American University of Beirut
Department of Computer Science
CMPS 211 - Spring 2017-18
Assignment 1
Propositional Logic

The following exercises are to be submitted on Wednesday, 28th of February at 11:55 PM. You are expected to submit a soft-copy to Moodle and another hard-copy during class. Late submissions will not be accepted

Excercise 1 (10 Points)

Which of these are propositions? What are the truth values of those that are propositions?

(a) $1/y = 5$.

(b) What's your name?

(c) I spent all my money on books.

(d) Do not pass! Just go!

(e) NAND gates are the primitive components of CPUs.

Solution:

(a) Not a proposition.

(b) Not a proposition.

(c) It is a proposition. Truth-value: if I spent all my money on books. False-value: if I didnt spend all my money on books.

(d) Not a proposition (an order).

(e) It is a proposition and its value is true.

Excercise 2 (8 Points)

Determine whether these biconditionals are True or False:

(a) 7*2=14 **if and only if** 3/0 is undefined.

(b) 2+2=5 **if and only if** Michael Jackson is still alive.

(c) 5*5=25 **if and only if** 1-1=2.

(d) 12 > 15 **if and only if** -15 > -12.

Solution:

(a) True since 7*2=14 is True and 3/0 is undefined is true, thus True if and only if True is True.

(b) True since 2+2=5 is False and Michael Jackson is still alive is False, thus False if and only if False is True.

(c) False since 5*5=25 is True and 1-1=2 is False, thus True if and only if False is False.

(d) True since 12 > 15 is False and -15 > -12 is False, thus False if and only if False is True.

Exercise 3 (8 Points)

Construct the truth tables for the following statements:

(a) $(q \wedge p) \vee (q \wedge \neg p)$

(b) $(p \vee q) \leftrightarrow (q \rightarrow \neg p)$

(c) $(p \leftrightarrow q) \vee (pXORq)$

(d) $(p \vee \neg r) \rightarrow (q \vee p)$

Solution:

(a)

q	p	$\neg p$	$q \wedge p$	$q \wedge \neg p$	$(q \wedge p) \vee (q \wedge \neg p)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	F	F
F	F	T	F	F	F

(b)

q	p	$\neg p$	$p \vee q$	$q \rightarrow \neg p$	$(p \vee q) \leftrightarrow (q \rightarrow \neg p)$
T	T	F	T	F	F
T	F	T	T	T	T
F	T	F	T	T	T
F	F	T	F	T	F

(c)

p	q	$p \leftrightarrow q$	$pXORq$	$(p \leftrightarrow q) \vee (pXORq)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

(d)

p	q	r	$\neg r$	$p \lor \neg r$	$q \lor r$	$(p \lor \neg r) \to (q \lor r)$
T	T	T	F	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	T	T
F	T	T	F	F	T	T
T	F	F	T	T	T	T
F	F	T	F	F	F	T
F	T	F	T	T	T	T
F	F	F	T	T	F	F

Exercise 4

Let

- p: "I am busy",
- q: "I am hungry",
- r: "I will order takeout",

Write the following statements in terms of p, q, and r.

(a) If I am hungry and busy then I will order takeout.
Solution: $q \land p \to r$

(b) I am not busy and I am not hungry.
Solution: $\neg p \land \neg q$

(c) I will order takeout only if I am busy.
Solution: $p \to r$

(d) I will order takeout if and only if I am hungry and busy.
Solution: $r \leftrightarrow (p \land q)$

Exercise 5

Translate the following into symbols using predicates and quantifiers of your choice:

(a) If you completed all program requirements then you're graduating this summer.
Solution:
let:
p: you completed all program requirements
q: you are graduating this summer

statement becomes: $p \to q$

(b) The difference between a real number and itself is zero.
Solution:
let:
$x, y \in \mathbb{R}$
P(x,y): x - y = 0

statement becomes: $\forall x P(x, x)$
or: $\forall x, y[P(x, y) \rightarrow x = y]$
or: $\forall x, y[x = y \rightarrow xP(x, y)]$

(c) Someone in class has an Instagram account but does not follow anyone from class.
Solution:
let:
$x, y \in$ students in class
I(x): x has an Instagram account
F(x,y): x follows y on Instagram
$\exists x \forall y I(x) \wedge [I(y) \rightarrow \neg F(x, y)]$

Exercise 6

Are the following pairs of statements logically equivalent? Show your work (Do not use truth tables)

(a) $(p \wedge q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$

Solution:
$(p \rightarrow q) \wedge (p \rightarrow r)$
$\equiv (\neg p \vee q) \wedge (\neg p \vee r)$
$\equiv \neg p \vee (q \wedge r)$
and: $p \rightarrow (q \wedge r) \equiv \neg p \vee (q \wedge r)$

(b) $(p \wedge q) \rightarrow (r \vee s)$ and $p \rightarrow (\neg q \vee r \vee s)$

Solution:
$(p \wedge q) \rightarrow (r \vee s) \equiv (p \vee \neg q \vee r \vee s)$
and: $p \rightarrow (\neg q \vee r \vee s) \equiv (p \vee \neg q \vee r \vee s)$

(c) $p \rightarrow (q \wedge r)$ and $(p \rightarrow q) \wedge (p \rightarrow r)$.
Solution:
$(p \rightarrow q) \wedge (p \rightarrow r)$
$\equiv (\neg p \vee q) \wedge (\neg p \vee r)$
$\equiv \neg p \vee (q \wedge r)$
and:
$p \rightarrow (q \wedge r) \equiv \neg p \vee (q \wedge r)$

(d) $(p \to r) \lor (q \to r)$ and $(p \land q) \to r$

Solution:
$(p \land q) \to r$
$\equiv \neg(p \land q) \lor r$
$\equiv \neg p \lor \neg q \lor r$
and:
$(p \to r) \lor (q \to r)$
$\equiv (\neg p \lor r) \lor (\neg q \lor r)$
$\equiv \neg p \lor \neg q \lor r$

Exercise 7

Negate the following statements:

(a) $\forall x P(x) \to Q(x)$

Solution: $\exists x P(x) \land \neg Q(x)$

(b) $(pXORq) \land (p \leftrightarrow q)$

Solution:
Using a truth table, you can show that the negation of $(pXORq)$ is $(p \leftrightarrow q)$ thus:
$(pXORq) \land (p \leftrightarrow q) \equiv (pXORq) \land \neg(pXORq)$ and:
$\neg((pXORq) \land \neg(pXORq)) \equiv \neg(pXORq) \lor (pXORq)$

(c) $\forall x \exists y P(x, y)$

Solution: $\exists x \forall y \neg P(x, y)$

(d) If I finish my homework, then if my friends are available, we will go to the movies. *(Negate this one in English)*

Solution: I finished my homework and my friends are available but we did not go to the movies.

Exercise 8

State the converse, contra-positive, and inverse of each of these conditional statements.

(a) If you practice a lot, you will become a good programmer.

Solution:
converse: If you are a good programmer, then you practice a lot.
contrapositive: if you are not a good programmer, then you do not practice.

inverse: if you do not practice a lot, then you will not become a good programmer.

(b) You cannot go out if you do not finish your homework.

Solution:
converse: If you cannot go out then you did not finish your homework.
contrapositive: If you can go out then you finished your homework.
inverse: if you finish your homework then you can go out.

Exercise 9 (8 Points)

For each of these sentences, determine whether an inclusive or, or an exclusive or, is intended. **Explain your answer**.

(a) To enter the country you need a passport or a voter registration card.

(b) Enrolling in CMPS211 or MATH201 is required.

(c) When you need it, you cant find it or it is not available.

(d) Dinner includes steak or chicken.

Solution:

(a) Inclusive (or) since you could have both.

(b) Inclusive (or) since you could be enrolled in both.

(c) Exclusive (or).You can pick only one choice.

(d) Exclusive (or).You can pick only one choice.

Exercise 10

Let:

- P(x) = x is a CMPS major
- Q(x) = x is taking CMPS 200

where the domain for x consists of all the students in the CMPS 211 class.

Express each of these statements using quantifiers, logical connectives, and Q(x) and P(x). Then express their negations in English and using quantifiers.

(a) All students in CMPS 211 class are CMPS major.
Solution: $\forall x P(x)$

(b) Some students in CMPS 211 class are not CMPS major.
Solution: $\exists x \neg P(x)$

(c) ALL CMPS students taking CMPS 211 are also taking CMPS 200.
Solution: $\forall x P(x) \wedge Q(x)$ or: $\forall x P(x) \rightarrow Q(x)$

Exercise 11 (5 Points)

Show that $p \leftrightarrow q$ is equivalent to $\neg(p \oplus q)$ using **both** truth table and equivalence relations.
Hint: You need first to express $p \leftrightarrow q$ and $p \oplus q$ using simple operators like $\neg$, $\vee$, & $\wedge$
Solution:
First method, by using a truth table:

q	p	$p \oplus q$	$\neg(p \oplus q)$	$p \leftrightarrow q$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

Second method:
Solution:

$$
\begin{aligned}
\neg(p \oplus q) &\equiv \neg((p \vee q) \wedge \neg(p \wedge q)) && \text{Implication (Table 7)} \\
&\equiv \neg((p \vee q) \wedge (\neg p \vee \neg q)) && \text{Implication (Table 7)} \\
&\equiv \neg(((p \vee q) \wedge \neg p) \vee ((p \vee q) \wedge \neg q)) && \text{Implication (Table 7)} \\
&\equiv \neg(((p \wedge \neg p) \vee (\neg p \wedge q)) \vee ((p \wedge \neg q) \vee (\neg q \wedge q))) && \text{Implication (Table 7)} \\
&\equiv \neg(((F) \vee (\neg p \wedge q)) \vee ((p \wedge \neg q) \vee (F))) && \text{Implication (Table 7)} \\
&\equiv \neg((\neg p \wedge q) \vee (p \wedge \neg q)) && \text{Implication (Table 7)} \\
&\equiv (p \vee \neg q) \wedge (\neg p \vee q)) && \text{Implication (Table 7)} \\
&\equiv (q \rightarrow p) \wedge (p \rightarrow q)) && \text{Implication (Table 7)} \\
&\equiv p \leftrightarrow q && \text{Implication (Table 7)}
\end{aligned}
$$

Exercise 12 (6 Points)

Use equivalence rules to show that the following compound proposition is a contradiction.
Do not use truth tables

(a) $((\neg p \vee q) \wedge (\neg q \vee r)) \wedge (p \wedge \neg r)$

Solution:

$$
\begin{array}{lll}
exp & \equiv (\neg p \lor q) \land ((\neg q \lor r) \land (p \land \neg r)) & \\
 & \equiv (\neg p \lor q) \land (((p \land \neg r) \land \neg q) \lor ((p \land \neg r) \land r)) & \text{Distributive Laws} \\
 & \equiv (\neg p \lor q) \land (((p \land \neg r) \land \neg q) \lor (p \land (\neg r \land r))) & \text{Distributive Laws} \\
 & \equiv (\neg p \lor q) \land (((p \land \neg r) \land \neg q) \lor (p \land (F))) & \text{Domination Laws} \\
 & \equiv (\neg p \lor q) \land (((p \land \neg r) \land \neg q) \lor (F)) & \text{Domination Laws} \\
 & \equiv (\neg p \lor q) \land (p \land \neg r \land \neg q) & \text{Domination Laws} \\
 & \equiv (\neg p \land (p \land \neg r \land \neg q)) \lor (q \land (p \land \neg r \land \neg q)) & \text{Distributive Laws} \\
 & \equiv ((\neg p \land p) \land (\neg r \land \neg q)) \lor ((q \land \neg q) \land (p \land \neg r)) & \text{Distributive Laws} \\
 & \equiv (F \land (\neg r \land \neg q)) \lor (F \land (p \land \neg r)) & \text{Domination Laws} \\
 & \equiv (F) \lor (F) & \text{Domination Laws}
\end{array}
$$

Exercise 13 **(13 Points)**

Use equivalence rules to show that the following compound propositions are tautologies. **Do not use truth tables**

(a) $(p \lor q) \land (\neg p \lor r) \to (q \lor r)$ (3 points)
Solution:

$$
\begin{array}{lll}
exp & \equiv \neg((p \lor q) \land (\neg p \lor r)) \lor (q \lor r) & \text{Implication (Table 7)} \\
 & \equiv (\neg(p \lor q) \lor \neg(\neg p \lor r)) \lor (q \lor r) & \text{De Morgan} \\
 & \equiv ((\neg p \land \neg q) \lor (\neg\neg p \land \neg r)) \lor (q \lor r) & \text{De Morgan} \\
 & \equiv ((\neg p \land \neg q) \lor (p \land \neg r)) \lor (q \lor r) & \text{Double Negation} \\
 & \equiv (\neg p \land \neg q) \lor ((p \land \neg r) \lor (q \lor r)) & \text{Associative Law} \\
 & \equiv (\neg p \land \neg q) \lor ((p \land \neg r) \lor (r \lor q)) & \text{Comm. Law} \\
 & \equiv (\neg p \land \neg q) \lor (((p \land \neg r) \lor r) \lor q) & \text{Assoc. Law} \\
 & \equiv (\neg p \land \neg q) \lor (q \lor (p \land \neg r) \lor r)) & \text{Comm. Law} \\
 & \equiv ((\neg p \land \neg q) \lor q) \lor ((p \land \neg r) \lor r) & \text{Assoc. Law} \\
 & \equiv (q \lor (\neg p \land \neg q)) \lor (r \lor (p \land \neg r)) & \text{Comm. Law} \\
 & \equiv ((q \lor \neg p) \land (q \lor \neg q)) \lor ((r \lor p) \land (r \lor \neg r)) & \text{Distributive Laws} \\
 & \equiv ((q \lor \neg p) \land T) \lor ((r \lor p) \land T) & \text{Negation Laws} \\
 & \equiv (q \lor \neg p) \lor (r \lor p) & \text{Identity Laws} \\
 & \equiv q \lor (\neg p \lor (r \lor p)) & \text{Assoc. Laws} \\
 & \equiv q \lor ((\neg p \lor p) \lor r) & \text{Assoc. and Comm.} \\
 & \equiv q \lor (T \lor r) & \text{Negation} \\
 & \equiv q \lor (r \lor T) & \text{Commutative Laws} \\
 & \equiv q \lor T & \text{Domination Laws} \\
 & \equiv T & \text{Domination Laws}
\end{array}
$$

(b) $(\neg q \land (p \to q)) \to \neg p$ (3 points)
Solution:

$$
\begin{aligned}
exp &\equiv \neg(\neg q \wedge (p \rightarrow q)) \vee \neg p && \text{Implication (Table 7)}\\
&\equiv \neg(\neg q \wedge (\neg p \vee q)) \vee \neg p && \text{Implication (Table 7)}\\
&\equiv (q \vee (p \wedge \neg q)) \vee \neg p && \text{De Morgan}\\
&\equiv q \vee (p \wedge \neg q) \vee \neg p && \text{Distributive Laws}\\
&\equiv q \vee ((p \vee \neg p) \wedge (\neg q \vee \neg p)) && \text{Assoc. Laws}\\
&\equiv q \vee ((T) \wedge (\neg q \vee \neg p)) && \text{Distributive Laws}\\
&\equiv q \vee (\neg q \vee \neg p) && \text{Distributive Laws}\\
&\equiv (q \vee \neg q) \vee \neg p && \text{Assoc. Laws}\\
&\equiv (T) \vee \neg p && \text{Domination Laws}\\
&\equiv (T)
\end{aligned}
$$

(c) $(\neg(p \rightarrow q) \rightarrow \neg q) \wedge ((p \wedge q) \rightarrow (p \rightarrow q))$ (7 points)
Solution: The following could be true if both $(\neg(p \rightarrow q) \rightarrow \neg q)$ is true and $((p \wedge q) \rightarrow (p \rightarrow q))$ is true.

$$
\begin{aligned}
exp1 &\equiv \neg(p \rightarrow q) \rightarrow \neg q\\
&\equiv \neg(\neg p \vee q) \rightarrow \neg q && \text{Implication (Table 7)}\\
&\equiv \neg(\neg(\neg p \vee q)) \vee \neg q && \text{Implication (Table 7)}\\
&\equiv (\neg p \vee q) \vee \neg q && \text{Negation}\\
&\equiv (\neg q \vee q) \vee \neg p && \text{Distributive Laws}\\
&\equiv (T) \vee \neg p && \text{Domination Laws}\\
&\equiv (T)
\end{aligned}
$$

$$
\begin{aligned}
exp2 &\equiv (p \wedge q) \rightarrow (p \rightarrow q)\\
&\equiv \neg(p \wedge q) \vee (p \rightarrow q) && \text{Implication (Table 7)}\\
&\equiv \neg(p \wedge q) \vee (\neg p \vee q) && \text{Implication (Table 7)}\\
&\equiv \neg p \vee \neg q \vee \neg p \vee q && \text{Distributive Laws}\\
&\equiv (\neg q \vee q) \neg p \vee \neg p && \text{Distributive Laws and Assoc.}\\
&\equiv (T) \neg p && \text{Domination Laws}\\
&\equiv (T) && \text{Domination Laws}
\end{aligned}
$$

Extra Practice **(Not To Be Submitted)**

From the textbook Discrete Mathematics and Its applications 7th Edition

- Sec. 1.1 (exercises $1 \rightarrow 25$)
- Sec. 1.2 (exercises $1 \rightarrow 8, 10 \rightarrow 22$)
- Sec. 1.3 (exercises $1 \rightarrow 16$)
- Sec. 1.4 (exercises $1 \rightarrow 25$)
- Sec. 1.5 (exercises $1 \rightarrow 26$)