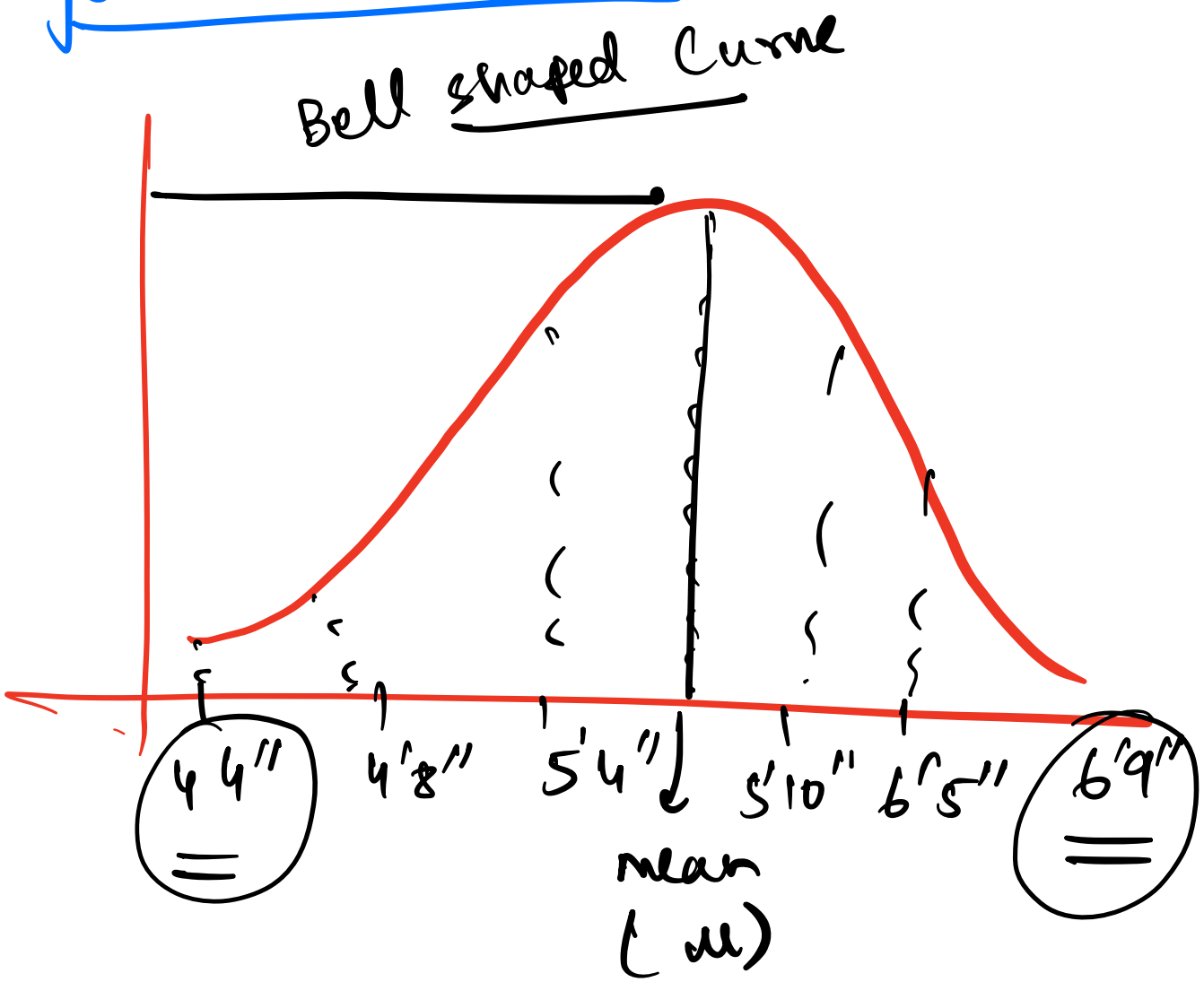
Prob Distri – 3

Agenda :-

1. Gaussian Distribution (Normal Distri)
2. Empirical Rule
3. z - score
4. PPF (Percent Point function)
5. Standard Normal Distri
6. Standardisation (some intuition)

* Gaussian Distribution

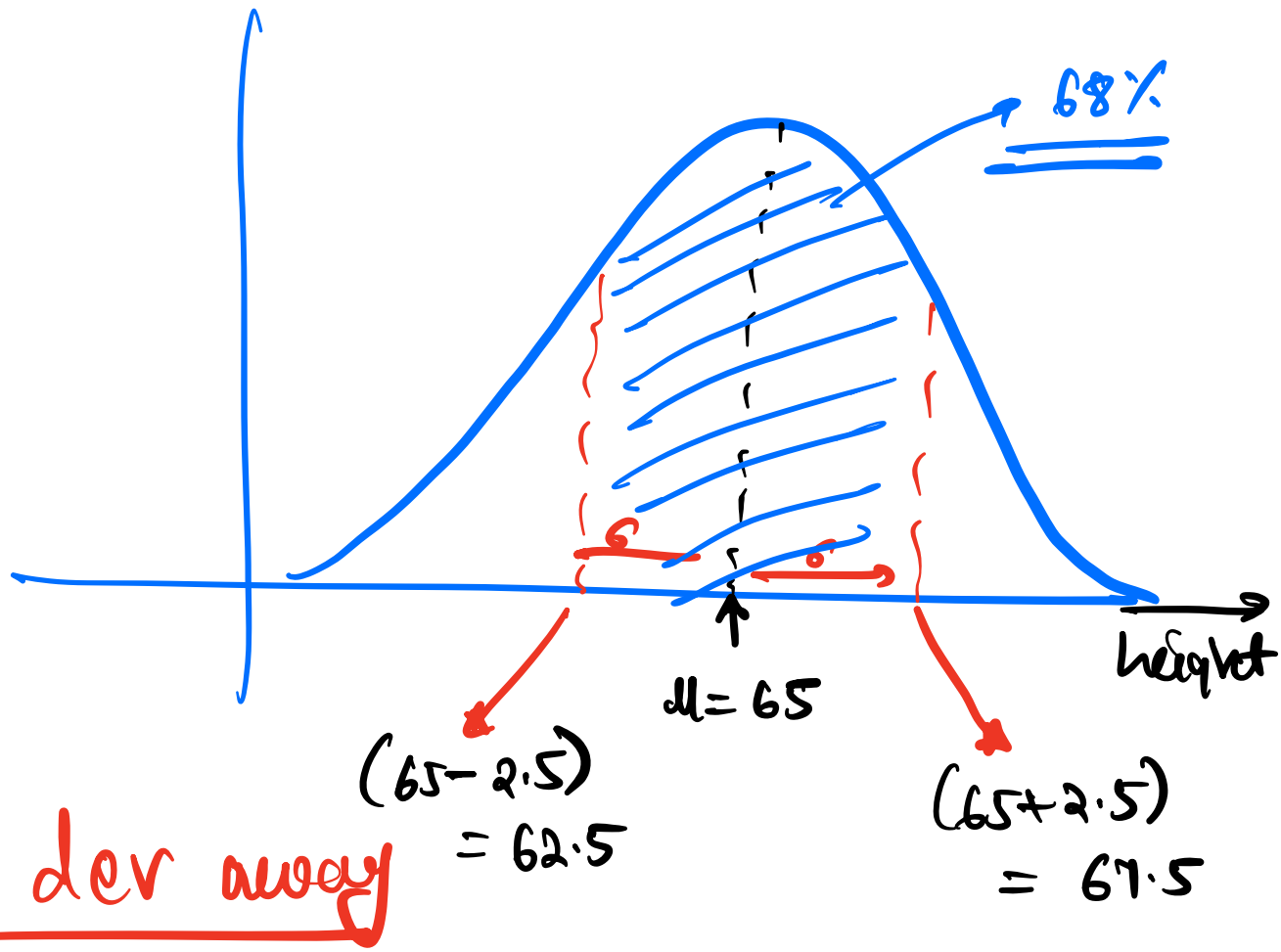


* Empirical Rule (68/95/99) Rule

- Mean Height (μ) = 65 inch

[1 ft = 12 inches]

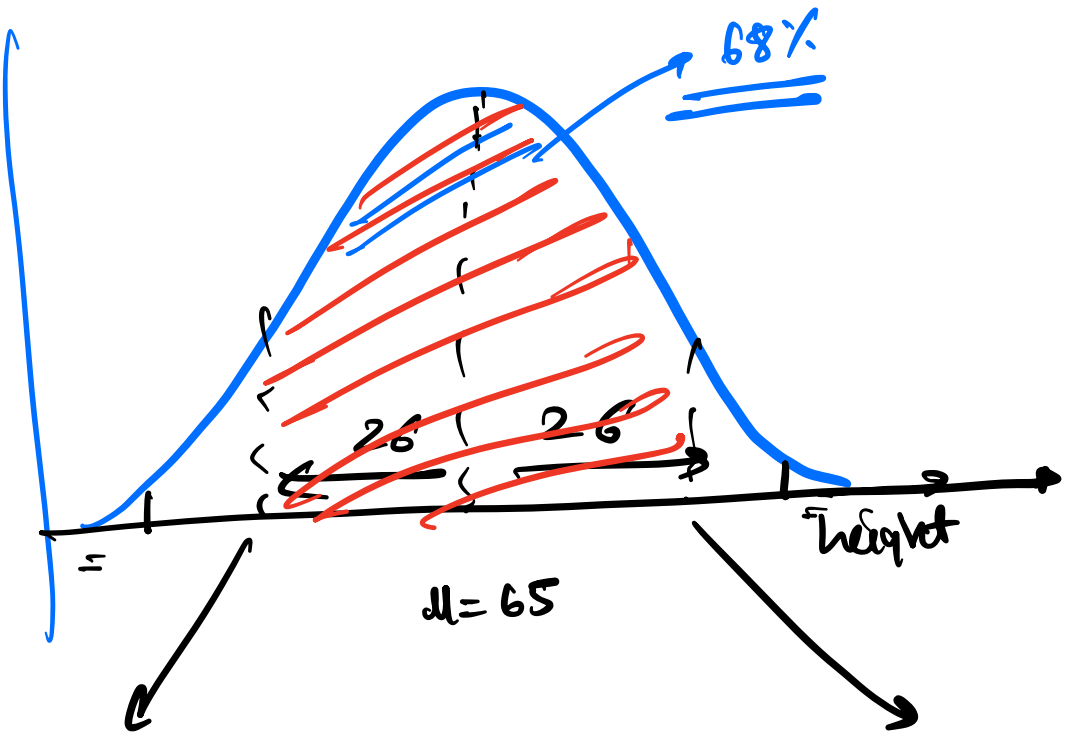
σ = 2.5 inch



1 std. dev away

$$P(\mu - \sigma < X < \mu + \sigma) = 0.68 \quad (68\%)$$

② Two-std der



$65 - 2 * 2.5$

$= 60$

$65 + 2 \times 2.5$

$= 70$

$$P\left(\mu - 2\sigma < x < \mu + 2\sigma\right) = 0.95$$

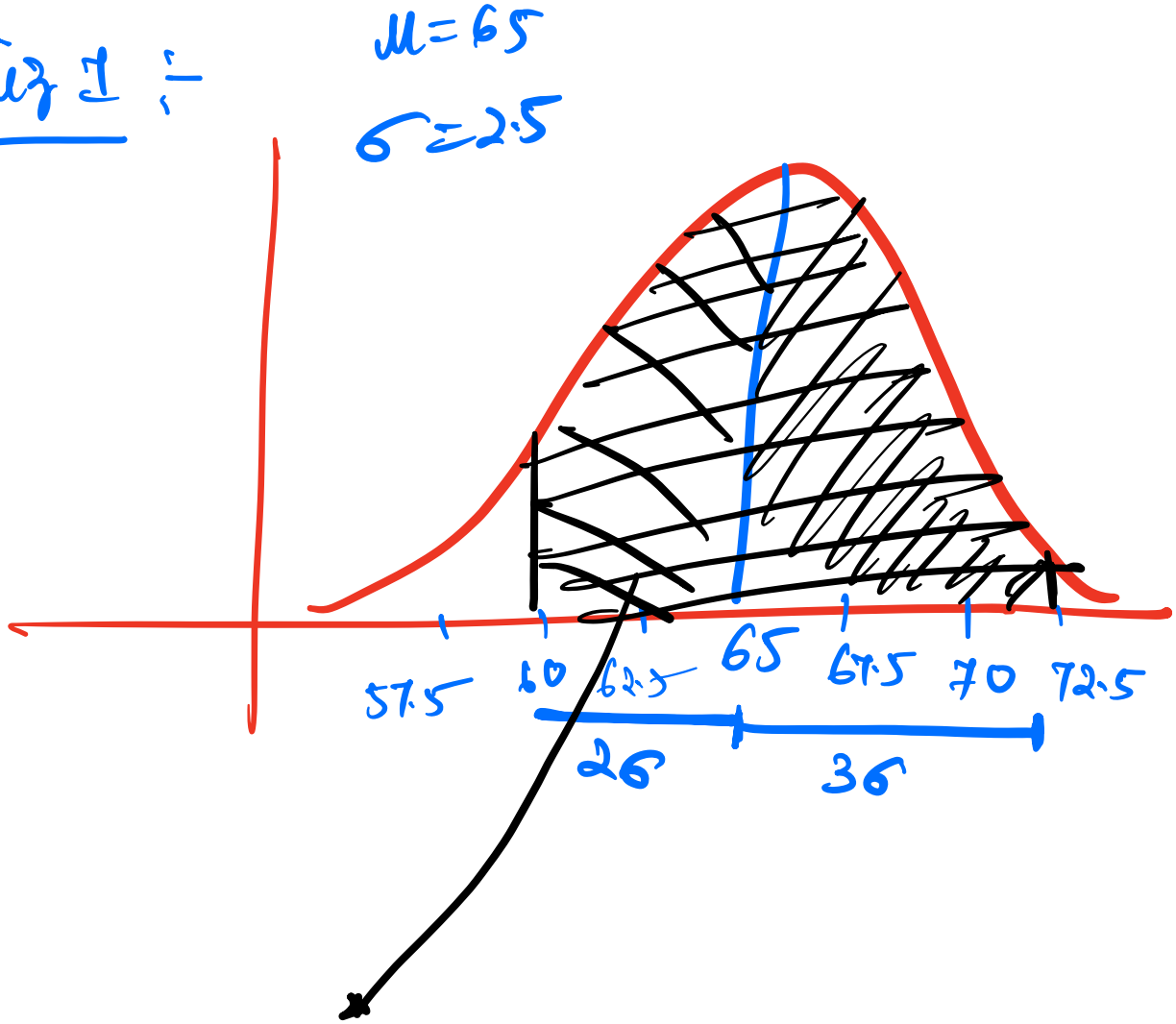
(95%)

★ 3 - std dev :-

$$P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.997$$

(99.7%)

Quiz 1 :-



Left → $\frac{95}{2} = 47.5\%$

+

Right → $\frac{99.7}{2} = 49.85\%$

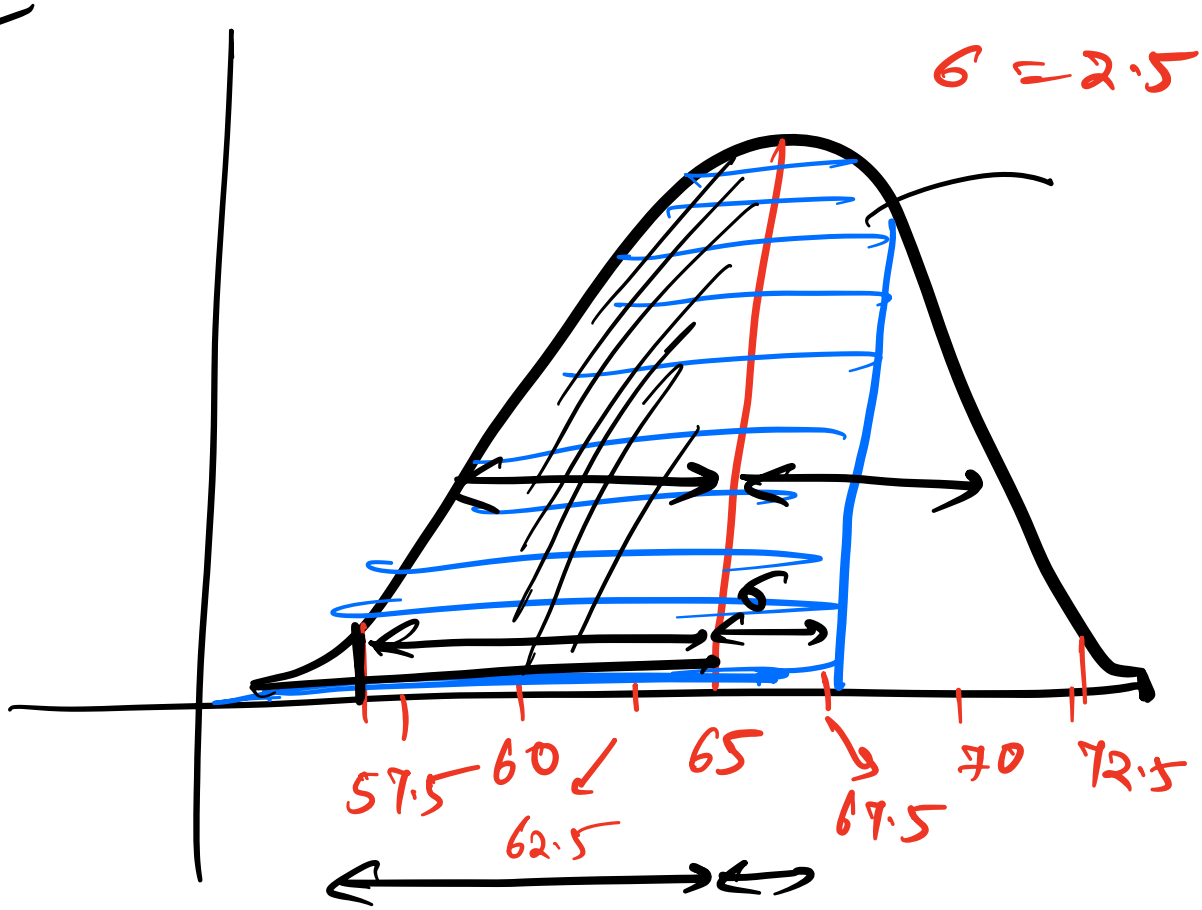
$= 97.35\%$

$= \frac{97.35}{100}$ fraction

Quiz 2:

$\mu = 65$

$\sigma = 2.5$



Left → $\frac{99.7}{2}$ ✗

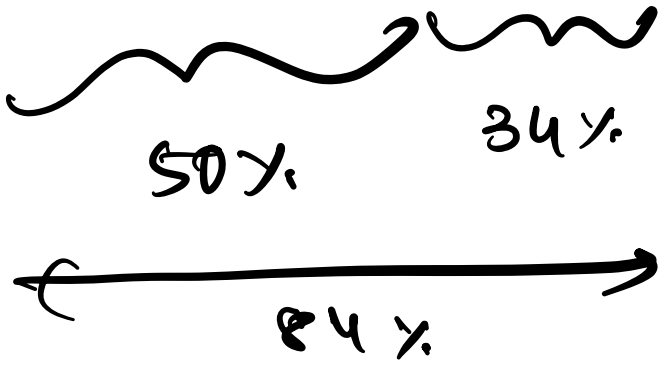
Total → 100%

Left → $\frac{100}{2} = 50\%$ (< 65)

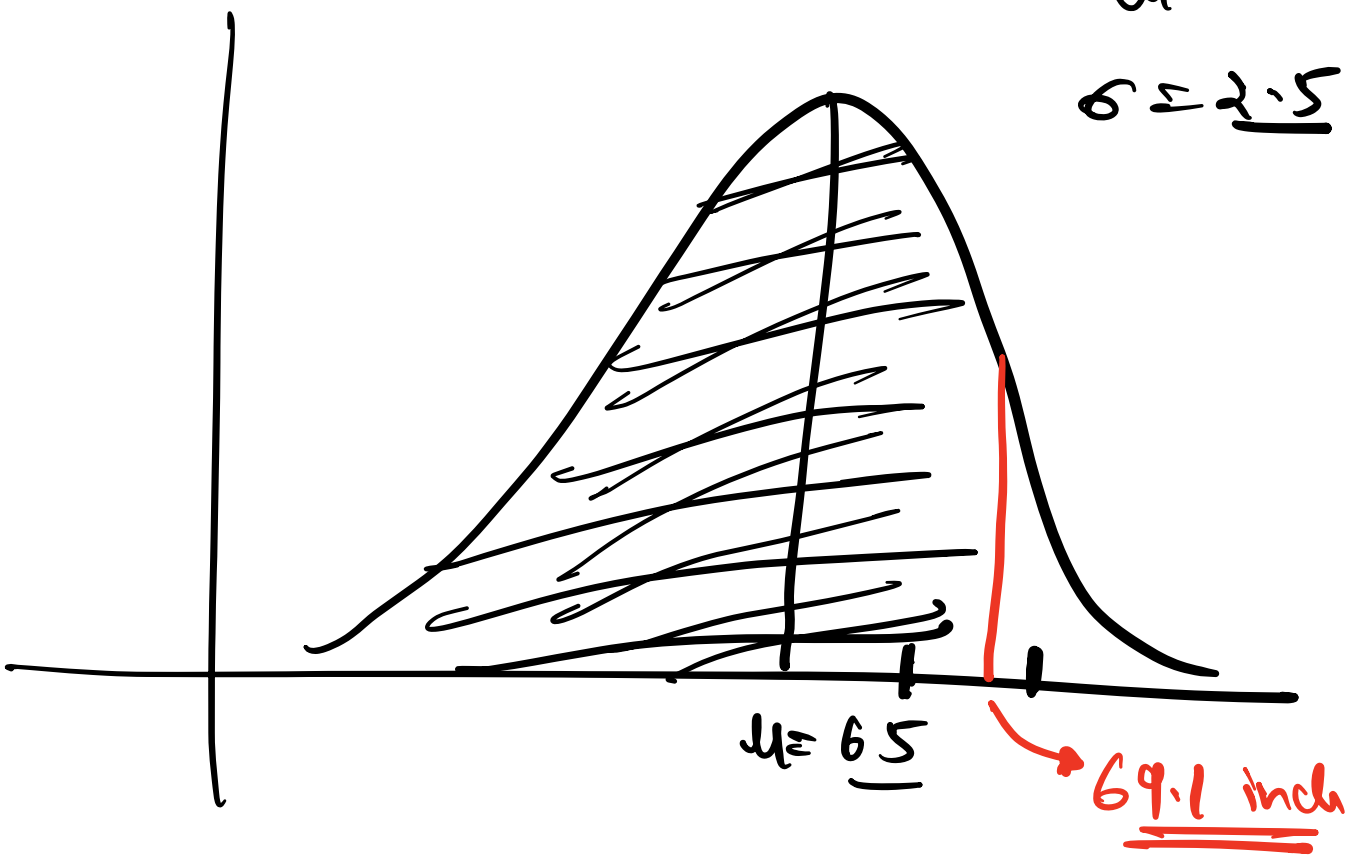
\+

Right → $\frac{68}{2} = 34\%$

84% → Read


50%
34%
84%

Z Score :-



$$70 = 65 + 2 * 2.5$$

$$70 = \mu + 2 * (SD)$$

$SD = \sigma$

$$67.5 = \mu + 1 * (SD)$$

$$69.1 = 65 + z * (SD)$$

no. of std dev away.

$$= \mu + 2 * (SD)$$

$$z = \frac{69.1 - 65}{2.5} = 1.64$$

z-stat / z-score

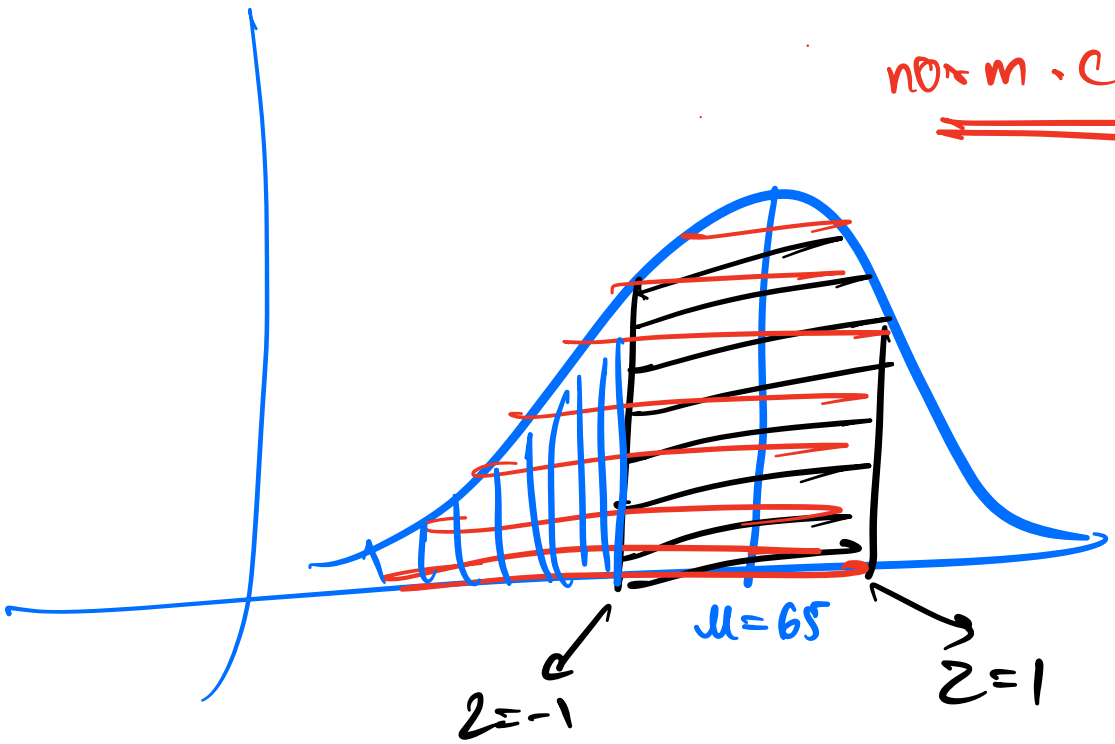
In general:

$$z = \frac{x - \mu}{\sigma}$$

⇒ 94.95% people are shorter than 69.1 inches

Z - table or Code

norm.cdf(1)

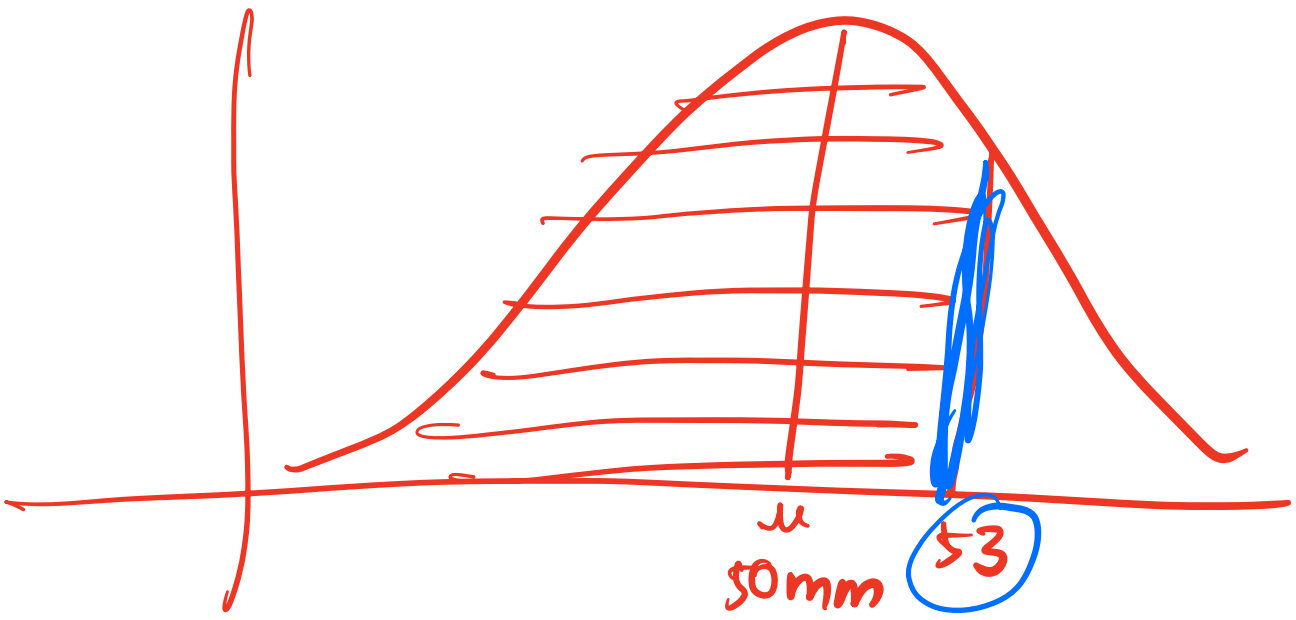


norm.cdf(1) − norm.cdf(-1)

Quiz 3:

$\mu = 50\,mm$

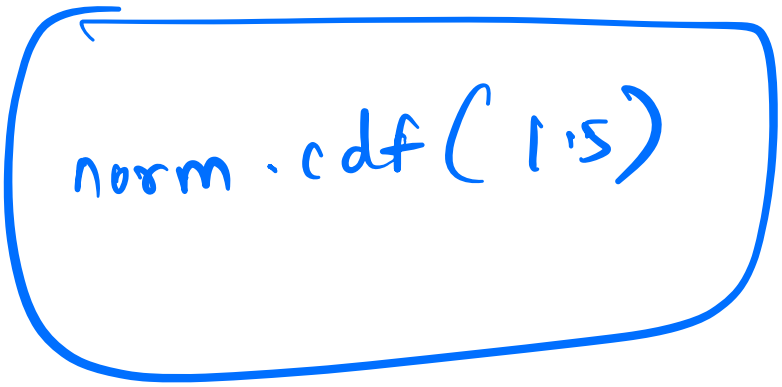
$\sigma = 2\,mm$



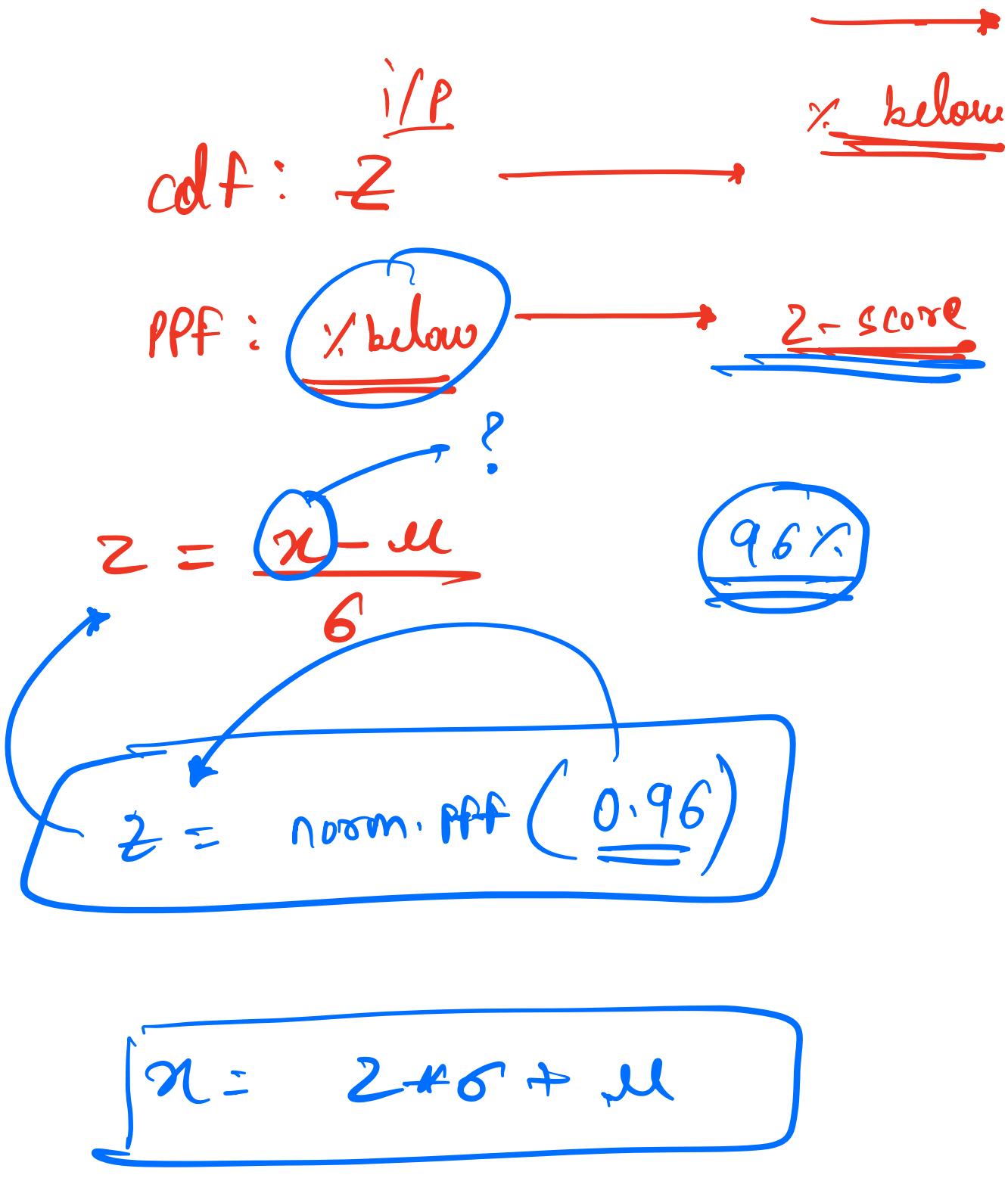
Z-score for 53 mm

$$Z = \frac{53 - 50}{2}$$

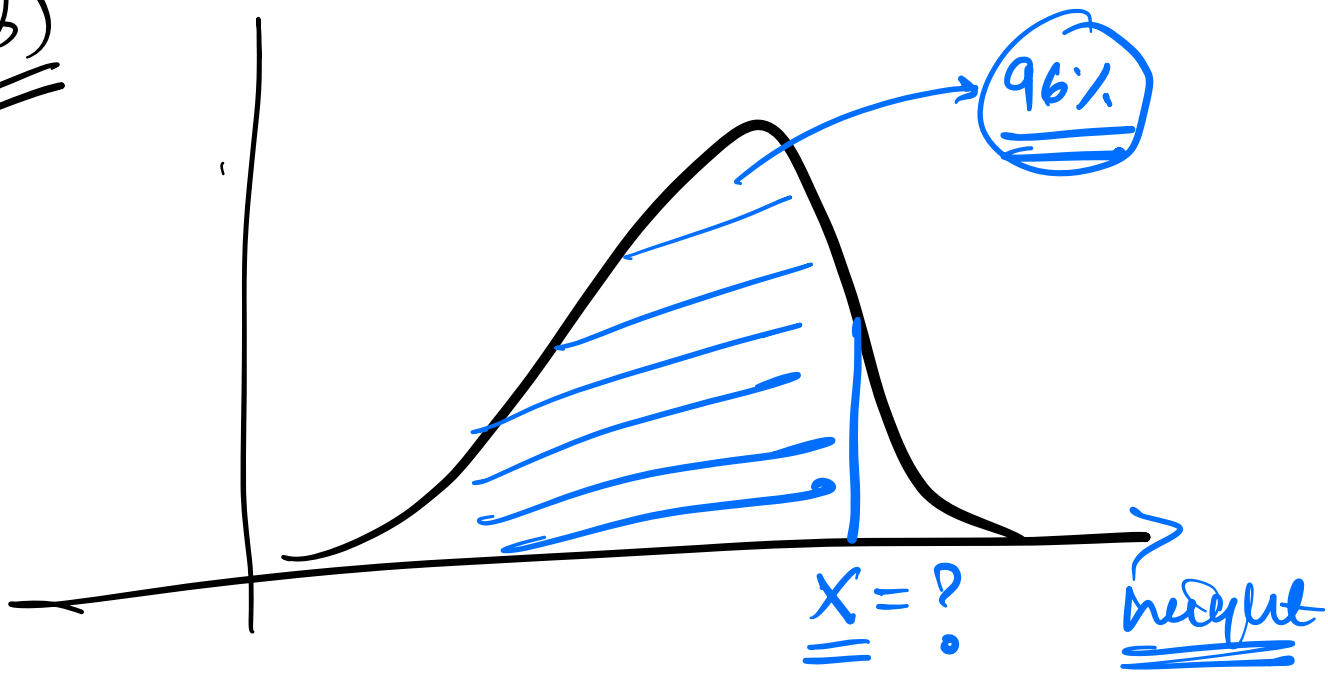
$$\boxed{Z = 1.5}$$


norm.cdf(1.5)

* PPF (Percent Point function)



(8)

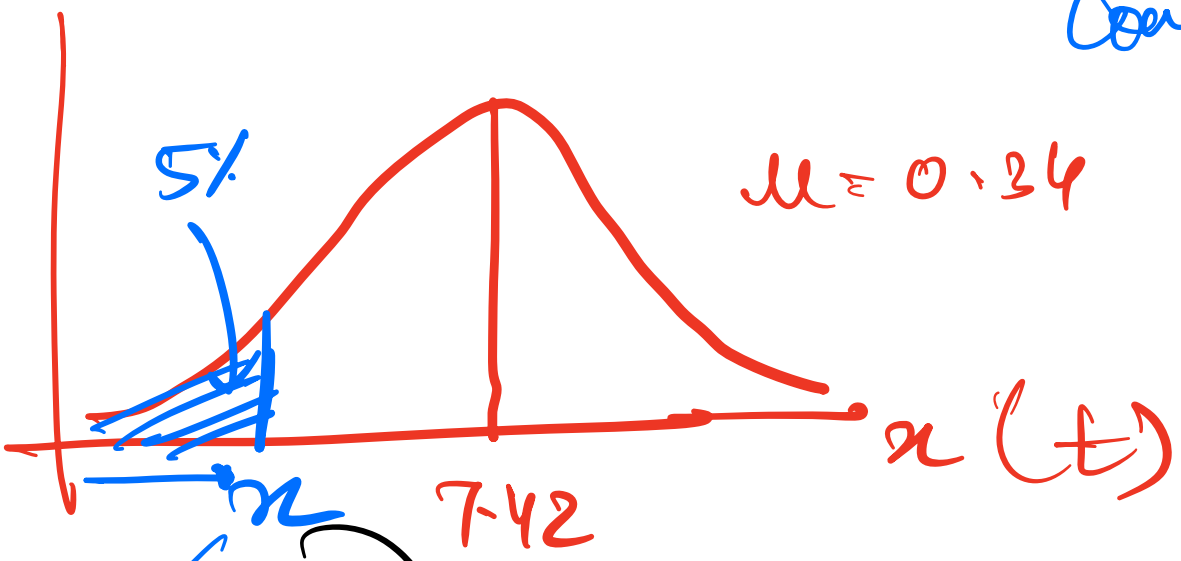


Quiz 4: Interview

- $\mu = 7.42$ s
- $\sigma = 0.34$ s

$d = 500$ m

Speed = ? s.t faster than 95% of cars competitors.



at this time, slower than 5%.

6.86 s then faster than 95%

$$\text{Speed} = \frac{d}{t}$$

$$= \frac{500}{6.86}$$

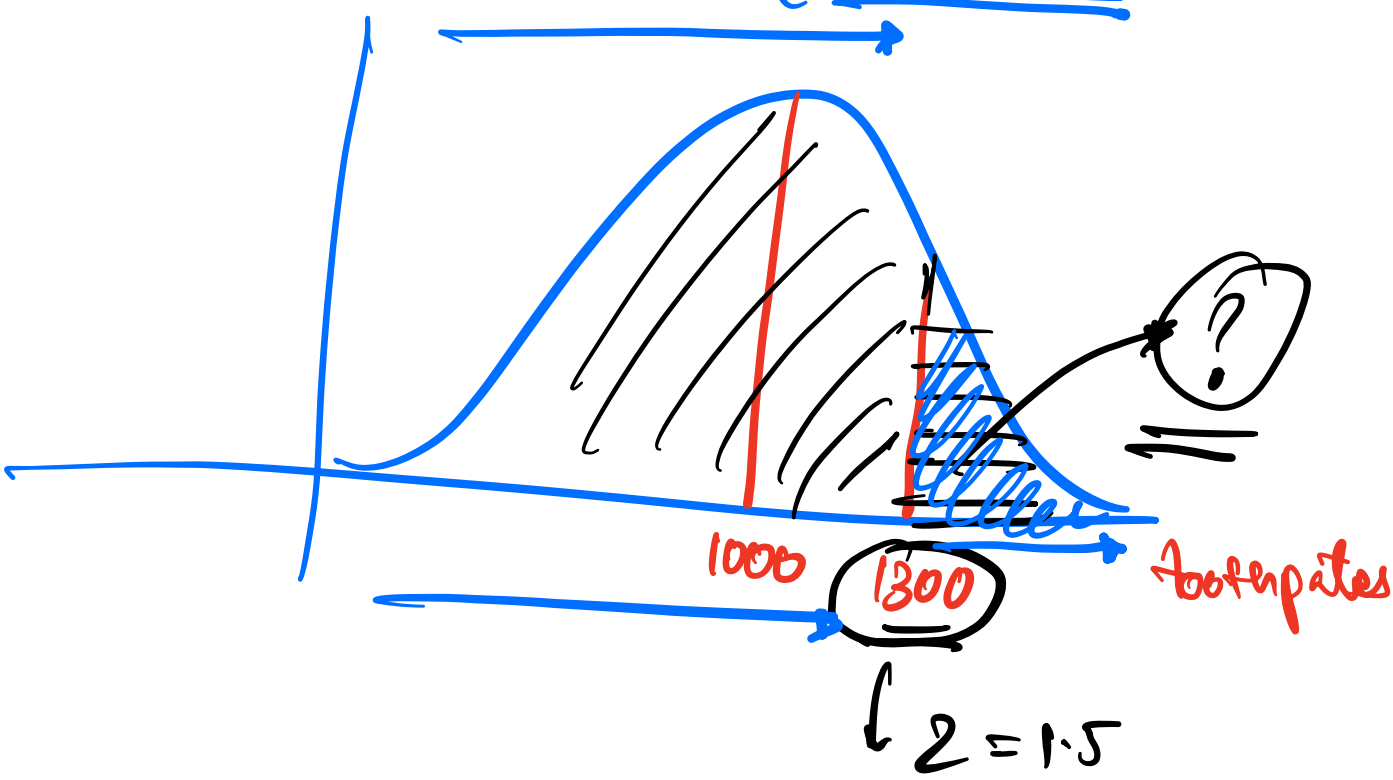
$$\approx 72.87 \text{ m/s}$$

(8) ∴ X : weekly sales

$\mu = 1000$

$\sigma = 200$

$P(X < 1300)$



$Z = 1.5$

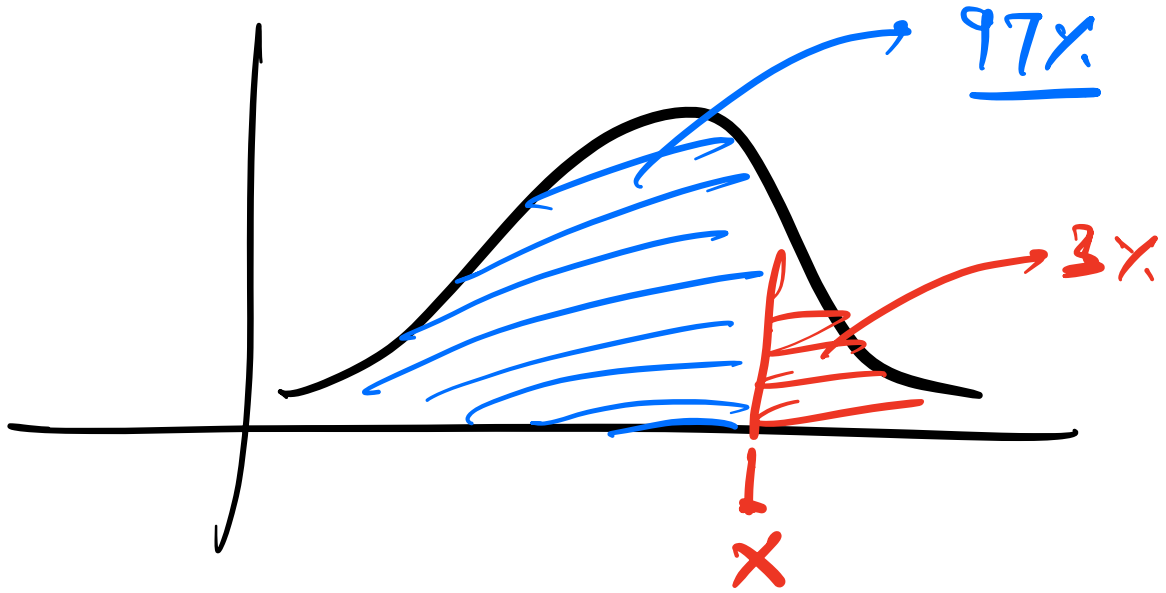
$P(X > 1300)$

$$Z = \frac{1300 - 1000}{200}$$

$$\boxed{Z = 1.5}$$

Reqd = $1 - \text{norm.cdf}(1.5)$

follow up Q



Z-score = $\text{norm.ppf}(0.97)$

$$Z = \frac{x - \mu}{\sigma}$$

$$x = Z * \sigma + \mu$$

$= 1000 + \text{ppf}(0.97) * 200$

= 1376.15 toothpicks

* Standard Normal Distn

$\mu = 0$

$\sigma = 1$

Standardization