Schedule of Events

Provide a three-year calendar or schedule of major collaborative activities, dissemination and publication activities, and any project milestones.

Project Milestones

Task: Develop the bcGAIM

- 6-8 months: Implement prior(s) for shape-constrained inference for 1^{st}-order and 2^{nd}-order random walks.
- 8-12 months: Implement prior(s) for shape-constrained inference for adaptive smoothing splines.
- 12-16 months: Write paper summarizing these results and submit for publication.
- 12-16 months: Develop an R package so these models (and the approximate inference models) are readily available.
- 16-24 months: Implement prior(s) for shape-constrained inference for additional smoothing functions as needed.

Task: Approximate Inference Algorithm

- 8-10 months: Develop, implement, and test approximation algorithms for $\pi(\theta|Y, \alpha)$ and $\pi(\theta|Y, \alpha)$ (for known $\pi(\alpha|Y)$) in Stan when the posterior distribution of (η, θ, α) is not log-concave, when s is a random walk or smoothing spline.
- 10-16 months: Implement the approximate inference algorithm outside of Stan. Compare estimation results to Stan. test
- 14-18 months: Fully characterize the bias introduced by the approximation algorithm through extensive simulation studies.
- 20-24 months: Write a paper summarizing these results and submit for publication.
- 8-10 months: Develop, implement, and test approximation algorithms for estimating the multi-modal posterior distribution $\pi(\alpha|Y)$.
- 10-16 months: Implement the approximate inference algorithm outside of Stan. Compare estimation results to Stan.
- 14-18 months: Fully characterize the bias introduced by the approximation algorithm through extensive simulation studies.
- 18-20 months: Apply both approximations to the multi-pollutant models across regions and mortality outcomes.
- 20-24 months: Write a paper summarizing these results and submit for publication.

- 20-24 months: Add approximate Bayesian inference models to the R package.
- 24-30 months: Extend the approximation algorithm to hierarchical models (across cities). Compare to the results obtained when fitting the hierarchical model from Stan (exact and approximate inference).
- 30-34 months: Write a paper summarizing these results and submit for publication.
- 30-34 months: Add hierarchical approximate inference model to the R package.

Task: Multi-Pollutant Application

- 12-13 months: Explore performance of bcGAIM in the multi-pollutant model across regions and mortality outcomes.
- 13-16 months: Iteratively refine the bcGAIM (including shape-constraining priors) for the multi-pollutant application.
- 16-24 months: Extend the multi-pollutant model to a hierarchical model that fits Canada-wide data, using the approximations for $\pi(\theta|Y, \alpha)$ and $\pi(\theta|Y, \alpha)$ if computationally neccesary.
- 24-28 months: Write a paper summarizing these results and submit for publication.
- 24-28 months: Add hierarhical bcGAIM model to the R package.

Task: COVID-19 Application

- 24-26 months: Identify COVID-19 confounders and data sets that may be used to fit a COVID-19 bcGAIM model.
- 26-28 months: Fit the bcGAIM model to COVID-19 mortality data. Use the hierarchical models if data is available.
- 28-32 months: Write a paper summarizing these results and submit for publication.

External Collaborations

- 12-16 months: Identify collaborators and epidemiological studies that may benefit from the applying the bcGAIM.
- 16-36 months: Work with collaborations on epidemological studies, as the opportunity arises.

Dissemination and Publication Activities

Year 2

- Submit Paper: Shape-constrained Bayesian inference with interpretable priors.
- Submit Paper: Approximate Bayesian inference for non-concave likelihoods (fix $\pi(\alpha|Y)$).
- Submit Paper: Approximate Bayesian inference for $\pi(\alpha|Y)$ and non-concave likelihoods.
- Dissemination: Discuss shape-constrained Bayesian inference at 1-2 conferences.

Year 3

- Submit paper: A multi-pollutant air quality index.
- Submit paper: The effects of multiple pollutant mixtures on COVID-19 mortality.
- Submit paper: A hierarchical extension to Approximate Bayesian inference with non-concave likelihoods.
- Dissemination: Discuss approximate Bayesian inference for non-concave likelihoods at 1-2 conferences.
- Dissemintation: Discuss the multi-pollutant air quality index at 1-2 conferences.
- Dissemintation: Discuss the hierarchical extensions to the multi-pollutant air quality index at 1-2 conferences.

Major Collaborative Activities

Question: What qualifies as a "major" activity?

The different components of the bcGAIM project are naturally related, which encourages research collaboration between team members. The bcGAIM is being developed in Stan in the first year, as is the first version of the approximate inference algorithm. Therefore, the bcGAIM model should be written in a way that faciliates incorporating these approximations. Moreover, the approximations should be developed knowing they will be implemented in Stan. In the second year, the two Stan models and the approximate inference algorithm will be extended to a hierarchical formulation. In the multi-pollutant model, the relative risks of time, temperature, and the pollutants are often very small making model hyper-parameters numerically difficult to estimate. Moreover, although the hierahichal structure is at the city-level, nearby cities differ in their distance from each other. Ideally, a hierahichal model should account for how the composition of a mixture of pollutants varies by distance. The numerical difficulties and more complicated hierahichal structure should also encourage strong collaboration at this stage of the project.

The third year is devoted to applications – applying the fully developed bcGAIM model to the multi-pollutant problem, COVID-19 data, and other epiodemiological applications that arise during the course of the project – as well as writing papers and producing a useful R package. There is again natural collaboration between those writing and maintaining the R package. The papers will be written independently, but ideally with feedback from the entire research term. They should be in a position to give good feedback as they colloborated on some level throughout the entire project.