CompSci 1DM3 - Discrete Math
Assignment 5

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Q1) number of passwords with at least one digit = total number of passwords − number of passwords without any digits

- number of uppercase letters: 26 (A-Z)
- number of lowercase letters: 26 (a-z)
- number of digits: 10 (0-9)

} 62 possible options for each character

total number of passwords: 62^6 (product rule)

number of passwords without any digits: 52^6 (product rule)

number of passwords with at least one digits: $62^6 - 52^6 = 37029625920$

So it takes 37029625920 units of time to check every possible character combinations.

Q2) To go from A to B, you must take 5 moves right (R) across 5 columns and 4 moves up (U) across 4 rows.

total number of moves = 5(R) + 4(U) = 9 moves

So we can choose 4 of these 9 moves to be up and the rest would be right.

$$C(n,r) = \frac{n!}{(n-r)!\,r!} \qquad C(9,4) = \frac{9!}{5!\,4!} = \frac{9 \times \cancel{8}^{2} \times 7 \times \cancel{6} \times \cancel{5!}}{\cancel{5!} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$$

$= 126$ ways.

So there is a total of 126 ways to get from the point A to the point B

Q3)

a) to guarantee a pair:

There are 13 different ranks at a deck of cards. So according to the pigeonhole principle, if we pick 13 cards it is possible each is a different rank but if we pick 14 cards, the 14th card must match one of the previous 13 cards.

min cards for a pair $= 13+1$
$= 14$

b) to guarantee three of a kind:

we can pick 26 cards and it is still possible to not have three of a kind, but if pick 27 cards, the 27th card must give us three of a kind.

min cards for three of a kind $= (13 \times 2)+1$
$= 27$ cards.

Q4)

we want to count the permutations of 12345 where 3 stays at position 3 and 1, 2, 4, and 5 are not in their original positions (derangements)

A_1: permutations where 1 is in its position
A_2: permutations where 2 is in its position

A_4: permutations where 4 is in its position.

A_5: permutations where 5 is in its position.

$N =$ total number of permutations of 4 digits $- |A_1 \cup A_2 \cup A_4 \cup A_5|$

$$|A_1 \cup A_2 \cup A_4 \cup A_5| = \sum |A_i| - \sum |A_i \cap A_j| + \sum |A_i \cap A_j \cap A_k| - |A_1 \cap A_2 \cap A_4 \cap A_5|$$

where $\sum |A_i| = 4 \times 3! = 24$

$\sum |A_i \cap A_j| = \binom{4}{2} \times 2! = 6 \times 2 = 12$

$\sum |A_i \cap A_j \cap A_k| = \binom{4}{3} \times 1! = 4 \times 1 = 4$

$|A_1 \cap A_2 \cap A_4 \cap A_5| = 1$

So $|A_1 \cup A_2 \cup A_4 \cup A_5| = 24 - 12 + 4 - 1 = 15$

total permutation of 4 digits $= 4! = 24$

final answer: $24 - 15 = 9$ so there are 9 permutations where they have 3 at its position and no other integer at its own position.

Q5) we use product rule for this question

step 1) we choose a consonant for the first position

: 5 Choices (B, C, D, F, G)

step 2) we chose anothe consonat for the last position

: 4 choices (one of them is chosen in step 1)

step 3) arrange the remaining 5 letters (2 vowels, 3 consonants) in the middle : $5! = 120$

total permutations : $120 \times 4 \times 5 = 2400$ final answer

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Q6)

a) we put a and z in a box beside eachother

- $2! = 2$ ways to arrange 'a' and 'z' in a box
- 25 items left to permute so $25!$
- Total permutations : $26!$

$$P(E) = \frac{|E|}{|S|} = \frac{2 \times 25!}{26!} = \frac{2 \times \cancel{25!}}{26 \times \cancel{25!}} = \frac{1}{13}$$

b) $\bar{E}$ = "a and b are not next to eachother"

E: "a and b are next to eachother"

$P(\bar{E}) = 1 - P(E)$

$P(E) = \frac{2 \cdot 25!}{26!} = \frac{1}{13}$ according to part a

$P(\bar{E}) = 1 - \frac{1}{13} = \frac{12}{13}$

c) "a and z are separated by at least 23 letters in the permutation"

there is only three valid positions for 'a' and 'z' (1,25), (1,26), and (2,26) and 'a' and 'z' can be swapped so there is $3 \times 2 = 6$ possible ways.

The 24 remaing letters can be permutated freely = 24!

$P(E) = \frac{|E|}{|S|} = \frac{6 \times 24!}{26!} = \frac{6}{26 \times 25} = \frac{3}{325}$

d) among the 3 letters {z,a,b} there 3! = 6 possible permutations. only 2 of these 6 permutations have 'z' before 'a' and 'b', which are "z a b" and "z b a".

$P(E) = \frac{|E|}{|S|} = \frac{2}{6} = \frac{1}{3}$

Q7) according to Bayes' Theorem

$P(D) = \frac{1}{10000} = 0.0001$ probability a person has the disease

$P(Pos|D) = 0.999 \rightarrow$ true positive

$P(Pos|\bar{D}) = 0.0002 \rightarrow$ false positive

$P(D|Pos) = ?$ probability someone has the disease given they test positive

Bayes theorem:

$$P(F|E) = \frac{P(E|F)P(F)}{P(E)}$$

$$P(D|Pos) = \frac{P(Pos|D) \cdot P(D)}{P(Pos)}$$

and $P(Pos) = P(Pos|D) \cdot P(D) + P(Pos|\bar{D}) \cdot P(\bar{D})$

$$P(Pos) = (0.999) \cdot (0.0001) + (0.0002) \cdot (1 - 0.0001)$$
$$= 0.00029988$$

$$P(D|Pos) = \frac{(0.999) \cdot (0.0001)}{0.00029988} \simeq 0.3331$$

The probability of someone who test positive actually has the disease is 33.3%

Q8)

a)

P = probability at least two people share the same birthday

$$P = 1 - \left(\frac{7}{7} \cdot \frac{6}{7} \cdot \frac{5}{7} \cdots \frac{7-n+1}{7}\right), \text{ when } n \leq 7$$

if $n > 7$, then:

$$P = 1$$

final answer: $P = 1$ when $n > 7$

by the pigeonhole principle, since there are only 7 days in a week, more than 7 people must result in at least two people sharing the same birthday. So the probability is 1.

b)

$1 - P(n) > \frac{1}{2}$, where $P(n) = \frac{7}{7} \cdot \frac{6}{7} \cdots \frac{7-n+1}{7}$

if we substitude values, the smallest number which results in $1 - P(n) > \frac{1}{2}$, would be 4

$$1 - P(4) = 1 - \left(\frac{7}{7} \cdot \frac{6}{7} \cdot \frac{5}{7} \cdot \frac{4}{7}\right) = 0.65 > 0.5$$

So $n = 4$