Non-linear regression for the motorcycle data

Nonparametric regression models

In this example X is the time since the motorcycle crash and Y is the acceleration of the driver's head. We will fit the semiparametric model

$$Y_i \sim \text{Normal}[g(X_i), \sigma^2]$$

where the mean function g is assumed to have spline basis representation

$$g(X) = \mu + \sum_{j=1}^{J} B_j(X)\beta_j.$$

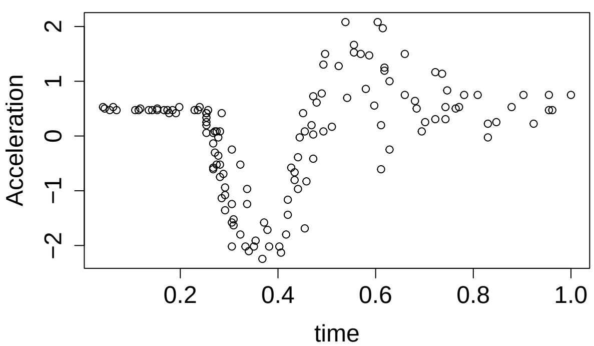
The remaining parameters have uninformative priors: $\mu \sim \text{Normal}(0, 100)$, $\beta_j \sim \text{Normal}(0, \sigma^2\tau^2)$, and $\sigma^2, \tau^2 \sim \text{InvGamma}(0.1, 0.1)$.

Load and plot the motorcylce data

```
library(MASS)

Y <- mcycle$accel
X <- mcycle$times

Y <- (Y-mean(Y))/sd(Y)
X <- X/max(X)

n <- length(Y)
n
```

```
## [1] 133
```

```
plot(X,Y,xlab="time",ylab="Acceleration",cex.lab=1.5,cex.axis=1.5)
```

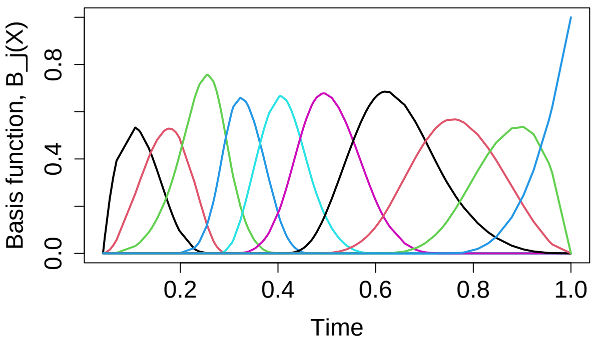


Set up a spline basis expansion

```
library(splines)
J <- 10        # Number of basis functions
B <- bs(X,J)   # Specify the basis functions
dim(B)
```

```
## [1] 133  10
```

```
matplot(X,B,type="l",
        xlab="Time",ylab="Basis function, B_j(X)",
        cex.lab=1.5,cex.axis=1.5,lty=1,lwd=2)
```




```
library(rjags)
```

```
## Loading required package: coda
## Linked to JAGS 4.3.1
## Loaded modules: basemod,bugs
```

```
Moto_model <- "model{

   # Likelihood
   for(i in 1:n){
      Y[i]    ~ dnorm(mean[i],taue)
      mean[i] <- mu + inprod(B[i,],beta[])
   }

   # Prior
   mu   ~ dnorm(0,0.01)
   taue ~ dgamma(0.1,0.1)
   for(j in 1:J){
    beta[j] ~ dnorm(0,taue*taub)
   }
   taub ~ dgamma(0.1,0.1)

  }"
```

Fit the model

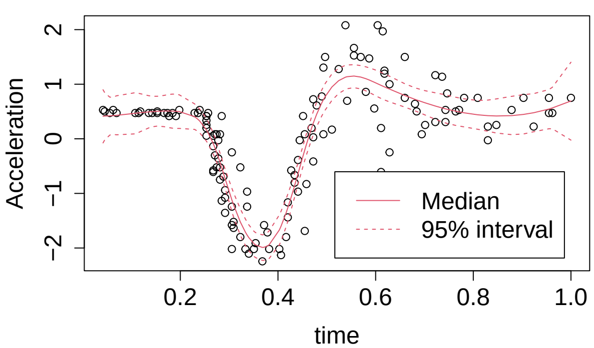
```
library(rjags)
   dat    <- list(Y=Y,n=n,B=B,J=J)
   init   <- list(mu=mean(Y),beta=rep(0,J),taue=1/var(Y))
   model  <- jags.model(textConnection(Moto_model),
                        inits=init,data = dat,quiet=TRUE)

   update(model, 10000, progress.bar="none")

   samp   <- coda.samples(model,
              variable.names=c("mean"),
              n.iter=20000, progress.bar="none")
```

Plot the fixed curve, g(X)

```
sum <- summary(samp)
names(sum)
```

```
## [1] "statistics" "quantiles"  "start"      "end"        "thin"
## [6] "nchain"
```

```
  q <- sum$quantiles

   plot(X,Y,xlab="time",ylab="Acceleration",
        cex.lab=1.5,cex.axis=1.5)

   lines(X,q[,1],col=2,lty=2) # 0.025 quantile (lower bound)
   lines(X,q[,3],col=2,lty=1) # 0.500 quantile (median)
   lines(X,q[,5],col=2,lty=2) # 0.975 quantile (upper bound)

   legend("bottomright",c("Median","95% interval"),
          lty=1:2,col=2,bg=gray(1),inset=0.05,cex=1.5)
```



Summary: The mean trend seems to fit the data well. However, the variance of the observations around the mean varies with X.

Heteroskedastic model

The variance is small for X near zero and increases with X. To account for this, we allow the log of the variance to vary with X following a second spline basis expansion:

$$Y_i \sim \text{Normal}[g(X_i), \sigma^2(X_i)]$$

where $g(X) = \mu + \sum_{j=1}^{J} B_j(X)\beta_j$ s modeled as above and $log[\sigma^2(X)] = \mu_2 + \sum_{j=1}^{J} B_j(X)\alpha_j$..

The parameters have uninformative priors $\mu_k \sim \text{Normal}(0, 100)$, $\beta_j \sim \text{Normal}(0, \sigma_b^2)$, $\alpha_j \sim \text{Normal}(0, \sigma_a^2)$, and $\sigma_a^2, \sigma_b^2 \sim \text{InvGamma}(0.1, 0.1)$.

```
library(rjags)
Moto_model2 <- "model{

   # Likelihood
   for(i in 1:n){
      Y[i]           ~ dnorm(mean[i],inv_var[i])
      mean[i]       <- mu1 + inprod(B[i,],beta[])
      inv_var[i]    <- 1/sig2[i]
      log(sig2[i]) <- mu2 + inprod(B[i,],alpha[])
   }

   # Prior
   mu1  ~ dnorm(0,0.01)
   mu2  ~ dnorm(0,0.01)
   for(j in 1:J){
     beta[j]  ~ dnorm(0,taub)
     alpha[j] ~ dnorm(0,taua)
   }
   taua ~ dgamma(0.1,0.1)
   taub ~ dgamma(0.1,0.1)

   # Prediction intervals
   for(i in 1:n){
     low[i]  <- mean[i] - 1.96*sqrt(sig2[i])
     high[i] <- mean[i] + 1.96*sqrt(sig2[i])
   }
 }"
```

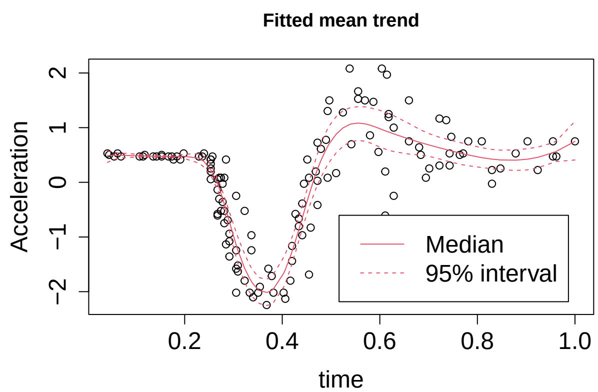
Fit the model

```
library(rjags)
   dat    <- list(Y=Y,n=n,B=B,J=J)
   init   <- list(mu1=mean(Y),beta=rep(0,J),
                  mu2=log(var(Y)),alpha=rep(0,J))
   model <- jags.model(textConnection(Moto_model2),
                       inits=init,data = dat, quiet=TRUE)

   update(model, 10000, progress.bar="none")
```

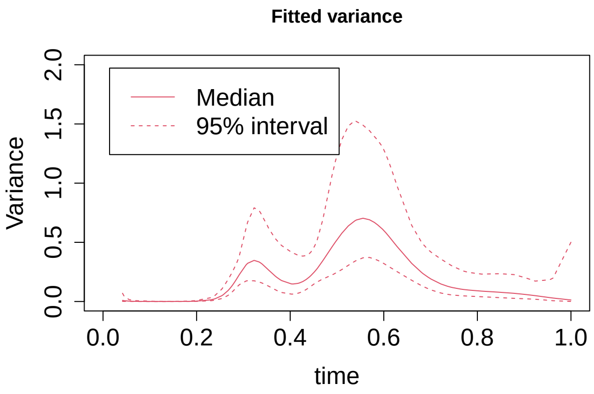
```
samp2  <- coda.samples(model,
            variable.names=c("mean","sig2","low","high"),
            n.iter=20000, progress.bar="none")
```

Plot the fixed curve, g(X)

```
q2 <- summary(samp2)$quantiles
   high <- q2[1:n+0*n,]
   low  <- q2[1:n+1*n,]
   mean <- q2[1:n+2*n,]
   sig2 <- q2[1:n+3*n,]


   plot(X,Y,xlab="time",ylab="Acceleration",
        main="Fitted mean trend",
        cex.lab=1.5,cex.axis=1.5)

   lines(X,mean[,1],col=2,lty=2) # 0.025 quantile (lower bound)
   lines(X,mean[,3],col=2,lty=1) # 0.500 quantile (median)
   lines(X,mean[,5],col=2,lty=2) # 0.975 quantile (upper bound)

   legend("bottomright",c("Median","95% interval"),
          lty=1:2,col=2,bg=gray(1),inset=0.05,cex=1.5)
```




```
plot(NA,xlim=c(0,1),ylim=c(0,2),
        xlab="time",ylab="Variance",
        main="Fitted variance",
        cex.lab=1.5,cex.axis=1.5)

   lines(X,sig2[,1],col=2,lty=2) # 0.025 quantile (lower bound)
   lines(X,sig2[,3],col=2,lty=1) # 0.500 quantile (median)
   lines(X,sig2[,5],col=2,lty=2) # 0.975 quantile (upper bound)

   legend("topleft",c("Median","95% interval"),
          lty=1:2,col=2,bg=gray(1),inset=0.05,cex=1.5)
```




```
plot(X,Y,xlab="time",ylab="Acceleration",
       main="95% prediction intervals (mn +- 2*sd)",
       cex.lab=1.5,cex.axis=1.5)

  lines(X,low[,3],col=2,lty=1) # 0.500 quantile (median)
  lines(X,high[,3],col=2,lty=1) # 0.500 quantile (median)
```