Ontario Virtual School

MHF 4U
TEST SUMMARY - TEST #4 NAME: Arjun Bally Manvir Sandher

Knowledge and Understanding	Inquiry	Communication	Application
/20	/20	/20	/15

*Note: 5 marks will be awarded for communication.

Communication throughout the test will be evaluated on 1) significant digits (avoid rounding off errors) 2) correct mathematical form 3) well-labelled diagrams 4) organized, neat and legible 5) Show all work with concluding statements.

Level 4	Level 3	Level 2	Level 1
communicates for different audiences and purposes with a high degree of effectiveness and uses conventions, vocabulary, and terminology of the discipline with a high degree of effectiveness	communicates for different audiences and purposes with considerable effectiveness and uses conventions, vocabulary, and terminology of the discipline with considerable effectiveness	communicates for different audiences and purposes with some effectiveness and uses conventions, vocabulary, and terminology of the discipline with some effectiveness	communicates for different audiences and purposes with limited effectiveness and uses conventions, vocabulary, and terminology of the discipline with limited effectiveness

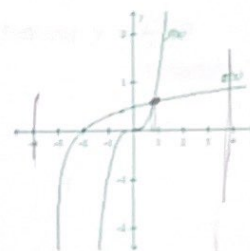## Part 1 Knowledge/Understanding (20 marks)

Multiple Choice: Please circle the most correct answer. Each question is worth one (1) mark each for a total of ten (10) marks.

1. If $f(x) = sin\ x$ and $g(x) = cos\ x$, what is the range of $(f + g)(x)$?

a) $\{y \epsilon R| -1 \le y \le 1\}$
b) $\{y \epsilon R | 0 \le y \le 1\}$
c) $\{y \epsilon R | 0 \le y \le \sqrt{2}\}$
d) $\{y \epsilon R| -\sqrt{2} \le y \le \sqrt{2}\}$

2. Suppose $f(x) = x^2$ and $g(x) = -x + 56$. What are the x-intercepts of the graph of $(f - g)(x)$?

$x^2 + x - 56 \rightarrow (x+8)(x-7)$

a) $-8\ and\ -7$
b) $-8\ and\ 7$
c) $8\ and\ -7$
d) $8\ and\ 7$

3. If $f = \{(-6,1), (-5,3), (3,9), (5,-2)\}$ and $g = \{(-6,4), (-5,3), (-1,5), (5,5)\}$, what is $f + g$?

a) $\{(-12,5), (-10,6), (2,14), (10,3)\}$
b) $\{(-6,1), (-6,4), (-5,3), (5,-2), (5,5)\}$
c) $\{(-12,5), (-10,6), (10,3)\}$
d) $\{(-6,5), (-5,6), (5,3)\}$

4. If $f(x) = \sqrt{2x + 6}$ and $g(x) = x + 3$, what is the domain of $(f + g)(x)$?

$\sqrt{2x+6} + x + 3$

a) $\{x \epsilon R\}$
b) $\{x \epsilon R | x \le -3\}$
c) $\{x \epsilon R | x \le -\frac{2}{6}\}$
d) $\{x \epsilon R | x \ge -3\}$

5. The number of deer in a forest as a function of the number of weeks that have passed can be modelled by the logistic function $D(t) = \frac{500}{1+499(0.6)^t}$. About how many deer are in the forest after 8 weeks have passed?

a) 44
b) 47
c) -50
d) 53

6. Given a function $f(x)$ and $g(x) = \frac{1}{3}x + 1$, how would the graph of $y = f(x)$ compare to the graph of $y = f(g(x))$?

a) vertically compressed and translated to the left

b) horizontally stretched and translated to the left *(circled)*

c) horizontal stretch and translated to the right

d) horizontally compressed and translated to the right

(handwritten: cosx -tanx)

7. If $f(x) = \cos x$ and $g(x) = \tan x$, which of these lines is an asymptote of the graph of $(f - g)(x)$?

a) $x = \frac{\pi}{4}$ b) $x = \frac{\pi}{2}$ *(circled)*   c) $x = \pi$ d) $x = 2\pi$

8. If $f(x) = \log 4x$ and $g(x) = x^2 - 2$, which of these expressions is equivalent to $(f \circ g)(4)$?

a) $\log^2 14$ *(circled)*   b) $\log^2 16 - 2$ c) $\log 4 + \log 14$ *(circled)*   d) $\log 4 \times \log 14$

9. In the graph shown, $f(x) = x^3$ and $g(x) = \log(x + 3)$. For what value of x is $x^3 = \log(x + 3)$ when $x \in [-4, 4]$

a) $x = -2.0$

b) $x = -0.8$

c) $x = 0.8$ *(circled)*

d) $x = 2.0$

(handwritten work:)
$f(g(x))$
$= \log^4 (x^2 - 2)$
$\log^4 (14)$
$= \log$

(handwritten: $x^2 - x - 6 > 5^x + 2$)

10. For which values of x is $x^2 - x - 6 > 5^x + 2$?

a) $\{x < -2.4\}$ *(circled)*   b) $\{x > -2.4\}$ c) $\{x < 2.4\}$ d) $\{x > 2.4\}$

Short Answer: show all your work from this point forward.

11. If $f(x) = \frac{2}{x}$ and $g(x) = x^3 + 2$, what is $y = g(x) - f(x)$ for $x = -4$?   (3 marks)

(handwritten:) $g(x) - f(x) = x^3 + 2 - \frac{2}{x}$, $(-4)^3 + 2 - \frac{2}{-4} = -64 + 2 + \frac{1}{2} = \frac{-123}{2}$

12. Given the function $f(x) = \sin x$ and $g(x) = \frac{1}{3}$ determine the range of the combined function $\frac{f(x)}{g(x)}$.   (2 marks)

(handwritten:) $\frac{\sin x}{\frac{1}{3}} = 3\sin x$ R: $\{y \in R \mid -3 \leq y \leq 3\}$

(handwritten graph)

13. Given the functions $f(x) = \cos x$ and $g(x) = 3$, determine the range of the combined function $y = f(x) + g(x)$.   (2 marks)

(handwritten:) $= \cos x + 3$ R: $\{y \in R \mid 2 \leq y \leq 4\}$

(handwritten graph)

14. If $f(x) = \frac{x+4}{2}$ and $g(x) = x^2 - 4$, what is $f(g(3))$?   (3 marks)

(handwritten:) $f(g(x)) = f(x^2 - 4) = \frac{x^2 - 4 + 4}{2} = \frac{x^2}{2}$

$f(g(3)) = \frac{3^2}{2} = \frac{9}{2}$ $\therefore f(g(3)) = \frac{9}{2}$

Part B: Thinking and Inquiry (20 marks)

1. The graph of $y = f(x) - g(x)$ is shown. Determine the approximate solution to the inequality
$f(x) \leq g(x)$ (3 marks)

$f(x) \leq g(x)$

$f(x) - g(x) \leq 0$

$\hookrightarrow y \leq 0$
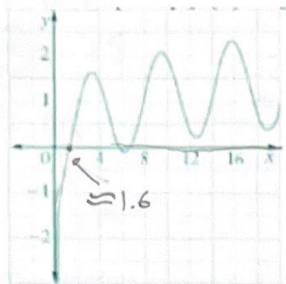
$y \leq 0$ when $x \leq 1.6$

$\therefore f(x) \leq g(x)$ when $x \leq 1.6$

≈ 1.6

2. If $f(3) = 0$, $f(-1) = 0$, $g(3) = 0$ and $g(4) = 0$ then what are the zero(s) of the function $y = \frac{f(x)}{g(x)}$? (3 marks)

$0 = \frac{f(x)}{g(x)}$

$f(x) = 0$

$\therefore$ zero is at $x = -1$

$f(3) = 0$ — undefined bc $g(3)$ is also 0

$\boxed{f(-1) = 0}$

3. Jack and Jill work at the same store. Jack's total weekly salary, in dollars, if he sells x items is given by $S = 110 + 5x$, and Jill's total weekly salary, in dollars, if she sells x items is given by $S = 90 + 6x$. If they sell the same number of items in a week, what is the minimum number of items they have to sell so that Jill's weekly salary is at least 300 more than Jack's? (4 marks)

$S_{Ja} = 110 + 5x$

$S_{Ji} = 90 + 6x$

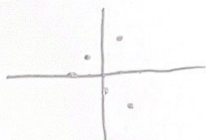$S_{Ji} \geq S_{Ja} + 300$

$90 + 6x \geq 110 + 5x + 300$

$x \geq 320$

$\therefore$ Jack and Jill have to sell atleast 320 items for Jill to have at least 300 more than Jack

4. Consider the table of values (below-left) for two functions. Complete the table of values (below-right) for the composite function $y = g(f(x))$. (5 marks)

x	$y = f(x)$	$y = g(x)$
-2	2	0
-1	0	1
0	1	-1
1	-2	2
2	-1	-2

x	$y = g(f(x))$
-2	-2
-1	-1
0	2
1	0
2	1

5. If $f(x) = \frac{2}{x-5}$ and $g(x) = 5^x$, find: $y = \frac{2}{x-5} \rightarrow x = \frac{2}{y-5} \rightarrow y-5 = \frac{2}{x}$

a) $f^{-1}(g(x))$

$y = \frac{2}{x} + 5$ (3 marks)

$= \frac{2}{g(x)} + 5$

$= \frac{2}{5^x} + 5$ $\therefore 5^{-1}(g(x)) = \frac{2}{5^x} + 5$

$\frac{2}{5^y} + 5 = x$

$\log 5^y = \log \frac{2}{x-5}$ $y = \frac{\log\left(\frac{2}{x-5}\right)}{\log 5}$

b) The domain and range of $f^{-1}(g(x))$ (2 marks)

$5^x \neq 0$

never can

happen anyway $\therefore D: \{x \in R\}$

so no restrictions $R: \{y \in R \mid y > 5\}$

Part C - Communication (15 marks)

1. Match each graph with its corresponding equation by writing the letter of the graph besides its corresponding equation.   (5 marks)

a.   b.   c.

d.   e.
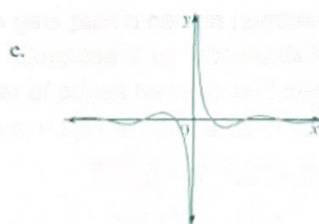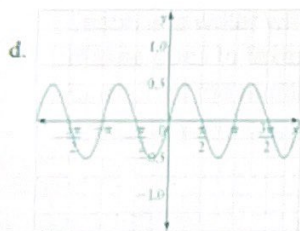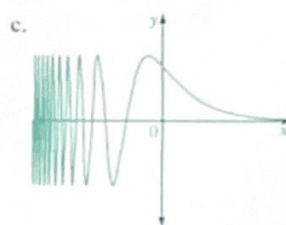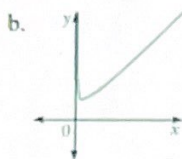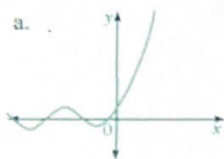
1. $y = \frac{\cos x}{x}$ e

2. $y = \sin(2^{-x})$ c

3. $y = 2^x + \sin x$ a

4. $y = \sin x \cos x$ b

5. $y = x + 1 - \log x$ d

2. The height, h, of two balls in metres, after t seconds are shown in the graph below.

a) Determine the approximate time interval for when the two balls were less than 4m apart (3 marks)

$$t \in R \mid 0 \le t \le 1.3 \}$$

the two ball's were less than 4m apart in the first 1.3s

b) When was the difference between the heights of the balls the greatest? (2 marks)

the difference in height was greatest at around 2.1sec (10m)
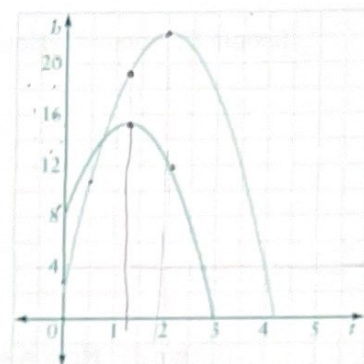
3. Given the graph of $f(x)$ and $g(x)$ as shown below. Sketch $y = \frac{1}{2}[f(x)g(x)]$ on the same grid. Plot at least 4 points. (5 marks)
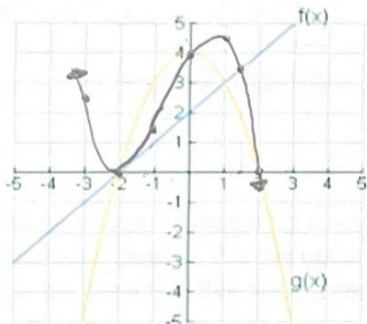
x	$f(x)$	$g(x)$	y
-1	1	3	3/2
0	2	4	4
1	3	3	9/2
2	4	0	0
1.5	3.5	2	3.5
3	5	-5	-25/2
-3	-1	-5	5/2
-2	0	0	0

Extra points to help sketch

Part D - Application (15 marks)

1. Jason is a writer who gets paid a certain number of dollars per page, but 15% of the money he makes goes to taxes. Suppose $d(p)$ represents the number of dollars Jason gets paid as a function of the number of pages he write and suppose $t(p)$ represents the amount of Jason's dollars that go to taxes. If $t(p) = 600$, what is the value of $d(p) - t(p)$? (5 marks)

$d(p) - t(p)$
$= 4000 - 600$
$= 3400$

$t(p) = 0.15 \, d(p)$
$d(p) = \dfrac{t(p)}{0.15}$

$d(p) = \dfrac{600}{0.15} = 4000$

∴ value of $d(p) - t(p)$ is $3400

2. Emily runs a bakery where she bakes and sells cupcakes. She bakes c cupcakes each week and sells them for 8.50−0.5c dollars each. Her costs include a fixed cost of $15 for ingredients plus $1.25 per cupcake made. Assuming Emily sells all the cupcakes she bakes:

a) Write an equation to represent her total weekly cost and another for weekly revenue. (3 marks)

Cost: $C(x) = 15 + 1.25x$

Revenue: $R(x) = $ price per cupcake $\times$ # of cupcakes

$$= x(8.5 - 0.5x)$$
$$R(x) = 8.5x - 0.5x^2$$

b) Determine how many cupcakes Emily should bake each week to make a profit. (3 marks)

$$C(x) > R(x)$$
$$15 + 1.25x > 8.5x - 0.5x^2$$
$$0.5x^2 - 7.25x + 15 > 0$$
$$x^2 - 14.5x + 30 > 0$$

$$x = \frac{14.5 \pm \sqrt{14.5^2 - 4(1)(30)}}{2}$$

$$= \frac{14 \pm 9.5}{2} = 11.75 \text{ or } 2.25 \text{ , round up because cupcakes come as wholes}$$

12 or 3

∴ Emily should bake between 3 and 12 cupcakes to make a profit

3. The rental fee for reserving a reception room at a banquet hall is $975, in addition to a charge of $39.95 per person. For the upcoming month, the banquet hall plans to provide a discount on the total bill. Describe this discounted cost as a function of the number of people attending. (2 marks)

$C(x) = 975 + 39.95x$ Let discounted amount be represented by d

$C(x) = (1-d)(975 + 39.95x)$ x is # of people

4. The function $f(x) = 0.13x$ represents the sales tax owed on a purchase with a selling price of x dollars, and the function $g(x) = 0.75x$ represents the sale price of an item with a price tag of x dollars during a 25% off sale. Write a function that represents the sales tax owed on an item with a price tag of x dollars during a 25% off sale. (2 marks)

sales tax owed is gonna be sales tax on price which is 0.75 (price)

$$f(g(x)) = f(0.75x) = 0.13(0.75x)$$

$$= 0.0975   ∴ Sales tax owed would be 0.0975x$$