COMPSCI 1DM3 - Assignment 4

Due date: March 17, 11:59 PM

1. (1 mark) Find the prime factorization of 45617.

2. (1 mark) Find $\gcd(2^{89}, 2^{346})$ by directly finding the largest divisor of both numbers.

3. (1 mark) Prove or disprove: A positive integer congruent to 1 modulo 4 cannot have a prime factor congruent to 3 modulo 4.

4. (1 mark) Prove or disprove: If $f(n) = n^2 - n + 17$, then $f(n)$ is prime for all positive integers n.

5. (1 mark) Prove or disprove: if p and q are prime numbers, then $pq + 1$ is prime.

6. (1 mark) Use the Euclidean Algorithm to find $\gcd(128, 729)$.

7. (2 marks) Express $\gcd(450, 120)$ as a linear combination of 120 and 450.

8. (2 marks) The inverse of a modulo m is an integer b such that $ab \equiv 1 \pmod{m}$. Find an inverse of 177 modulo 919. Explain how you found the inverse.

9. (2 marks) Use your method of finding inverses from the last question to solve the linear congruence $31x \equiv 57 \pmod{61}$.

10. (2 marks) Take any three-digit integer, reverse its digits, and subtract. For example, $742 - 247 = 495$. The difference is divisible by 9. Prove that this must happen for all three-digit numbers abc.

11. (2 marks) Use induction to prove that $\sum_{j=n}^{2n-1}(2j + 1) = 3n^2$ for all positive integers n.

12. (2 marks) Let $a_1 = 2$, $a_2 = 9$, and $a_n = 2a_{n-1} + 3a_{n-2}$ for $n \geq 3$. Use induction to show that $a_n \leq 3^n$ for all positive integers n.

13. (2 marks) Use mathematical induction to prove that $4 \mid (9^n - 5^n)$ for all $n \geq 0$.