

Deep # Deep - 2 layer N.N

→ ~~Shallow~~ Shallow Vs Deep is a matter of depth ie no of hidden layers

→ Logistic reg. is a 1 layer N.N

→

Forward propagation in Deep Network

For a single training eg :-
for 1st hidden layer

$$\rightarrow \text{old } z^{[1]} = W^{[1]}X + b^{[1]}$$

$$\rightarrow a^{[1]} = g^{[1]}(z^{[1]})$$

for 2nd hidden layer

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = g^{[2]}(z^{[2]})$$

... and soon

\rightarrow In O/P layer

$$z^{[last]} = W^{[last]}a^{[last-1]} + b^{[last]}$$

$$a^{[last]} = g^{[last]}(z^{[last]}) = \hat{y}$$

general rule

$$z^{[l]} = W^{[l]}a^{[l-1]} + b^{[l]}$$

$$a^{[l]} = g^{[l]}(z^{[l]})$$

if vectorized:

$$z^{[1]} = W^{[1]}X + b^{[1]}$$

$$(X = A^{[0]})$$

$$A^{[1]} = g^{[1]}(z^{[1]})$$

$$z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

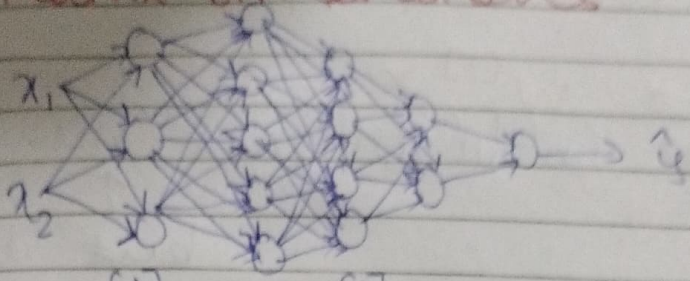
$$A^{[2]} = g^{[2]}(z^{[2]})$$

... and so on

for
let's
last
no
way
to
eg
do
without
for loop

$$z^{[2]} = \begin{bmatrix} \vdots \\ z^{[2]}(1) & \dots & z^{[2]}(m) \\ \vdots \end{bmatrix}$$

Matrix Dimensions



$$\begin{aligned} n^{[0]} &= 2 \\ n^{[1]} &= 3 \\ n^{[2]} &= 5 \\ n^{[3]} &= 4 \\ n^{[4]} &= 2 \\ n^{[5]} &= 1 \end{aligned}$$

$$z^{[1]} = W^{[1]} X + b$$

$$z^{[1]} = (3, 1) = (n^{[1]}, 1) \quad b = (n^{[1]}, 1)$$

$$X = (2, 1) = (n^{[0]}, 1) = (3, 1)$$

$$W = (3, 2) = (n^{[1]}, n^{[0]})$$

$$z^{[1]} = W^{[1]} X + b$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$z^{[2]} = W^{[2]} a^{[1]} + b^{[2]}$$

$$\begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow \\ (5 \times 1) & (5 \times 3) & (3, 1) & (5, 1) \end{matrix}$$

$$\rightarrow W^{[l]} = W^{[l]} = (n^{[l]}, n^{[l-1]})$$

$\rightarrow dw$ shld be same dimension as w

$$ie \quad dw^{[l]} = (n^{[l]}, n^{[l-1]})$$

$$\rightarrow db^{[l]} = [n^{[l]} - 1] \text{ --- same as } b$$

$$\rightarrow a^{[l]} = g^{[l]}(z^{[l]})$$

$\therefore z$ & a shld hv same dimension $= (n^{[l]}, 1)$

For m examples

$$z^{[l]} = W^{[l]} x + b^{[l]}$$

$$(n^{[l]}, m) \quad (n^{[l]}, n^{[l-1]}) \quad (n^{[l-1]}, m) \quad (n^{[l]}, 1)$$

but by python broadcasting it is duplicated to $(n^{[l]}, m)$

$$\therefore z^{[l]}, A^{[l]} : (n^{[l]}, m)$$

$$dz^{[l]}, dA^{[l]} : (n^{[l]}, m) \text{ (same as } z \text{ \& } A)$$

Why deep networks work well
 \rightarrow Detects simple things like edges first and then builds them into more complex network things.

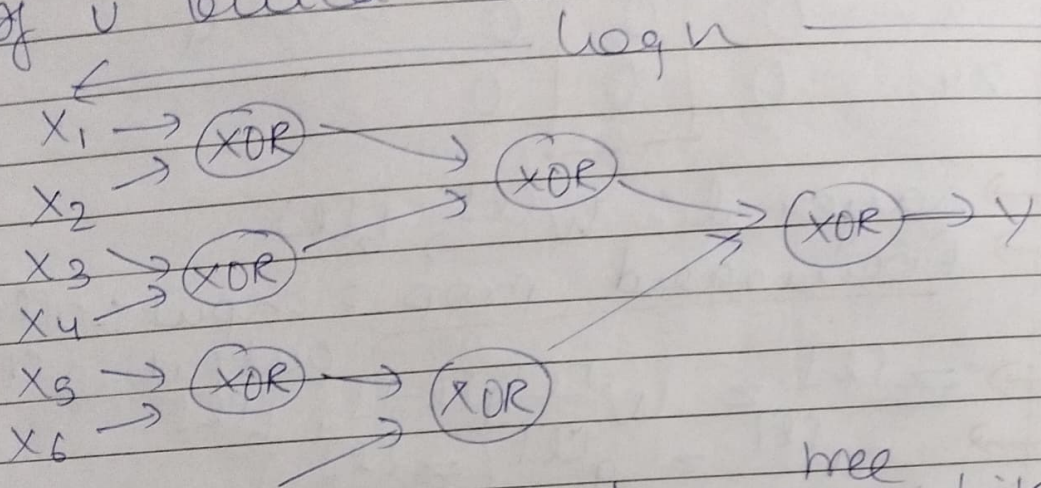
★ Circuit theory and deep learning
 These are functions you can compute with a "small" n
 L -layer deep neural network that shallower networks require exponentially more hidden units to compute

\rightarrow Circuit theory requires the thinking about what types of functions you can compute with different

AND, OR and NOT gates.

→ If we are trying to compute $X_1 \text{ XOR } X_2 \text{ XOR } \dots \text{ XOR } X_n$ if we have n features

If we build XOR tree like this:

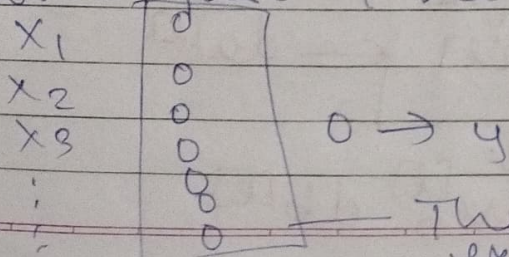


We can build a ~~circuit~~ ^{tree} like this

→ To compute XOR The depth of network will be on the order ~~of~~ $\log n$

→ no of nodes/circuit components/gates in the network is not that large. We don't need that many gates in order to compute exclusive ~~XOR~~ OR

→ If we are forced to compute with just 1 hidden layer



This layer will be ^{exponentially} large (2^{n-1})

Building blocks of deep neural networks

x_1	0	0	0
x_2	0	0	0
x_3	0	0	0
x_4	0	0	0

0 \rightarrow 1

\rightarrow Layer l : $W^{[l]}, b^{[l]}$

$\star \rightarrow$ Forward prop: Input: $a^{[l-1]}$
output: $a^{[l]}$

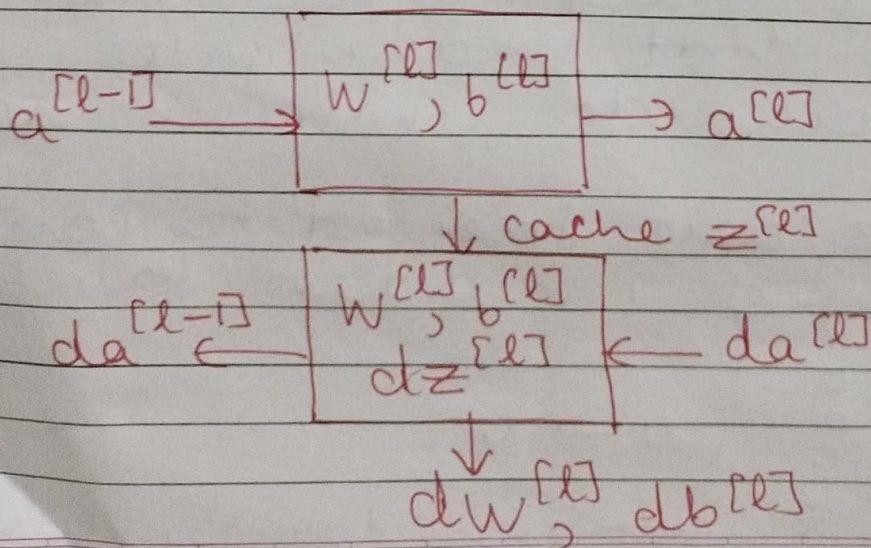
$\rightarrow z^{[l]} = W^{[l]} a^{[l-1]} + b^{[l]}$

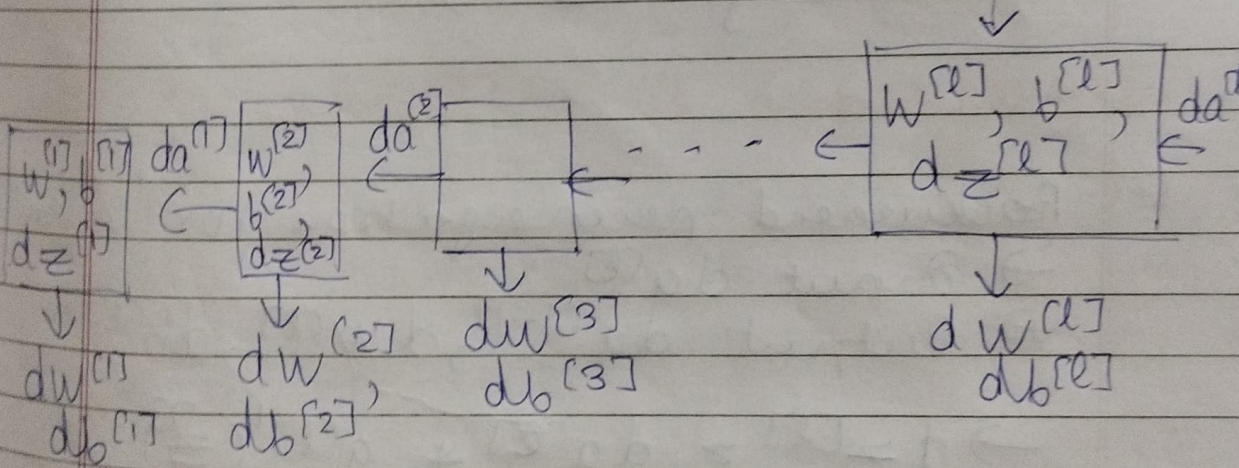
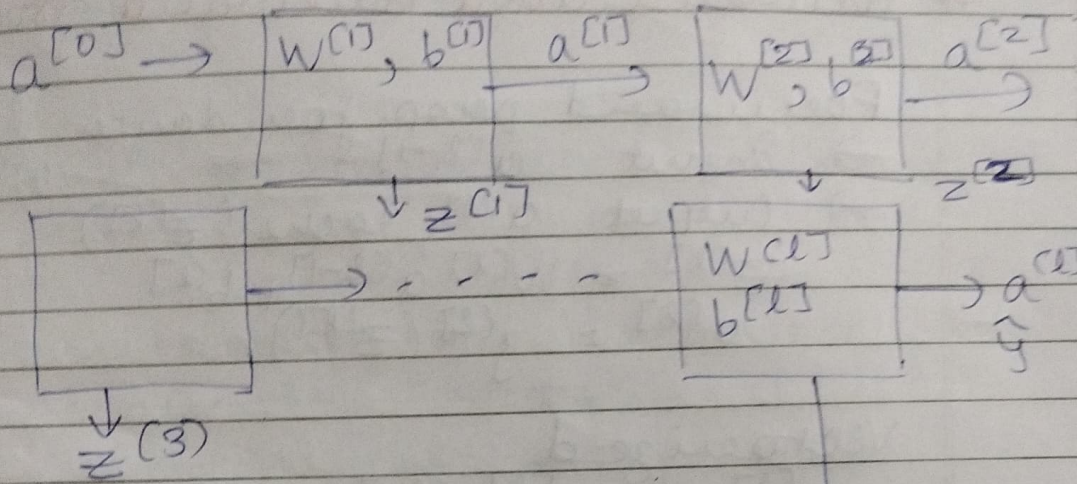
$\rightarrow a^{[l]} = g^{[l]}(z^{[l]})$

\rightarrow also cache $z^{[l]}$ as storing $z^{[l]}$ is useful for back propagation

$\star \rightarrow$ Backward prop: Input: $da^{[l]}$, cache
Output: $da^{[l-1]}$

Layer l





$$W^{[l+1]} = W^{[l+1]} - \alpha dz^{[l]} a^{[l]}$$

$$b^{[l+1]} = b^{[l+1]} - \alpha dz^{[l]}$$

→ It is useful to store $z^{[1]}$, $W^{[1]}$ & $b^{[1]}$ in cache during forward pass to be used in backward pass.

Forward and Backward Propagation

Forward prop. for layer l

→ Input $a^{[l-1]}$

→ Output $a^{[l]}$, cache $(z^{[l]})$ $W^{[l]}, b^{[l]}$

$$z^{[l]} = W^{[l]} a^{[l-1]} + b^{[l]}$$

$$a^{[l]} = g^{[l]}(z^{[l]})$$

Vectorized

$$z^{[l]} = W^{[l]} A^{[l-1]} + b^{[l]}$$

$$A^{[l]} = g^{[l]}(z^{[l]})$$

Backward propagation for layer l

→ Input $da^{[l]}$

→ Output $da^{[l-1]}, dW^{[l]}, db^{[l]}$

$$\rightarrow dz^{[l]} = da^{[l]} * g^{[l]'}(z^{[l]})$$

↑
element

wise multiplication

$$\rightarrow dW^{[l]} = dz^{[l]} a^{[l-1]T}$$

$$\rightarrow db^{[l]} = dz^{[l]}$$

$$\rightarrow da^{[l-1]} = W^{[l]T} dz^{[l]}$$

Vectorized:

$$dz^{[l]} = dA^{[l]} * g^{[l]'}(z^{[l]})$$

$$dW^{[l]} = \frac{1}{m} dz^{[l]} \cdot A^{[l-1]T}$$

⊗

$$db^{[l]} = \frac{1}{m} \text{np.sum}(dz^{[l]}, \text{axis}=1)$$

Keepdims = True)

$$dA^{[l-1]} = W^{[l]T} \cdot da^{[l]}$$

- In ~~an~~ forward pass for layer 0 we pass in the parameters x
- Similarly for backward pass we pass $da^{[l]}$ which is
- $$da^{[l]} = \frac{y}{-a} + \frac{(1-y)}{(1-a)}$$

Parameters Vs Hyperparameters

Parameters : $W^{[1]}, b^{[1]}, W^{[2]}, b^{[2]}, \dots$

Hyperparameters :

- learning rate $= \alpha$
- iterations $=$

→ hidden layers $= l$

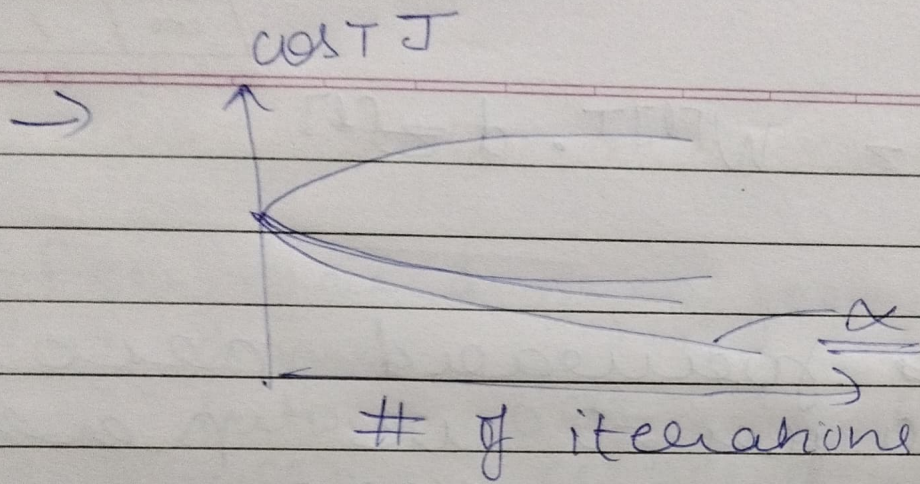
→ hidden units $n^{[1]}, n^{[2]}, \dots$

choice of activation functions

These determine final value of parameters

* Applying deep learning is an empirical process

- We keep trying diff values of cost function x and observe cost function



→ I just have to try out diff values of hyperparameters and see what works best
It is an empirical process