

# Logistic regression for NBA clutch free throws

## Generalized linear models

The NBA clutch free throws data set has three variables for player  $i = 1, \dots, 10$ .

1.  $Y_i$  is the number clutch free throws made.
2.  $N_i$  is the number clutch free throws attempted
3.  $q_i$  is the proportion of the non-clutch free throws made.

We model these data as

$$Y_i \sim \text{Binomial}(N_i, p_i),$$

where  $p_i$  is the true probability of making a clutch shot. The objective is to explore the relationship between clutch and overall percentages,  $p_i$  and  $q_i$ . We do this using two logistic regression models:

$$\text{logit}(p_i) = \beta_1 + \beta_2 \text{logit}(q_i)$$

$$\text{logit}(p_i) = \beta_1 + \text{logit}(q_i)$$

In both models, we select uninformative priors  $\beta_j \sim \text{Normal}(0, 10^2)$ .

In the first model,  $p_i = q_i$  if  $\beta_1 = 0$  and  $\beta_2 = 1$ . In the second model,  $p_i = q_i$  if  $\beta_1 = 0$ . Therefore, we compare the posteriors of the  $\beta_j$  to these values to analyze the relationship between  $p_i$  and  $q_i$ .

## Load the Data

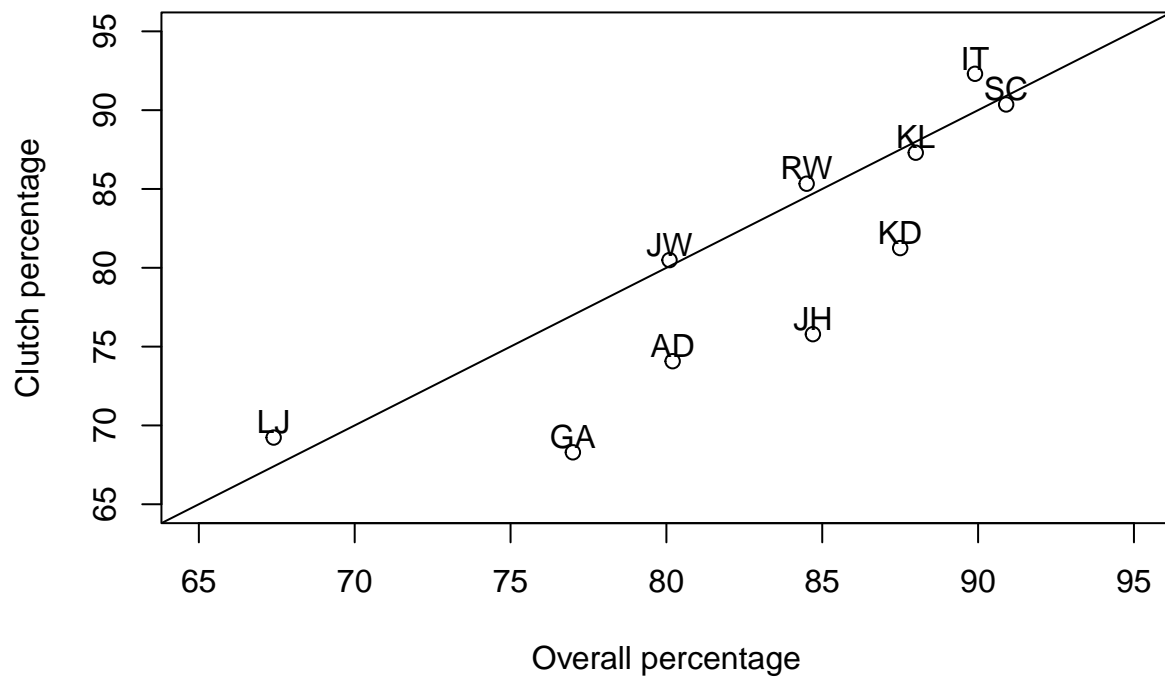
```
set.seed(0820)

Y = c(64, 72, 55, 27, 75, 24, 28, 66, 40, 13)
N = c(75, 95, 63, 39, 83, 26, 41, 82, 54, 16)
q = c(0.845, 0.847, 0.880, 0.674, 0.909, 0.899, 0.770, 0.801, 0.802, 0.875)

X = log(q)-log(1-q) # X = logit(q)
```

## Plot the Data

```
inits = c("RW", "JH", "KL", "LJ", "SC", "IT", "GA", "JW", "AD", "KD")
plot(100*q, 100*Y/N,
     xlim=100*c(0.65, 0.95), ylim=100*c(0.65, 0.95),
     xlab="Overall percentage", ylab="Clutch percentage")
text(100*q, 100*Y/N+1, inits)
abline(0, 1)
```



## Fit the first model in JAGS

```
library(rjags)

## Loading required package: coda

## Linked to JAGS 4.3.1

## Loaded modules: basemod,bugs

data  = list(Y=Y,N=N,X=X)
params = c("beta")

model_string = textConnection("model{
  # Likelihood
  for(i in 1:10){
    Y[i]      ~ dbinom(p[i],N[i])
    logit(p[i]) = beta[1] + beta[2]*X[i]
  }
  # Priors
  beta[1] ~ dnorm(0,0.01)
  beta[2] ~ dnorm(0,0.01)
}
```

```

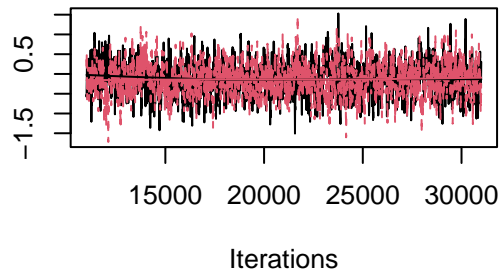
})

model = jags.model(model_string,data = data, n.chains=2,quiet=TRUE)
update(model, 10000, progress.bar="none")
samples1 = coda.samples(model, variable.names=params, thin=5, n.iter=20000, progress.bar="none")

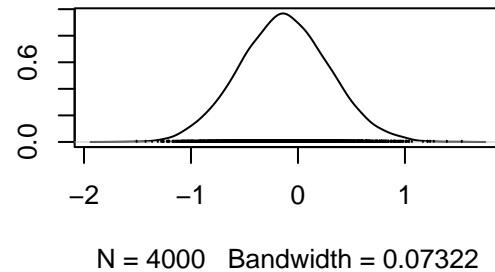
plot(samples1)

```

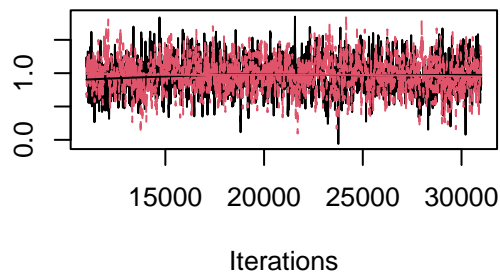
**Trace of beta[1]**



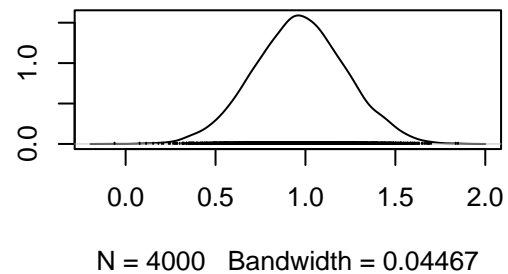
**Density of beta[1]**



**Trace of beta[2]**



**Density of beta[2]**



```
summary(samples1)
```

```

##
## Iterations = 11005:31000
## Thinning interval = 5
## Number of chains = 2
## Sample size per chain = 4000
##
## 1. Empirical mean and standard deviation for each variable,
##    plus standard error of the mean:
##
##           Mean      SD Naive SE Time-series SE
## beta[1] -0.1276 0.4168 0.004660      0.013679
## beta[2]  0.9696 0.2544 0.002844      0.008418
##
## 2. Quantiles for each variable:
##

```

```
##           2.5%    25%    50%    75%  97.5%
## beta[1] -0.9357 -0.4101 -0.1297 0.1506 0.6993
## beta[2]  0.4732  0.7983  0.9695 1.1390 1.4717
```

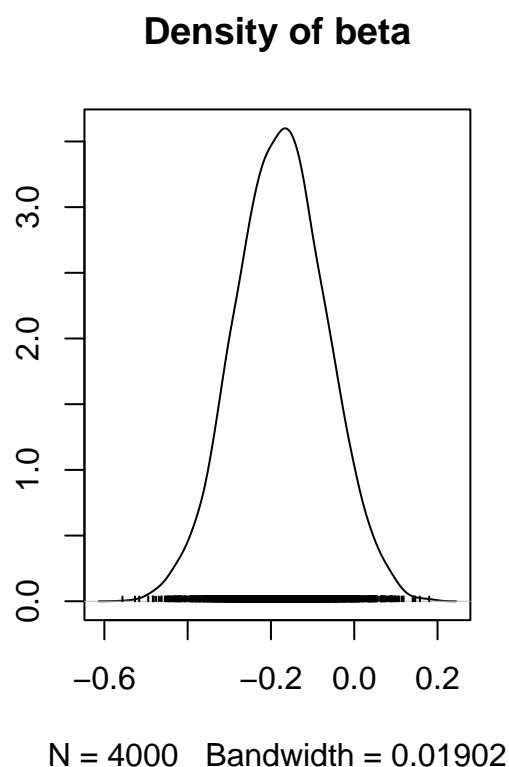
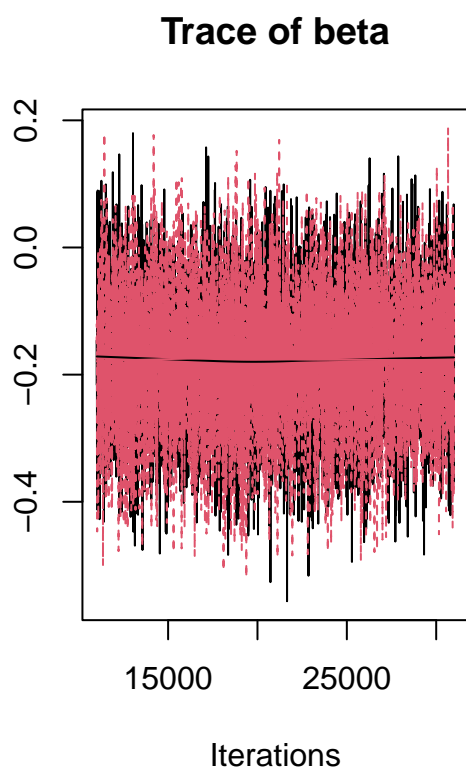
```
b1 = c(samples1[[1]][,1],samples1[[2]][,1])
b2 = c(samples1[[1]][,2],samples1[[2]][,2])
```

## Fit the second model in JAGS

```
model_string = textConnection("model{
  # Likelihood
  for(i in 1:10){
    Y[i] ~ dbinom(p[i],N[i])
    logit(p[i]) = beta + X[i]
  }
  # Priors
  beta ~ dnorm(0,0.01)
}")

model = jags.model(model_string,data = data, n.chains=2,quiet=TRUE)
update(model, 10000, progress.bar="none")
samples2 = coda.samples(model, variable.names=params, thin=5, n.iter=20000, progress.bar="none")
b3 = c(samples2[[1]],samples2[[2]])

plot(samples2)
```



```
summary(samples2)
```

```
##
## Iterations = 11005:31000
## Thinning interval = 5
## Number of chains = 2
## Sample size per chain = 4000
##
## 1. Empirical mean and standard deviation for each variable,
##    plus standard error of the mean:
##
##      Mean          SD      Naive SE Time-series SE
##      -0.17629      0.10825      0.00121      0.00121
##
## 2. Quantiles for each variable:
##
##      2.5%      25%      50%      75%      97.5%
##      -0.38918 -0.25018 -0.17578 -0.10319  0.03737
```

Plot the posterior densities from both models

```
d1 = density(b1,from=-1,to=2)
d2 = density(b2,from=-1,to=2)
```

```

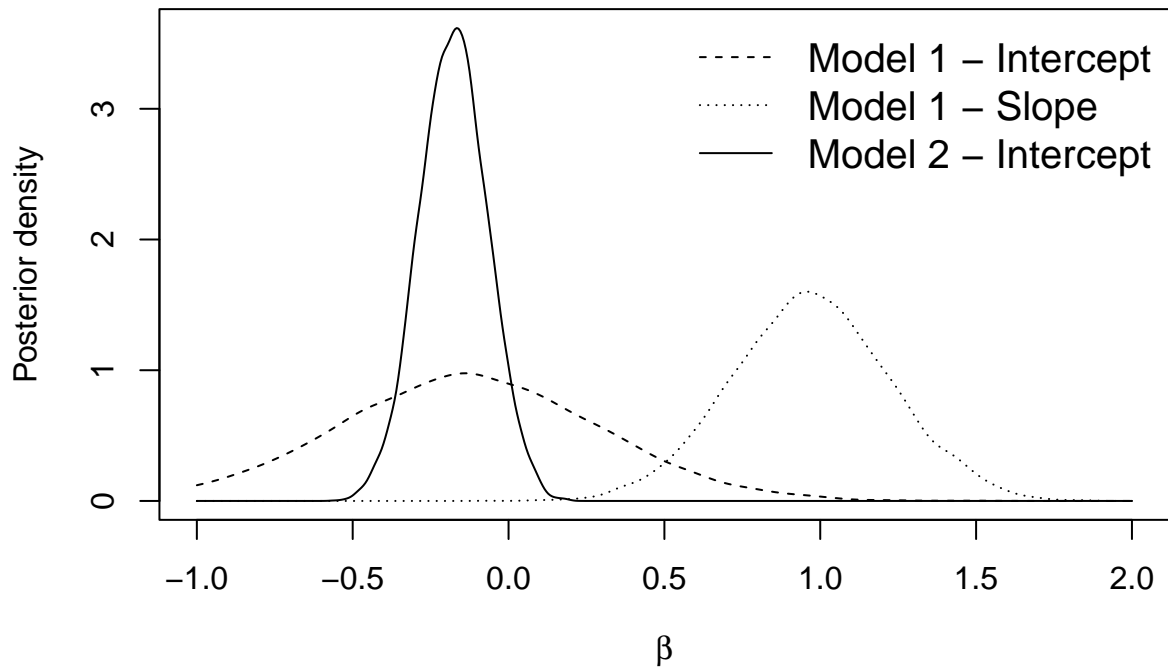
d3 = density(b3,from=-1,to=2)

mx = max(c(d1$y,d2$y,d3$y))

plot(d3$x,d3$y,type="l",xlim=c(-1,2),ylim=c(0,mx),xlab=expression(beta),ylab="Posterior density")
lines(d1$x,d1$y,lty=2)
lines(d2$x,d2$y,lty=3)

legend("topright",c("Model 1 - Intercept","Model 1 - Slope","Model 2 - Intercept"),
      bty="n",lty=c(2,3,1),cex=1.25)

```



Summary: In the second model, we find that  $\beta_1$  is negative with posterior probability around 0.95. If  $\beta_1$  is negative this implies that the clutch probability is less than the overall probability. Therefore, there is some evidence that free throw percentage decreases in clutch situations.