Experimental Statistics for Engineers I

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Problem 1: A family consisting of three persons D, E, F goes to a medical clinic that always has a doctor at each station 1, 2, and 3. During a certain week, each member of the family visits the clinic once and is assigned at random to a station. The experiment consists of recording the station number for each member. One outcome is (1,2,1) for D to station 1, E to station 2, and F to station 1.

(a) List all the 27 outcomes of the sample space.

For station 1: $\{$(1,1,1), (1,1,2), (1,1,3), (1,2,1), (1,2,2), (1,2,3), (1,3,1), (1,3,2), (1,3,3)$\}$

For station 2: $\{$(2,1,1), (2,1,2), (2,1,3), (2,2,1), (2,2,2), (2,2,3), (2,3,1), (2,3,2), (2,3,3)$\}$

For station 3: $\{$(3,1,1), (3,1,2), (3,1,3), (3,2,1), (3,2,2), (3,2,3), (3,3,1), (3,3,2), (3,3,3)$\}$

Totally, there 27 outcomes.

(b) List all outcomes in the event that all three members go the same station (event A)

$\{$(1,1,1), (2,2,2), (3,3,3)$\}$

(c) List all outcomes in the event that all members go to different stations (event B)

$\{$(1,2,3), (1,3,2), (2,1,3), (2,3,1), (3,1,2), (3,2,1)$\}$

(d) List all outcomes in the event that no one goes to station 2 (event C).

$\{$(1,1,1), (1,1,3), (1,3,1), (1,3,3), (3,1,1), (3,1,3), (3,3,1), (3,3,3)$\}$

(e) List the outcomes of $A \cup B$,$A \cup C$,$A \cap C$, A' ,$B \cap C$

$A \cup B$= $\{$(1,1,1), (2,2,2), (3,3,3), (1,2,3), (1,3,2), (2,1,3), (2,3,1), (3,1,2), (3,2,1) $\}$

$A \cup C$=$\{$ (1,1,1), (2,2,2), (3,3,3), (1,1,3), (1,3,1), (1,3,3), (3,1,1), (3,1,3), (3,3,1) $\}$

$A \cap C$=$\{$ (1,1,1), (3,3,3) $\}$

A'=$\{$(1,1,2), (1,1,3), (1,2,1), (1,2,2), (1,2,3), (1,3,1), (1,3,2), (1,3,3), (2,1,1), (2,1,2), (2,1,3), (2,2,1), (2,2,3), (2,3,1), (2,3,2), (2,3,3), (3,1,1), (3,1,2), (3,1,3), (3,2,1), (3,2,2), (3,2,3), (3,3,1), (3,3,2)$\}$

$B \cap C$= $\emptyset$

Problem 2: For customers purchasing a refrigerator at a certain appliance store, let A be the event that the refrigerators was manufactured in the U.S., B be the event that the refrigerator had an icemaker, and C be the event that the customer purchased an extended warranty. Relevant probabilities are

$$P(A) = 0.75, P(B|A) = 0.9, P(B|A') = 0.8, P(C|A \cap B) = 0.8, P(C|A \cap B') = 0.6,$$

$$P(C|A' \cap B) = 0.7, P(C|A' \cap B') = 0.3$$

(a) Describe the event $A \cap B \cap C$ in words; compute $P(A \cap B \cap C)$.

It is event which describes that refrigerators was manufactured in U.S., and had a icemaker and customers purchased extended warranty.

We know that condition probability formula is

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}.$$

Using it we can write the following,

$$P(C|A \cap B) = \frac{P(C \cap A \cap B)}{P(A \cap B)} = \frac{P(A \cap B \cap C)}{P(A \cap B)}$$

Note that we use the fact that $P(A \cap B) = P(B \cap A)$. Hence,

$$P(A \cap B \cap C) = P(C|A \cap B) \cdot P(A \cap B) = 0.8 \cdot 0.675 = 0.54.$$

where

$$P(A \cap B) = P(B \cap A) = P(B|A) \cdot P(A) = 0.9 \cdot 0.75 = 0.675$$

(b) Describe the event $B \cap C$ in words; compute $P(B \cap C)$.

This event describes that refrigerators had a icemaker and customers purchased extended warranty.

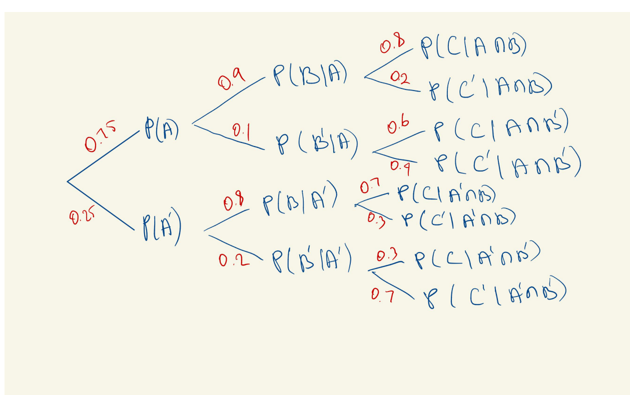

Figure 1: Probability tree

From decision tree given in Figure 1, one can observe that

$$P(A' \cap B \cap C) = P(A') \cdot P(B|A') \cdot P(C|A' \cap B) = 0.25 \cdot 0.8 \cdot 0.7 = 0.14$$

We see that $B \cap C$ exist in both $A \cap B \cap C$ and $A' \cap B \cap C$, thus

$$P(B \cap C) = P(A \cap B \cap C) + P(A' \cap B \cap C) = 0.64 + 0.14 = 0.68$$

(c) Describe the event C in words; compute P(C).

C is an event that describes customer purchased an extended warranty.

Using tree diagram in Figure 1, we get

$$P(C) = P(A \cap B \cap C) + P(A \cap B' \cap C) + P(A' \cap B \cap C) + P(A' \cap B' \cap C) =$$

$$0.75 \cdot 0.9 \cdot 0.8 + 0.75 \cdot 0.1 \cdot 0.6 + 0.25 \cdot 0.8 \cdot 0.7 + 0.25 \cdot 0.2 \cdot 0.3 = 0.74.$$

(d) Describe the event $A|B \cap C$ in words; compute $P(A|B \cap C)$.

Probability of the refrigerators was manufactured in the U.S given that the refrigerator had an icemaker, and the customer purchased an extended warranty.

Using part (a) and (b), we get

$$P(A|B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)} = \frac{0.54}{0.68} = 0.7941$$

Problem 3: Suppose the diagram of an electrical system is given in diagram below. What is the probability that the system works? Assume the components fail independently.

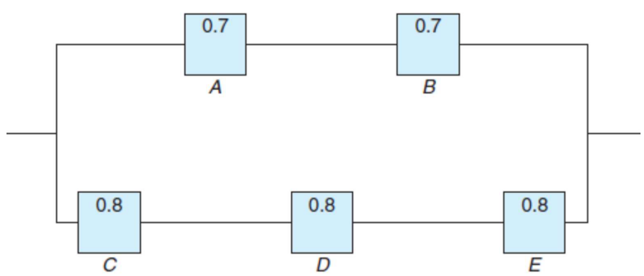

Figure 2: Probability tree

The probability of that system works is

$$P(A \cap B) \cup P(C \cap D \cap E) = P(A \cap B) + P(C \cap D \cap E) - P[(A \cap B) \cap (C \cap D \cap E)]$$

$$= 0.7 \cdot 0.7 + 0.8 \cdot 0.8 \cdot 0.8 - 0.7 \cdot 0.7 \cdot 0.8 \cdot 0.8 \cdot 0.8 = 0.75112$$

where we assumed that event (A,B) are independent, also we assume that event (C,D,E) are independent .

Problem 4: Police plan to enforce speed limits by using radar traps at 4 different locations within the city limits. The radar traps at each of the locations L1, L2, L3 and L4 are operated 40%, 30%, 20%, and 30% of the time. If a person who is speeding on his way to work has probabilities of 0.2, 0.1, 0.5, and 0.2, respectively, of passing through these locations, and we assume a driver will only pass through one of the locations, what is the probability that he/she will receive a speeding ticket?

Since those events are mutually exclusive , the probability

$$P(T) = \sum_{i=1}^{4} P(T|L_i)P(L_i) =$$

$$P(L1 \cap T) + P(L2 \cap T) + P(L3 \cap T) + P(L4 \cap T) = 0.4 \cdot 0.2 + 0.3 \cdot 0.1 + 0.2 \cdot 0.5 + 0.3 \cdot 0.2 = 0.27.$$

Problem 5: A construction company employs 2 sales engineers. Engineer 1 does the work in estimating cost for 70% of jobs bid by the company. Engineer 2 does the work for 30% of jobs bid by the company. It is known that the error rate for engineer 1 is such that 0.02 is the probability of an error when he does the work, whereas the probability of an error in the work of engineer 2 is 0.04. Suppose a bid arrives and a serious error occurs in estimating cost. Which engineer would you guess did the work? Explain and show all work.

$E1$- engineer 1 does work
$E2$-engineer 2 does work
E-error
$P(E1) = 0.7$, $P(E2) = 0.3$, $P(E|E1) = 0.02$, $P(E|E2) = 0.04$.
We are wanted to find: $P(E1|E)$ and $P(E2|E)$

$$P(E1|E) = \frac{P(E1 \cap E)}{P(E)} = \frac{0.7 \cdot 0.02}{0.7 \cdot 0.02 + 0.3 \cdot 0.04}$$

$$P(E2|E) = \frac{P(E2 \cap E)}{P(E)} = \frac{0.3 \cdot 0.04}{0.7 \cdot 0.02 + 0.3 \cdot 0.04}$$

$$P(E1|E) > P(E2|E)$$

Hence,the answer is engineer 1.