Roll No.: | | | | | | | | | | | | | | Date: | | | | | | |

NORTHWEST ACCREDITATION COMMISSION, USA
HIGH SCHOOL DIPLOMA (Sr. Secondary/12TH) 2022-2023

Subject- MATHEMATICS **Question Paper No. :** M T 4 5

Subject Code : ME1207 **Question Paper Code:** M S 7 1

Total Time: 03.00 Hours.	**Total Marks: 100**

GENERAL INSTRUCTIONS

1. OPENING AND CHECKING OF THE QUESTION-BOOKLET

Break open the seal of the Question-Booklet only when the announcement is made by the Invigilator. After breaking the seal and before attempting the questions, student should immediately check for:

a) The number of the printed page in the Question-Booklet is the same as mentioned on the cover page of the Booklet and

b) Any printing error in the Booklet pages, if any.

Any discrepancy or error should be brought to the notice of the Invigilator who will then replace the Booklet. No additional time will be given for this.

2. No student, without the permission of the Superintendent or the Invigilator concerned, is to leave his/ her seat or the Examination Room.

3. FILLING UP THE REQUIRED INFORMATION ON QUESTION-BOOKLET AND ANSWER SHEET

After breaking open the seal and checking the Booklet, student should:

a) Fill up the **Question Paper No.** and **Question Paper Code** (mentioned on the cover of Question-Booklet) in the space provided on the First Answer Sheet.

b) Fill up his/her Roll Number on the First Answer Sheet and on each Supplementary Answer Sheet, if taken.

c) Student should mention the total number of **Supplementary Answer Sheet**, if taken, in the space provided on the First Answer Sheet and also fill up the Serial Number mentioned on each **Supplementary Answer Sheet** along with his/her Roll Number in the register maintained by the Invigilator. Student must tie all the Answer Sheets with the thread provided by the Invigilator.

4. INSTRUCTIONS ABOUT QUESTION PAPER

This Question Paper is divided into three Sections – A, B and C. All Sections are compulsory. Attempt all Sections as per instructions.

a) Section A contains 8 questions which are very short carrying 3 marks each.

b) Section B contains 10 questions which are short carrying 4 marks each.

c) Section C contains 6 questions which are long carrying 6 marks each.

5. Student found in possession of Cellular Phone / Mobile Phone / Pager or any other Communication Device and/or any Book/Note whether using or not, will be liable to be debarred for taking examination(s) either permanently or for specified period or/and dealt with as per law or/and ordinance of the School/SERI according to the nature of offence, or/and he/she may be proceeded against and shall be liable for prosecution under the relevant provision of the Statutory Law.

This Question Paper **MUST** be attached with Answer Sheet

THE ANSWER SHEET IS TO BE RETURNED ON COMPLETION OF THE TEST

SECTION A

Total number of questions: 8	Marks allocated to each question: 3	Total marks: 24

Question 1. Write the number of all possible matrices of order 2×2 with each entry 1, 2 or 3.

Question 2. If A = $\begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$, then write the cofactor of the element a_{21} of its 2nd row.

Question 3. Let R is the equivalence relation in the set A = {0,1, 2, 3, 4, 5} given by R = {(a, b) : 2 divides (a – b)}. Write the equivalence class [0].

Question 4. The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represent the two sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of triangle ABC. Find the length of the median through A.

Question 5. Maximize and minimize Z = x + 2y subject to the constraints

$x + 2y \geq 100$

$2x - y \leq 0$

$2x + y \leq 200$

$x, y \geq 0$

Solve the above LPP graphically.

Question 6. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Question 7. The total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$ in rupees. Find the marginal revenue when x = 5, where by marginal revenue we mean the rate of change of total revenue with respect to the number of items sold at an instant.

Question 8. If $\sin(\sin^{-1}\frac{1}{5} + \cos^{-1}x) = 1$, then find the value of x.

SECTION B

Total number of questions: 10	Marks allocated to each question: 4	Total marks: 40

Question 9. Find: $\int \frac{3x+5}{x^2+3x-18}\,dx$.

Question 10. Find the shortest distance between the lines $\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$.

Question 11. If the function f(x) given by

$$f(x)=\begin{cases}3ax+b, & \text{if } x>1\\ 11, & \text{if } x=1\\ 5ax-2b, & \text{if } x<1\end{cases}$$

is continuous at x = 1, then find the values of a and b.

Question 12. Find the differential equation of family of circles touching Y-axis at the origin.

Question 13. Using properties of determinants, prove that

$$\begin{vmatrix}a^2+2a & 2a+1 & 1\\ 2a+1 & a+2 & 1\\ 3 & 3 & 1\end{vmatrix} = (a-1)^3$$

Question 14. Write the principal value of $\tan^{-1}(1) + \cos^{-1}(-\frac{1}{2})$.

Question 15. Two groups are competing for the positions of the Board of Directors of a corporation. The probabilities that the first and second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced way by the second group.

Question 16. Find matrix A such that

$$\begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix} A = \begin{bmatrix}-1 & -8\\ 1 & -2\\ 9 & 22\end{bmatrix}$$

Question 17. The length x of a rectangle is decreasing at the rate of 5 cm/min and the width y is increasing at the rate of 4 cm/min. When x = 8 cm and y = 6 cm, find the rate of change of
(a) the perimeter.
(b) area of rectangle.

Question 18. Using integration, find the area of Δ ABC, the coordinates of whose vertices are A (2, 5), B(4, 7) and C(6, 2).

SECTION C

Total number of questions: 6	Marks allocated to each question: 6	Total marks: 36

Question 19. Prove that $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$.

OR

If $y = e^x \sin x$, then prove that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$.

Question 20. If N denotes the set of all natural numbers and R be the relation on N × N defined by (a, b) R (c, d), if ad (b + c) = bc (a + d). Show that R is an equivalence relation.

OR

Evaluate $\int_1^3 (2x^2 + 5x)$ dx as a limit of a sum.

Question 21. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 2 & -1 & 0 \end{bmatrix}$ and $A^3 - 6A^2 + 7A + kI_3 = 0$, find the value of k.

OR

Find the differential equation of family of circles touching X-axis at the origin.

Question 22. If A and B are two independent events such that $P(\bar{A} \cap B) = \frac{2}{15}$ and $(A \cap \bar{B}) = \frac{1}{6}$, then find P (A) and P (B).

OR

Using integration, prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by x = 0, x = 4, y = 4 and y = 0 into three equal parts.

Question 23. Using matrices, solve the following system of equations.

$x - y + 2z = 7$

$3x + 4y - 5z = -5$

and $2x - y + 3z = 12$

OR

If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a vector $\vec{c}$, such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.

Question 24. Prove that y = $\frac{4\ sin\ \theta}{2+\cos\theta}$ – θ is an increasing function in (0, $\frac{\pi}{2}$).

OR

A dietician wishes to mix two types of foods in such a way that the vitamin contents of mixture contains atleast 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C, while food II contains 1 unit per kg of vitamin. A and 2 units per kg of vitamin C. It costs ₹ 5 per kg to purchase food I and ₹ 7 per kg to purchase food II. Find the minimum cost of such a mixture. Formulate above as an LPP and solve it graphically.

END OF THE QUESTION PAPER