COMPSCI 1DM3 - Assignment 5

Due date: April 7, 11:59 PM

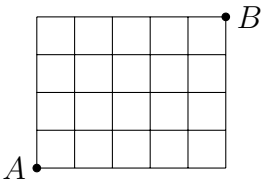
For all subsequent questions, the following are required:

- ☐ ***Initial formula*** *with variables*
- ☐ *Substitution of* ***values into formula***
- ☐ *final* ***numeric simplification***
- ☐ *brief* ***explanation*** *(1-2 sentences each)*

1. (2 points) Each user has a password 6 characters long where each character is an uppercase letter, a lowercase letter, or a digit. Each password must contain at least one digit. How long will it take to check every possible character combination, if each check takes one unit of time.

2. (2 points)

The figure at the right shows a 4-block by 5-block grid of streets. Find the number of ways in which you can go from point A to point B, where at each stage you can only go right or up. (You are not allowed to go left or down.) For example, one allowable route from A to B is:
`Right, Right, Up, Right, Up, Up, Right, Right, Up`.



3. (2 points) You pick cards one at a time without replacement from an ordinary deck of 52 playing cards. What is the minimum number of cards you must pick in order to guarantee that you get:

(a) a pair (for example, two kings or two 5s).

(b) three of a kind (for example, three 7s).

4. (1 point) How many permutations of 12345 are there that leave 3 in the third position but leave no other integer in its own position? Must use the inclusion-exclusion principle.

5. (2 points) How many permutations of the seven letters A, B, C, D, E, F, G do not have vowels on the ends? Assume without replacement.

6. (4 points) What is the probability of these events when we randomly select a permutation of the 26 lowercase letters of the English alphabet?

(a) a and z are next to each other in the permutation.

(b) a and b are not next to each other in the permutation.

(c) a and z are separated by at least 23 letters in the permutation.

(d) z precedes both a and b in the permutation.

7. (4 points) Suppose that one person in 10,000 people has a rare genetic disease. There is an excellent test for the disease; 99.9% of people with the disease test positive and only 0.02% who do not have the disease test positive.

What is the probability that someone who tests positive has the genetic disease?

8. (3 points) Must use the Pidgeonhole Principle in explanation.

(a) What is the probability that in a group of n people chosen uniformly randomly, there are at least two born on the same day of the week?

(b) How many people chosen at random are needed to make the probability greater than $\frac{1}{2}$ that there are at least two people born on the same day of the week?