# CompSci 1DM3
# Assignment 4

**Q1)** $45617 = (11^2).(13).(29)$

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**Q2)** $\gcd(2^{89}, 2^{346})$

largest divisor of $2^{89}$ is $2^{89}$

largest divisor of $2^{346}$ is $2^{346}$

because both of them are powers of two

then $\gcd(2^{89}, 2^{346}) = 2^{\min(89,346)}$

therefore the $\gcd(2^{89}, 2^{346})$ is $2^{89}$

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**Q3)** let $x$ be 33:

we have $33 \equiv 1 \pmod 4$

$33 = 3.(11)$ → prime factor of 33

and we know that $11 \equiv 3 \pmod 4$

So the claim is False!

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Q4) $f(n) = n^2 - n + 17$

if $n = 17$ then $f(n) = (17)^2 - 17 + 17$

$f(n) = 17 \times 17$ which is divisible by 17 so it is not a prime number $\longrightarrow$ False

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Q5) if $p = 5$ and $q = 11$

then $pq + 1 = (5)(11) + 1 = 56$

and 56 is divisible by 2, 4, and 13

so it is not prime $\longrightarrow$ False

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Q6) $\gcd(128, 729)$

$729 = 5 \cdot 128 + 89$

$128 = 1 \cdot 89 + 39$

$89 = 2 \cdot 39 + 11$

$39 = 3 \cdot 11 + 6$

$11 = 1 \cdot 6 + 5$

$6 = 1 \cdot 5 + 1$ $\rightarrow$ last non-zero remainder

$5 = 5 \cdot 1 + 0$

So the $\gcd(128, 729)$ is 1

**Q7)** $\gcd(450,120)$

$450 = 3.120 + 90$

$120 = 1.90 + 30 \rightarrow \gcd$

$90 = 3.30 + 0$

therefore: $30 = 120 - 1.90$

$30 = 120 - 1.(450 - 3.120)$

$30 = 120 + 3.120 - 1.450$

$$\boxed{30 = 4.120 - 1.450}$$

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**Q8)** $\gcd(177, 919)$

$919 = 5.177 + 34$

$177 = 5.34 + 7$

$34 = 4.7 + 6$

$7 = 1.6 + 1 \rightarrow \gcd$

$6 = 6.1 + 0$

therefore:

$1 = 7 - 1.6$

$1 = 7 - 1.(34 - 4.7) = 5.7 - 1.34$

$1 = 5.(177 - 5.34) - 1.34$

$1 = 5.177 - 26.34$

$1 = 5.177 - 26.(919 - 5.177)$

$1 = 5.177 - 26(919) + 130.177$

$1 = 135.177 - 26.919$

So the coefficients are 135 and $-26$, therefore 135 is the inverse of 177 modulo 99.

$177 \times 135 \equiv 1 \pmod{919}$

Q9) $31x \equiv 57 \pmod{61}$

first, we have to find the inverse of 31 modulo 61

we have to find $\gcd(31,61)$ first.

$61 = 1 \cdot 31 + 30$

$31 = 1 \cdot 30 + 1 \rightarrow \gcd$

$30 = 30 \cdot 1 + 0$

therefore:

$1 = 31 - 1 \cdot 30$

$1 = 31 - 1 \cdot (61 - 1 \cdot 31)$

$1 = 2 \cdot 31 - 61$

So the coefficients are 2 and $-1$

and the inverse of 31 modulo 61 is 2

if we multiply both sides of the congruence by the inverse we get:

$2 \times 31x \equiv 2 \times 57 \pmod{61}$

as we know $2 \times 31 \equiv 1 \pmod{61}$ so therefore

$x \equiv 2 \times 57 \pmod{61}$

$x \equiv 114 \pmod{61}$

$x \equiv 53 \pmod{61}$

Q10) imagine we have a three digit number

$$abc \xrightarrow{s} abc = (a \times 100) + (b \times 10) + c$$

$$abc = 100a + 10b + c$$

and the number cba is going to be:

$$cba = 100c + 10b + a$$

therefore $abc - cba = (100a + 10b + c) - (100c + 10b + a)$

$$abc - cba = 99a - 99c$$

$$abc - cba = 99(a - c)$$

$$abc - cba = 9 \times 11 \times (a - c)$$

So it's always divisible by 9

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Q11) prove $\sum_{j=n}^{2n-1} (2j+1) = 3n^2$ for all positive integers.

base case: for $n=1$, the sum has only one term:

$\sum_{j=1}^{1} (2j+1) = 2(1)+1 = 3$ and the righ hand side

of the equation is $3(1)^2 = 3$

therefore it's true for the base case

inductive hypothesis: we assume the statement is true for some positive integer $n=k$

$$\sum_{j=k}^{2k-1} (2j+1) = 3k^2$$

inductive step:

prove the statement holds for $n = K+1$, meaning:

$$\sum_{j=K+1}^{2(K+1)-1} (2j+1) = 3(K+1)^2$$

we expand the left side:

$$\sum_{j=K+1}^{2(K+1)-1} (2j+1) = \sum_{j=K}^{2K-1} (2j+1) + \sum_{j=2K}^{2K+1} (2j+1)$$

from the inductive hypothesis:

$$\sum_{j=K+1}^{2(K+1)-1} (2j+1) = 3K^2$$

therefore we would have:

$$\sum_{j=K+1}^{2(K+1)-1} (2j+1) = 3K^2 + \left(2(2K)+1\right) + \left(2(2K+1)+1\right)$$

$$- (2K+1) = 3K^2 + 6K + 3$$

$$= 3(K+1)^2 \quad \therefore \text{ proven!}$$

Q12) $a_1 = 2$, $a_2 = 9$, and $a_n = 2a_{n-1} + 3a_{n-2}$ $n \geq 3$

prove $a_n \leq 3^n$ for for all positive integers $n$

base case $n=1$:

$a_1 = 2$ , $2 \leq 3^1$ ✓

$a_2 = 9$ , $9 \leq 3^2$ ✓

$\therefore$ the base case is true

induction hypothesis:

we assume

$a_n \leq 3^n$

inductive step: we have to prove that

$a_{n+1} \leq 3^{n+1}$ :

$a_{n+1} = 2a_n + 3a_{n-1}$

$a_{n+1} \leq 2(3^n) + 3(3^{n-1})$

$a_{n+1} \leq 2(3^n) + \cancel{3}\left(\frac{3^n}{\cancel{3}}\right)$

$a_{n+1} \leq 2(3^n) + 3^n$

$a_{n+1} \leq (2+1)3^n$

$a_{n+1} \leq 3(3^n)$

$a_{n+1} \leq 3^{n+1}$

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Q13) prove $4 \mid (9^n - 5^n)$ for all $n \geq 0$

base case: $n=0$

$4 \mid (9^0 - 5^0)$

$4 \mid 0$ True ✓

induction hypothesis:

we assume the statement is true for $n$

$4 \mid (9^n - 5^n)$ so $9^n - 5^n = 4m$

induction step:

now we have to prove its true for $n+1$ (next page!)

$4 \mid (9^{n+1} - 5^{n+1})$

$\equiv 4 \mid (9.9^n - 5.5^n)$

$9^{n+1} - 5^{n+1} = (9.9^n - 9.5^n) + (9.5^n - 5.5^n)$

$9^{n+1} - 5^{n+1} = 9(9^n - 5^n) + 5^n(9-5)$

$9^{n+1} - 5^{n+1} = 9\underbrace{(9^n - 5^n)}_{4m} + 4.5^n$

$9^{n+1} - 5^{n+1} = 9(4m) + 4.5^n$

$9^{n+1} - 5^{n+1} = 4(9m) + 4.5^n$

$9^{n+1} - 5^{n+1} = \underbrace{4(9m + 5^n)}_{\text{divisible by } 4}$

So we conclude that $4 \mid 9^{n+1} - 5^{n+1}$ therefore the statement $4 \mid 9^n - 5^n$ for $n \geq 0$ is proven to be true.