Analysis of fatigue behavior in 2nd order octahedral metamaterials with hierarchical configuration upon uniaxial compression using numerical simulations

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Hierarchical metamaterial structures have drawn the attention of scientific and engineering community nowadays due to their unique characteristics that cannot be found in nature. The 2nd order octahedral metamaterials, as one of the unique structures, have shown improved stiffness, strength and recoverability upon uniaxial compression [1]. The research has been taken further in the present study, in order to account for the effect of fatigue on the hierarchical structures. This has been done by subjecting the structures to fatigue loading using different stress ratios and observing the life of the metamaterials using readily available numerical simulation. The hierarchy and the length of the unit cells were constantly changed to observe the effect on the life of the structures. In addition, the cross sectional areas of the beams were taken into account by considering the loading on structures with rectangular and triangular cross-sectional areas and producing results on biaxiality indication, damage tolerance and fatigue sensitivity of the samples. The results showed that increasing the hierarchy decreases the fatigue life of the metamaterials to a certain degree. The similar is true when increasing the length of the unit cells. In addition, increasing the cross-sectional area of the beams reduced the fatigue life of structures dramatically. As such, it was concluded that an optimal condition would be decreasing the order of hierarchy, the length of the unit cells and the cross-sectional area. These findings provide important insights for understanding the effect of fatigue on hierarchical metamaterials and their practicality in real-life applications.

1. Introduction

Metamaterials are the class of materials that are artificially created and can exhibit properties that cannot be found in nature. Due to these characteristics, the same metamaterial can exhibit different properties by tuning its structure. For example, by restructuring the metamaterial that is comprised of identical unit cells, the property of the material can change from bend-dominant to stretch-dominant and reverse [2]. Hierarchical metamaterials are one of the more prominent types, since they are ultra-lightweight and show linear scaling in stiffness, strength and recoverability with higher relative density [1]. These type of structures exist in nature such as in bones [1] and are significant to researchers and engineers due to their damage tolerance and robustness. However, their application currently is limited due to difficulty in fabrication process.

Considerable research has been undertaken into analysing the mechanical properties of hierarchical metamaterials such as stiffness, strength, and strain energy density. The research shows that these properties have been analysed by subjecting the structures to uniaxial compression or tension [1]. While the mechanical properties are vital for engineering practices, they are not enough in order to make sure that the metamaterial structures will not fracture or fail due to real word effects such as and fatigue. Limited research is available on failure analysis of metamaterial structures and how to prevent it, hence further studies must be conducted. Fatigue behaviour as one of such areas plays important role in practice relative to failure mechanisms. While there are studies available on fatigue behaviour of different types of metamaterials [3-9], there is a limited research available on the fatigue behaviour of hierarchical 2nd order octahedral metamaterials.

Hierarchical metamaterials are comprised of identical unit cells that make up the entire structure. These unit cells can be of various forms such as honeycomb [10,11], kirigami[12], cubic [13], quasi-random [14] and octahedral [1] forms. By joining the identical unit cells together, the metamaterial can be of different hierarchical scales. This scale defines the number of unit cells in the structure. For example, a structure with 3rd order of hierarchy has more unit elements than one with 2nd order of hierarchy. The research into the unit cells show that the octahedral unit cells are the most efficient in terms of their mechanical properties such as stiffness, strength, recoverability and the strain energy density [1,15]. However, the effect of fatigue loading on these unit cells is not explored throughout, which adds novelty to the current research. Particularly, limited information is available on the fatigue behaviour of octahedral metamaterials. Therefore, the analysis in this paper focuses on the fatigue behaviour of the 2nd order octahedral metamaterials with hierarchical configurations by subjecting them to alternating uniaxial compression.

The metamaterial structures in the present study are analysed in terms of their fatigue life, damage tolerance, fatigue sensitivity and biaxiality indication. This allows to assess the effect of fatigue loading from various aspects. The analysis is computational in nature, i.e. the octahedral structures are designed computationally and analysed on a simulation software ANSYS. The scale of hierarchy of metamaterials is varied in order to see the fatigue behaviour during scaling. In addition, the length of the unit cells and their cross sectional areas are varied in order to compare the fatigue results and assess the fatigue behaviour from various these perspectives. Since the beams, the octahedral unit cell is comprised of, are identical, the length of the unit cells refers to the length of the individual beam. The same is true about the cross sectional area of the unit cells.

These structures are subjected to different stress ratios in order to analyse the life of structures through different alternating stress ranges. In addition, the structures are subjected to quasistatic analysis in order to compare the results with the existing research. This analysis mainly considers the stiffness and stress distribution aspect of the structures in order to validate the present study analysis.

2. Methodology

Initially, the octahedral metamaterials are designed on Solidworks software. These structures are designed with various configurations in terms of the length of the unit cells, hierarchical scale and the cross-sectional area of the unit cell beams. The dimensions of the cells are in microns, since the presents study is an extension of the work accomplished by L. Meza where such dimensions are used. Since comparison will be made with the existing research in terms of quasistatic analysis, these dimensions were used. The length of the unit cells, denoted as L, ranges from 4 µm to 8 µm which is appropriate enough to contrast the effect of the unit cell length on the fatigue behaviour. The hierarchical scale, denoted as N, ranges from N=4 to N=10. This allows comparison of structures at different scales in order to draw appropriate conclusions. As such, a metamaterial with N=10 has more unit cells than a metamaterial with N=8. The N configuration was assigned in this manner due to the design method. When creating the structure, the unit cells that make up the base of the metamaterial were designed first. The number of unit cells making up the rest of the metamaterial, therefore, were dependent on the number of cells at the base of the metamaterial. As such, it was decided, to use the number of base unit cells to denote N hence denoting the scale of hierarchy. The L and N configurations that were used for the analysis can be seen in figure 1 a.

The cross-sectional areas of unit cells were designed of two types: rectangular and triangular cross-sectional areas. The rectangular cross sectional area has dimensions of 0.25 by 0.25 µm. On the other hand, the triangular area has an area of 0.065 μm^2 and is an equilateral triangle. These areas were kept low in order to avoid complications during assembly stage. This allowed seamless construction of unit cells. These unit cells can be seen in Figure 1 b. Hence, the full design of the hierarchical metamaterial involved assembling the identical unit cells together which is shown in figure 1c.

Finally, the designed structures were transported to numerical simulation software Ansys for fatigue analysis. This software allows to simulate fatigue loading using several steps. However, the material of choice for the metamaterial structures had to be chosen first. For the research purposes structural steel was chosen.

The meshing of the structures involved using Modeler tool in Ansys. The mesh type used for the analysis was quadratic tetrahedral mesh which allows to obtain more accurate results than if the default linear mesh was used. Due to how complex the metamaterial structures were in design stage, the mesh type was not changed, however their sizing and quality have been tuned (mesh type was experimented with). The sizing of the mesh was set to fine which increased the number of meshes and decreased the size. The quality of the mesh was set to high. The smoothing of the mesh cells was set to high as well in order to have better size transition between mesh cells. The growth rate of the cells was set at 1.2 which is an acceptable standard. Since the effect of fatigue on the entire metamaterial structure is the purpose in this research, every section of the metamaterial was important to analysis. As such, one type of mesh was used throughout the entire structure with high quality. An example of the meshing used is shown in figure 1 d.

The fatigue loading in the research was performed using uniaxial loading. Therefore, the load was applied on top of the structure while the base of the metamaterial was fixed. The load types used for the analysis were cyclic fully reversed and zero- based loading respectively. This means that the load varied between $-\sigma$ to $+\sigma$ and 0 to $+\sigma$ respectively.

In addition, in order to produce results from fatigue loading, mean stress theory had to be chosen in the software. This theory affects the results of fatigue loading making it important aspect of the simulations [16]. The studies show that most of the experimental data on fatigue fall between Goodman and Gerber mean stress theories [17]. Therefore, for the fatigue analysis, the Goodman theory was chosen. This theory does not predict the cycles to failure, however it can be used to see if there is a probability for fatigue failure. According to this theory, the endurance of the material drops linearly to 0 as it approaches ultimate strength of the material. This reflects the experimental findings conducted on various materials [16].

In terms of the results generated from simulation, fatigue life, biaxiality indication, fatigue sensitivity and damage tolerance were analysed. Fatigue life shows the number of cycles until the structure fails. Biaxiality coefficient indicates the areas that are subjected to pure bending, pure uniaxial stress and pure biaxial state [18]. This is shown by the variation in the coefficient values between -1 and 1 in the results; the coefficient of -1 indicates pure shear loading, the coefficient of 0 indicates pure uniaxial loading and +1 indicates a state of biaxiality [18]. Fatigue sensitivity produces an S-N curve between applied stress and life of the structure. The applied stress is shown in terms of coefficients where coefficient of 1 indicates the actual applied stress and coefficient of 0.5 indicates half of the stress. The damage tolerance indicates how damaged different areas of the structure are. This allows to see which are the most sensitive parts of octahedral structures.

The quasistatic analysis was also conducted using Ansys software. In this case, the method adopted by L. Meza was used. In order to calculate the stiffness of the metamaterials, the structure was subjected to compressive vertical deformation up to 50% of the original height. Using that, the stress distribution could be generated using equivalent Von Misses stress criterion option in Ansys. The reaction force due to compressive deformation was calculated which was done similar to the technique used by L. Meza [1,19]. Knowing the reaction force and the amount of vertical deformation the stiffness could be calculated. The results of stress distribution and stiffness then were compared to the ones by L. Meza.

(a)

L=4 μm	L=6 μm	L=8 μm
N=4	N=4	N=4
N=6	N=6	N=6
N=8	N=8	N=8
N=10	N=10	N=10

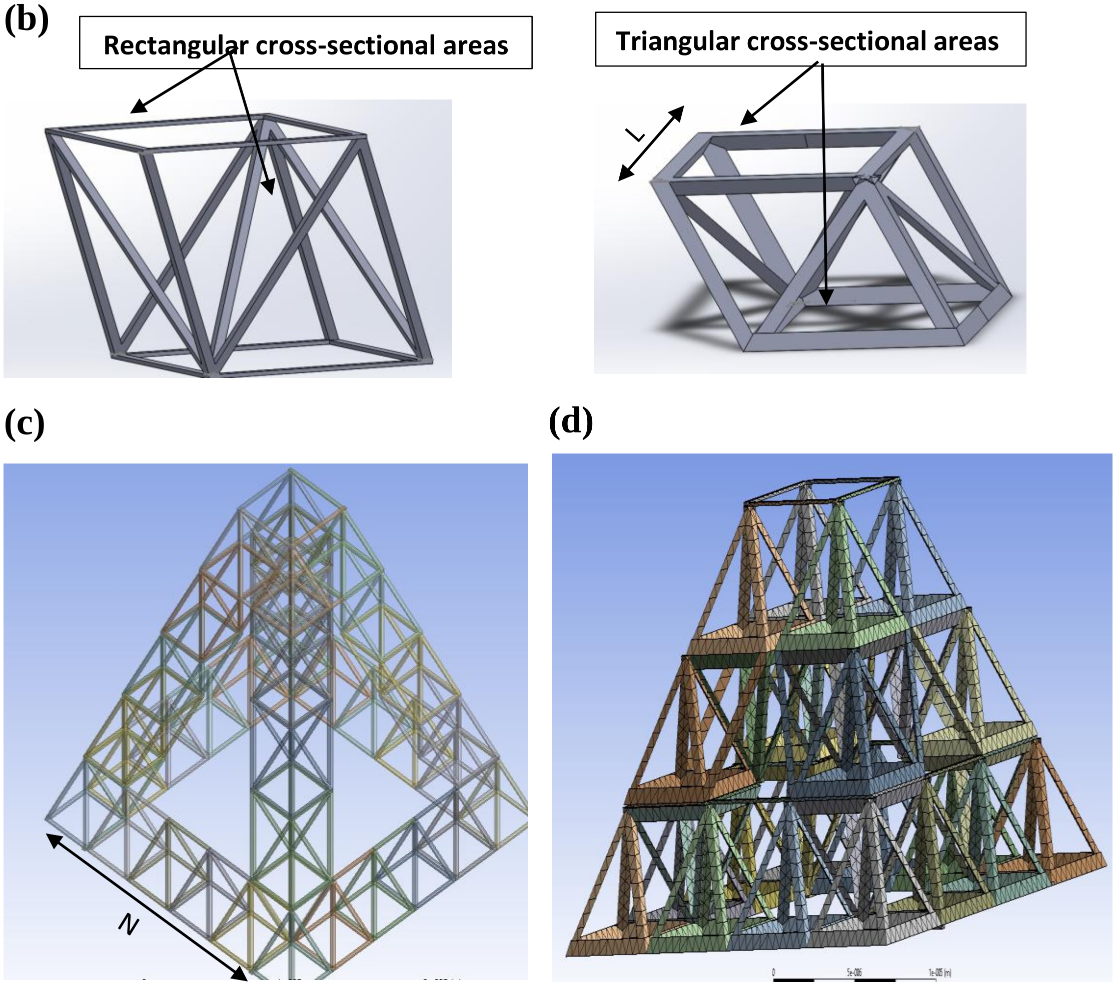


Figure 1. Octahedral unit cells: (a) Octahedral metamaterial configurations with L denoting unit cell length and N denoting the hierarchical scale. **(**b) The unit cell: left- rectangular cross-sectional area, right-triangular cross-sectional area. (c) Assembled metamaterial using identical unit cells. (d) Meshed metamaterial with quadratic tetrahedral mesh cells.

3. Results and Discussion

3.1 Quasistatic analysis in terms of stress distribution and stiffness.

The results of the quasistatic analysis are compared with papers written by L. Meza [1] and A. Saigal [19]. This analysis was conducted in order to generate the results for stiffness and equivalent stress distribution. This analysis is of importance since the above-mentioned researchers analysed their respective metamaterial structures in terms of stiffness and stress distribution, thus allowing to compare and validate the present numerical simulations.

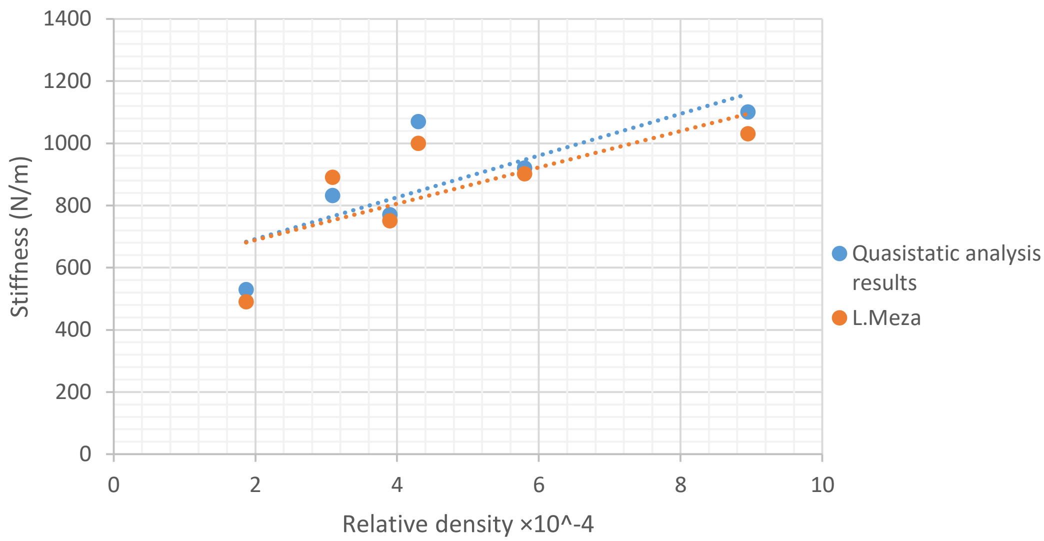
The results of quasistatic analysis were generated using hollow ceramic to compare with the stiffness results by L.Meza. Since, Ansys does not provide the data for ceramic used by L. Meza, it was essential to manually input the mechanical properties. These properties were researched using the references provided in the author's paper [20-26]. The stiffness results were plotted against the relative density of the structures. The relative density is defined as the density of the metamaterial structure relative to the density of the material it is made of. Higher the relative density of the metamaterial, lower the scale of hierarchy which is also stated by L. Meza in his paper [1]. Therefore, by comparing the analysis, it can be seen that the stiffness of the structures increases with increased relative density and reduced scale of hierarchy. In addition, the results by L. Meza are very close to the results obtained in the simulations which validates the quasistatic analysis of stiffness.

In terms of stress distribution, the metamaterial configurations were subjected to uniaxial stress, in order to analyse the stress distribution across the entire octahedron-of-octahedral. The results were compared with the stress distribution conducted by L. Meza. It has been found that the highest stress concentration is located at axially oriented beams. The stress concentration at this location ranged from 55 to 88% of the entire load, which supports the conclusion drawn by L. Meza in his paper where the axially oriented beams supported up to 91 % of the load. This is shown in figure 2b. In addition, the unit cells located at the base of the metamaterial, were largely underused which is similar to metamaterial models designed by L. Meza. An example can be seen in figure 2b.

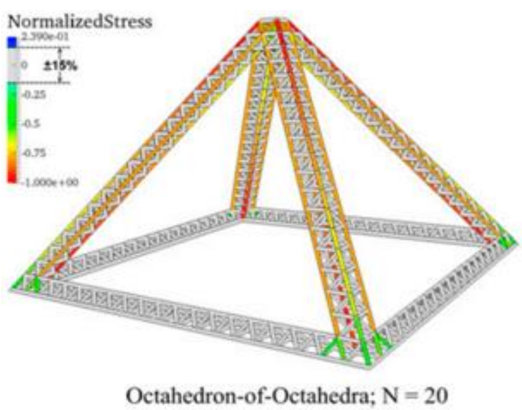
The results of quasistatic analysis were also compared with the research conducted by A. Saigal. Since, Ansys provides extensive data on structural steel, this material was used for the present research. It can be seen that increase in the scale of hierarchy results in higher stiffness which agrees with results of stiffness by A. Saigal. The main difference with this research, is that the research by A. Saigal used acrylate PR25 as the main material for octahedral metamaterial fabrication. This resulted in different values of stiffness.

In addition, the stiffness results were compared with the research conducted by J.J. Berger, where the author used 3rd order hierarchical octahedral metamaterials for analysis in addition to other metamaterials. The results by the author showed an increase in stiffness with increased relative density. This meant, that the structures with lower scale of hierarchy dominated. This further goes to validate the quasistatic analysis.

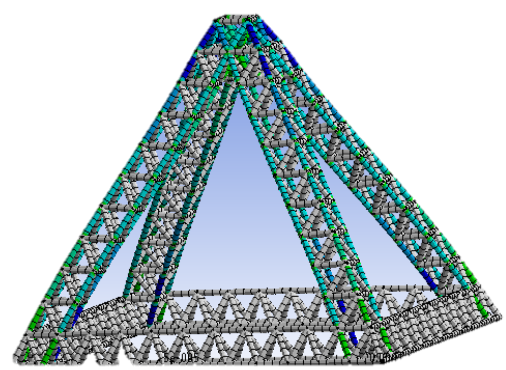
(a)



(b)

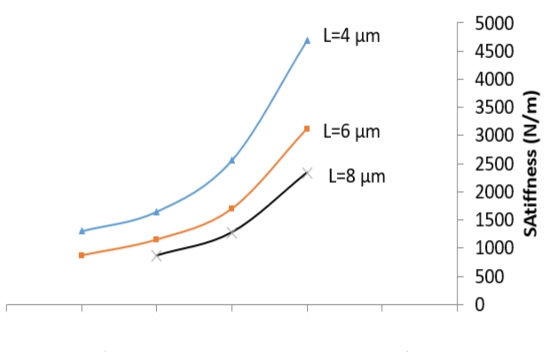


Stress distribution by L. Meza

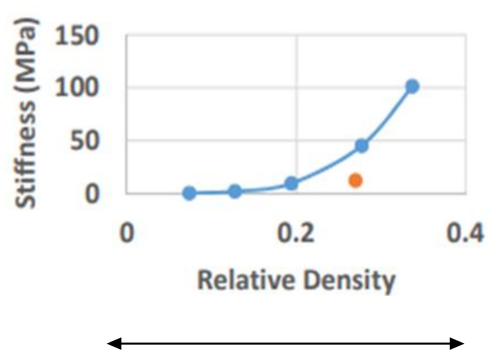


Stress distribution in present study

(c)



Structural steel test material



A. Saigal

Figure 2. a) The results of the stiffness versus relative density from present research and the existing one. b) The results of stress distribution in the metamaterial structures from existing and present research. c) Results of stiffness versus scale of hierarchy for structural steel and A. Saigal results

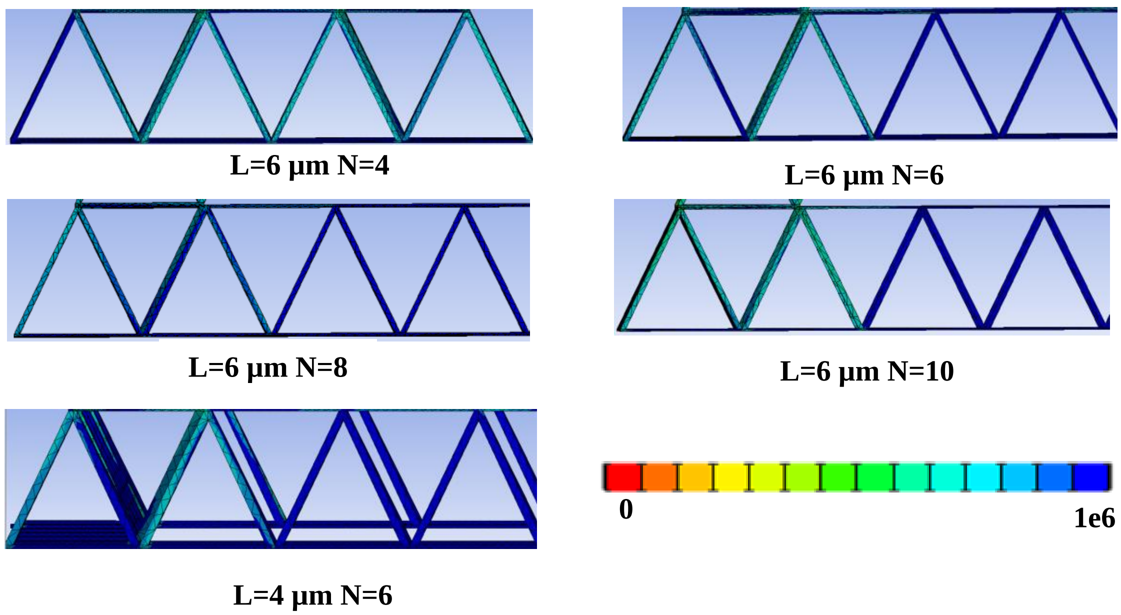
3.2. Fully reversed loading fatigue analysis

The fatigue life of the octahedral structures in the simulation showed unique characteristics in terms of the life of axially oriented beams and the base of the structures. It has been found that the maximum life that metamaterial structures could have at any particular point was 10^6 cycles. This is true for all design configurations. However, the overall life at different sections did vary

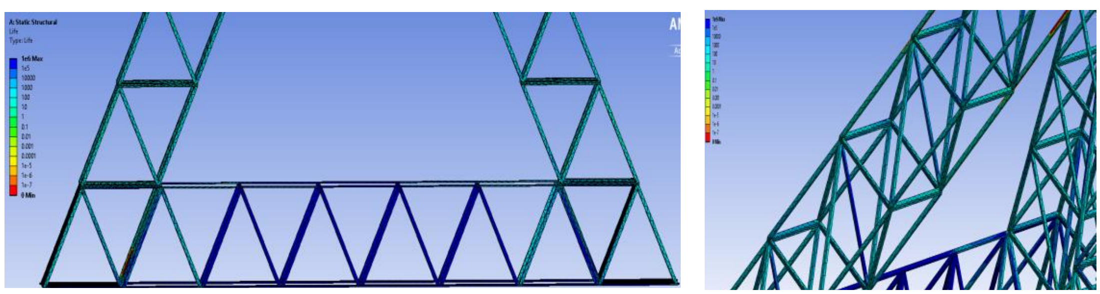
Figure 3 a shows the base of metamaterials that have the same unit cell length of 6 µm, while the scale of hierarchy is varied. It can be seen that increasing the number of unit cells at the base of the structures increases their life cycles, which can potentially indicate that less of the load is distributed around the base, this increasing the stress on the top and the axially oriented beams. If the base unit cells can have more stress concentration, the axially oriented beams would have less load concentration, thus making the metamaterial more durable by being able to hold more stress. This is supported by considering the results of fatigue sensitivity in figure 3 c. It can be seen that increasing the scale of hierarchy decreases the overall life of structures. Therefore, increased number of base unit cells negatively affects the fatigue durability, since more load is concentrated on axially oriented beams rather than the base. By comparing the effects by changing the unit cell lengths in figure 3 a, the stress load on base unit cells is lower with decreased unit cell length. This shows that lower length of the beams, would result in more durable structures. Figure 3 b shows that most of the stress is concentrated on axially oriented beams, thus indicating that, the main failure due to fatigue loading will originate at axially oriented beams while the base cells would be mostly untouched. This is true for all configurations tested. However, it must be noted that increasing the scale of hierarchy from N= 10, resulted on little variations in the fatigue sensitivity results. Therefore, the fatigue durability of octahedral metamaterial of 2nd order does decrease with increased scale of hierarchy to a certain degree, while staying constant on average with large hierarchy scales.

Figure 3 d shows the metamaterial structure with the configuration L=8 µm N=4 at fatigue load of 1 GPa. Blue colour indicates pure shear loading, green indicates the uniaxial state and the red colour indicates the state of pure biaxiality. It can be seen from the figure that most of the structure has a green contour which from the simulations indicates a state of uniaxiality. However, the axially oriented beams in the structure indicate that there are conditions of pure shearing which can cause a crack initiation in a structure. This shows that during fatigue loading the axially oriented beams were subjected pure shearing as well. The slip band intrusion and extrusion are one of the main reasons that can cause crack initiation which leads to fatigue failure [27]. Since the designed structures do not have pre-existing fractures, slip band extrusion and intrusion are the mostly likely cause of fatigue crack initiation in the analysis. This phenomenon occurs due to shearing of planes of atoms on top of one another which can mean that the pure shear loading can cause a crack initiation in the metamaterial structures. This is true for all tested configurations Final analysis that is of an importance is the damage tolerance. This simulation allows to identify which regions in the metamaterial are most vulnerable to fatigue failure. From the simulations it has been found that the regions that receive the most damage are the axially oriented beams and top of the metamaterial structures. This does agree with the findings by L. Meza where it is noted that the axially oriented beams absorb most of the load.

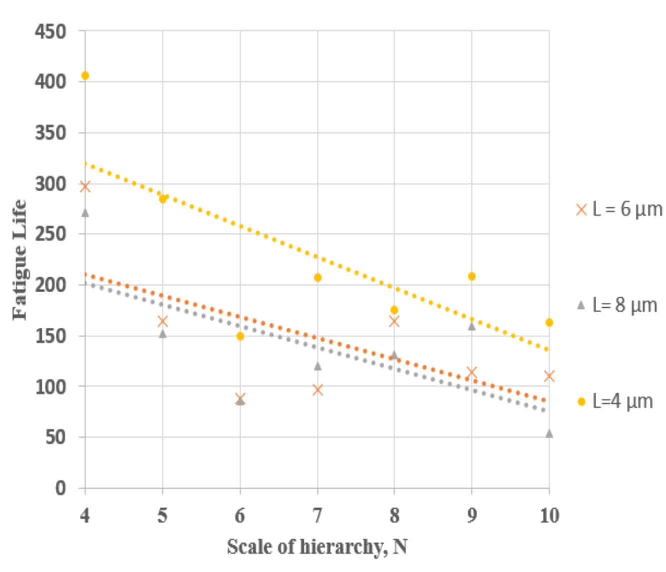
(a)


L=6 µm N=4
L=6 µm N=6
L=6 µm N=8
L=6 µm N=10
L=4 µm N=6
0
1e6

(b)



(c)


Fatigue Life
450
400
350
300
250
200
150
100
50
0
4
5
6
7
8
9
10
Scale of hierarchy, N
L = 6 µm
L= 8 µm
L=4 µm

(d)

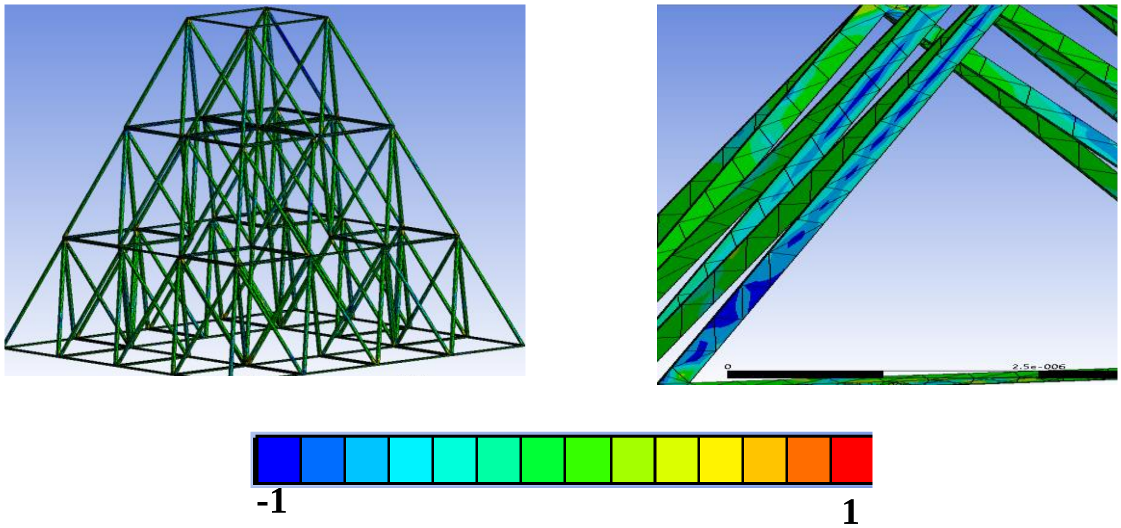
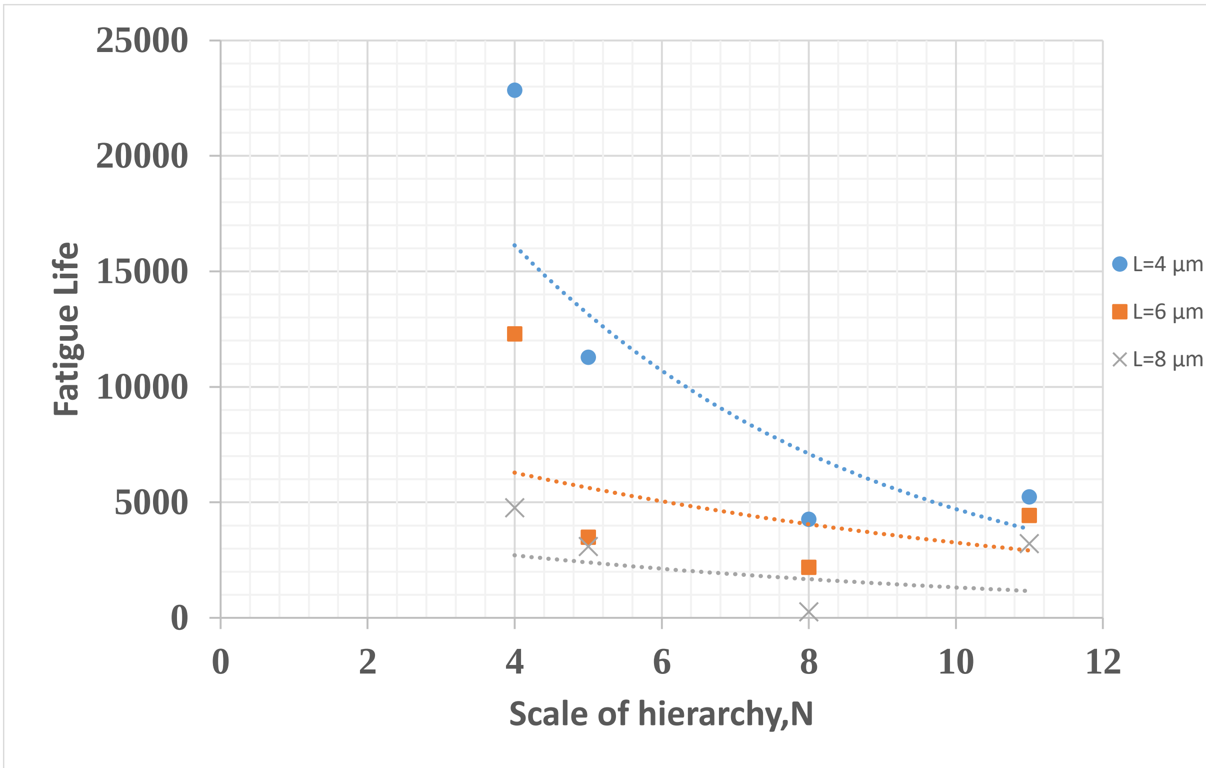


Figure 3. a) Life of the base unit cells for metamaterials with unit cell lengths at 4 µm and 6µm. b) Life of the axially oriented beams under high fatigue load. c) Fatigue sensitivity results for structural steel for all configurations. d) Biaxiality indication for a metamaterial with configuration L=8 µm N=4

3.3. Zero-based loading fatigue analysis

The metamaterial configurations were also tested for zero-based load conditions. This for undertaken to see if there is a difference in fatigue behaviour between configurations when the alternating load ratio was changed. The results in figure 4 a show the fatigue life cycles of structures with increase scale of hierarchy under 350 MPA compression. At 1GPa the life of the structures dropped to 0. This showed that the stress ratio does affect the results of fatigue durability to a certain extent. As a result, the applied load was decreased in order to compare fatigue life results for different metamaterial configurations. The applied load was reduced to 350 MPa. It can be seen from figure 4 a, that increase in scale of hierarchy results in drop of fatigue durability. However, as with fully-reversed loading, with hierarchy above N=10, the results of fatigue life did not vary considerably and the difference was considerably small. This is supported by comparing the results of fatigue stress simulations in figure 4 b. Metamaterials with configurations L= 6 µm N=4 and L= 8 µm N= 4 were subjected to stress at 1GPa. The figure shows that while the structure with unit cell length of 6 µm did not sustain a significant damage, the metamaterial with the unit cell length of 8 µm completely failed at axially oriented beams while the base cells remained intact.

(a)



(b)

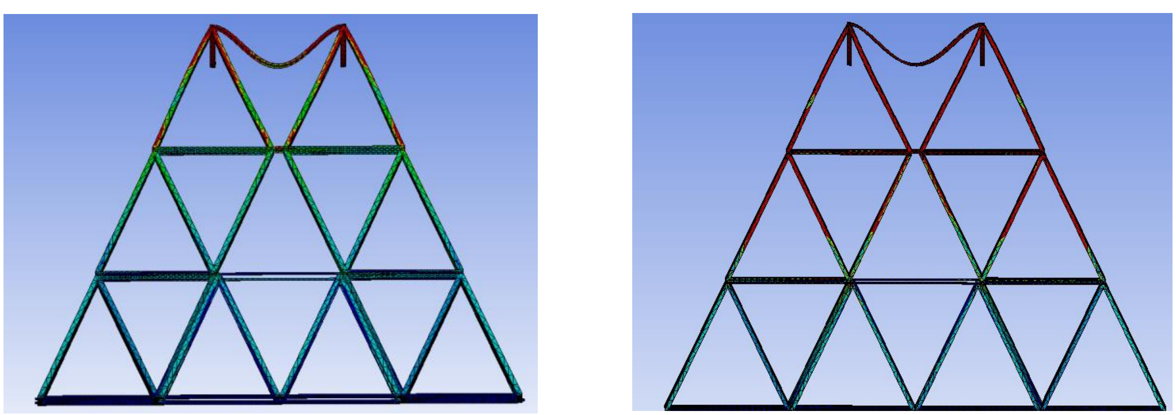


Figure 4. a) The results of the fatigue sensitivity for structural steel. b) Fatigue stress simulation of metamaterials with configurations L= 6 μm N=4 and L= 8 μm N= 4.

3.4. Effect of cross-sectional area change

The effect of increased unit cell length and the scale of hierarchy has been analysed so far. However, during the design of unit cells, the cross sectional area had to be accounted for. As such, it was decided to compare cross-sectional areas, to analyse the effect. For a different cross-sectional area, the triangular shape was chosen. In order to see, how the cross-sectional shape affects the results, the total area of the cross-section was kept to 0.025 µm similar to the rectangular cross-section. This would allow to analyse the effect of the cross-sectional shape without changing the value of the area.

Figure 5 a shows the results of fatigue life analysis for metamaterial structures with the unit cell length at 6 µm, while the comparison is shown between the shapes of the cross-section. The analysis was conducted under fully cyclic load at 100 MPa. It can be seen that changing the shape of the cross section has a dramatic effect on the results of the simulation. The drop in the life cycle from rectangular to triangular cross-section is significant, differing by a factor of 100. This clearly shows that during the design of the metamaterials, the shape of the cross-section has to be accounted for. This can be explained stress distribution relative to the cross-sectional areas. For a rectangular shape, the stress-distribution is more stable due to even distribution of stress between 4 corners. However, the stress concentration on triangular shapes is concentrated on three points, thus decreasing the overall life cycle of the metamaterials. In addition, it can be noted that the change in the shape of the structures does not affect fatigue behaviour of structures with respect to hierarchy scaling; the fatigue life still decreases with increased order of hierarchy.

Figure 5 b shows, the effect of fatigue loading on the axially oriented breams which does concur with the results mentioned in previous sections. The structures were subjected to stress of 1 GPa. It can be seen that the stress distribution on axially oriented beams is similar when comparing two different metamaterials. Hence the shape of the cross-sections does not have noticeable effect on the fatigue behaviour of axially oriented beams. One significant point to note is that the top section of the metamaterials was modified compared to previous sections of this report. This was done for both rectangle and triangle, for ease of comparison. This can be seen in figure 5 c.

In addition, to above analysis, the change in the cross-sectional area was analysed in terms of the overall area value. The shape of the cross-section was kept constant, while the area was varied. The analyst was conducted for rectangular cross sections for two different values, i.e. A= 0.0625µm and A=0.1225 µm. The unit cell length was kept to 6 µm. It can be seen from the figure that the small increase in the cross-sectional area of the structures, has a dramatic effect on the fatigue life of the structures. With increased area, the life drops which makes the structures with higher cross-sectional areas less durable to fatigue loading.

The metamaterial with higher cross-sectional area has an increased life cycle at scale of hierarchy equal to 8, relative to hierarchy at scale 6. Such deviation can be possible due to uncertainties in numerical simulations, however the increase of the scale to 10 decreases the life to 0. This shows that the life cycle of structures still drops dramatically, despite an increase at scale 8.

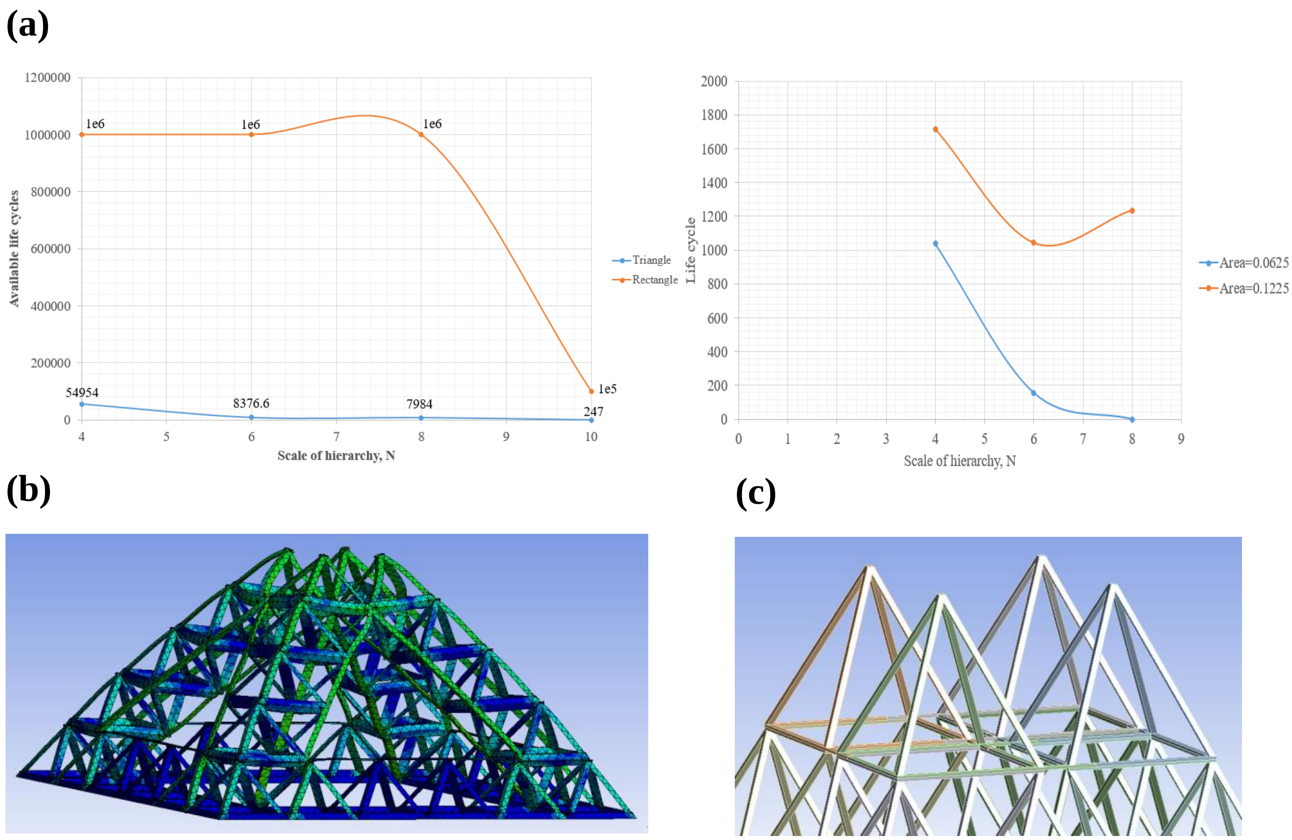


Figure 5. a) The results of fatigue life of metamaterials with the same vale of cross-sectional area but varied shape. b) Results of fatigue life for metamaterials with the same shape of the cross section but different area values.

4. Conclusion

In the present study, 2nd order octahedral metamaterials were analysed by changing the scale of their hierarchy. The numerical analysis was conducted in order to see the effect of fatigue on these structures. Throughout the process, the stress ratio, the material and the scale of hierarchy was changed. In addition, the cross-sectional area of unit cells was investigated to observe its impact on the overall fatigue behaviour. As a result, several conclusions were drawn out:

1. The increase in the scale of hierarchy resulted in the reduction of fatigue life of all metamaterials. However, the drop rate in fatigue durability would decrease with higher scale of hierarchy
2. Increase in unit cell length decreased the fatigue life of structures, increasing the overall damage of metamaterials with high hierarchy.
3. The reduction in stress ratio, increased the impact of unit cell length on fatigue behaviour by several orders of magnitude.
4. The increase in cross-sectional area of unit cells had a dramatic effect on the fatigue life of structures, resulting in less durable metamaterials against fatigue loading.
5. The change in the shape of the cross-section, while keeping the same area value of the beams, reduced the fatigue durability of metamaterial.

To design optimal octahedral metamaterials, the unit cell length has to be reduced, the amplitude of the fatigue load has to be decreased, the cross –sectional area of the beams making the entire metamaterial has to be decreased and the shape of the cross-sections needs to be accounted for. Future work has to be conducted in this field to account for more octahedral metamaterial configurations and their fatigue behaviours.

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