Summary

$$\begin{aligned}\alpha_1 y + \beta_1 z &= \gamma_1 \\ \alpha_2 y + \beta_2 z &= \gamma_2\end{aligned} \quad \longleftrightarrow \quad Ax = b$$

Step 1: Order the equations as you want them and stack the unknowns into a vector such as $x = \begin{bmatrix} y \\ z \end{bmatrix}$

Step 2: Move all constants to the RHS and stack into a vector $b = \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix}$

Step 3: Form A by a_{ij} is coeff. multiplying x_j in the ith equation

$$\begin{aligned}\alpha_1 y + \beta_1 z &= \gamma_1 \\ \alpha_2 y + \beta_2 z &= \gamma_2\end{aligned} \quad \longleftrightarrow \quad \underbrace{\begin{bmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} y \\ z \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix}}_{b}$$

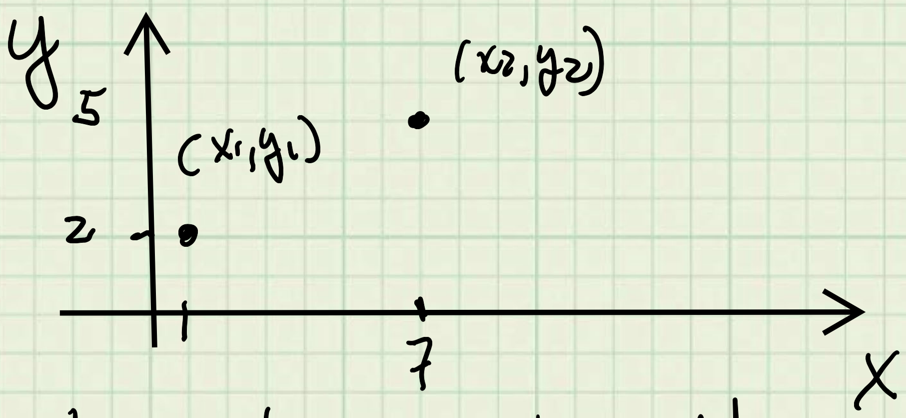
IF we define $x = \begin{bmatrix} z \\ y \end{bmatrix}$, what are A and b?

$$\begin{aligned} \alpha_1 y + \beta_1 z &= \gamma_1 \\ \alpha_2 y + \beta_2 z &= \gamma_2 \end{aligned} \quad \overset{?}{\longleftrightarrow} \quad \begin{bmatrix} \beta_1 & \alpha_1 \\ \beta_2 & \alpha_2 \end{bmatrix} \begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix}$$

"Missing Coefficients"

$$\begin{array}{ll} & 2x_1 + 3x_2 + 5x_3 = 1 \\ +0x_3 & x_1 - x_2 = 2 \\ +0x_1 & x_2 + 4x_3 = 3 \end{array} \longleftrightarrow \begin{bmatrix} 2 & 3 & 5 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Now, we build both the equations and the matrix vector representation of them



Fit a line to the points

$$y_i = m x_i + b$$

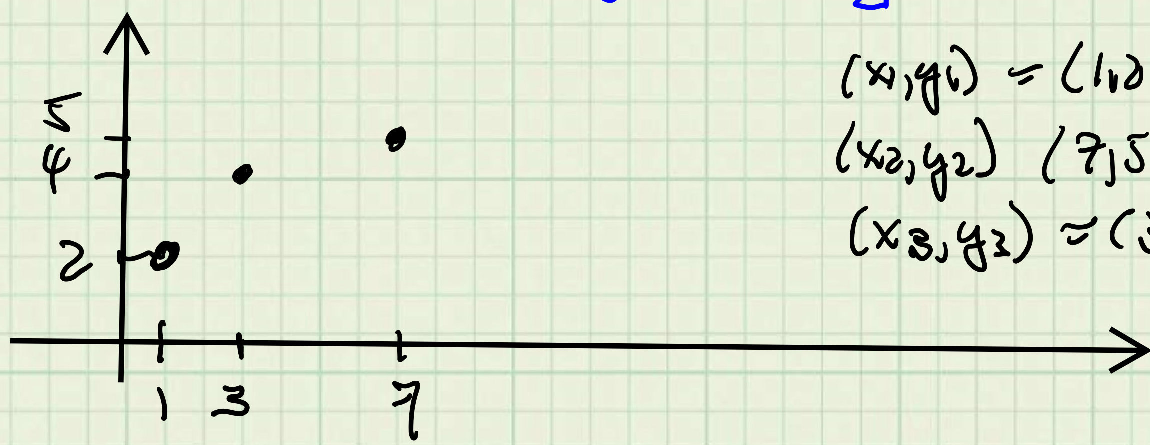
$$2 = m \cdot 1 + b \qquad (x_1, y_1) = (1, 2)$$

$$5 = m \cdot 7 + b \qquad (x_2, y_2) = (7, 5)$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

Vector of unknowns $\alpha = \begin{bmatrix} m \\ b \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 \\ 7 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$



$(x_1, y_1) = (1, 2)$
(x_2, y_2) $(7, 5)$
$(x_3, y_3) = (3, 4)$

$$y_i = m x_i + b$$

$$\underset{Y}{\begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix}} = \begin{bmatrix} 1 & 1 \\ 7 & 1 \\ 3 & 1 \end{bmatrix} \underset{\alpha}{\begin{bmatrix} m \\ b \end{bmatrix}}$$

Matrix Determinant

- Let A be an $n \times n$ matrix of real numbers.
- $n \times n \Rightarrow$ square

Fact 1: The matrix determinant, denoted $\det(A)$ is a real number.

Fact 2: A square system of linear equations

$$Ax = b$$

(n-equations, n-unknowns) has a unique solution for any choice of b if, and only if, $\det(A) \neq 0$.

Fact 3: When $\det(A) = 0$,

$Ax = b$ may have EITHER no solutions OR an infinite number of solutions.

Chapter 7 will deal with these cases.

Fact 4: Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then

$$\boxed{\det(A) = a \cdot d - b \cdot c}$$