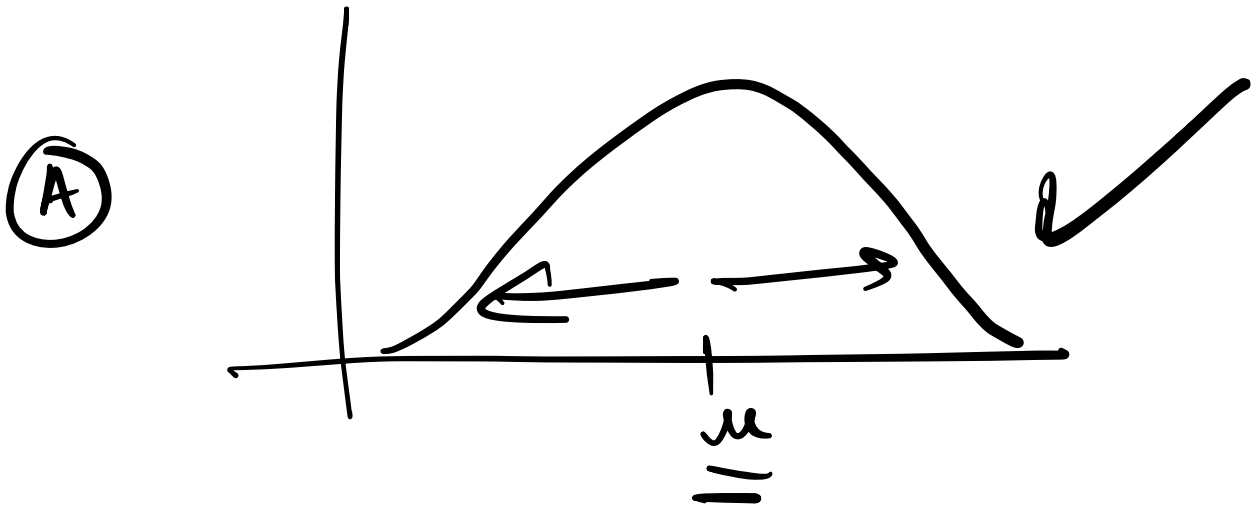
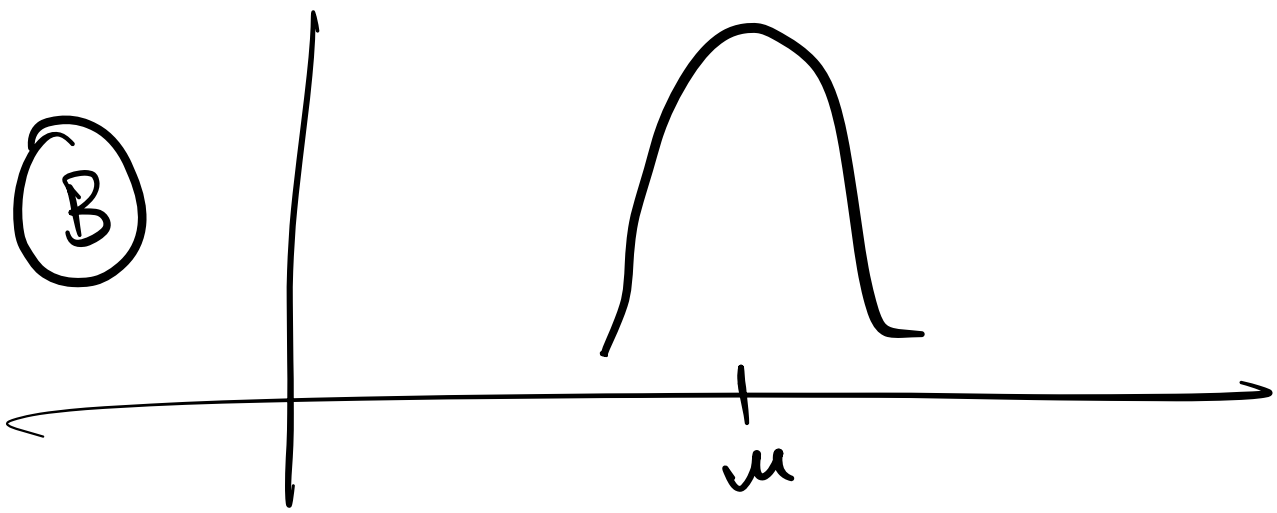
Agenda :-

1. Variance & Std. deviation
2. Empirical vs Theoretical Prob
3. Expectation ≈ weighted avg
4. Binomial Distri
5. Bernoulli Distri

① Variance :





Height game :-

Guessed = 68 inch

Actual = 64 inch

error = (68 - 64) = 4 inch.

± ✗

Guessed = 63 inch

Actual = 68 inch

error = (68 - 63) = 5 inch

$$\text{Error} = (\text{Actual} - \text{Guessed})^2$$

$$= (64 - 68)^2$$

$$= (-4)^2 = 16$$

$$\text{Error1} = (H_1 - \mu)^2$$

μ → mean

$$\text{Error2} = (H_2 - \mu)^2$$

$$\vdots$$

$$\text{Error10} = (H_{10} - \mu)^2$$

$$\text{Variance} = \frac{\text{Error1} + E_2 + E_3 \cdots E_{10}}{10}$$

$$= \frac{(H_1 - \mu)^2 + (H_2 - \mu)^2 + \cdots + (H_{10} - \mu)^2}{10}$$

$$\text{variance } (\sigma^2) = \frac{\sum_{i=1}^{n} (H_i - \mu)^2}{n}$$ [above n = 10]

Std deviation

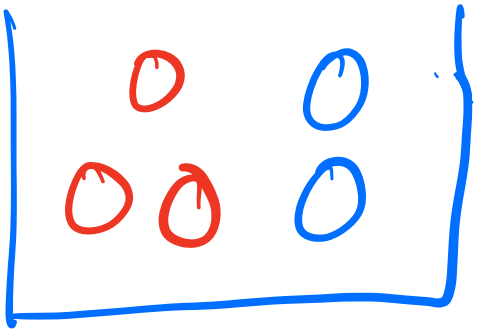
$$SD = \sqrt{\text{variance}}$$

$$= \sqrt{\sigma^2}$$

$$= \sigma = \sqrt{\frac{\sum_{i=1}^{n} (H_i - \mu)^2}{n}}$$

Case study :- (Empirical vs Theoritical prob)

Casino



all red win 150 Rs

4 → 0 →

else lose 10 Rs

RV: X : # of Red Balls Picked

$X \rightarrow \{0, 1, 2, 3, 4\}$

Empirical approach.

$$\text{mean} = \frac{4*(1312) + 3*(3450) + 2*(3488) + 1*(1525) + 0*(225)}{10000}$$

$$= \boxed{2.41}$$

$$\text{mean} = \frac{4*(1312) + 3*(3450) + 2*(3488) + 1*(1525) + 0*(225)}{10000}$$

$$E(X) = 4 * \frac{(1312)}{10K} + 3 * \frac{(3450)}{10K} \ldots$$

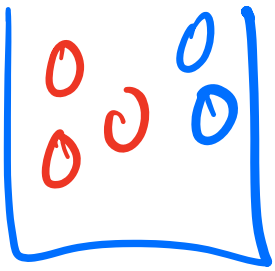
$X = 4$ $\quad$ $P(4)$

$$E(X) = \sum x_i * P(X = x_i)$$

$$= 0 * P(X=0) + 1 * P(X=1) + 2 * P(X=2) + 3 * P(X=3) + 4 * P(X=4)$$

Empirical

Theoretical approach :-



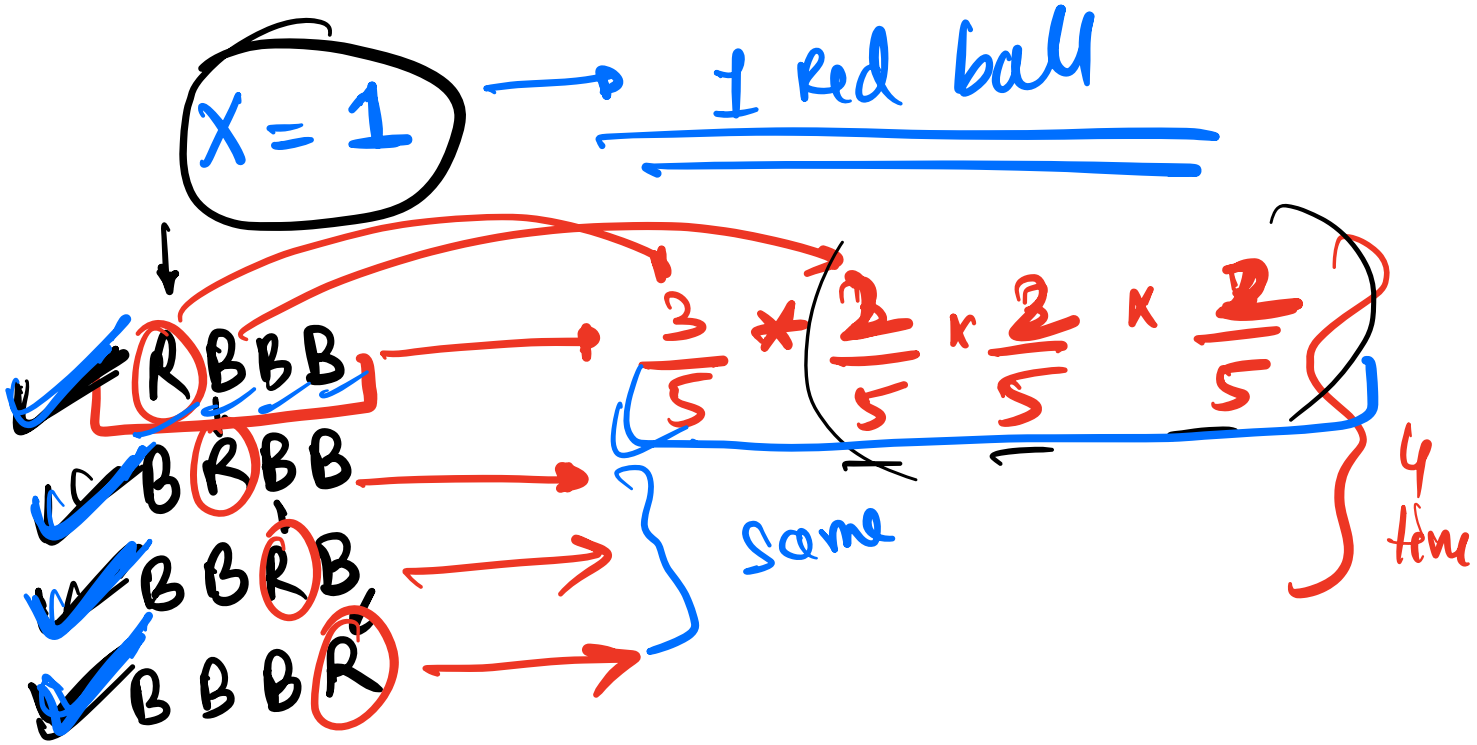
R → Red ball

B → Blue ball.

$P(R) = 3/5$

$P(B) = 2/5$

In Case of Casino :-



getting exactly 1 R ball when you pick 4 ball

$P(HH) = \frac{1}{2} \times \frac{1}{2}$

$$P(X=1) = 4 * \left(\frac{3}{5}\right) * \left(\frac{2}{5}\right)^3$$

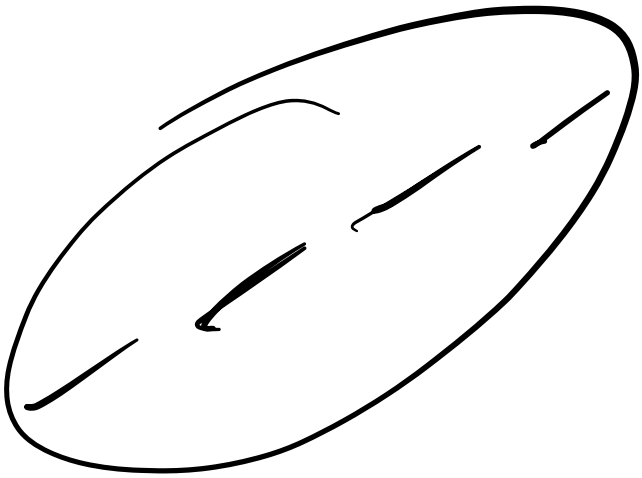
$$= {}^4C_1 \times (3/5) \times (2/5)^3$$

$X = 2$

RRBB
RBRB
RBBR and so on..

$$P(X=2) = {}^4C_2 * \left(\frac{3}{5}\right)^2 * \left(\frac{2}{5}\right)^2$$

$$P(X=K) = {}^{4}C_{K} * \left(\frac{3}{5}\right)^{K} * \left(\frac{2}{5}\right)^{4-K}$$



Binomial Distn

p → prob of success

$1-p$ → " failure

n → trials

$$P(X=k) = {}^{n}C_{k} \left(p\right)^{k} * \left(1-p\right)^{n-k}$$

RV:

$Y \longrightarrow$ Amount

$$Y = \{150, -10\}$$

$$P(Y=150) = P(X=4)$$

$$P(Y=-10) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$P(Y=-10) = 1 - P(X=4)$$

Expected value

$$= \sum y_i * P(Y = y_i)$$

$$E(Y) = 150 * P(Y = 150) + (-10) * P(Y = -10)$$

Quiz 1 :-

$$n = 10$$

$$p = \left(\frac{1}{4}\right)$$

① $P(x = 4)$ X

② $P(x >= 4)$

$$P(x >= 4)$$

$$= 1 - P(x < 4)$$

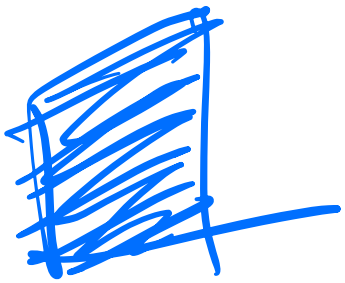
$$= 1 - \left[P(x=0) + P(x=1) + P(x=2) + P(x=3) \right]$$

pmf

OR

Ans = 1 − binom.cdf(n=10, P=1/4, K=3)

$P(X <= 3)$ $X <= K$

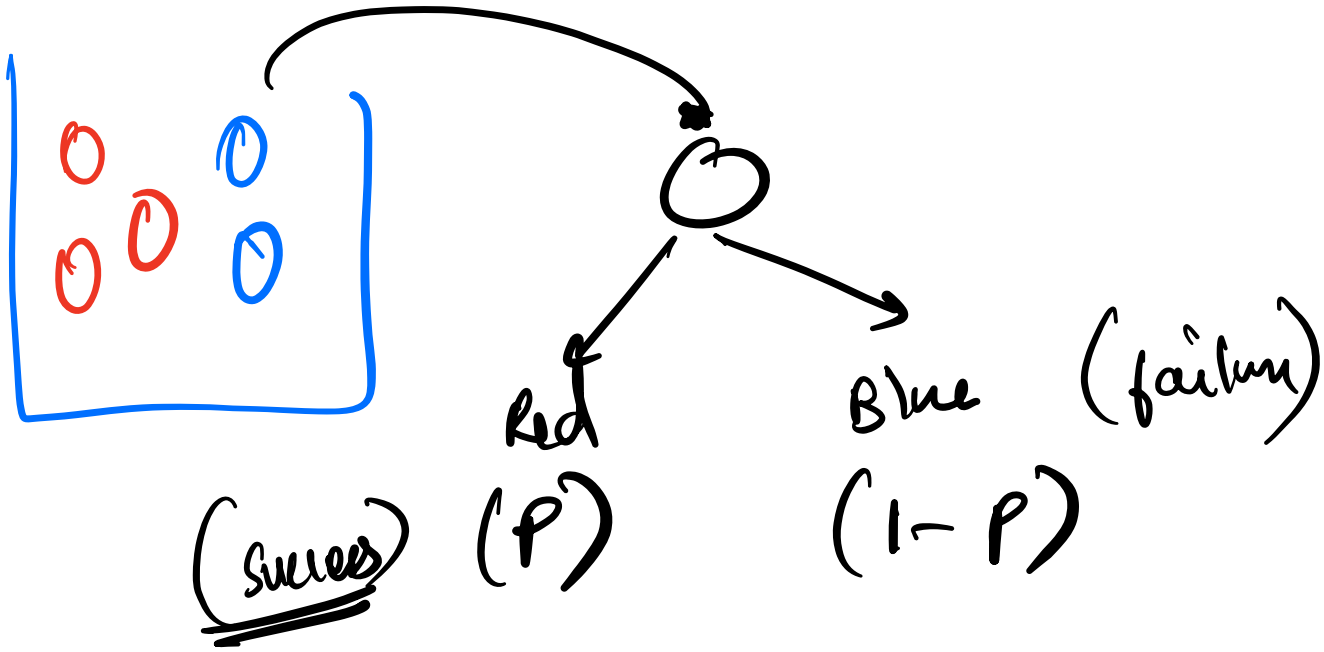


pmf → $P(X = k)$

11.00

* Bernoulli Distri

(Binomial Distri where n = 1)



$P(H) + P(T) = 1$

$P(H) = 1 - P(T)$

$P(R) \rightarrow 2/5$

$P(B) = 1 - 3/5$

$= 2/5$