American University of Beirut

CMPS 211

Discrete Structures

Spring 2023-2024

Quiz # 1 (Duration: 70 mins)

ID:

Name:

Section: 1 (MWF 8-9), 2 (TR 8:00-9:15)

<u>This exam is closed books, closed notes.</u>

<u>Part I: Multiple choice questions</u> (6 pts each)

1. If p denotes that "it is winter" and q denotes "it snows" then what is the best formalization of the sentence:

   It only snows in winter.

   a) $p \rightarrow q$
   b) $q \rightarrow p$
   c) $q \leftrightarrow p$
   d) $p \land q$

2. Assuming $p \rightarrow q$ is true and $r \rightarrow s$ is false which of the following statements is known to be true?

   a) $\neg r \land q$
   b) q
   c) r
   d) None of the above

3. Is the following statement true?

   $$(a \leftrightarrow b) \leftrightarrow c \equiv a \leftrightarrow (b \leftrightarrow c)$$

   a) Yes, because both propositions give the same truth values in every row of the truth table for the given equivalence.
   b) No, because there is a row in the truth table for the given equivalence in which the left-hand side proposition is true, and the right-hand side proposition is false (but not the other way around).
   c) No, because there is a row in the truth table for the given equivalence in which the right-hand side proposition is true, and the left-hand side formula is false (but not the other way around).
   d) No, because there is a row in the truth table for the given equivalence in which the left-hand side proposition is true and the right-hand side proposition, and there is a row in the truth table for the given equivalence in which the right-hand side proposition is true, and the left-hand side proposition is false.

4. $(p \rightarrow (q \lor r)) \rightarrow ((p \land q) \rightarrow r)$ is a
   a) Satisfiable but not valid
   b) Valid
   c) Contradiction
   d) None of the above

5. $\forall x P(x) \Leftrightarrow \neg \exists x \neg P(x)$ is:
   a) True
   b) False

6. If the universe of discourse is the set of natural numbers N (all integers ≥ 0) and E(n)= ``n is even" then the following statement:

   $$\forall n \exists m ((\neg E(n) \land n \neq m) \rightarrow E(n + m))$$

   translated to English would give:
   a) For any two distinct natural numbers, if one of them is odd then their sum is even.
   b) There exists an odd natural number whose sum with every other distinct natural number is even.

c) For each odd natural number there is another natural number where their sum is even.
d) There exist two distinct natural numbers such that one of them is odd and their sum is even.

7. Which of the following formulas does express that there is exactly one element of some domain D that has property P(x)?
   a) $\exists x \forall y\, [P(y) \leftrightarrow y = x]$
   b) $\exists x[P(x) \land \forall y\, [y \neq x \rightarrow \neg P(y)]]$
   c) $\exists x P(x) \land \forall x_1, x_2 [P(x_1) \land P(x_2) \rightarrow x_1 = x_2]$
   d) All of the above.

8. Determine which of these compound propositions is satisfiable.
   a) $(p \lor \neg q) \land (\neg p \lor q) \land (\neg p \lor \neg q)$
   b) $(p \rightarrow q) \land (p \rightarrow \neg q) \land (\neg p \rightarrow q) \land (\neg p \rightarrow \neg q)$
   c) $(p \leftrightarrow q) \land (\neg p \leftrightarrow q)$
   d) None of the above statements is satisfiable

Part II: Show your work.

1. (15 pts) Prove that if ab is irrational, then at least one of a and b are also irrational. Show the entire flow of your proof and justify the proof methodology. (7 pts on methodology choice and why you chose it, 8 pts on the actual solution).

2. (15 pts) Prove that for all real numbers x , if $x > 1$ or $x < -1$, then $|x| > 1$. (7 pts on methodology choice and why you chose it, 8 pts on the actual solution).

3. (15 pts) Let m and n be integers. Then mn is odd if and only if m and n are both odd. (7 pts on methodology choice and why you chose it, 8 pts on the actual solution).

4. (15 pts) Given the following set of premises

    a. $p \rightarrow (q \wedge r)$
    b. $q \rightarrow \neg r$
    c. $\neg p \rightarrow r$

   Prove $\neg p \wedge \neg q$ using the rules of inference