

✓ Content

- Combinatorics
- Permutations
- Permutation : Generic Approach
- Combinations

Suppose we have 2 True/False questions. In how many ways can they be solved?

For question 1, we have two possible outcomes:

- True
- False

Similarly, for question 2 as well.

Since we need to solve both questions, will we add or multiply their number of possible outcomes?

We will **multiply** since we have to solve question **1 AND 2**

Instead, if we had to solve question **1 OR 2**, we would've **added**.

This is because, when considering Event 1 AND Event 2, we are talking about two **independent events**.

- Solving question 1 is independent from solving question 2
- Hence, we need to multiply to consider their combined effect.

Therefore, we can solve them in $2 * 2 = 4$ ways:

- True, True
- True, False
- False, True
- False, False

✓ Permutation and Combination

What is a permutation?

- When talking about permutations, we mean **arrangement of objects**.
- Therefore, as with arranging objects, the most important thing is **order** is which they are arranged.

- This means that $(i, j) \neq (j, i)$

Formal Definition: A permutation is an arrangement of items or elements in a specific order, where the order of the arrangement matters.

The second aspect is **Combinations**

What is a combination?

- Combination is **Selection of objects**.
- Over here, the order of objects **does not matter**.
 - This means that $(i, j) = (j, i)$

Formal Definition: A combination is selection of items or elements where the order of the arrangement does not matter.

Permutation.

↳ Arrangement of object

→ Order matters

$$\begin{pmatrix} a \\ i, j \end{pmatrix} \neq \begin{pmatrix} a \\ j, i \end{pmatrix}$$

$$a, b \neq b, a$$

Combination

↳ Selection of object

→ Order doesn't matter

$$\begin{pmatrix} a \\ i, j \end{pmatrix} = \begin{pmatrix} a \\ j, i \end{pmatrix}$$

$$a, b = b, a$$

✓ Permutation : Generic Formula

Q1. How would we arrange N object, given that there only 3 slots?

Since there are 3 slots for N objects, the no. of ways in which we can arrange them is ${}^N P_3$

i.e. ${}^N P_3 = N \cdot (N - 1) \cdot (N - 2)$

Q2. How would we arrange N object, given that there only 4 slots?

$${}^N P_4 = N \cdot (N - 1) \cdot (N - 2) \cdot (N - 3)$$

We can observe a pattern between the no. of slots/blanks, and the last term of the above expressions

Q3. Then how would we arrange N object, given that there are k slots available?

This can be found using:

$${}^N P_k = N(N - 1)(N - 2)(N - 3) \dots (N - (k - 1)) = N(N - 1)(N - 2)(N - 3) \dots (N - k$$

Let's re-write this equation by multiplying and dividing by same expression, as:

$${}^N P_k = N(N - 1)(N - 2)(N - 3) \dots (N - (k - 1)) = N(N - 1)(N - 2)(N - 3) \dots (N - k$$

As we know, we can write this in the form of factorial as: ${}^N P_k = \frac{N!}{(N-k)!}$

✓ Combinations

- Combinations, in simple terms, are all the different ways you can choose a certain number of items from a group, where the order in which you pick them doesn't matter.
- It's like making a sandwich with different ingredients – the combination is the unique mix of ingredients you choose, regardless of the order you add them.

In the language of combinatorics, this number of ways of **selecting** is known as **Combination**.

Similarly, we can write the **general formula** for combinations in terms of permutations as:

$${}^n C_k = \frac{{}^n P_k}{k!}$$

We can further expand it as: ${}^n C_k = \frac{n!}{k!(n-k)!}$
