EFFICIENCY CHANGE IN ELEMENTARY SCHOOLS IN BEAVERTON SCHOOL DISTRICT

(A MALMQUIST ANALYSIS)

PROJECT REPORT

By

Kamal Thapa

Instructor: Dr. Timothy R. Anderson

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Abstract

We analyze the productivity changes of 34 elementary schools in Beaverton School District across the academic years 2018-19, 2020-21, and 2021-22. Utilizing the Malmquist productivity index as our methodological framework, we aim to estimate the changes in the total factor productivity which can be decomposed into two main components: pure technical efficiency change (PTEC) and technological change (TC). Our approach involves one input variable (total expenditure) and four output variables (number of disabled students, number of free/reduced meal recipients, number of ELA proficients, and number of Maths proficients). Utilizing these variables allow us to measure how effectively these schools are spending to achieve both equity and academic success. The findings indicate that, on average, the regression in total factor productivity from the academic year 2018-19 to 2020-21 is predominantly associated with technological change, while managerial efficiency exhibits a minimal impact on total factor productivity during this period. In contrast, the progression in total factor productivity from the academic year 2020-21 to 2021-22 is more closely linked to technological change, with managerial efficiency contributing minimally to the overall progress.

Keywords: *elementary school, pure technical efficiency change, technological change, total factor productivity, Malmquist productivity index, DEA*

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1 Introduction

Education, particularly within the context of elementary schools, occupies a pivotal role as a foundational building block for individual and societal development. The early years of education represent a critical phase in shaping young minds, imparting essential skills such as literacy, numeracy, and critical thinking. Beyond personal growth, education assumes profound significance for communities and nations, acting as a catalyst for social progress, economic advancement, and civic engagement. In this era of rapid globalization and technological advancement, the quality of elementary education assumes paramount importance, as it not only equips individuals with the tools to navigate the complexities of the modern world but also contributes to the overall vitality of a nation, which, in turn, is strongly associated with economic growth.

In the pursuit of educational excellence, the concept of benchmarking emerges as an integral strategy. Benchmarking schools allows for a systematic evaluation of their performance, enabling educational leaders to identify effective practices, areas for improvement, and innovative approaches. The utilization of Data Envelopment Analysis (DEA) as a benchmarking tool provides a rigorous framework for assessing the efficiency and effectiveness of elementary schools. DEA's inherent ability to accommodate multiple inputs and outputs without requiring predetermined weights makes it particularly well-suited for evaluating educational institutions [1].

1.1 Quality Education Model (QEM)

Oregon's Quality Education Model is one such program which helps to identify best practices in delivering a quality K-12 education to all students and determine the resources needed to provide an optimal public education system. In doing so, it aims to rectify the disparities in educational resources and outcome, thereby fostering an equitable learning landscape. Through consistent updates and enhancements, the model incorporates contemporary best practices, empowering it to effectively assess and analyze education policy proposals in its biennial reports. This continuous improvement process is geared towards elevating the model into a research-oriented tool, providing valuable insights to educators and policymakers. By comprehensively understanding the level of investment required to achieve desired educational goals, this model facilitates informed decision-making, ultimately advancing the quality and fairness of education across the board [2].

Quality Education Model report from 2022 (13th edition) suggests that there are three major areas for best practices which can be listed as follows:

- **Focus on equity centered practices**: Quality Education Commission believes focusing on equity is important to achieve lasting educational gains by promoting Oregon's most marginalized students namely students of color, first time English language learners, students with disabilities, and students from low-income families. The aim is to reduce disparities and improve equity.
- **Support student success**: Quality Education Model emphasizes on promoting student success by running a highly effective school system through various means such as implementing newest research, optimal use of technologies and so on.
- **Target investment where most needed**: Quality Education Model suggests to target uses of resources such that it incorporates a focus on equity and increased support for traditionally marginalized students.

Quality Education Model is a "professional judgment model" that proposes a set of hypothetical prototype schools. These schools serve as the unit of analysis for evaluating costs. In other words, each hypothetical school helps to estimate how much would it cost to run a highly effective school system using a set of inputs required. Since we will be focusing primarily on elementary schools, a prototype elementary school defined by the Quality Education Model has the following attributes [2]:

Elementary School- 360 Students

- All-day kindergarten
- Class size average: 20
- 1 librarian per school
- 1 school nurse per school
- 1 PE and music specialist per school
- 1 Family Resource Staffer per school
- Computers for students and staff

1.2 Beaverton School District

The Beaverton School District is a prominent educational institution known for its commitment to providing quality education to students in the Beaverton area. It serves as a hub for diverse communities, catering to a wide range of student needs and aspirations. With a strong emphasis on academic excellence and holistic development, the district strives to prepare students for lifelong success. It caters to students from various areas, including Beaverton, Hillsboro, Aloha, and unincorporated neighborhoods of Portland. The district has a rich history, with its roots dating back to 1876 when the Beaverton Elementary School District 48 was established. Over time, it merged with other elementary districts and eventually unified with the Beaverton High School District to create the current unified school district. With an enrollment of approximately 39,180 students as of 2022, the Beaverton School District is the third-largest school district in the state. It operates a total of 34 elementary schools, 9 middle schools, and 6 high schools, including the newest addition, Mountainside High School, which opened in 2017.

The Beaverton School District has made significant strides in managing resources effectively to support student achievement. It employs over 2,100 teachers and approximately 4,458 staff members to provide quality education. With a budget of $622.8 million for the 2022-2023 school year, the district continues to invest in various educational initiatives to enhance learning experiences for its diverse student population. The Beaverton School District is committed to meeting the diverse needs of its students, providing equitable education, and ensuring their success in their academic journey. Through ongoing efforts and strategic planning, the district aims to continue its mission of delivering high-quality education and preparing students for lifelong success [3].

In conjunction with the Quality Education Model's efforts to explore optimal approaches in Oregon's public education landscape, our study has chosen to focus on Beaverton School District and its elementary schools as the subject of our efficiency analysis through DEA.

1.3 Impact of COVID Pandemic in Elementary Schools

The COVID-19 pandemic brought about profound disruptions in elementary education, prompting a transition from traditional in-person classrooms to online learning. This shift, though crucial for continuity, brought disparities in technology access, particularly affecting lower-income students with limited internet connectivity at home [4]. The impact on elementary school education, however, extends beyond access challenges. The pandemic-induced closures and the subsequent shift to virtual learning led to a decline in learning progress, especially affecting younger students' skill development and cognitive growth. This decline, exemplified by a notable drop in Maths scores, poses challenges for elementary school students, necessitating targeted remediation policies to address the widening educational inequalities [5].

The repercussions of the pandemic on elementary education are multifaceted, encompassing not only academic setbacks but also concerning trends in students' mental health. It highlights a significant rise in anxiety and depression among elementary students, emphasizing the need for comprehensive support measures [6]. Furthermore, data reveals persistent academic gaps among elementary students, with the interruption of in-person learning creating a compounding effect that requires substantial intervention for recovery. Efforts, including federal funding for interventions like tutoring and summer school, have not yet bridged the gap, underscoring the ongoing challenges in aligning the scale of unfinished learning [7]. These findings collectively emphasize the urgent need for targeted strategies, including increased technology access, mental health support, and tailored academic recovery programs, to address the diverse challenges faced by elementary students in the wake of the COVID-19 pandemic.

1.4 Scope and Objective of Study

This study aims to assess the efficiency change of elementary schools in Beaverton School District using DEA-based Malmquist productivity index analysis. The primary focus lies on assessing efficiency changes across three pivotal academic years: 2018-19 (pre-COVID pandemic), 2020-21 (first period of COVID pandemic) and 2021-22 (second period of COVID pandemic). The objective is to quantify and analyze the efficiency shifts within these time-frames and understand the impact of COVID-19 pandemic on elementary school performance in the district. This will also enable us to better understand how elementary schools adapted and handled the challenges brought by the pandemic.

The rest of this report is organized as follows. Section 2 presents a literature review on benchmarking tools and use of Malmquist productivity index analysis in education. Section 3 outlines the data source, data processing and selection of input, and output variables. Section 4 presents input oriented VRS DEA model and Malmquist productivity index used for efficiency analysis. Section 5 delves into the interpretation and discussion of the results obtained. Section 6 provides concluding remarks and Section 7 discusses the limitations of this study and suggests possible areas for future work. Finally, Section 8 provides efficiency results in Appendix using several benchmarking packages.

2 Literature Review

2.1 Methods of Evaluation

Efficiency estimation in various fields often relies on two distinct methods: the parametric Stochastic Frontier Analysis (SFA) and the non-parametric Data Envelopment Analysis (DEA). SFA takes a different route. It employs a stochastic procedure, meaning it accounts for uncertainty in its evaluations. This makes SFA well-suited for cases where data might have inherent variability or randomness [8]. However, SFA introduces complexity, especially when dealing with scenarios that involve multiple inputs and outputs. The interplay of these various factors can challenge the precise determination of efficiency scores, necessitating careful consideration of model assumptions and data characteristics [8].

On the other hand, DEA takes a deterministic approach, systematically enveloping observations within a predetermined framework to quantify efficiency levels [8]. By doing so, it offers valuable insights into how effectively tasks are being accomplished and identifies potential benchmarks for improvement. This method proves particularly useful in situations where a direct comparison of inputs and outputs is needed with minimal assumptions and without explicit information on either inputs or outputs[8] [9] [10]. DEA was initially introduced by Charnes, Cooper & Rhodes [11] as the CCR model which assumes Constant Return to Scale (CRS) and later extended by Banker, Charnes & Cooper (1984) as the BCC model which employs Variable Returns to Scale (VRS) [12]. CRS scale means that changes in inputs have a proportional impact on outputs whereas VRS scale assumes that changes in inputs do not have a proportional impact on outputs. As a non-parametric linear programming method, DEA enables the comparison of efficiencies among various units under evaluation, termed as Decision Making Units (DMUs). The performance of these DMUs is evaluated based on the concept of efficiency and productivity, represented as the ratio of total outputs to total inputs. Efficiency scores are estimated in relation to the top-performing DMU (or DMUs), which receives a score of 1, while other DMUs' performance falls between 0 and 1 relative to this best performance. Thus by providing frontier locations, DEA helps to identify benchmarks for inefficient units while offering areas for improvement to enhance overall efficiency [8] [13].

2.2 Malmquist Productivity Index Analysis in Education

The Malmquist productivity index has been a pivotal tool in evaluating educational efficiency, technology change, and overall productivity across diverse contexts. Rahimian et al. studied on Iranian universities spanning the years 2004 to 2007, employing a combination of DEA and the Malmquist productivity index. Their approach considered three inputs- number of students, university professors, and employees- and two outputs- number of educated individuals and research outputs. The results show a notable opportunity for improvement among several private educational institutions, emphasizing these institutions to focus on enhancing their research activities and increase performance levels, particularly through emphasis on publications [14]. Similarly, in their study, Wang et al. highlighted the pivotal role of universities in national development, particularly addressing challenges encountered by the top 8 institutions in New Zealand. Despite their global acclaim, declining rankings are attributed to government funding policies leading to insufficient resources. Through the application of the Malmquist productivity index model, the research analyses technical efficiency, technological change, and overall productivity from 2013 to 2018. The results reveal a "no-change" scenario, prompting universities to enhance internal factors and adapt to technological advancements for sustainable progress [15].

Pietrzak et al. studied to determine changes in efficiency of humanities faculties using Malmquist productivity index across 14 universities. The study utilized output-oriented Malmquist productivity index for 2008-09 to 2014-15. The results show an average total factor productivity value of 28%, indicating increased faculty productivity. The technological growth exhibited minimal influence on the index, maintaining an average value of around 1. The primary driving factor for increased productivity of faculties was identified as changes in technical efficiency [16]. Zhang et al. proposed a DEA approach to evaluate the performance of higher education resource utilization in provincial level in China. To assess the productivity change of each province, Malmquist productivity index was used for period 2005 to 2015. Results show that higher education in China experienced overall gains in productivity, primarily driven by technological advancements, with policy recommendations emphasizing the need for balanced development and improved resource utilization, especially in less developed provinces [17].

Afonso et al. analyzed the productivity changes in basic and secondary education for 24 governorates in Tunisia over the period 2004 to 2008 using Malmquist productivity index to estimate the changes in total factor productivity. The study used four input variables- teacher-to-student ratio, classes-to-student ratio, schools-to-inhabitants ratio, and education expenditure per student- and two output variables that measure success rate of baccalaurate exam and rate of non-doubling in the 9th year. The results show that on average, changes in total factor productivity growth during the period 2004-2008 linked more to the changes in technology, while the impact of managerial efficiency on total factor productivity was insignificant [18]. Mostoli et al. studied on 120 ninth grade female students using Malmquist productivity index at different time intervals to evaluate the development of the educational process in increasing mathematical literacy. The initial use of the output-oriented CCR model allowed for the determination of students' performance coefficients, followed by the application of the Malmquist productivity index to compare productivity evaluations at two distinct points: end of the training course in December and the end of the year in June. The results show that students in the experimental group, exposed to problem-solving and realistic mathematics, demonstrated an increased overall productivity evaluation factor after completing the training compared to the others [19].

3 Data

3.1 Data Processing

The data used in this study was extracted from the official website of the Oregon Department of Education (ODE) for three academic years 2018-19, 2020-21, and 2021-22 for 34 elementary schools in Beaverton School District. The study excluded the academic year 2019-20 due to unavailability of data. Additionally, there were some missing data for ELA and Maths proficiency in the academic year 2020-21. To address this, we calculated average percentage drop for ELA and Maths proficients from academic year 2018-19 to 2020-21. On average, ELA proficients experienced a decline of approximately 90%, while Maths proficients dropped by around 89%. This observed trend guided our approach to fill in the missing proficiency data for the academic year 2020-21. To ensure data accuracy and reliability, a series of data pre-processing and cleaning steps were undertaken. Essential R library packages, including `dplyr`, `stringr`, and `readr`, were employed for this purpose. The GitHub repository containing all the data processing files can be accessed at Kamaljthapa.

The data for academic year 2018-19 is presented in Table 1

Table 1: Data for Academic Year 2018-19

School Name	Total Student	Total Expenditure	No. of Students with Disability	No. of Free/Reduced Lunch Recipeients	No. of ELA Proficients	No. of Maths Proficients
Aloha-Huber	925	12632610	130	701	273	260
Barnes	623	8792172	94	410	109	57
Beaver	643	9899004	123	371	172	141
Bethany	530	6696406	64	76	214	198
Bonny	653	8159241	53	66	276	269
Cedar	425	5682892	43	54	157	143
Chehalem	495	7898631	85	295	105	90
Cooper	478	7178193	67	82	163	145
Elmonica	721	9511353	80	336	186	159
Errol	457	6810338	83	169	140	122
Findley	701	8817527	43	39	367	358
Fir	385	6306070	54	189	115	84
Greenway	328	6022002	63	221	67	50
Hazeldale	453	7724995	96	233	138	105
Hiteon	661	9241286	113	182	220	218
Jacob	731	9334173	44	40	355	350
Kinnaman	658	10407798	119	432	172	123
McKay	295	5407526	80	169	63	52
McKinley	565	9162499	74	341	118	84
Montclair	304	4529157	37	68	125	99
Nancy	657	7379215	66	125	237	196
Oak	568	7717841	63	92	219	188
RaleighHills	549	8653730	105	238	207	168
ReleighPark	369	5970724	67	142	125	106
Ridgewood	415	6782238	75	82	155	140
Rock	564	7419737	57	122	215	195
Sato	627	2683854	44	81	232	238
Scholls	563	7997753	91	95	209	195
Sexton	550	7434903	66	97	188	189
Springville	868	11208460	96	138	377	374
Terra	361	5691396	62	124	122	107
Vose	651	9588145	72	504	156	142
West	349	5521967	67	37	131	121
William	449	7775287	68	424	64	60
Mean	**546**	**7707033**	**75**	**199**	**182**	**163**
Std. Dev.	**154**	**1990634**	**24**	**158**	**80**	**85**
Max. Value	**925**	**12632610**	**130**	**701**	**377**	**374**
Min. Value	**295**	**2683854**	**37**	**37**	**63**	**50**

The data for academic year 2020-21 is presented in Table 2.

Table 2: Data for Academic Year 2020-21

School Name	Total Student	Total Expenditure	No. of Students with Disability	No. of Free/Reduced Lunch Recipeients	No. of ELA Proficients	No. of Maths Proficients
Aloha-Huber	842	12877122	127	635	27*	29*
Barnes	516	7656996	78	331	11*	6*
Beaver	712	11114002	129	371	7	16*
Bethany	450	6465263	59	73	19	26
Bonny	569	8102428	52	74	21	18
Cedar	346	5364718	35	43	22	20
Chehalem	422	7218284	68	244	10*	10*
Cooper	390	6050854	59	64	10	13
Elmonica	466	6907531	47	218	10	11
Errol	335	5156133	54	120	14*	5
Findley	541	7070379	28	30	49	49
Fir	341	5615165	58	160	14	4
Greenway	302	5786872	58	205	3	6*
Hazeldale	434	7032349	92	206	6	12*
Hiteon	543	7997196	98	141	15	15
Jacob	641	8920616	39	35	48	50
Kinnaman	549	9338559	105	336	17*	14*
McKay	291	5466262	76	150	6*	6*
McKinley	625	9388804	82	387	12*	9*
Montclair	252	4315456	31	48	15	8
Nancy	510	7061759	57	120	22	11
Oak	476	7278385	58	79	14	13
RaleighHills	437	7025425	79	201	21*	18*
ReleighPark	309	5225224	59	129	12	12*
Ridgewood	358	5621720	54	72	9	18
Rock	429	6070228	43	78	23	15
Sato	670	8877474	54	91	31	41
Scholls	588	8647866	89	82	21*	9
Sexton	477	7013869	63	75	13	15
Springville	833	10919702	75	105	50	42
Terra	306	4858299	65	107	5	8
Vose	696	10545369	84	471	5	16*
West	277	4499023	45	37	13*	8
William	484	9070555	83	343	6*	7*
Mean	**483**	**7369408**	**67**	**172**	**17**	**16**
Std. Dev.	**154**	**2058994**	**24**	**143**	**12**	**12**
Max. Value	**842**	**12877122**	**129**	**635**	**50**	**50**
Min. Value	**252**	**4315456**	**28**	**30**	**3**	**4**

* Calculated based on average percentage change from academic year 2018-19 to 2020-21

The data for academic year 2021-22 is presented in Table 3.

Table 3: Data for Academic Year 2021-22

School Name	Total Student	Total Expenditure	No. of Students with Disability	No. of Free/Reduced Lunch Recipeients	No. of ELA Proficients	No. of Maths Proficients
Aloha-Huber	869	14575280	113	633	171	183
Barnes	506	8956710	76	314	47	32
Beaver	705	12224309	127	368	126	92
Bethany	399	6471875	56	58	149	135
Bonny	630	9415429	51	74	261	249
Cedar	369	6070325	37	41	152	131
Chehalem	401	7773039	61	213	80	64
Cooper	435	8951142	66	66	123	111
Elmonica	448	6826964	50	203	99	79
Errol	353	5748729	39	119	68	60
Findley	510	7470182	26	12	230	233
Fir	358	6433931	58	169	87	58
Greenway	296	6540453	48	197	48	34
Hazeldale	420	8033818	97	194	92	73
Hiteon	517	8598041	94	130	144	124
Jacob	613	9232867	31	25	256	245
Kinnaman	514	10126221	108	309	78	51
McKay	282	6202423	54	142	44	30
McKinley	606	10875640	73	360	64	53
Montclair	293	5146980	27	55	74	63
Nancy	521	7737861	58	116	173	135
Oak	512	8253395	57	85	150	146
RaleighHills	345	6688993	42	153	102	85
ReleighPark	327	5972842	59	132	62	65
Ridgewood	378	6743362	57	73	116	97
Rock	435	6628720	44	75	130	118
Sato	761	10742457	61	86	273	265
Scholls	630	9838519	95	79	195	186
Sexton	481	7981198	77	69	153	133
Springville	774	10631983	62	91	289	271
Terra	295	5640633	68	92	88	78
Vose	695	11851703	91	462	98	85
West	300	5532104	48	37	94	78
William	510	10292904	82	354	57	43
Mean	**485**	**8241501**	**64**	**164**	**129**	**114**
Std. Dev.	**155**	**2238622**	**25**	**140**	**69**	**71**
Max. Value	**869**	**14575280**	**127**	**633**	**289**	**271**
Min. Value	**282**	**5146980**	**26**	**12**	**44**	**30**

3.2 Selection of Inputs and Outputs

The criteria for selection of inputs and outputs are subjective and lack specific guidelines for this process [13]. Typically, inputs encompass the resources or factors employed by DMUs to generate outputs, while outputs signify the outcomes or results produced by the DMUs using these inputs. DEA studies commonly employ “lesser-the-better” measures for inputs, indicating that lower values correspond to heightened efficiency. Conversely, “more-the-better” measures are used for outputs, with higher values signifying enhanced performance [20]. This approach enables DEA to evaluate DMU efficiency by minimizing inputs or maximizing outputs.

Generally inputs and outputs are chosen based on the objective of the study. In our study, we seek to assess the overall performance of elementary schools in Beaverton School District, considering the challenge of budget constraints while prioritizing processes that support student success and equitable practices in elementary schools. Therefore, we chose the following inputs and outputs that aligns with the three key focus areas- expenditure, equity, and performance as outlined by Quality Education Model.

3.2.1 Input

- **Total school expenditure**: It refers to the overall amount of money spent on various aspects of operation and educational programs. This expenditure covers resources, personnel, facilities, and services aimed at providing education and support to students and faculties in categories such as *Direct Classroom Expenditures*, *Classroom Support*, *Building Support*, *Central Support*, and *Other Expenditures*.

3.2.2 Outputs

- **Number of students with disability**: This represents the number of students who had an Individualized Education Program at any time during that school year. It indicates the school's commitment to inclusivity and providing individualized support for diverse learners.
- **Number of free/discounted lunch recipients**: This indicates the number of students eligible for free or discounted lunch under the National School Lunch Program at any time during that school year. This information is used as an indicator of the school's socio-economic diversity and the proportion of students from low-income families.
- **Number of ELA proficients**: This refers to the number of students proficient in English, language and arts. Students are proficient when they attain a level of English language skill necessary to independently produce, interpret, collaborate on, and succeed on grade-level content-related academic tasks in English. This is indicated on *ELPA21* by attaining a profile of Level 4 or higher in all domains. It signifies a school's focus on language development, academic preparedness, support for English language learners, effective teaching, and readiness for future success.
- **Number of Maths proficients**: This reflects the number of students proficient in Mathematics. Students are proficient when they can apply math skills and knowledge to real world contexts and to problem-solving. It quantifies the student performances and reflects their mastery of mathematical concepts in a school.

Thus, based on QEM, above output variables can be categorized into two groups:

Equity Centered Measures- Number of students with disability and number of free/discounted lunch recipients

Performance Based Measures- Number of ELA proficients and number of Maths proficients

Figure 1 shows the DEA model used for this study.

4 Methodology

4.1 DEA model

DEA analysis needs to clearly identify what is to be achieved from the analysis and based on that DEA models can typically adopt two main forms: input-oriented or output-oriented [20]. Input orientation, also known as input minimization or contraction, aims to minimize inputs while still achieving the given output

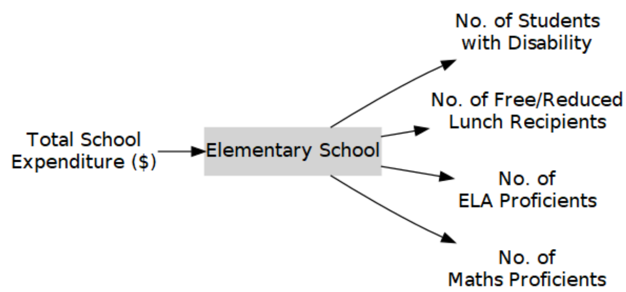

Figure 1: DEA Input-Output Diagram (BSD)

levels. On the other hand, output orientation, also known as output maximization or expansion, seeks to maximize outputs without requiring any increase in observed input values. When dealing with inflexible inputs not fully under control, an output-based approach would be more suitable. However, in cases where outputs are determined by management goals rather than maximizing performance, an input-based approach may be more appropriate [13].

Another option in DEA analysis is the choice between constant returns to scale (CRS) and variable returns to scale (VRS). The concern about returns to scale revolves around how schools' outputs are affected when they alter the quantity of inputs used in their production process [21]. CRS assumes no significant relationship between the scale of operations and school efficiency, meaning large schools can be just as efficient as small ones in converting inputs to outputs. The efficiency score under CRS corresponds to technical efficiency (TE) which measures efficiency due to input and output configuration as well as the scale of operation. In contrast, VRS implies that an increase in input would lead to a disproportionate increase in output. VRS is preferred when there is a significant correlation between the size of DMUs and efficiency in a large sample [13]. The efficiency score under VRS corresponds to pure technical efficiency (PTE) which is a measure of efficiency without the scale efficiency. For our study, there are schools of varying operational sizes. As a result, we have chosen a VRS scale assuming there may be a significant relationship between the scale of operation and school efficiency.

Based on Quality Education Model report [2], elementary schools in Beaverton School District aim to minimize the total school expenditure (because of budget constraint) while promoting school performance, equity centered practices and fulfillment of socio-economic needs. Since the input variable, *total school expenditure*, can be adjusted and controlled to a certain extent, this study uses input orientation VRS DEA model for efficiency assessment.

The input-oriented VRS envelopment model for calculating efficiency can be formulated as the following linear programming problem:

$$
\begin{aligned}
\text{minimize } & \theta \\
\text{subject to } & \sum_{j=1}^{n} x_{i,j}\lambda_j \leq \theta x_{i,k} \; \forall \, i \\
& \sum_{j=1}^{n} y_{r,j}\lambda_j \geq y_{r,k} \; \forall \, r \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0 \; \forall \, j
\end{aligned}
$$

4.2 Malmquist Productivity Index (MPI)

The Malmquist productivity index (MPI) captures the progress or regress in the efficiency of a DMU and the progress or regress of frontier technology over time. Initially introduced by Malmquist in 1953 as a measure of standard of living, it was subsequently applied to production theory in 1982 by Christensen and Diewert [22]. In 1989, Farr et al., utilized DEA to calculate the Malmquist index, and by 1994, the index was broken down into two factors: efficiency and technology. This technical efficiency values obtained from DEA equations are computed as a distance function [19]. This DEA-based MPI measures the total factor productivity by handling multiple inputs and outputs with minimal assumptions and without explicit information on either inputs or outputs.

To illustrate the MPI, Figure 2 depicts a DMU at period 1 and period 2. Calculating the MPI for a specific DMU involves computing the geometric mean of two ratios. These ratios are derived from the efficiencies of the DMU at period 1 and 2 for both frontiers.

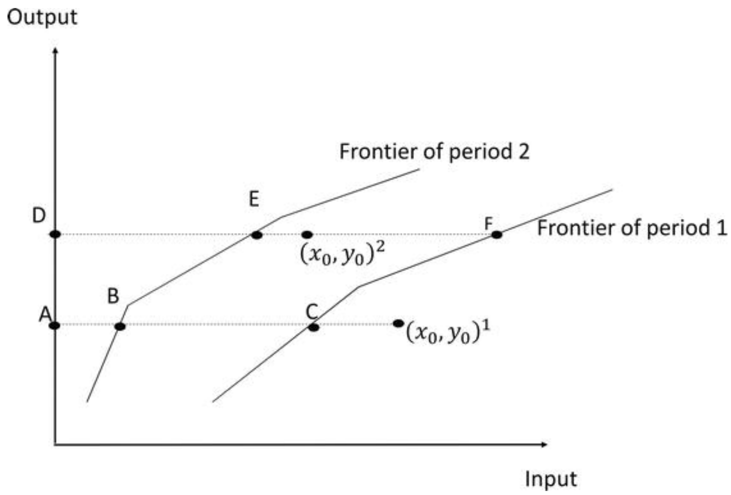

Figure 2: Vectors of Inputs and Outputs in Different Periods

MPI can be explained by decomposing it into two factors- Catch-up effect pertaining to efficiency change and Frontier-shift effect associated with technological change.

4.2.1 Catch-up Effect

The catch-up effect from period 1 to 2 is measured as:

$$\text{Catch-up} = \frac{\text{Efficiency of } (x_0, y_0)^2 \text{ with respect to period 2 frontier}}{\text{Efficiency of } (x_0, y_0)^1 \text{ with respect to period 1 frontier}}$$

$$\text{Catch-up} = \frac{\text{DE}/\text{D}{x_0}^2}{\text{AC}/\text{A}{x_0}^1} \qquad [Eq-1]$$

In Figure 2, DE represents the distance between points D and E, while D${x_0}^2$ symbolizes the distance between D and the abscissa of the point $(x_0,\, y_0)^2$. Similarly, AC signifies the distance between points A and C, and and A${x_0}^1$ is the distance between A and abscissa of the point $(x_0,\, y_0)^1$.

(Catch-up) $>$ 1 indicates progress in relative efficiency from period 1 to 2 while (Catch-up) = 1 and (Catch-up) $<$ 1 indicate no change and regress in efficiency respectively.

4.2.2 Frontier-shift Effect

The change of $(x_0,\, y_0)^1$ from frontier of period 1 to frontier of period 2 is called the Frontier-shift effect at $(x_0,\, y_0)^1$ and it can be evaluated as:

$$\mathrm{F1} = \frac{\mathrm{AC}}{\mathrm{AB}} \quad [Eq-2]$$

If numerator and denominator are divided by the distance between A and abscissa of point $(x_0,\, y_0)^1$ in Eq-2, then:

$$\mathrm{F1} = \frac{\mathrm{AC}/\mathrm{A}{x_0}^1}{\mathrm{AB}/\mathrm{A}{x_0}^1} \qquad [Eq-3]$$

$$\mathrm{F1} = \frac{\text{Efficiency of } (x_0, y_0)^1 \text{ with respect to period 1 frontier}}{\text{Efficiency of } (x_0, y_0)^1 \text{ with respect to period 2 frontier}} \qquad [Eq-4]$$

The change of $(x_0,\, y_0)^2$ from frontier of period 1 to frontier of period 2 is called the Frontier-shift effect at $(x_0,\, y_0)^2$ and it can be expressed as:

$$\mathrm{F2} = \frac{\mathrm{DF}}{\mathrm{DE}} \quad [Eq-5]$$

$$\mathrm{F2} = \frac{\mathrm{DF}/\mathrm{D}{x_0}^2}{\mathrm{DE}/\mathrm{D}{x_0}^2} \qquad [Eq-6]$$

$$\mathrm{F2} = \frac{\text{Efficiency of } (x_0, y_0)^2 \text{ with respect to period 1 frontier}}{\text{Efficiency of } (x_0, y_0)^2 \text{ with respect to period 2 frontier}} \qquad [Eq-7]$$

Using F1 and F2, the Frontier-shift effect can be defined as the geometric mean as follows:

$$\text{Frontier-shift} = \sqrt{\mathrm{F1} \cdot \mathrm{F2}} \quad [Eq-8]$$

A value of (Frontier-shift) > 1 signifies progress in the frontier technology around DMU from period 1 to 2. Conversely, (Frontier-shift) $= 1$ and (Frontier-shift) < 1 indicate the status-quo and regress in the frontier technology respectively.

4.2.3 Malmquist Productivity Index (MPI)

The Malmquist productivity index (MPI) is computed as the product of Catch-up and Frontier shift terms as:

$$\mathrm{MPI} = \text{Catch-up} \cdot \text{Frontier-shift} \quad [Eq-9]$$

Using Eq-1, 2, 5, and 8, Eq-9 can be re-written as:

$$\mathrm{MPI} = \frac{\mathrm{A}{x_0}^1}{\mathrm{D}{x_0}^2}\sqrt{\frac{\mathrm{DF} \cdot \mathrm{DE}}{\mathrm{AC} \cdot \mathrm{AB}}} \quad [Eq-10]$$

In Eq-10, first term signifies the relative change in performance, while the second represents the relative change in frontier used to evaluate these performances.

Eq-1 can also be expressed using notation for efficiency score of DMU as follows:

$$\text{Catch-up} = \frac{\delta^2((x_0, y_0)^2)}{\delta^1((x_0, y_0)^1)} \quad [Eq - 11]$$

In Eq-11, $\delta^2((x_0,\, y_0)^2)$ denotes the efficiency of DMU observed in period 2 measured by the frontier technology 2 and $\delta^1((x_0,\, y_0)^1)$ denotes the efficiency of DMU observed in period 1 measured by the frontier technology 1. δ^1 refers to the frontier efficiency of period 1 and δ^2 refers to frontier efficiency of period 2.

Likewise, the Frontier-shift and can be re-written as:

$$\text{Frontier-shift} = \sqrt{\frac{\delta^1((x_0, y_0)^1)}{\delta^2((x_0, y_0)^1)} \cdot \frac{\delta^1((x_0, y_0)^2)}{\delta^2((x_0, y_0)^2)}} \quad [Eq - 12]$$

Similarly, MPI can be expressed as:

$$\text{MPI} = \sqrt{\frac{\delta^1((x_0, y_0)^2)}{\delta^1((x_0, y_0)^1)} \cdot \frac{\delta^2((x_0, y_0)^2)}{\delta^2((x_0, y_0)^1)}} \quad [Eq - 13]$$

Eq-13 provides interpretation of MPI as the geometric mean of the two efficiency ratios: one representing the efficiency change measured by the period 1 technology and the other efficiency change measured by the period 2 technology. Also, MPI comprises four terms: $\delta^1((x_0,\, y_0)^1)$, $\delta^2((x_0,\, y_0)^2)$, $\delta^1((x_0,\, y_0)^2)$, and $\delta^2((x_0,\, y_0)^1)$. The first two are associated to the measurements within the same time period with $t = 1$ or $t = 2$, while the last two are employed for intertemporal comparison.

Alternatively, VRS MPI can be expressed as the product of Pure Technical Efficiency Change (PTEC) and Technology Change (TC) as shown below:

$$\text{MPI} = \text{PTEC} \cdot \text{TC} \quad [Eq - 14]$$

MPI > 1 indicates progress in the total factor productivity of the DMU from period 1 to 2, while MPI $= 1$ and MPI < 1 indicate the status-quo and deterioration in the total factor productivity respectively.

5 Results and Discussion

In order to solve our MPI model, we have utilized `MultiplierDEA` package where functions are provided for calculating efficiency using multiplier DEA.

5.1 Academic Year 2018-19 and 2020-21

Table 4: MPI Results for Academic Year 2018-19 and 2020-21 (IO-VRS)

	E1819-T1819	E2021-T1819	E2021-T2021	E1819-T2021	PTEC	TC	MPI
Aloha-Huber	1.000	0.930	1.000	6.624	1.000	0.375	0.375
Barnes	0.933	0.873	1.000	3.596	1.072	0.476	0.510
Beaver	1.000	1.102	1.000	4.552	1.000	0.492	0.492
Bethany	0.672	0.591	1.000	5.648	1.488	0.265	0.395
Bonny	0.616	0.406	0.695	6.555	1.129	0.234	0.264
Cedar	0.472	0.500	0.968	4.787	2.049	0.226	0.463
Chehalem	0.854	0.745	0.917	3.555	1.075	0.441	0.474
Cooper	0.638	0.631	0.843	4.633	1.322	0.321	0.424
Elmonica	0.716	0.683	0.914	4.837	1.276	0.333	0.425
Errol	0.881	0.691	1.000	4.302	1.136	0.376	0.427
Findley	1.000	0.380	1.000	8.334	1.000	0.213	0.213
Fir	0.687	0.734	1.000	4.032	1.456	0.353	0.515
Greenway	0.818	0.798	1.000	3.249	1.223	0.448	0.548
Hazeldale	0.934	0.947	1.000	4.174	1.071	0.460	0.493
Hiteon	1.000	0.911	1.000	5.371	1.000	0.412	0.412
Jacob	0.904	0.301	1.000	8.025	1.106	0.184	0.204
Kinnaman	0.963	0.894	1.000	4.552	1.039	0.435	0.452
McKay	1.000	0.934	1.000	3.326	1.000	0.530	0.530
McKinley	0.737	0.801	0.918	3.640	1.246	0.420	0.523
Montclair	0.593	0.622	1.000	4.911	1.688	0.274	0.462
Nancy	0.677	0.528	0.852	6.220	1.259	0.260	0.327
Oak	0.574	0.514	0.728	5.747	1.268	0.266	0.337
RaleighHills	0.973	0.800	1.000	5.402	1.028	0.380	0.390
ReleighPark	0.748	0.749	1.000	4.299	1.337	0.361	0.483
Ridgewood	0.782	0.612	0.954	4.508	1.220	0.334	0.407
Rock	0.518	0.442	0.899	5.589	1.735	0.214	0.370
Sato	1.000	0.388	0.919	7.862	0.919	0.232	0.213
Scholls	0.882	0.734	0.925	5.313	1.049	0.363	0.381
Sexton	0.619	0.588	0.789	4.830	1.273	0.309	0.394
Springville	1.000	0.461	1.000	8.334	1.000	0.235	0.235
Terra	0.711	0.879	1.000	4.211	1.407	0.385	0.542
Vose	0.964	0.825	0.954	4.450	0.989	0.433	0.428
West	0.806	0.613	1.000	4.257	1.240	0.341	0.423
William	1.000	0.776	0.878	2.980	0.878	0.544	0.478
Average	**0.814**	**0.688**	**0.946**	**5.080**	**1.205**	**0.351**	**0.412**

Note:

1819 = Academic Year 2018-19 and 2021 = Academic Year 2020-21

E1819-T1819 = Efficiency for academic year 2018-19 with reference technology from academic year 2018-19

E2021-T1819 = Efficiency for academic year 2020-21 with reference technology from academic year 2018-19

E2021-T2021 = Efficiency for academic year 2020-21 with reference technology from academic year 2020-21

E1819-T2021 = Efficiency for academic year 2018-19 with reference technology from academic year 2020-21

The results show that among the 34 elementary schools evaluated during the academic year 2018-19, only 8 were deemed efficient as illustrated by E1819-T1819. These 8 schools build up the technology frontier for

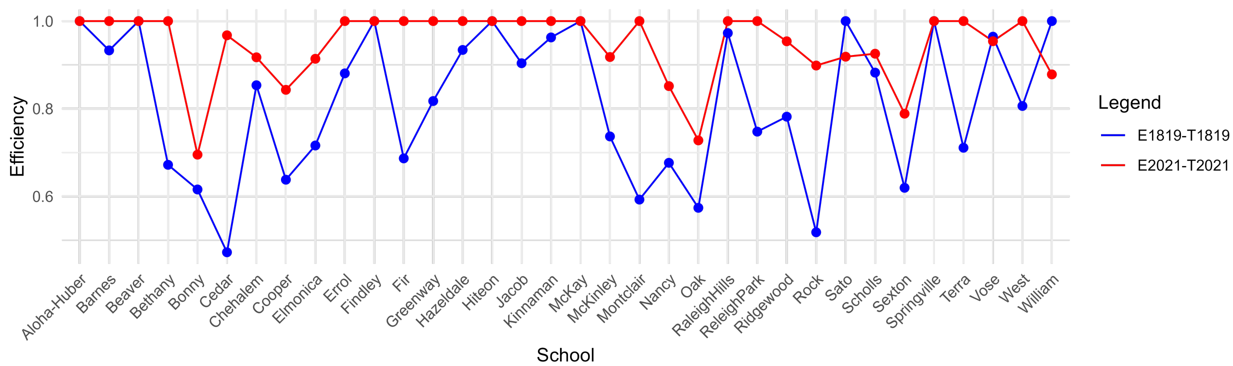

Figure 3: Efficiency of Schools in Academic Years 2018-19 and 2020-21

this academic year. In contrast, in the academic year 2020-21, the number of efficient schools increased to 19, as depicted by E2021-T2021. These 19 schools now establish the technology frontier for this academic year. Aloha-Huber, Beaver Acres, Findley, Hiteon, McKay, and Springville elementary schools maintained their 100% efficiency in both years, as indicated in Table 4, which presents the overall MPI results for the two academic years. Overall, the average efficiency level of schools underwent a notable improvement, rising from 81.4% in the academic year 2018-19 to 94.6% in 2020-21. Figure 3 below shows line graph for efficiency of schools for these two academic years.

Notably, some schools underwent significant changes in either 2018-19 or 2020-21 academic year. Sato and William Walker elementary schools, which achieved 100% efficiency in 2018-19, experienced decreases to 91.9% and 87.8% efficiency, respectively, in the academic year 2020-21. This is also supported by the pure technical efficiency change (PTEC) values being less than 1 which implies regress in operational efficiency. Table 5 shows input and outputs for schools that have efficiency loss from academic year 2018-19 to 2020-21. The comparison of inputs and outputs for schools in the year they were efficient with the year they lost their efficiency dramatically, revealed that their ELA and Maths proficients reduced significantly along with huge increase in school expenditures.

Table 5: School with Efficiency Loss: 2018-19 and 2020-21 Input and Outputs

School	Tot_Exp	Dis_Stu	Meal_Elig	ELA_Prof	Math_Prof	A_Year	Efficiency
Sato	2683854	44	81	232	238	2018-19	1.000
Sato	8877474	54	91	31	41	2020-21	0.919
William	7775287	68	424	64	60	2018-19	1.000
William	9070555	83	343	6*	7*	2020-21	0.878

* Calculated based on average percentage change from academic year 2018-19 to 2020-21

Conversely, Barnes, Bethany, Errol Hassell, Fir Grove, Greenway, Hazeldale, Jacob Wismer, Kinnaman, Montclair, Raleigh-Hills, Raleigh-Park, Terra Linda, and West Tualatin View elementary schools improved their efficiency, progressing from 93.3%, 67.2%, 88.1%, 68.7%, 81.8%, 93.4%, 90.4%, 96.3%, 59.3%, 97.3%, 74.8%, 71.1% and 80.6% in academic year 2018-19 to 100% in 2020-21. This improvement is evident from the PTEC values exceeding 1 which implies progress in operational efficiency. Table 6 shows input and outputs for schools that have efficiency gain from academic year 2018-19 to 2020-21. The comparison of input and outputs for schools in the year they were inefficient with the year they gained their efficiency, revealed that almost all schools significantly reduced their total expenditures.

Table 6: School with Efficiency Gain: 2018-19 and 2020-21 Input and Outputs

School	Tot_Exp	Dis_Stu	Meal_Elig	ELA_Prof	Math_Prof	A_Year	Efficiency
Barnes	8792172	94	410	109	57	2018-19	0.933
Barnes	7656996	78	331	11*	6*	2020-21	1.000
Bethany	6696406	64	76	214	198	2018-19	0.672
Bethany	6465263	59	73	19	26	2020-21	1.000
Errol	6810338	83	169	140	122	2018-19	0.881
Errol	5156133	54	120	14*	5	2020-21	1.000
Fir	6306070	54	189	115	84	2018-19	0.687
Fir	5615165	58	160	14	4	2020-21	1.000
Greenway	6022002	63	221	67	50	2018-19	0.818
Greenway	5786872	58	205	3	6*	2020-21	1.000
Hazeldale	7724995	96	233	138	105	2018-19	0.934
Hazeldale	7032349	92	206	6	12*	2020-21	1.000
Jacob	9334173	44	40	355	350	2018-19	0.904
Jacob	8920616	39	35	48	50	2020-21	1.000
Kinnaman	10407798	119	432	172	123	2018-19	0.963
Kinnaman	9338559	105	336	17*	14*	2020-21	1.000
Montclair	4529157	37	68	125	99	2018-19	0.593
Montclair	4315456	31	48	15	8	2020-21	1.000
RaleighHills	8653730	105	238	207	168	2018-19	0.973
RaleighHills	7025425	79	201	21*	18*	2020-21	1.000
ReleighPark	5970724	67	142	125	106	2018-19	0.748
ReleighPark	5225224	59	129	12	12*	2020-21	1.000
Terra	5691396	62	124	122	107	2018-19	0.711
Terra	4858299	65	107	5	8	2020-21	1.000
West	5521967	67	37	131	121	2018-19	0.806
West	4499023	45	37	13*	8	2020-21	1.000

* Calculated based on average percentage change from academic year 2018-19 to 2020-21

Technological change (TC) refers to the changes in technology or the adoption of new methods and practices that impact productivity. It is notable in Table 4 that TC values, encompassing all schools, including the one that improved technical efficiency from academic year 2018-19 to 2020-21, are less than 1 with an average value of 0.351. This suggests that the elementary schools are experiencing a decrease in productivity due to technological factors. Given the impact of COVID-19 pandemic in the academic year 2020-21, this signifies that schools were not equipped to handle the sudden change in the academic environment. During this first year of COVID period, schools encountered several challenges, including limited technology infrastructure, a digital literacy gap, issues with internet and device access, interactive and hands-on learning challenges, teacher preparedness, student engagement and motivation, as well as student assessment and evaluation.

Table 7 shows the forward frontier shifts represented by $\left[\frac{E1819-T1819}{E1819-T2021}\right]$ and backward frontier shifts represented by $\left[\frac{E2021-T1819}{E2021-T2021}\right]$. The forward frontier shifts represent how schools in academic year 2018-19 think the technology frontier has moved (shifted) between periods 2018-19 and 2020-21. The backward frontier shifts represent how schools in academic year 2020-21 think the technology frontier has moved (shifted) between periods 2020-21 and 2018-19. Both of these shifts provide schools' behavior in academic year 2018-19 and 2020-21, with respect to the frontier in both time periods [23]. Further, if we take the geometric mean of these two shifts, we will obtain the TC value as illustrated in Table 4.

Table 7: Forward and Backward Frontier Shifts for Academic Years 2018-19 and 2020-21

	Forward Frontier Shift	Backward Frontier Shift
	$\frac{E1819-T1819}{E1819-T2021}$	$\frac{E2021-T1819}{E2021-T2021}$
Aloha-Huber	0.151	0.930
Barnes	0.259	0.873
Beaver	0.220	1.102
Bethany	0.119	0.591
Bonny	0.094	0.584
Cedar	0.099	0.517
Chehalem	0.240	0.812
Cooper	0.138	0.749
Elmonica	0.148	0.747
Errol	0.205	0.691
Findley	0.120	0.380
Fir	0.170	0.734
Greenway	0.252	0.798
Hazeldale	0.224	0.947
Hiteon	0.186	0.911
Jacob	0.113	0.301
Kinnaman	0.211	0.894
McKay	0.301	0.934
McKinley	0.202	0.872
Montclair	0.121	0.622
Nancy	0.109	0.620
Oak	0.100	0.707
RaleighHills	0.180	0.800
ReleighPark	0.174	0.749
Ridgewood	0.173	0.642
Rock	0.093	0.492
Sato	0.127	0.422
Scholls	0.166	0.793
Sexton	0.128	0.745
Springville	0.120	0.461
Terra	0.169	0.879
Vose	0.217	0.864
West	0.189	0.613
William	0.336	0.883

Case 1: If $\left[\frac{E1819-T1819}{E1819-T2021}\right] < 1$ and $\left[\frac{E2021-T1819}{E2021-T2021}\right] < 1$ in Table 7 then TC value in Table 4 must also be less than 1, indicating the school has a negative shift and the technology of school declines. In other words, this shift towards negative facet indicates an unfavorable strategy change [23] [24]. This observation matches for 33 schools based on Table 4 and 7, suggesting these schools struggled hugely to adapt the challenges posed by the COVID-19 pandemic during academic year 2020-21.

Case 2: If $\left[\frac{E1819-T1819}{E1819-T2021}\right] < 1$ but $\left[\frac{E2021-T1819}{E2021-T2021}\right] > 1$ in Table 7 then TC value in Table 4 can be greater or less than 1. However, we can certainly conclude that the school moves from a negative shift facet towards a positive shift facet. Moreover, there is a change in the trade-off between the two inputs. Based on Table 4 and 7, this result is true for Beaver Acres and its TC value is less than 1, indicating that the change resulting from the positive shift facet is less than that of the negative shift facet; therefore, on average, the technology of school declines [23] [24].

Consequently, all 34 schools obtained an MPI value of less than 1 (see Table 4) with an average value of 0.412, indicating decline in the total factor productivity. This result is primarily linked to negative technological shifts and operational efficiency change for the respective schools. This result sheds light on the substantial challenges and adverse consequences faced by schools during the first year of COVID pandemic, sharply contrasting with the conditions prevalent in the pre-COVID period. Overall, these MPI values provide a quantitative understanding of the pandemic's impact on school performances.

We will select three random schools, namely- Cedar Mill, Jacob Wismer and Terra Linda, to interpret the MPI results based on Table 4. We can extend the interpretation of results to other schools in a similar way.

- Cedar Mill, with a PTEC value of 2.049 and TC value of 0.226, has an MPI value of 0.463. This means that there is nearly 104.9% improvement in technical (or operational) efficiency separate from the general overall operating conditions while almost 77.4% of the school's deteriorated performance is attributed to the general regress of technology or challenging operating conditions. Despite the overly optimistic operational efficiency progress, there is overall 53.7% regress in total factor productivity, resulted primarily due to significant deterioration in operating conditions during first year of COVID pandemic.
- Jacob Wismer exhibits a PTEC value of 1.106 and a TC value of 0.184, resulting in an MPI value of 0.204. This signifies almost 10.6% improvement in technical (or operational) efficiency which is independent of general overall operating conditions. However, there is approximately 81.6% of the school's performance decline due to the general regress of technology or challenging operating conditions. As a result, there is overall 79.6% decline in total factor productivity, primarily stemming by the extremely unfavorable operating conditions.
- Terra Linda has a PTEC value of 1.407 and TC value of 0.385 resulting an MPI value of 0.542. This means that there is almost 40.7% improvement in technical (or operational) efficiency separate from the overall operating conditions while approximately 61.5% of the school's deteriorated performance is due to the general regress of technology or challenging operating conditions. Despite the a significant operational efficiency progress, there is overall 45.8% regress in total factor productivity, primarily resulting from a substantial deterioration in operating conditions during first year of COVID pandemic.

Let's now delve into some intriguing results from Table 4. Upon superficial observation, it appears misleading to witness all 34 schools exhibiting exceptional efficiency in academic year 2018-19 with respect to the technology frontier in 2020-21, with E1819-T2021 values almost greater than 300% for each schools. A closer examination of inputs and outputs in Table 8 for top 10 efficient schools shows that these schools demonstrate either a remarkable reduction in output or a substantial increase in input during the academic year 2020-21 compared to 2018-19. In other words, the reference technology frontier in 2020-21 lies significantly below these schools in academic year 2018-19. As a result, these schools display an extra-ordinary efficiency (super-efficiency) in academic year 2018-19 with reference to the technology frontier in academic year 2020-21. This further highlights the challenging circumstances and adverse effects experienced by schools during the first year of COVID pandemic compared to the pre-COVID period.

Table 8: Top 10 Schools with Exceptional Efficiency in 2018-19 w.r.t. Technology Frontier in 2020-21: Input and Outputs

School	Tot_Exp	Dis_Stu	Meal_Elig	ELA_Prof	Math_Prof	A_Year	E1819-T2021
Findley	8817527	43	39	367	358	2018-19	8.334
Findley	7070379	28	30	49	49	2020-21	8.334
Springville	11208460	96	138	377	374	2018-19	8.334
Springville	10919702	75	105	50	42	2020-21	8.334

Table 8: Top 10 Schools with Exceptional Efficiency in 2018-19 w.r.t. Technology Frontier in 2020-21: Input and Outputs *(continued)*

School	Tot_Exp	Dis_Stu	Meal_Elig	ELA_Prof	Math_Prof	A_Year	E1819-T2021
Jacob	9334173	44	40	355	350	2018-19	8.025
Jacob	8920616	39	35	48	50	2020-21	8.025
Sato	2683854	44	81	232	238	2018-19	7.862
Sato	8877474	54	91	31	41	2020-21	7.862
Aloha-Huber	12632610	130	701	273	260	2018-19	6.624
Aloha-Huber	12877122	127	635	27*	29*	2020-21	6.624
Bonny	8159241	53	66	276	269	2018-19	6.555
Bonny	8102428	52	74	21	18	2020-21	6.555
Nancy	7379215	66	125	237	196	2018-19	6.220
Nancy	7061759	57	120	22	11	2020-21	6.220
Oak	7717841	63	92	219	188	2018-19	5.747
Oak	7278385	58	79	14	13	2020-21	5.747
Bethany	6696406	64	76	214	198	2018-19	5.648
Bethany	6465263	59	73	19	26	2020-21	5.648
Rock	7419737	57	122	215	195	2018-19	5.589
Rock	6070228	43	78	23	15	2020-21	5.589

Note:
E1819-T2021 = Efficiency for academic year 2018-19 with reference technology from academic year 2020-21
* Calculated based on average percentage change from academic year 2018-19 to 2020-21

5.2 Academic Year 2020-21 and 2021-22

Table 9: MPI Results for Academic Year 2020-21 and 2021-22 (IO-VRS)

	E2021-T2021	E2122-T2021	E2122-T2122	E2021-T2122	PTEC	TC	MPI
Aloha-Huber	1.000	4.673	1.000	2.191	1.000	1.460	1.460
Barnes	1.000	2.404	1.000	1.209	1.000	1.410	1.410
Beaver	1.000	3.507	1.000	2.116	1.000	1.287	1.287
Bethany	1.000	4.444	1.000	0.856	1.000	2.279	2.279
Bonny	0.695	6.163	1.000	0.672	1.439	2.524	3.631
Cedar	0.968	4.759	1.000	0.977	1.034	2.171	2.243
Chehalem	0.917	3.053	0.919	1.075	1.002	1.684	1.687
Cooper	0.843	3.530	0.715	0.914	0.848	2.134	1.809
Elmonica	0.914	3.579	1.000	1.003	1.094	1.806	1.976
Errol	1.000	3.132	1.000	1.132	1.000	1.664	1.664
Findley	1.000	5.750	1.000	0.730	1.000	2.807	2.807
Fir	1.000	3.437	1.000	1.123	1.000	1.749	1.749
Greenway	1.000	2.754	1.000	1.198	1.000	1.516	1.516
Hazeldale	1.000	3.200	1.000	1.141	1.000	1.675	1.675
Hiteon	1.000	4.098	1.000	1.022	1.000	2.002	2.002
Jacob	1.000	6.073	0.960	0.593	0.960	3.266	3.135
Kinnaman	1.000	2.783	1.000	1.104	1.000	1.588	1.588

Table 9: MPI Results for Academic Year 2020-21 and 2021-22 (IO-VRS) *(continued)*

	E2021-T2021	E2122-T2021	E2122-T2122	E2021-T2122	PTEC	TC	MPI
McKay	1.000	1.381	0.974	1.227	0.974	1.075	1.047
McKinley	0.918	2.512	0.879	1.084	0.958	1.555	1.489
Montclair	1.000	3.550	1.000	1.204	1.000	1.717	1.717
Nancy	0.852	4.871	1.000	0.828	1.174	2.238	2.628
Oak	0.728	3.984	0.844	0.758	1.160	2.128	2.469
RaleighHills	1.000	3.672	0.958	1.069	0.958	1.894	1.815
ReleighPark	1.000	2.921	1.000	1.138	1.000	1.602	1.602
Ridgewood	0.954	3.939	0.875	0.973	0.918	2.100	1.927
Rock	0.899	4.021	0.930	0.892	1.035	2.087	2.161
Sato	0.919	6.274	0.968	0.628	1.053	3.080	3.245
Scholls	0.925	4.787	1.000	0.853	1.081	2.279	2.463
Sexton	0.789	4.350	0.973	0.796	1.234	2.105	2.598
Springville	1.000	6.656	1.000	0.569	1.000	3.419	3.419
Terra	1.000	3.506	1.000	1.187	1.000	1.718	1.718
Vose	0.954	3.119	0.971	1.099	1.018	1.670	1.700
West	1.000	3.854	1.000	1.192	1.000	1.798	1.798
William	0.878	2.445	0.943	1.057	1.074	1.468	1.576
Average	**0.946**	**3.917**	**0.968**	**1.047**	**1.030**	**1.969**	**2.038**

Note:
2021 = Academic Year 2020-21 and 2122 = Academic Year 2021-22
E2021-T2021 = Efficiency for academic year 2020-21 with reference technology from academic year 2020-21
E2122-T2021 = Efficiency for academic year 2021-22 with reference technology from academic year 2020-21
E2122-T2122 = Efficiency for academic year 2021-22 with reference technology from academic year 2021-22
E2021-T2122 = Efficiency for academic year 2020-21 with reference technology from academic year 2021-22

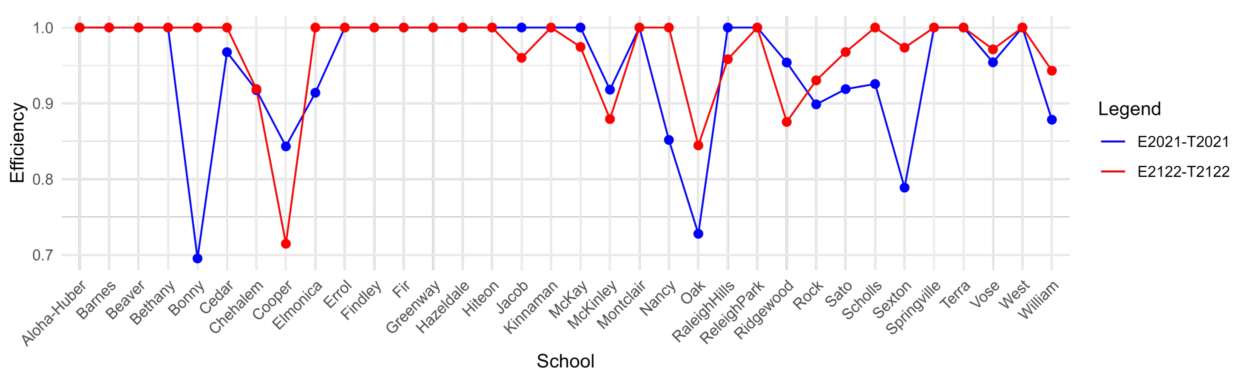

Figure 4: Efficiency of Schools in Academic Years 2020-21 and 2021-22

Results from Table 9 shows that there are 19 efficient schools in academic year 2020-21 exactly matching the results obtained from Table 4 as indicated by E2021-T2021. Similarly, there are 21 efficient schools in academic year 2021-22 as represented by E2122-T2122. These 21 schools constitute the technology frontier for this academic year. Among these, 16 schools from academic year 2020-21 also maintained their 100% efficient in 2021-22. However, three schools- Jacob Wismer, McKay, and Raleigh-Hills elementary schools- which achieved 100% efficiency in 2020-21 experienced slight declines to 96%, 97.4%, and 95.8% respectively.

This observation aligns with PTEC values being less than 1 which implies regress in operational efficiency. Overall, the average efficiency level of schools underwent a notable improvement, rising from 94.6% in the academic year 2020-21 to 104.7% in 2021-22. These efficiencies for both academic years are visually depicted in the line graph presented in Figure 4.

Table 10 shows input and outputs for schools that have efficiency loss from academic year 2020-21 to 2021-22. The comparison of inputs and outputs for schools in the year they were efficient with the year they lost their efficiency indicates a notable decrease in either the number of disabled students and students eligible for free meals or a substantial increase in school expenditures.

Table 10: School with Efficiency Loss: 2020-21 and 2021-22 Input and Outputs

School	Tot_Exp	Dis_Stu	Meal_Elig	ELA_Prof	Math_Prof	A_Year	Efficiency
Jacob	8920616	39	35	48	50	2020-21	1.000
Jacob	9232867	31	25	256	245	2021-22	0.960
McKay	5466262	76	150	6*	6*	2020-21	1.000
McKay	6202423	54	142	44	30	2021-22	0.974
RaleighHills	7025425	79	201	21*	18*	2020-21	1.000
RaleighHills	6688993	42	153	102	85	2021-22	0.958

* Calculated based on average percentage change from academic year 2018-19 to 2020-21

On the other hand, Bonny Slope, Cedar Mill, Elmonica, Nancy Ryles, and Scholls Heights elementary schools improved their efficiency, advancing from 69.5%, 96.8%, 91.4%, 85.2%, and 92.5% in academic year 2020-21 to 100% in 2021-22. This improvement is reflected in the PTEC values exceeding 1 which implies progress in operational efficiency. Table 11 shows input and outputs for schools that have efficiency gain from academic year 2020-21 to 2021-22. The comparison of input and outputs for schools in the year they were inefficient with year they gained their efficiency reveals that, despite an increase in expenditures, they also substantially increased their outputs.

Table 11: School with Efficiency Gain: 2020-21 and 2021-22 Input and Outputs

School	Tot_Exp	Dis_Stu	Meal_Elig	ELA_Prof	Math_Prof	A_Year	Efficiency
Bonny	8102428	52	74	21	18	2020-21	0.695
Bonny	9415429	51	74	261	249	2021-22	1.000
Cedar	5364718	35	43	22	20	2020-21	0.968
Cedar	6070325	37	41	152	131	2021-22	1.000
Elmonica	6907531	47	218	10	11	2020-21	0.914
Elmonica	6826964	50	203	99	79	2021-22	1.000
Nancy	7061759	57	120	22	11	2020-21	0.852
Nancy	7737861	58	116	173	135	2021-22	1.000
Scholls	8647866	89	82	21*	9	2020-21	0.925
Scholls	9838519	95	79	195	186	2021-22	1.000

* Calculated based on average percentage change from academic year 2018-19 to 2020-21

Examining the TC column in Table 9, encompassing all 34 schools, shows values greater than 1. The average TC value, standing at 1.969, sharply contrasts with the result we obtained for academic year 2018-19 and 2020-21. This suggests that the elementary schools experienced an increase in productivity due to technological factors. These schools demonstrated increased resilience in adapting to the challenges posed by the COVID-19 pandemic during the second period, as they successfully navigated obstacles such as limited technology infrastructure, a digital literacy gap, issues related to internet and device accessibility along with transition to in-person learning.

Let's explore more into forward and backward frontier shifts aspects of TC to examine the technological change in details in Table 12. The forward frontier shifts represent how schools in academic year 2020-21 think the technology frontier has moved (shifted) between periods 2020-21 and 2021-22, indicated by $\left[\frac{E2021-T2021}{E2021-T2122}\right]$. The backward frontier shifts represent how school in academic year 2021-22 thinks the technology frontier has moved (shifted) between periods 2020-21 and 2021-22, depicted by $\left[\frac{E2122-T2021}{E2122-T2122}\right]$. Both of these shifts provide schools' behavior in academic year 2020-21 and 2021-22, with respect to the frontier in both time periods [23]. Further, if we take the geometric mean of these two shifts, we will obtain the TC value as illustrated in Table 9.

Table 12: Forward and Backward Frontier Shifts for Academic Years 2020-21 and 2021-22

	Forward Frontier Shift $\frac{E2021-T2021}{E2021-T2122}$	Backward Frontier Shift $\frac{E2122-T2021}{E2122-T2122}$
Aloha-Huber	0.456	4.673
Barnes	0.827	2.404
Beaver	0.473	3.507
Bethany	1.169	4.444
Bonny	1.034	6.163
Cedar	0.990	4.759
Chehalem	0.854	3.323
Cooper	0.922	4.940
Elmonica	0.911	3.579
Errol	0.884	3.132
Findley	1.370	5.750
Fir	0.890	3.437
Greenway	0.835	2.754
Hazeldale	0.876	3.200
Hiteon	0.978	4.098
Jacob	1.686	6.326
Kinnaman	0.906	2.783
McKay	0.815	1.417
McKinley	0.846	2.857
Montclair	0.831	3.550
Nancy	1.028	4.871
Oak	0.959	4.718
RaleighHills	0.936	3.833
ReleighPark	0.879	2.921
Ridgewood	0.980	4.500
Rock	1.008	4.323
Sato	1.464	6.483
Scholls	1.085	4.787
Sexton	0.991	4.469
Springville	1.756	6.656
Terra	0.842	3.506
Vose	0.868	3.212
West	0.839	3.854

Table 12: Forward and Backward Frontier Shifts for Academic Years 2020-21 and 2021-22 *(continued)*

	Forward Frontier Shift	Backward Frontier Shift
	$\frac{E2021-T2021}{E2021-T2122}$	$\frac{E2122-T2021}{E2122-T2122}$
William	0.831	2.593

Case 1: If $\left[\frac{E2021-T2021}{E2021-T2122}\right] > 1$ and $\left[\frac{E2122-T2021}{E2122-T2122}\right] > 1$ in Table 12 then TC value in Table 9 must be greater than 1 indicating that school has a positive shift and the technology of school progresses. Based on Table 9 and 12, this result is true for 9 schools- Bethany, Bonny Slope, Findley, Jacob Wismer, Nancy Ryles, Rock Creek, Sato, Scholls Heights, and Springville. This TC value exceeding 1 indicates an overall progress in school technology and operating conditions [23] [24].

Case 2: If $\left[\frac{E2021-T2021}{E2021-T2122}\right] < 1$ but $\left[\frac{E2122-T2021}{E2122-T2122}\right] > 1$ in Table 12 then TC value in Table 9 can be greater or less than 1. However, we can certainly conclude that the school moves from a negative shift facet towards a positive shift facet. Moreover, there is a change in the trade-off between the two inputs. Based on Table 9 and 12, this result is true for remaining 25 schools with TC values greater than 1, indicating that the change resulting from the positive shift facet is larger than that of the negative shift facet; on average, the technology of school progresses [23] [24].

As a result, each of the 34 schools achieved an MPI value exceeding 1 (see Table 9) with an average value of 2.038. This indicates a remarkable progress in the total factor productivity from first year of COVID pandemic to second year. The progress is attributed to positive shifts in technology and enhancement in operational efficiency within the respective schools. It highlights the resilience and adaptability demonstrated by schools to cope with substantial challenges during the second period of COVID pandemic. Overall, these MPI values provide a quantitative performance of schools in the second period of the COVID pandemic compared to the first period.

We will select same three schools we selected in earlier analysis, namely- Cedar Mill, Jacob Wismer and Terra Linda, to interpret the MPI results based on Table 9. We can extend the interpretation of results to other schools in a similar way.

- Cedar Mill, with a PTEC value of 1.034 and TC value of 2.171, attains an MPI value of 2.243. This implies a distinct 3.4% improvement in technical (or operational) efficiency, independent of general overall operating conditions while almost 117.1% of the school's improved performance is attributed to the general progress of technology or operating conditions. As a result, there is 124.3% progress in total factor productivity during the second period of COVID pandemic.
- Jacob Wismer has a PTEC value of 0.96 and a TC value of 3.266, resulting in an MPI value of 3.135. This signifies almost 4% deterioration in technical (or operational) efficiency, independent of general overall operating conditions. However, approximately 226.6% of the school's performance improvement is due to the general progress of technology or operating conditions. Consequently, there is overall 213.5% progress in total factor productivity, primarily propelled by the extraordinary progress of technology or operating conditions.
- Terra Linda has a PTEC value of 1 and TC value of 1.798, leading to an MPI value of 1.798. This indicates that there is no change in technical (or operational) efficiency which is independent of overall operating conditions while approximately 79.8% of the school's improved performance is due to the general progress of technology or operating conditions. Despite no change in operational efficiency, there is overall 79.8% progress in total factor productivity, primarily driven by a substantial improvement in operating conditions during second year of COVID pandemic.

Let's explore some interesting results from Table 9. It appears that all 34 schools exhibit exceptional efficiency in academic year 2021-22 with respect to the technology frontier in 2020-21, with E2122-T2021 values almost greater than 200% for each schools. However, a more in-depth examination of inputs and outputs in Table 13 for top 10 efficient schools shows that these schools demonstrate a remarkable increase in outputs with minimal increase in input during the academic year 2021-22 compared to 2020-21. In other words, the reference technology frontier in 2020-21 lies significantly below these schools in academic year 2021-22. As a result, these schools display an extra-ordinary efficiency (super-efficiency) in academic year 2021-22 with reference to the technology frontier in academic year 2020-21. This highlights the remarkable rebound of school performances from first phase to second phase of COVID pandemic.

Table 13: Top 10 Schools with Exceptional Efficiency in 2021-22 Compared to 2020-21: Input and Outputs

School	Tot_Exp	Dis_Stu	Meal_Elig	ELA_Prof	Math_Prof	A_Year	E2122-T2021
Springville	10919702	75	105	50	42	2020-21	6.656
Springville	10631983	62	91	289	271	2021-22	6.656
Sato	8877474	54	91	31	41	2020-21	6.274
Sato	10742457	61	86	273	265	2021-22	6.274
Bonny	8102428	52	74	21	18	2020-21	6.163
Bonny	9415429	51	74	261	249	2021-22	6.163
Jacob	8920616	39	35	48	50	2020-21	6.073
Jacob	9232867	31	25	256	245	2021-22	6.073
Findley	7070379	28	30	49	49	2020-21	5.750
Findley	7470182	26	12	230	233	2021-22	5.750
Nancy	7061759	57	120	22	11	2020-21	4.871
Nancy	7737861	58	116	173	135	2021-22	4.871
Scholls	8647866	89	82	21*	9	2020-21	4.787
Scholls	9838519	95	79	195	186	2021-22	4.787
Cedar	5364718	35	43	22	20	2020-21	4.759
Cedar	6070325	37	41	152	131	2021-22	4.759
Aloha-Huber	12877122	127	635	27*	29*	2020-21	4.673
Aloha-Huber	14575280	113	633	171	183	2021-22	4.673
Bethany	6465263	59	73	19	26	2020-21	4.444
Bethany	6471875	56	58	149	135	2021-22	4.444

Note:
E2122-T2021 = Efficiency for academic year 2021-22 with reference technology from academic year 2020-21
* Calculated based on average percentage change from academic year 2018-19 to 2020-21

The majority of schools in MPI results for academic year 2020-21 and 2021-22 show that they are efficient with reference to different year frontier technologies, as indicated by E2122-T2021 and E2021-T2122 values of 1 or greater. These schools in academic year 2020-21 not only far exceeded the technology frontier in 2021-22 but also, reciprocally, in the academic year 2021-22, exceeded the technology frontier in 2020-21. This interesting observation is presented in Table 14.

Further, if we closely examine Table 14, the majority of these schools are efficient with the same-year frontier, or at least greater than 90% as reflected by E2021-T2021 and E2122-T2122 values. This implies that these schools are positioned directly on or in very close proximity to their respective frontiers. In comparison to other-year reference frontiers, they lie above these frontiers which is evident from the values of E2122-T2021 and E2021-T2122 being greater than 1. This implies that these schools can be considered as super-efficient schools with respect to other-year reference technology frontiers.

Table 14: Efficient Schools for Different Year Frontiers: E2122-T2021 and E2021-T2122

	E2021-T2021	E2122-T2021	E2122-T2122	E2021-T2122	PTEC	TC	MPI
Aloha-Huber	1.000	4.673	1.000	2.191	1.000	1.460	1.460
Barnes	1.000	2.404	1.000	1.209	1.000	1.410	1.410
Beaver	1.000	3.507	1.000	2.116	1.000	1.287	1.287
Chehalem	0.917	3.053	0.919	1.075	1.002	1.684	1.687
Elmonica	0.914	3.579	1.000	1.003	1.094	1.806	1.976
Errol	1.000	3.132	1.000	1.132	1.000	1.664	1.664
Fir	1.000	3.437	1.000	1.123	1.000	1.749	1.749
Greenway	1.000	2.754	1.000	1.198	1.000	1.516	1.516
Hazeldale	1.000	3.200	1.000	1.141	1.000	1.675	1.675
Hiteon	1.000	4.098	1.000	1.022	1.000	2.002	2.002
Kinnaman	1.000	2.783	1.000	1.104	1.000	1.588	1.588
McKay	1.000	1.381	0.974	1.227	0.974	1.075	1.047
McKinley	0.918	2.512	0.879	1.084	0.958	1.555	1.489
Montclair	1.000	3.550	1.000	1.204	1.000	1.717	1.717
RaleighHills	1.000	3.672	0.958	1.069	0.958	1.894	1.815
ReleighPark	1.000	2.921	1.000	1.138	1.000	1.602	1.602
Terra	1.000	3.506	1.000	1.187	1.000	1.718	1.718
Vose	0.954	3.119	0.971	1.099	1.018	1.670	1.700
West	1.000	3.854	1.000	1.192	1.000	1.798	1.798
William	0.878	2.445	0.943	1.057	1.074	1.468	1.576

Note:

2021 = Academic Year 2020-21 and 2122 = Academic Year 2021-22

E2021-T2021 = Efficiency for academic year 2020-21 with reference technology from academic year 2020-21

E2122-T2021 = Efficiency for academic year 2021-22 with reference technology from academic year 2020-21

E2122-T2122 = Efficiency for academic year 2021-22 with reference technology from academic year 2021-22

E2021-T2122 = Efficiency for academic year 2020-21 with reference technology from academic year 2021-22

5.3 Evaluation of DEA-Based MPI Packages

This study utilizes `MultiplierDEA` package to obtain the MPI results. In addition to this, Appendix in Section 8 explores on the results obtained from other DEA packages. The `Benchmarking` package shows `Inf` in E1819-T2021 column, pertaining to infeasibility, for efficiency of schools in academic year 2018-19 against the technology frontier in 2020-21 (see Table 18 in Appendix) . A similar observation arises in E2122-T2021 column for efficiency of schools in academic year 2021-22 against the technology frontier in 2020-21 (see Table 19 in Appendix). This is likely due to substantial drop in output variables in COVID year 2020-21 compared to 2018-19 and 2021-22 (see Table 1, 2, and 3). As a result, these schools in academic year 2018-19 and 2021-22 out-lie so high against the frontier in 2020-21 that they demonstrate an extraordinary level of super-efficiency. Consequently, these schools in academic year 2018-19 and 2021-22 fail to hit the technology frontier target in 2020-21 resulting in an infeasible solution. Similar observation holds true for `DJL` package and the same interpretation is applicable (see Table 20 and 21 in Appendix).

In contrast to the two-factor decomposition of MPI by the `MultiplierDEA` package, `deaR` package decomposes it into three factors- PTEC, TC, and SEC (scale efficiency change). Thus, we were able to obtain PTEC, TC, SEC and MPI values using this package (see Table 22 and 23 in Appendix). However, these TC and MPI values obtained from `deaR` package for input oriented VRS is rather consistent with results with `MultiplierDEA` package for CRS scale (see Table 24 and 25 in Appendix). This demonstrates that the `deaR` package likely fails to accommodate input oriented VRS model given the extreme nature of data for this study. Nevertheless, a thorough investigation into this approach is warranted.

On the other hand, `MultiplierDEA` package proves handy in incorporating the extreme nature of the dataset in this study and also fairly handles the schools posing as super-efficient, offering meaningful results.

6 Conclusion

In this study, we employ an input-oriented VRS Malmquist productivity index to evaluate the productivity change of 34 elementary schools in Beaverton School District over the academic years 2018-19, 2020-21, and 2021-22. Our approach involves utilizing total school expenditure as the input variable while the number of disabled students, the number of free/reduced meal recipients, the number of ELA proficients, and the number of Maths proficients as the four outputs variables. This allows us to measure how effectively these schools are spending to accomplish both equity and academic success.

The decomposition of the MPI into PTEC and TC from academic years 2018-19 to 2020-21 allows us to conclude that the total factor productivity, reflected by an average MPI value of 0.412, is primarily associated with the negative shift in technology of schools with an average TC value of 0.351. This underscores the significant struggle of schools to adapt the challenges posed by the COVID-19 pandemic during the first year. Despite an average managerial efficiency score of 1.205, as reflected in the PTEC value, it does not have an important effect on the total factor productivity growth.

In contrast, examining the productivity changes from academic years 2020-21 to 2021-22, the total factor productivity is reflected in an average MPI value of 2.038 which is primarily related to the movement of school technology from negative shift facet to positive shift facet with an average TC value of 1.969. This highlights the resilience and adaptive strategies employed by schools during the second year of COVID pandemic. Moreover, with an average PTEC value of 1.030, managerial efficiency does not appear to exert a substantial impact on total factor productivity growth compared to technological change.

7 Limitations and Future Work

This study extensively relied on the information that was publicly available information from the official website of the Oregon Department of Education (ODE). The simple model created in this study represent how effectively schools spend in promoting equity and academic success. While this study aims to assess the efficiency change in elementary schools using Malmquist productivity index, there are few limitations and areas that could be explored in future research.

DEA model assumption: As it is often said that no DEA models are correct but few are useful, the DEA models discussed in this study are far from perfect. One pivotal aspect to consider is the selection of input and output variables, as this can have considerable influence over calculated efficiencies. This study assumes a significant relationship between the scale of operation and school efficiency. Future studies can improve DEA model by examining other data variables and various return to scales.

Exclusion of academic year 2019-20: Our study does not incorporate data from the academic year 2019-20 due to unavailability of ELA and Maths proficients information. Including this academic year, especially considering it as the initial year of the COVID-19 pandemic, could provide valuable insights into the efficiency change during this critical period.

Data gaps in academic year 2020-21: Some elementary schools in our study have missing data on ELA and Maths proficients for the academic year 2020-21. While we mitigated this by calculating the average percentage drop in proficients from academic year 2018-19 to 2020-21, incorporating actual data from these schools would offer a more accurate depiction of efficiency change.

Exploration beyond Beaverton School District: Given that our study is confined to the Beaverton School District, future research could explore efficiency change in elementary schools across different districts.

8 Appendix

8.1 Results Using Envelopment Model

8.1.1 Efficiency Result for Academic Year 2018-19

Table 15: School Efficiency for Academic Year 2018-19 Based on Envelopment Model (IO-VRS)

	θ^{VRS}
Aloha-Huber	1.000
Barnes	0.933
Beaver	1.000
Bethany	0.672
Bonny	0.616
Cedar	0.472
Chehalem	0.854
Cooper	0.638
Elmonica	0.716
Errol	0.881
Findley	1.000
Fir	0.687
Greenway	0.818
Hazeldale	0.934
Hiteon	1.000
Jacob	0.904
Kinnaman	0.963
McKay	1.000
McKinley	0.737
Montclair	0.593
Nancy	0.677
Oak	0.574
RaleighHills	0.973
ReleighPark	0.748
Ridgewood	0.782
Rock	0.518
Sato	1.000
Scholls	0.882
Sexton	0.619
Springville	1.000
Terra	0.711
Vose	0.964
West	0.806
William	1.000

8.1.2 Efficiency Result for Academic Year 2020-21

Table 16: School Efficiency for Academic Year 2020-21 Based on Envelopment Model (IO-VRS)

	θ^{VRS}
Aloha-Huber	1.000
Barnes	1.000
Beaver	1.000
Bethany	1.000
Bonny	0.695
Cedar	0.968
Chehalem	0.917
Cooper	0.843
Elmonica	0.914
Errol	1.000
Findley	1.000
Fir	1.000
Greenway	1.000
Hazeldale	1.000
Hiteon	1.000
Jacob	1.000
Kinnaman	1.000
McKay	1.000
McKinley	0.918
Montclair	1.000
Nancy	0.852
Oak	0.728
RaleighHills	1.000
ReleighPark	1.000
Ridgewood	0.954
Rock	0.899
Sato	0.919
Scholls	0.925
Sexton	0.789
Springville	1.000
Terra	1.000
Vose	0.954
West	1.000
William	0.878

8.1.3 Efficiency Result for Academic Year 2021-22

Table 17: School Efficiency for Academic Year 2021-22 Based on Envelopment Model (IO-VRS)

	θ^{VRS}
Aloha-Huber	1.000
Barnes	1.000
Beaver	1.000

Table 17: School Efficiency for Academic Year 2021-22 Based on Envelopment Model (IO-VRS) *(continued)*

	θ^{VRS}
Bethany	1.000
Bonny	1.000
Cedar	1.000
Chehalem	0.919
Cooper	0.715
Elmonica	1.000
Errol	1.000
Findley	1.000
Fir	1.000
Greenway	1.000
Hazeldale	1.000
Hiteon	1.000
Jacob	0.960
Kinnaman	1.000
McKay	0.974
McKinley	0.879
Montclair	1.000
Nancy	1.000
Oak	0.844
RaleighHills	0.958
ReleighPark	1.000
Ridgewood	0.875
Rock	0.930
Sato	0.968
Scholls	1.000
Sexton	0.973
Springville	1.000
Terra	1.000
Vose	0.971
West	1.000
William	0.943

8.2 MPI Results using `Benchmarking` Package

8.2.1 Academic Year 2018-19 and 2020-21

Table 18: Benchamrking Pacakge Results (IO-VRS): Academic Year 2018-19 and 2020-21

School	E1819-T1819	E2021-T1819	E2021-T2021	E1819-T2021	PTEC	TC	MPI
Aloha-Huber	1.000	0.930	1.000	Inf	1.000	0	0
Barnes	0.933	0.873	1.000	Inf	1.072	0	0
Beaver	1.000	1.102	1.000	Inf	1.000	0	0
Bethany	0.672	0.591	1.000	Inf	1.488	0	0
Bonny	0.616	0.406	0.695	Inf	1.129	0	0

Table 18: Benchamrking Pacakge Results (IO-VRS): Academic Year 2018-19 and 2020-21 *(continued)*

School	E1819-T1819	E2021-T1819	E2021-T2021	E1819-T2021	PTEC	TC	MPI
Cedar	0.472	0.500	0.968	Inf	2.049	0	0
Chehalem	0.854	0.745	0.917	Inf	1.075	0	0
Cooper	0.638	0.631	0.843	Inf	1.322	0	0
Elmonica	0.716	0.683	0.914	Inf	1.276	0	0
Errol	0.881	0.691	1.000	Inf	1.136	0	0
Findley	1.000	0.380	1.000	Inf	1.000	0	0
Fir	0.687	0.734	1.000	Inf	1.456	0	0
Greenway	0.818	0.798	1.000	Inf	1.223	0	0
Hazeldale	0.934	0.947	1.000	Inf	1.071	0	0
Hiteon	1.000	0.911	1.000	Inf	1.000	0	0
Jacob	0.904	0.301	1.000	Inf	1.106	0	0
Kinnaman	0.963	0.894	1.000	Inf	1.039	0	0
McKay	1.000	0.934	1.000	Inf	1.000	0	0
McKinley	0.737	0.801	0.918	Inf	1.246	0	0
Montclair	0.593	0.622	1.000	Inf	1.688	0	0
Nancy	0.677	0.528	0.852	Inf	1.259	0	0
Oak	0.574	0.514	0.728	Inf	1.268	0	0
RaleighHills	0.973	0.800	1.000	Inf	1.028	0	0
ReleighPark	0.748	0.749	1.000	Inf	1.337	0	0
Ridgewood	0.782	0.612	0.954	Inf	1.220	0	0
Rock	0.518	0.442	0.899	Inf	1.735	0	0
Sato	1.000	0.388	0.919	Inf	0.919	0	0
Scholls	0.882	0.734	0.925	Inf	1.049	0	0
Sexton	0.619	0.588	0.789	Inf	1.273	0	0
Springville	1.000	0.461	1.000	Inf	1.000	0	0
Terra	0.711	0.879	1.000	Inf	1.407	0	0
Vose	0.964	0.825	0.954	Inf	0.989	0	0
West	0.806	0.613	1.000	Inf	1.240	0	0
William	1.000	0.776	0.878	Inf	0.878	0	0

Note:

1819 = Academic Year 2018-19 and 2021 = Academic Year 2020-21

E1819-T1819 = Efficiency for academic year 2018-19 with reference technology from academic year 2018-19

E2021-T1819 = Efficiency for academic year 2020-21 with reference technology from academic year 2018-19

E2021-T2021 = Efficiency for academic year 2020-21 with reference technology from academic year 2020-21

E1819-T2021 = Efficiency for academic year 2018-19 with reference technology from academic year 2020-21

8.2.2 Academic Year 2020-21 and 2021-22

Table 19: Benchamrking Pacakge Results (IO-VRS): Academic Year 2020-21 and 2021-22

School	E2021-T2021	E2122-T2021	E2122-T2122	E2021-T2122	PTEC	TC	MPI
Aloha-Huber	1.000	Inf	1.000	Inf	1.000	NaN	NaN
Barnes	1.000	Inf	1.000	1.209	1.000	Inf	Inf

Table 19: Benchamrking Pacakge Results (IO-VRS): Academic Year 2020-21 and 2021-22 *(continued)*

School	E2021-T2021	E2122-T2021	E2122-T2122	E2021-T2122	PTEC	TC	MPI
Beaver	1.000	Inf	1.000	Inf	1.000	NaN	NaN
Bethany	1.000	Inf	1.000	0.856	1.000	Inf	Inf
Bonny	0.695	Inf	1.000	0.672	1.439	Inf	Inf
Cedar	0.968	Inf	1.000	0.977	1.034	Inf	Inf
Chehalem	0.917	Inf	0.919	1.075	1.002	Inf	Inf
Cooper	0.843	Inf	0.715	0.914	0.848	Inf	Inf
Elmonica	0.914	Inf	1.000	1.003	1.094	Inf	Inf
Errol	1.000	Inf	1.000	1.132	1.000	Inf	Inf
Findley	1.000	Inf	1.000	0.730	1.000	Inf	Inf
Fir	1.000	Inf	1.000	1.123	1.000	Inf	Inf
Greenway	1.000	Inf	1.000	1.198	1.000	Inf	Inf
Hazeldale	1.000	Inf	1.000	1.141	1.000	Inf	Inf
Hiteon	1.000	Inf	1.000	1.022	1.000	Inf	Inf
Jacob	1.000	Inf	0.960	0.593	0.960	Inf	Inf
Kinnaman	1.000	Inf	1.000	1.104	1.000	Inf	Inf
McKay	1.000	1.381	0.974	1.227	0.974	1.075	1.047
McKinley	0.918	Inf	0.879	1.084	0.958	Inf	Inf
Montclair	1.000	Inf	1.000	1.204	1.000	Inf	Inf
Nancy	0.852	Inf	1.000	0.828	1.174	Inf	Inf
Oak	0.728	Inf	0.844	0.758	1.160	Inf	Inf
RaleighHills	1.000	Inf	0.958	1.069	0.958	Inf	Inf
ReleighPark	1.000	Inf	1.000	1.138	1.000	Inf	Inf
Ridgewood	0.954	Inf	0.875	0.973	0.918	Inf	Inf
Rock	0.899	Inf	0.930	0.892	1.035	Inf	Inf
Sato	0.919	Inf	0.968	0.628	1.053	Inf	Inf
Scholls	0.925	Inf	1.000	0.853	1.081	Inf	Inf
Sexton	0.789	Inf	0.973	0.796	1.234	Inf	Inf
Springville	1.000	Inf	1.000	0.569	1.000	Inf	Inf
Terra	1.000	Inf	1.000	1.187	1.000	Inf	Inf
Vose	0.954	Inf	0.971	1.099	1.018	Inf	Inf
West	1.000	Inf	1.000	1.192	1.000	Inf	Inf
William	0.878	Inf	0.943	1.057	1.074	Inf	Inf

Note:
2021 = Academic Year 2020-21 and 2122 = Academic Year 2021-22
E2021-T2021 = Efficiency for academic year 2020-21 with reference technology from academic year 2020-21
E2122-T2021 = Efficiency for academic year 2021-22 with reference technology from academic year 2020-21
E2122-T2122 = Efficiency for academic year 2021-22 with reference technology from academic year 2021-22
E2021-T2122 = Efficiency for academic year 2020-21 with reference technology from academic year 2021-22

8.3 MPI Results using DJL Package

8.3.1 Academic Year 2018-19 and 2020-21

Table 20: DJL Pacakge Results (IO-VRS): Academic Year 2018-19 and 2020-21

School	PTEC	TC	MPI
Aloha-Huber	1.000	0	0
Barnes	1.072	0	0
Beaver	1.000	0	0
Bethany	1.488	0	0
Bonny	1.129	0	0
Cedar	2.049	0	0
Chehalem	1.075	0	0
Cooper	1.322	0	0
Elmonica	1.276	0	0
Errol	1.136	0	0
Findley	1.000	0	0
Fir	1.456	0	0
Greenway	1.223	0	0
Hazeldale	1.071	0	0
Hiteon	1.000	0	0
Jacob	1.106	0	0
Kinnaman	1.039	0	0
McKay	1.000	0	0
McKinley	1.246	0	0
Montclair	1.688	0	0
Nancy	1.259	0	0
Oak	1.268	0	0
RaleighHills	1.028	0	0
ReleighPark	1.337	0	0
Ridgewood	1.220	0	0
Rock	1.735	0	0
Sato	0.919	0	0
Scholls	1.049	0	0
Sexton	1.273	0	0
Springville	1.000	0	0
Terra	1.407	0	0
Vose	0.989	0	0
West	1.240	0	0
William	0.878	0	0

8.3.2 Academic Year 2020-21 and 2021-22

Table 21: DJL Pacakge Results (IO-VRS): Academic Year 2020-21 and 2021-22

School	PTEC	TC	MPI
Aloha-Huber	1.000	1.000000e+00	1.000000e+00
Barnes	1.000	9.094782e+14	9.094782e+14
Beaver	1.000	1.000000e+00	1.000000e+00
Bethany	1.000	1.081039e+15	1.081039e+15

Table 21: DJL Pacakge Results (IO-VRS): Academic Year 2020-21 and 2021-22 *(continued)*

School	PTEC	TC	MPI
Bonny	1.439	1.016770e+15	1.462703e+15
Cedar	1.034	9.949404e+14	1.028359e+15
Chehalem	1.002	9.638778e+14	9.654092e+14
Cooper	0.848	1.135980e+15	9.628144e+14
Elmonica	1.094	9.546104e+14	1.044535e+15
Errol	1.000	9.400967e+14	9.400967e+14
Findley	1.000	1.170679e+15	1.170679e+15
Fir	1.000	9.436079e+14	9.436079e+14
Greenway	1.000	9.136136e+14	9.136136e+14
Hazeldale	1.000	9.361227e+14	9.361227e+14
Hiteon	1.000	9.891560e+14	9.891560e+14
Jacob	0.960	1.325180e+15	1.272167e+15
Kinnaman	1.000	9.518669e+14	9.518669e+14
McKay	0.974	1.075000e+00	1.047000e+00
McKinley	0.958	9.812333e+14	9.397637e+14
Montclair	1.000	9.114118e+14	9.114118e+14
Nancy	1.174	1.013959e+15	1.190649e+15
Oak	1.160	1.065959e+15	1.236907e+15
RaleighHills	0.958	9.882577e+14	9.468989e+14
ReleighPark	1.000	9.375502e+14	9.375502e+14
Ridgewood	0.918	1.058041e+15	9.709913e+14
Rock	1.035	1.040729e+15	1.077480e+15
Sato	1.053	1.229857e+15	1.295431e+15
Scholls	1.081	1.041839e+15	1.125711e+15
Sexton	1.234	1.009101e+15	1.245533e+15
Springville	1.000	1.325164e+15	1.325164e+15
Terra	1.000	9.177541e+14	9.177541e+14
Vose	1.018	9.454375e+14	9.623891e+14
West	1.000	9.158548e+14	9.158548e+14
William	1.074	9.385895e+14	1.007804e+15

8.4 MPI Results using `deaR` Package

8.4.1 Academic Year 2018-19 and 2020-21

Table 22: deaR Pacakge Results (IO-VRS): Academic Year 2018-19 and 2020-21

	PTEC	TC	SEC	MPI
Aloha-Huber	1.000	0.496	1.000	0.496
Barnes	1.072	0.598	0.964	0.618
Beaver	1.000	0.527	1.034	0.544
Bethany	1.488	0.265	1.151	0.455
Bonny	1.129	0.218	1.494	0.368

Table 22: deaR Pacakge Results (IO-VRS): Academic Year 2018-19 and 2020-21 *(continued)*

	PTEC	TC	SEC	MPI
Cedar	2.049	0.234	0.889	0.427
Chehalem	1.075	0.566	0.920	0.560
Cooper	1.322	0.352	1.106	0.515
Elmonica	1.276	0.450	0.747	0.429
Errol	1.136	0.423	1.061	0.510
Findley	1.000	0.139	2.077	0.289
Fir	1.456	0.427	0.942	0.586
Greenway	1.223	0.599	0.875	0.642
Hazeldale	1.071	0.498	1.122	0.598
Hiteon	1.000	0.403	1.309	0.528
Jacob	1.106	0.156	1.809	0.312
Kinnaman	1.039	0.527	1.026	0.562
McKay	1.000	0.613	1.060	0.650
McKinley	1.246	0.537	0.960	0.643
Montclair	1.688	0.260	0.965	0.423
Nancy	1.259	0.272	1.190	0.408
Oak	1.268	0.291	1.108	0.408
RaleighHills	1.028	0.409	1.227	0.516
ReleighPark	1.337	0.413	1.003	0.554
Ridgewood	1.220	0.357	1.141	0.497
Rock	1.735	0.246	0.984	0.420
Sato	0.919	0.182	0.958	0.160
Scholls	1.049	0.361	1.219	0.461
Sexton	1.273	0.320	1.143	0.466
Springville	1.000	0.222	1.750	0.389
Terra	1.407	0.416	1.042	0.610
Vose	0.989	0.533	0.966	0.509
West	1.240	0.380	0.997	0.469
William	0.878	0.684	0.985	0.592

8.4.2 Academic Year 2020-21 and 2021-22

Table 23: deaR Pacakge Results (IO-VRS): Academic Year 2020-21 and 2021-22

	PTEC	TC	SEC	MPI
Aloha-Huber	1.000	1.391	1.000	1.391
Barnes	1.000	1.049	0.975	1.023
Beaver	1.000	1.271	1.045	1.328
Bethany	1.000	2.095	1.001	2.098
Bonny	1.439	2.243	1.040	3.355
Cedar	1.034	2.433	1.093	2.748
Chehalem	1.002	1.364	0.990	1.353
Cooper	0.848	1.693	1.011	1.450
Elmonica	1.094	1.491	1.242	2.025

Table 23: deaR Pacakge Results (IO-VRS): Academic Year 2020-21 and 2021-22 *(continued)*

	PTEC	TC	SEC	MPI
Errol	1.000	1.616	0.812	1.312
Findley	1.000	3.399	1.000	3.399
Fir	1.000	1.510	1.016	1.534
Greenway	1.000	1.229	0.922	1.134
Hazeldale	1.000	1.333	1.000	1.333
Hiteon	1.000	1.570	1.001	1.572
Jacob	0.960	3.186	1.087	3.325
Kinnaman	1.000	1.164	1.035	1.205
McKay	0.974	1.214	0.812	0.960
McKinley	0.958	1.152	0.979	1.080
Montclair	1.000	2.074	0.821	1.703
Nancy	1.174	1.988	1.029	2.403
Oak	1.160	1.815	1.040	2.189
RaleighHills	0.958	1.679	0.879	1.415
ReleighPark	1.000	1.433	0.928	1.330
Ridgewood	0.918	1.849	0.996	1.691
Rock	1.035	2.172	0.988	2.221
Sato	1.053	2.577	1.040	2.824
Scholls	1.081	1.725	1.043	1.945
Sexton	1.234	1.747	0.987	2.129
Springville	1.000	2.511	1.094	2.746
Terra	1.000	1.483	1.000	1.483
Vose	1.018	1.258	1.018	1.303
West	1.000	1.777	0.937	1.665
William	1.074	1.085	0.981	1.143

8.5 MPI Results using `MultiplierDEA` Package (IO-CRS)

8.5.1 Academic Year 2018-19 and 2020-21

Table 24: MultiplierDEA Package Results (IO-CRS): Academic Year 2018-19 and 2020-21

	E1819-T1819	E2021-T1819	E2021-T2021	E1819-T2021	TEC	TC	MPI
Aloha-Huber	1.000	0.919	1.000	3.732	1.000	0.496	0.496
Barnes	0.927	0.871	0.958	2.356	1.034	0.598	0.618
Beaver	0.907	0.830	0.937	2.897	1.034	0.527	0.544
Bethany	0.583	0.557	0.999	4.611	1.713	0.265	0.455
Bonny	0.396	0.391	0.668	4.881	1.687	0.218	0.368
Cedar	0.462	0.398	0.841	3.986	1.822	0.234	0.427
Chehalem	0.835	0.742	0.826	2.342	0.989	0.566	0.560
Cooper	0.569	0.595	0.832	3.277	1.462	0.352	0.515
Elmonica	0.715	0.609	0.682	3.161	0.953	0.450	0.429
Errol	0.768	0.679	0.925	3.144	1.205	0.423	0.510

Table 24: MultiplierDEA Package Results (IO-CRS): Academic Year 2018-19 and 2020-21 *(continued)*

	E1819-T1819	E2021-T1819	E2021-T2021	E1819-T2021	TEC	TC	MPI
Findley	0.481	0.242	1.000	6.006	2.077	0.139	0.289
Fir	0.667	0.727	0.915	2.904	1.372	0.427	0.586
Greenway	0.816	0.784	0.873	2.039	1.070	0.599	0.642
Hazeldale	0.832	0.851	1.000	2.857	1.202	0.498	0.598
Hiteon	0.746	0.747	0.976	3.509	1.309	0.403	0.528
Jacob	0.440	0.267	0.880	5.488	2.001	0.156	0.312
Kinnaman	0.906	0.841	0.966	2.840	1.066	0.527	0.562
McKay	0.943	0.867	1.000	2.176	1.060	0.613	0.650
McKinley	0.720	0.789	0.862	2.284	1.196	0.537	0.643
Montclair	0.498	0.438	0.811	3.982	1.628	0.260	0.423
Nancy	0.550	0.514	0.825	4.634	1.498	0.272	0.408
Oak	0.498	0.486	0.700	4.094	1.406	0.291	0.408
RaleighHills	0.793	0.766	1.000	3.638	1.261	0.409	0.516
ReleighPark	0.716	0.728	0.961	3.180	1.342	0.413	0.554
Ridgewood	0.675	0.586	0.939	3.298	1.392	0.357	0.497
Rock	0.492	0.432	0.840	4.181	1.707	0.246	0.420
Sato	1.000	0.371	0.880	12.796	0.880	0.182	0.160
Scholls	0.694	0.628	0.887	3.771	1.278	0.361	0.461
Sexton	0.541	0.548	0.788	3.668	1.455	0.320	0.466
Springville	0.522	0.419	0.914	4.853	1.750	0.222	0.389
Terra	0.682	0.816	1.000	3.219	1.466	0.416	0.610
Vose	0.947	0.805	0.906	2.966	0.956	0.533	0.509
West	0.740	0.610	0.915	3.423	1.236	0.380	0.469
William	0.983	0.772	0.851	1.906	0.865	0.684	0.592

Note:

1819 = Academic Year 2018-19 and 2021 = Academic Year 2020-21

E1819-T1819 = Efficiency for academic year 2018-19 with reference technology from academic year 2018-19

E2021-T1819 = Efficiency for academic year 2020-21 with reference technology from academic year 2018-19

E2021-T2021 = Efficiency for academic year 2020-21 with reference technology from academic year 2020-21

E1819-T2021 = Efficiency for academic year 2018-19 with reference technology from academic year 2020-21

8.5.2 Academic Year 2020-21 and 2021-22

Table 25: MultiplierDEA Package Results (IO-CRS): Academic Year 2020-21 and 2021-22

	E2021-T2021	E2122-T2021	E2122-T2122	E2021-T2122	TEC	TC	MPI
Aloha-Huber	1.000	2.315	1.000	1.196	1.000	1.391	1.391
Barnes	0.958	1.220	0.934	1.136	0.975	1.049	1.023
Beaver	0.937	1.841	0.980	1.091	1.045	1.271	1.328
Bethany	0.999	3.322	1.000	0.756	1.001	2.095	2.098
Bonny	0.668	4.000	1.000	0.532	1.496	2.243	3.355
Cedar	0.841	3.613	0.950	0.540	1.130	2.433	2.748
Chehalem	0.826	1.791	0.819	0.971	0.992	1.364	1.353

Table 25: MultiplierDEA Package Results (IO-CRS): Academic Year 2020-21 and 2021-22 *(continued)*

	E2021-T2021	E2122-T2021	E2122-T2122	E2021-T2122	TEC	TC	MPI
Cooper	0.832	1.983	0.713	0.808	0.857	1.693	1.450
Elmonica	0.682	2.395	0.926	0.793	1.358	1.491	2.025
Errol	0.925	1.902	0.751	0.897	0.812	1.616	1.312
Findley	1.000	4.501	1.000	0.390	1.000	3.399	3.399
Fir	0.915	2.212	0.930	0.955	1.016	1.510	1.534
Greenway	0.873	1.431	0.805	1.026	0.922	1.229	1.134
Hazeldale	1.000	1.994	1.000	1.123	1.000	1.333	1.333
Hiteon	0.976	2.506	0.978	1.015	1.001	1.570	1.572
Jacob	0.880	4.001	0.919	0.378	1.044	3.186	3.325
Kinnaman	0.966	1.548	1.000	1.103	1.035	1.164	1.205
McKay	1.000	1.343	0.791	1.152	0.791	1.214	0.960
McKinley	0.862	1.277	0.808	1.027	0.938	1.152	1.080
Montclair	0.811	2.102	0.666	0.595	0.821	2.074	1.703
Nancy	0.825	3.245	0.997	0.679	1.208	1.988	2.403
Oak	0.700	2.622	0.844	0.660	1.206	1.815	2.189
RaleighHills	1.000	2.397	0.842	1.009	0.842	1.679	1.415
ReleighPark	0.961	1.833	0.892	0.962	0.928	1.433	1.330
Ridgewood	0.939	2.486	0.858	0.796	0.914	1.849	1.691
Rock	0.840	2.830	0.859	0.587	1.023	2.172	2.221
Sato	0.880	3.667	0.964	0.504	1.096	2.577	2.824
Scholls	0.887	2.860	1.000	0.852	1.127	1.725	1.945
Sexton	0.788	2.766	0.960	0.744	1.219	1.747	2.129
Springville	0.914	3.922	1.000	0.569	1.094	2.511	2.746
Terra	1.000	2.438	1.000	1.108	1.000	1.483	1.483
Vose	0.906	1.686	0.939	1.028	1.036	1.258	1.303
West	0.915	2.452	0.857	0.828	0.937	1.777	1.665
William	0.851	1.249	0.896	1.007	1.054	1.085	1.143

Note:

2021 = Academic Year 2020-21 and 2122 = Academic Year 2021-22

E2021-T2021 = Efficiency for academic year 2020-21 with reference technology from academic year 2020-21

E2122-T2021 = Efficiency for academic year 2021-22 with reference technology from academic year 2020-21

E2122-T2122 = Efficiency for academic year 2021-22 with reference technology from academic year 2021-22

E2021-T2122 = Efficiency for academic year 2020-21 with reference technology from academic year 2021-22

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