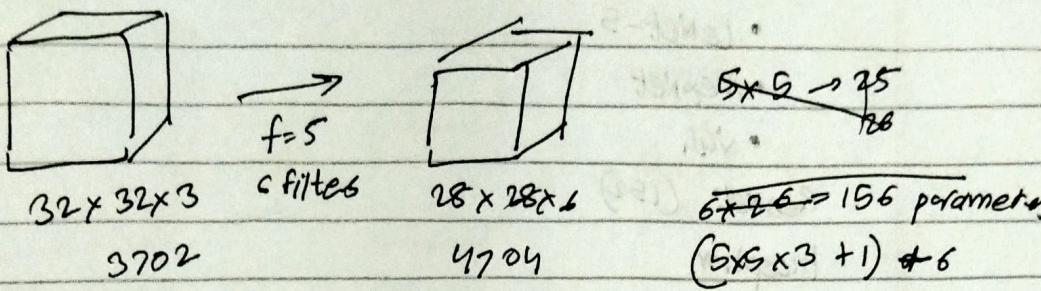
No parameters to learn

CNN Example

Neural Network Example (LeNet-5)



Why convolutions?



$$3702 \times 4704 \approx 14M$$

- Parameter sharing A feature detector in one part helpful in another part or image
- Sparsity of connections Depends on only a small number of inputs

Training set $(x^{(1)}, y^{(1)}) - (x^{(m)}, y^{(m)})$

$$\text{cost } J = \frac{1}{m} \sum_{i=1}^m L(y^{(i)}, f(x^{(i)}))$$

Use Gradient descent

test

$$6y + 2p - 9 = 6y$$

$$2p - 9 = 0$$

$$2p = 9$$

$$p = 4.5$$

$$\frac{66 - 3}{3} = 22$$

$$63 + 2p - 9 = 0$$

$$\frac{63 - 7}{2} = 28$$

$$66 - 3$$

$$2p = 6$$

$$\frac{64 - 9}{2} = 28$$