

Analysis of the 2016 US Presidential Election

The data for this analysis come from Tony McGovern. The response variable, Y_i , is the percentage change in Republican (GOP) support from 2012 to 2016, i.e.,

$$100 \left(\frac{\% \text{ in 2016}}{\% \text{ in 2012}} - 1 \right),$$

in county $i = 1, \dots, n$.

The $p = 10$ covariates X_{ij} are county-level census variables obtained from Kaggle are:

Population, percent change - April 1, 2010 to July 1, 2014

Persons 65 years and over, percent, 2014

Black or African American alone, percent, 2014

Hispanic or Latino, percent, 2014

High school graduate or higher, percent of persons age 25+, 2009-2013

Bachelor's degree or higher, percent of persons age 25+, 2009-2013

Homeownership rate, 2009-2013

Median value of owner-occupied housing units, 2009-2013

Median household income, 2009-2013

Persons below poverty level, percent, 2009-2013

For a county in state s , we assume the linear model

$$Y_i = \beta_{0s} + \sum_{j=1}^p X_{ij} \beta_{sj} + \varepsilon_i,$$

where β_{js} is the effect of covariate j in state s . We compare three models for the β_{js} .

1. Constant slopes: $\beta_{js} = \beta_j$ for all counties.
2. Varying slopes with uninformative priors: $\beta_{js} \sim \text{Normal}(0, 100)$
3. Varying slopes with informative priors: $\beta_{js} \sim \text{Normal}(\mu_j, \sigma_j^2)$.

In the third model, the means μ_j and variances σ_j^2 are given prior and estimated from the data, therefore information is pooled across states via the prior. The three methods are compared using DIC, and final results are compared across models.

```
library(choroplethr)
```

```
## Loading required package: acs
```

```
## Loading required package: stringr
```

```
## Loading required package: XML
```

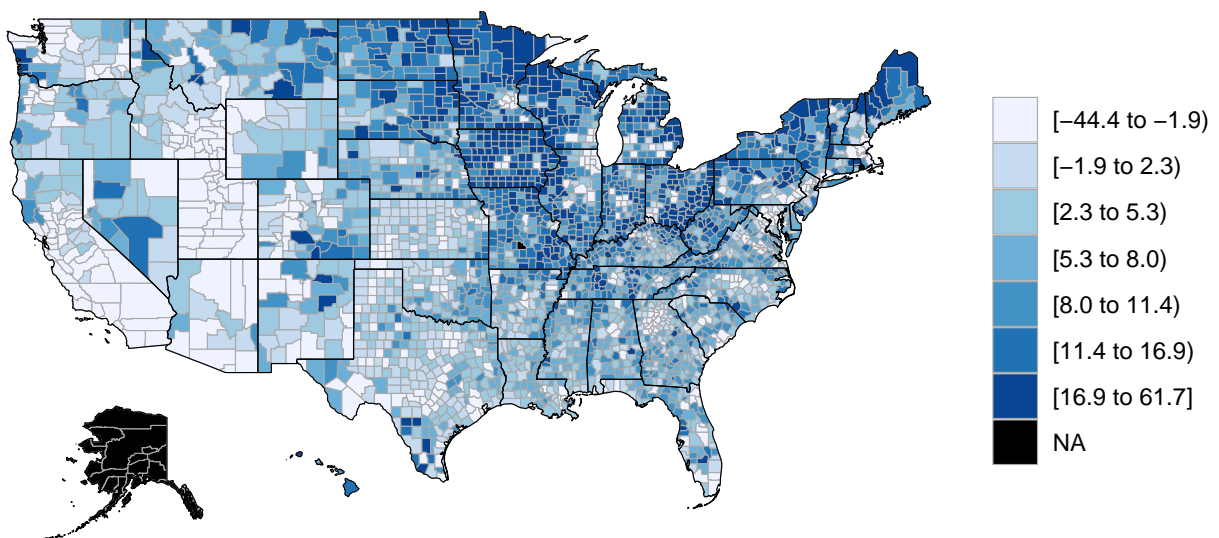
```
##
## Attaching package: 'acs'

## The following object is masked from 'package:base':
##
##      apply

load("/Users/kamaladadashova/Documents/DoctoralCourses/Applied Bayesian Statistics/Lecture Notes with A
X      <- scale(X)      # Standardize the covariates
X      <- cbind(1,X)
short <- c("Intercept", "Pop change", "65+", "African American",
           "Hispanic", "HS grad", "Bachelor's",
           "Homeownership rate", "Home value",
           "Median income", "Poverty")
names <- c("Intercept", as.character(names[1:11,2]))
colnames(X) <- short

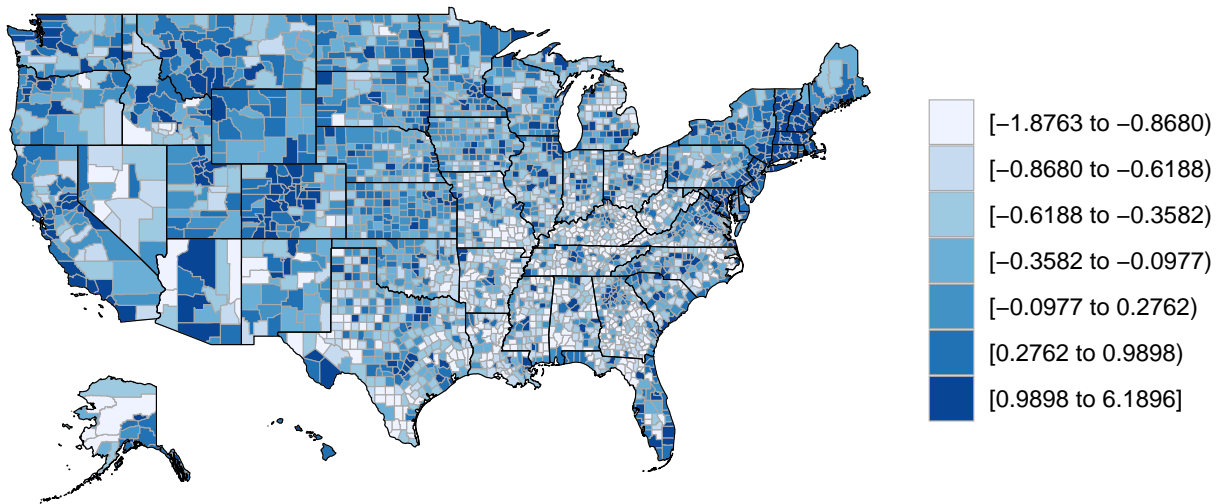
# Define a function to make county maps
county_plot <- function(fips,Y,main="",units=""){
  library(choroplethr)
  temp <- as.data.frame(list(region=fips,value=Y))
  suppressWarnings(county_choropleth(temp,title=main,legend=units))
}
county_plot(fips,Y,main="Percent increase in GOP support",units="")
```

Percent increase in GOP support



```
county_plot(fips,X[,7],main=names[7],units="")
```

Bachelor's degree or higher, percent of persons age 25+, 2009–2013



```
# Remove AK, HI and DC due to missing data
state <- as.character(all_dat[,3])
AKHI <- state=="AK" | state=="HI" | state=="DC"
fips <- fips[!AKHI]
Y <- Y[!AKHI]
X <- X[!AKHI,]
state <- state[!AKHI]

# Assign a numeric id to the counties in each state
st <- unique(state)
id <- rep(NA,length(Y))
for(j in 1:48){
  id[state==st[j]]<-j
}

n <- length(Y) # number of counties
N <- 48 # number of states
p <- ncol(X) # number of covariates

set.seed(0820)
iters <- 50000
burn <- 10000
```

Constant slopes

```
modell1_string <- "model{  
  
  # Likelihood  
  for(i in 1:n){  
    Y[i] ~ dnorm(mu[i],taue)  
    mu[i] <- inprod(X[i,],beta[])  
  }  
  # Priors  
  for(j in 1:p){beta[j] ~ dnorm(0,0.01)}  
  taue ~ dgamma(0.1,0.1)  
  sig <- 1/sqrt(taue)  
  
  # WAIC calculations  
  for(i in 1:n){  
    like[i] <- dnorm(Y[i],mu[i],taue)  
  }  
}"
```

```
library(rjags)
```

```
## Loading required package: coda
```

```
## Linked to JAGS 4.3.1
```

```
## Loaded modules: basemod,bugs
```

```
# Load the model  
dat <- list(Y=Y,n=n,X=X,p=p)  
init <- list(beta=rep(0,p))  
modell1 <- jags.model(textConnection(modell1_string),n.chains=2,  
                      inits=init,data = dat,quiet=TRUE)  
  
# Generate samples  
update(modell1, burn, progress.bar="none")  
samp1 <- coda.samples(modell1,  
                      variable.names="beta",  
                      n.iter=iters, progress.bar="none")  
  
# Compile results  
ESS1 <- effectiveSize(samp1)  
out1 <- summary(samp1)$quantiles  
rownames(out1)<-short  
  
# Compute DIC  
dic1 <- dic.samples(modell1,n.iter=iters,progress.bar="none")  
  
# Compute WAIC  
waic1 <- coda.samples(modell1,  
                      variable.names=c("like"),  
                      n.iter=iters, progress.bar="none")  
like1 <- waic1[[1]]
```

```
fbar1 <- colMeans(like1)
P1 <- sum(base::apply(log(like1),2,var))
WAIC1 <- -2*sum(log(fbar1))+2*P1
```

Slopes as fixed effects

```
model2_string <- "model{

  # Likelihood
  for(i in 1:n){
    Y[i] ~ dnorm(mnY[i],taue)
    mnY[i] <- inprod(X[i,],beta[id[i],])
  }

  # Slopes
  for(j in 1:p){for(i in 1:N){
    beta[i,j] ~ dnorm(0,0.01)
  }}

  # Priors
  taue ~ dgamma(0.1,0.1)

  # WAIC calculations
  for(i in 1:n){
    like[i] <- dnorm(Y[i],mnY[i],taue)
  }
}"

library(rjags)

# Load the model
dat <- list(Y=Y,n=n,N=N,X=X,p=p,id=id)
init <- list(beta=matrix(0,N,p))
model2 <- jags.model(textConnection(model2_string),n.chains=2,
  inits=init,data = dat,quiet=TRUE)

# Generate samples
update(model2, burn, progress.bar="none")
samp2 <- coda.samples(model2,
  variable.names="beta",
  n.iter=iters, progress.bar="none")

# Compile results
ESS2 <- effectiveSize(samp2)
sum <- summary(samp2)$stat
post_mn2 <- matrix(sum[,1],N,p)
post_sd2 <- matrix(sum[,2],N,p)

# Compute DIC
```

```

dic2    <- dic.samples(model2,n.iter=iters,progress.bar="none")

# Compute WAIC
waic2   <- coda.samples(model2,
  variable.names=c("like"),
  n.iter=iters, progress.bar="none")
like2   <- waic2[[1]]
fbar2   <- colMeans(like2)
P2      <- sum(base::apply(log(like2),2,var))
WAIC2   <- -2*sum(log(fbar2))+2*P2

```

Slopes as random effects

```

model3_string <- "model{

  # Likelihood
  for(i in 1:n){
    Y[i]      ~ dnorm(mnY[i],taue)
    mnY[i] <- inprod(X[i,],beta[id[i],])
  }

  # Random slopes
  for(j in 1:p){
    for(i in 1:N){
      beta[i,j] ~ dnorm(mu[j],taub[j])
    }
    mu[j]      ~ dnorm(0,0.01)
    taub[j]    ~ dgamma(0.1,0.1)
  }

  # Priors
  taue ~ dgamma(0.1,0.1)

  # WAIC calculations
  for(i in 1:n){
    like[i]    <- dnorm(Y[i],mnY[i],taue)
  }
}"

library(rjags)

# Load the model
dat    <- list(Y=Y,n=n,N=N,X=X,p=p,id=id)
init   <- list(beta=matrix(0,N,p))
model3 <- jags.model(textConnection(model3_string),n.chains=2,
  inits=init,data = dat,quiet=TRUE)

# Generate samples
update(model3, burn, progress.bar="none")

```

```
samp3 <- coda.samples(model3,
  variable.names="beta",
  n.iter=iters, progress.bar="none")

# Compile results
ESS3 <- effectiveSize(samp3)
sum <- summary(samp3)$stat
post_mn3 <- matrix(sum[,1],N,p)
post_sd3 <- matrix(sum[,2],N,p)

# Compute DIC
dic3 <- dic.samples(model3,n.iter=iters,progress.bar="none")

# Compute WAIC
waic3 <- coda.samples(model3,
  variable.names=c("like"),
  n.iter=iters, progress.bar="none")
like3 <- waic3[[1]]
fbar3 <- colMeans(like3)
P3 <- sum(base::apply(log(like3),2,var))
WAIC3 <- -2*sum(log(fbar3))+2*P3
```

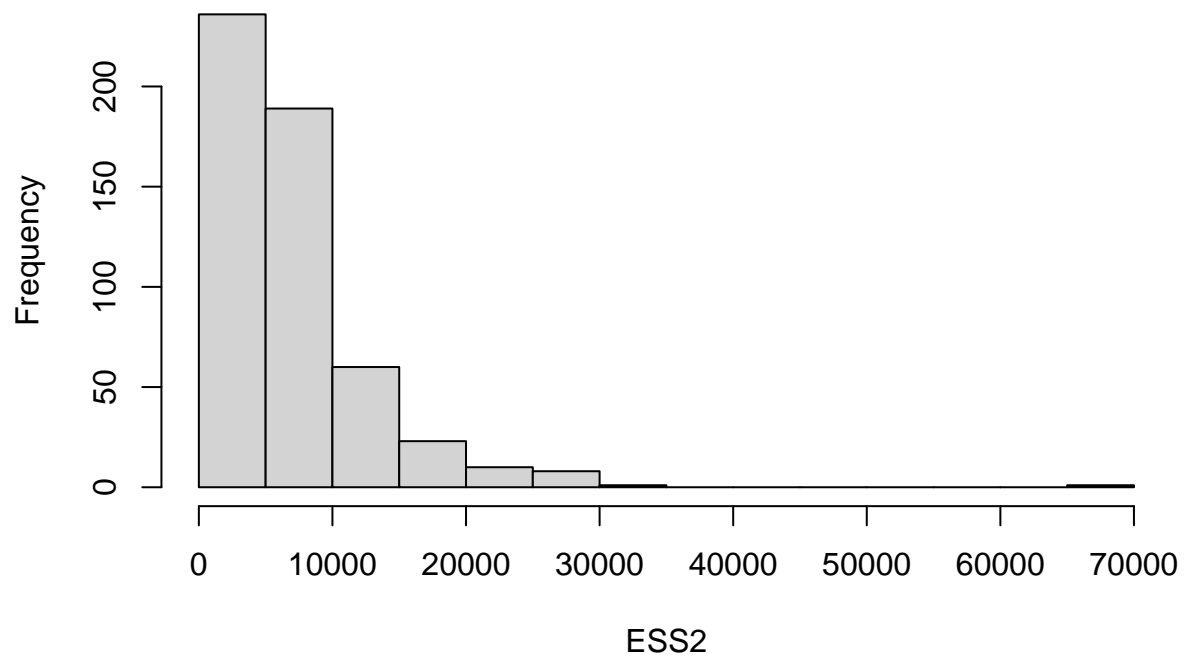
Check convergence

ESS1

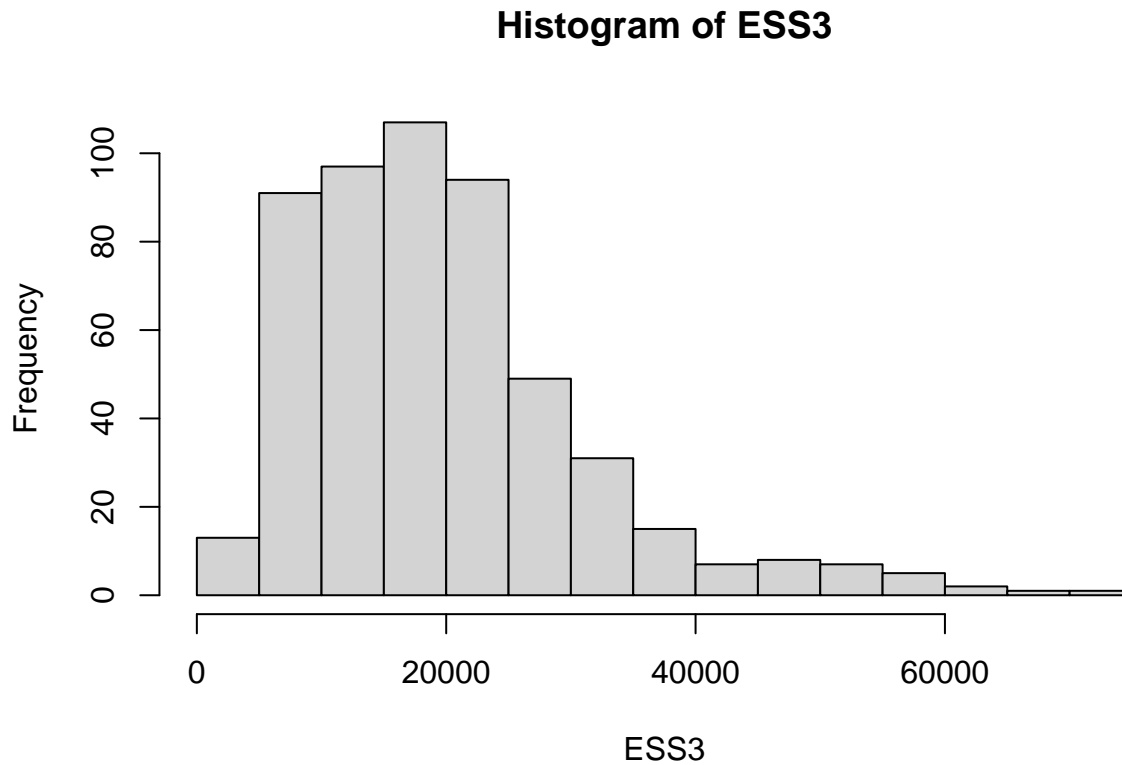
```
##      beta[1]      beta[2]      beta[3]      beta[4]      beta[5]      beta[6]      beta[7]
## 100000.000  54783.888  18098.126  41377.224  29620.781  16623.860  16510.458
##      beta[8]      beta[9]      beta[10]      beta[11]
##   23658.552   17431.601    8569.037   11775.625
```

`hist(ESS2)`

Histogram of ESS2



```
hist(ESS3)
```

Summary: The effective sample size is large for all parameters and all models, therefore it seems the MCMC algorithm has converged.

Summarize the non-spatial model

```
library(kableExtra)
kbl(round(out1,2))
```

	2.5%	25%	50%	75%	97.5%
Intercept	6.41	6.58	6.67	6.76	6.93
Pop change	-1.45	-1.25	-1.14	-1.03	-0.82
65+	0.53	0.79	0.93	1.06	1.32
African American	-1.89	-1.67	-1.56	-1.45	-1.23
Hispanic	-2.40	-2.18	-2.06	-1.95	-1.73
HS grad	1.25	1.58	1.75	1.92	2.25
Bachelor's	-6.72	-6.37	-6.19	-6.01	-5.67
Homeownership rate	-0.38	-0.12	0.01	0.15	0.40
Home value	-1.98	-1.68	-1.52	-1.36	-1.05
Median income	1.13	1.62	1.87	2.13	2.61
Poverty	0.91	1.28	1.47	1.67	2.04

Summary: All but one (home ownership rate) of the covariates have 95% interval that excludes zero. GOP support tended to increase in counties with

decreasing population high proportion of seniors and high school graduates low proportions of African Americans and Hispanics High income but low home value High poverty rate

Compare models with DIC

```
dic1
```

```
## Mean deviance: 21300
## penalty 11.99
## Penalized deviance: 21312
```

```
dic2
```

```
## Mean deviance: 18484
## penalty 455.6
## Penalized deviance: 18939
```

```
dic3
```

```
## Mean deviance: 18604
## penalty 238.1
## Penalized deviance: 18842
```

Summary: The first model with constant slopes is the simplest but fits the data poorly and thus has highest DIC. The second model with different slopes in each state has the best fit (smallest mean deviance), but is too complicated and has large p_D . The final model has fairly small mean deviance and p_D , and thus balances fit and complexity to give the smallest DIC.

Compare models with WAIC

```
WAIC1; P1
```

```
## [1] 21334.81
## [1] 20.00944
```

```
WAIC2; P2
```

```
## [1] 18969.98
## [1] 405.1366
```

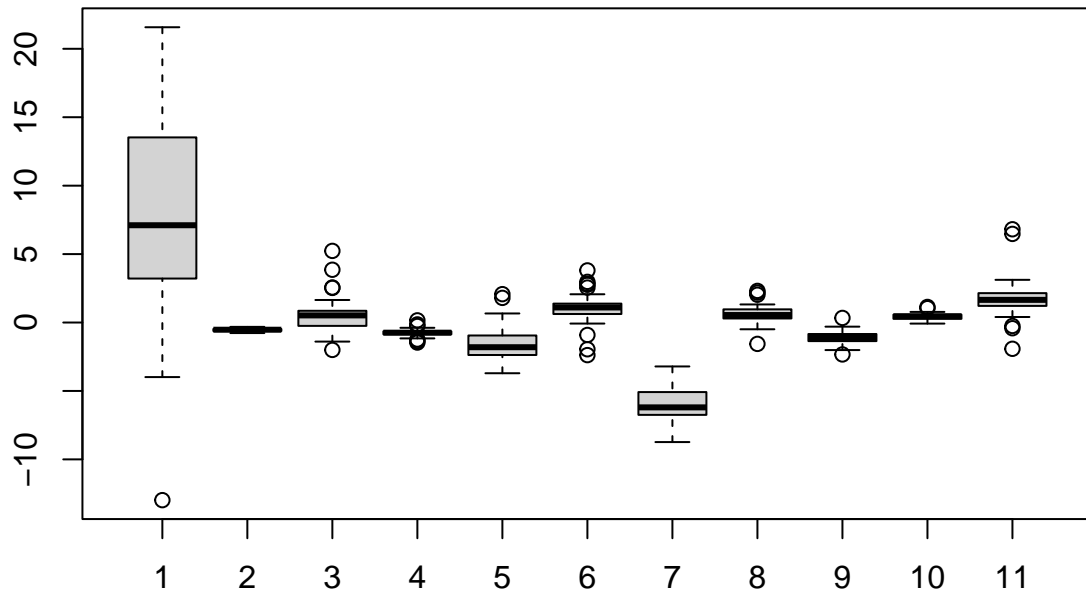
```
WAIC3; P3
```

```
## [1] 18909.09
## [1] 259.9894
```

Summary: WAIC agrees with DIC. Both prefer Model 3 with the regression coefficients treated as random effects.

Explore the results of the final model

```
boxplot(post_mn3)
```

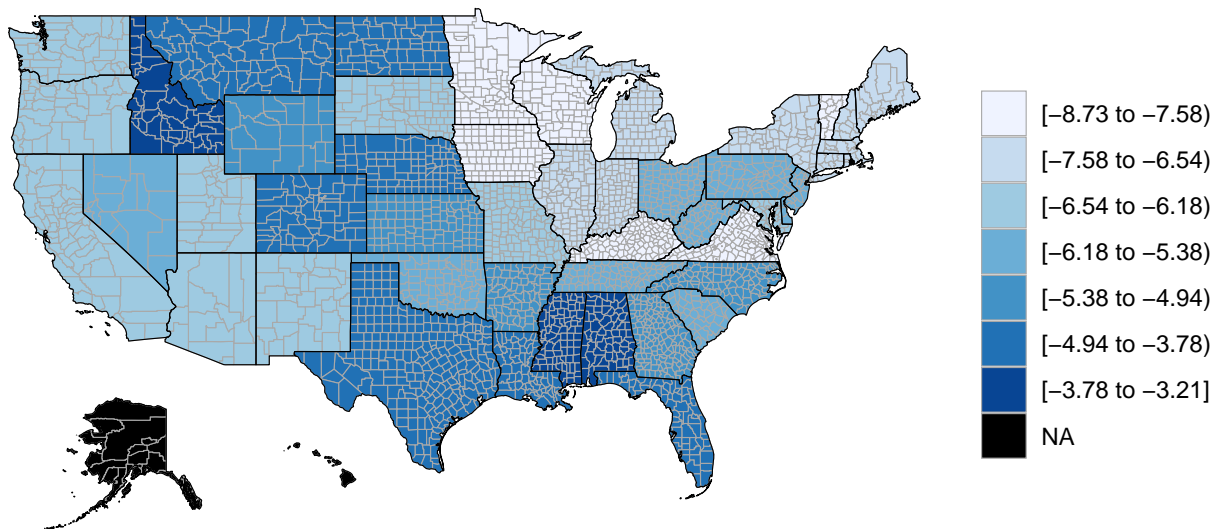


Summary: The effect of the proportion of college graduates varies the most across states

Explore the three estimate of the effects of college graduates

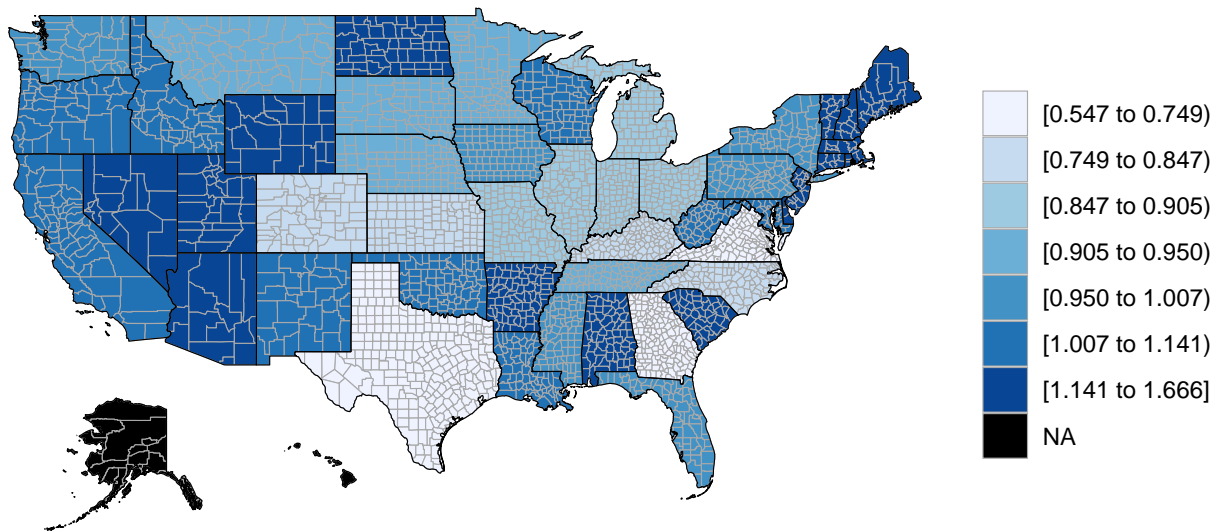
```
# Posterior mean
county_plot(fips, post_mn3[id,7],
            main="Proportion of college graduates - posterior mean")
```

Proportion of college graduates – posterior mean



```
# Posterior sd  
county_plot(fips,post_sd3[id,7],  
main="Proportion of college graduates - posterior SD")
```

Proportion of college graduates – posterior SD



Summary: The effect of the proportion of college graduates is the strongest (most negative) in the midwest.