# Задания к уроку №10

1. $\int \frac{x^4+x^2-6x}{x^3}dx = \int \frac{x(x^3+x-6)}{x \cdot x^2} dx =$

$= \int \frac{x^3}{x^2} + \frac{x}{x^2} - \frac{6}{x^2} dx =$

$= \int x\, dx + \int \frac{1}{x} dx - \int \frac{6}{x^2} dx =$

$= \frac{x^2}{2} + \ln(|x|) + \frac{6}{x} + C, \ C \in \mathbb{R}$

2. $\int \cos 2x dx \Rightarrow \int \frac{\cos(t)}{2} dt =$

$= \frac{1}{2} \int \cos(t)\, dt = \frac{1}{2} \sin(t) \Rightarrow$

$= \frac{1}{2} \sin(2x) = \frac{\sin(2x)}{2} + C,$

$C \in \mathbb{R}$

$$3.\ \int \frac{dx}{(3x+2)^4} \Rightarrow \int \frac{1}{(3x+2)^4}\,dx \Rightarrow$$

$$\Rightarrow \int \frac{1}{3t^4}\,dt = \frac{1}{3}\int \frac{1}{t^4}\,dt =$$

$$= \frac{1}{3}\left(-\frac{1}{3t^3}\right) = \frac{1}{3}\left(-\frac{1}{3(3x+2)^3}\right) =$$

$$= -\frac{1}{9(3x+2)^3} + C,\ C \in \mathbf{R}$$

$$4.\ \int \frac{5x-1}{\sqrt{4-x^2}}\,dx = \int \frac{5x}{\sqrt{4-x^2}} - \frac{1}{\sqrt{4-x^2}}\,dx$$

$$= \int \frac{5x}{\sqrt{4-x^2}}\,dx - \int \frac{1}{\sqrt{4-x^2}}\,dx =$$

$$= -5\sqrt{4-x^2} - \arcsin\left(\frac{x}{2}\right) + C,\ C \in \mathbf{R}$$

$$5.\ \int \frac{dx}{\sqrt{x}(1+\sqrt{x})} = \int \frac{1}{\sqrt{x}\,(1+\sqrt{x})}\,dx \Rightarrow$$

$$\Rightarrow t = 1+\sqrt{x} \Rightarrow \int \frac{2}{t}\,dt \Rightarrow$$

$$\frac{a}{x}\,dx = a \cdot \ln(|x|) \Rightarrow 2\ln(|t|) \Rightarrow$$

$$\Rightarrow 2\ln(|1+\sqrt{x}|) + C,\ C \in \mathbf{R}$$

$$6.\ \int x^2 \cos x\,dx \Rightarrow \begin{array}{l} u = x^2 \\ dv = \cos(x)\,dx \end{array}$$

$$\Rightarrow \begin{array}{l} du = 2x\,dx \\ v = \sin(x) \end{array} \Rightarrow x^2 \cdot \sin(x) - \int \sin(x) \cdot 2x\,dx =$$

$$= x^2 \cdot \sin(x) - 2 \cdot \int \sin(x) \cdot x\,dx =$$

$$= x^2 \cdot \sin(x) - 2 \cdot \int x \cdot \sin(x)\,dx =$$

$$= x^2 \cdot \sin(x) - 2\left(x(-\cos(x)) - \int -\cos(x)\,dx\right) =$$

$$= x^2 \cdot \sin(x) - 2\left(x(-\cos(x)) + \int \cos(x)\,dx\right) =$$

$$= x^2 \cdot \sin(x) - 2\left(x(-\cos(x)) + \sin(x)\right) =$$

$$= x^2 \cdot \sin(x) + 2x \cdot \cos(x) - 2\sin(x) + C,\ C \in \mathbf{R}$$

7. $\int \arctan x dx$ $= \int \arctan(x) \cdot 1 dx$

$\Rightarrow u = \arctan(x);\ dv = 1 dx \Rightarrow$

$du = \dfrac{1}{1+x^2} dx;\ v = x \Rightarrow$

$\Rightarrow \arctan(x) \cdot x - \int x \cdot \dfrac{1}{1+x^2} dx \Rightarrow$

$\Rightarrow \arctan(x) \cdot x - \int \dfrac{1}{1+x^2} dx \Rightarrow$

$\Rightarrow \arctan(x) \cdot x - \int \dfrac{1}{2t} dt =$

$= \arctan(x) \cdot x - \dfrac{1}{2} \cdot \int \dfrac{1}{t} dt =$

$= \arctan(x) \cdot x - \dfrac{1}{2} \cdot \ln(|t|) \Rightarrow$

$\Rightarrow \arctan(x) \cdot x - \dfrac{1}{2} \cdot \ln(1+x^2) + C,$

$C \in \mathbb{R}$

8. $\int \frac{dx}{(x-1)^5}$ $= \int \frac{1}{(x-1)^5}\, dx \Rightarrow$

$\Rightarrow \int \frac{1}{t^5}\, dt \Rightarrow \frac{1}{x^n}\, dx = \frac{1}{(n-1)\cdot x^{n-1}}$,

где $n \neq 1$

$\Rightarrow -\frac{1}{4t^4} \Rightarrow -\frac{1}{4(x-1)^4} + C$,

$C \in \mathbb{R}$

9. $\int \frac{(x+6)dx}{x^2-2x+17}$ $\Rightarrow \int \frac{x+6}{x^2-2x+17}\, dx =$

$= \int \frac{\frac{1}{2}\cdot(2x-2)+7}{x^2-2x+17} = \int \frac{\frac{1}{2}\cdot(2x-2)}{x^2-2x+17}\, dx +$

$+ \int \frac{7}{x^2-2x+17}\, dx =$

$= \frac{1}{2}\cdot \ln(|x^2-2x+17|) + \frac{7\cdot \arctan\left(\frac{x-1}{4}\right)}{4} + C$,

$C \in \mathbb{R}$

10. $\int \frac{xdx}{(x^2-1)(x^2+1)} \Rightarrow \int \frac{x}{(x^2-1)(x^2+1)}\,dx$

$= \int \frac{1}{4(x-1)}\,dx + \int \frac{1}{4(x+1)}\,dx - \int \frac{x}{2(x^2+1)}\,dx =$

$= \frac{1}{4}\ln(|x-1|) + \frac{1}{4}\ln(|x+1|) - \frac{1}{4}\ln(x^2+1) + C$

$C \in \mathbf{R}$

11. $\frac{\sqrt[3]{x}dx}{\sqrt[3]{x^2}-\sqrt{x}} \Rightarrow \frac{\sqrt[3]{x}}{\sqrt[3]{x^2}-\sqrt{x}}\,dx =$

$= \int \frac{\sqrt[3]{x}}{x^{\frac{2}{3}} - \sqrt{x}}\,dx = \int \frac{1}{6x^{5/6}} \cdot \frac{6x^{\frac{2}{3}}}{\sqrt[6]{x}-1}\,dx \Rightarrow$

$\Rightarrow u = \sqrt[6]{x} \Rightarrow \frac{du}{dx} = \frac{1}{6x^{5/6}} \Rightarrow dx = 6x^{5/6}du$

$\Rightarrow \sqrt{x} = u^3;\ x^{\frac{2}{3}} = u^4;\ x^{\frac{7}{6}} = u^7 \Rightarrow$

$\Rightarrow 6\int \frac{u^4}{u-1}\,du \Rightarrow \int \frac{u^4}{u-1}\,du \Rightarrow$

$$\Rightarrow v = u - 1 \Rightarrow \frac{dv}{du} = 1 \Rightarrow du = dv \Rightarrow$$

$$\Rightarrow u^4 = (v+1)^4 \Rightarrow \int \frac{(v+1)^4}{v}\, dv \Rightarrow$$

$$= \int \left( v^3 + 4v^2 + 6v + \frac{1}{v} + 4 \right) dv =$$

$$= \int v^3 dv + 4\int v^2 dv + 6\int v\, dv + \int \frac{1}{v} dv + 4\int 1\, dv \Rightarrow$$

1) $\int v^3 dv = \frac{v^4}{4}$ $(n = 3)$

2) $\int v^2 dv = \frac{v^2}{2}$ $(n = 2)$

3) $\int v\, dv = \frac{v^2}{2}$ $(n = 1)$

4) $\int \frac{1}{v} dv = \ln(v)$

5) $\int 1\, dv = v$ (от константы)

$$\Rightarrow \ln(v) + \frac{v^4}{4} + \frac{4v^3}{3} + 3v^2 + 4v \Rightarrow$$

$\Rightarrow$ заменяем на $u$ $\Rightarrow$

$$= 4(u-1) + \frac{(u-1)^4}{4} + \frac{4(u-1)^3}{3} +$$

$$+ 3(u-1)^2 + \ln(u-1) =$$

$$= 24\left(\sqrt[6]{x} - 1\right) + \frac{3\left(\sqrt[6]{x} - 1\right)^4}{2} + 8\left(\sqrt[6]{x} - 1\right)^3 +$$

$$+ 18\left(\sqrt[6]{x} - 1\right)^2 + 6\ln\left(\sqrt[6]{x} - 1\right) =$$

$$= \frac{3x^{\frac{2}{3}}}{2} + 2\sqrt{x} + 3\sqrt[3]{x} + 6\left(\sqrt[6]{x} + \ln\left(\left|\sqrt[6]{x} - 1\right|\right)\right) + C$$

$$C \in \mathbf{R}$$

12. $\int \sqrt[3]{x} \cdot \sqrt[3]{1 + 3 \cdot \sqrt[3]{x^2}}\,dx \Rightarrow$

$$\Rightarrow \int \sqrt[3]{3x^{\frac{2}{3}} + 1} \cdot \sqrt[3]{x}\,dx \Rightarrow$$

$$\Rightarrow u = 3x^{\frac{2}{3}} + 1 \Rightarrow \frac{du}{dx} = \frac{2}{\sqrt[3]{x}} \Rightarrow dx = \frac{\sqrt[3]{x}}{2}du$$

$$\Rightarrow \frac{1}{6}\int \left(u^{\frac{4}{3}} - \sqrt[3]{u}\right) du \Rightarrow$$

$$\Rightarrow \int \left(u^{4/3} - \sqrt[3]{u}\right) du = \int u^{\frac{4}{3}} du - \int \sqrt[3]{u}\, du$$

$$1)\ \int u^{\frac{4}{3}} du = \frac{3u^{7/3}}{7} \quad \left(n = \frac{4}{3}\right)$$

$$2)\ \int \sqrt[3]{u}\, du = \frac{3u^{4/3}}{4} \quad \left(n = \frac{1}{3}\right) \Rightarrow$$

$$\Rightarrow \frac{3u^{7/3}}{7} - \frac{3u^{4/3}}{4} \Rightarrow$$

$$\Rightarrow \frac{1}{6} \int \left(u^{\frac{4}{3}} - \sqrt[3]{u}\right) du = \frac{u^{\frac{7}{3}}}{14} - \frac{u^{4/3}}{8}$$

$$\Rightarrow \frac{\left(3x^{\frac{2}{3}} + 1\right)^{\frac{7}{3}}}{14} - \frac{\left(3x^{\frac{2}{3}} + 1\right)^{\frac{4}{3}}}{8} + C$$

$$C \in \mathbb{R}$$

13. $\displaystyle\int \frac{dx}{\sin x \cdot \sin 2x} \Rightarrow \int \frac{1}{\sin(x) \cdot \sin(2x)} dx$

$$= \int \frac{1}{2\cos(x) \cdot \sin^2(x)} dx =$$

$$= \frac{1}{2} \int \frac{1}{\cos(x) \cdot \sin^2(x)} \, dx =$$

$$= \int \csc^2(x) \cdot \sec(x) \, dx =$$

$$= \int \left(\cot^2(x) + 1\right) \cdot \sec(x) \, dx =$$

$$= \int \left( \sec(x) + \cot(x) \cdot \sec(x) \right) dx =$$

$$= \int \sec(x) \, dx + \int \cot(x) \cdot \sec(x) \, dx =$$

$$\Rightarrow \left[ \begin{array}{l} 1) \int \sec(x) \, dx = \ln\left(\operatorname{tg}(x) + \sec(x)\right) \\ 2) \int \cot(x) \cdot \sec(x) \, dx = -\csc(x) \end{array} \right]$$

$$= \frac{\ln\left(\left|\operatorname{tg}(x) + \sec(x)\right|\right) - \csc(x)}{2} + C$$

$$C \in \mathbb{R}$$

14. $\int \cos 5x \cdot \cos 3x dx =$

$$= \int \frac{\cos(8x) + \cos(2x)}{2} dx =$$

$$= \frac{1}{2}\int \cos(8x) dx + \frac{1}{2}\int \cos(2x) dx$$

1) $u = 8x \Rightarrow \frac{du}{dx} = 8 \Rightarrow dx = \frac{1}{8} du \Rightarrow$

$$\Rightarrow \frac{1}{8}\int \cos(u) du = \frac{\sin(u)}{8} \Rightarrow$$

$$\Rightarrow \frac{\sin(8x)}{8}$$

2) $u = 2x \Rightarrow \frac{du}{dx} = 2 \Rightarrow dx = \frac{1}{2} du \Rightarrow$

$$\Rightarrow \frac{1}{2}\int \cos(u) du = \sin(u) \Rightarrow$$

$$\Rightarrow \frac{\sin(2x)}{2}$$

$$\frac{1}{2}\int \cos(8x)dx + \frac{1}{2}\int \cos(2x)\,dx =$$

$$= \frac{\sin(8x)}{16} + \frac{\sin(2x)}{4} =$$

$$= \frac{\sin(8x) + 4\sin(2x)}{16} + C\,;\ C \in \mathbb{R}$$

15. $\int_0^\pi (2x + \sin 2x)dx$ $=$

$$= \int \sin(2x)\,dx + 2\int x\,dx$$

1) $u = 2x \rightarrow \frac{du}{dx} = 2 \Rightarrow dx = \frac{1}{2}du \Rightarrow$

$$\Rightarrow \frac{1}{2}\int \sin(u)\,du = -\frac{\cos(2x)}{2}$$

2) $\int x\,dx = \frac{x^2}{2}$ $(n=1)$

$$= \int \sin(2x)\,dx + 2\int x\,dx = x^2 - \frac{\cos(2x)}{2}$$

$$\Rightarrow \left(x^2 - \frac{\cos(2x)}{2}\right)\Bigg|_0^{\pi} = \pi^2 - \frac{\cos(2\pi)}{2} -$$

$$-\left(0^2 - \frac{\cos(2\cdot 0)}{2}\right) = \pi^2 \simeq 9{,}87$$

16. $\int_{\frac{1}{2}}^{1} \sqrt{4x-2}$

$$\Rightarrow t = 4x-2 \Rightarrow \int \frac{1}{4}\cdot\sqrt{t}\,dt \Rightarrow$$

$$\Rightarrow \frac{1}{4}\int \sqrt{t}\,dt = \frac{1}{4}\cdot\frac{2\sqrt{t}\cdot|t|}{3} \Rightarrow$$

$$\Rightarrow \frac{1}{4}\cdot\frac{2\sqrt{4x-2}\cdot|4x-2|}{3} =$$

$$\Rightarrow \frac{\sqrt{4x-2}\cdot|4x-2|}{6}\Bigg|_{\frac{1}{2}}^{1} =$$

$$= \frac{\sqrt{4\cdot 1-2}\cdot|4\cdot 1-2|}{6} - \frac{\sqrt{4\cdot\frac{1}{2}-2}\cdot\left|4\cdot\frac{1}{2}-2\right|}{6} =$$

$$= \frac{\sqrt{2}}{3} \simeq 0{,}47$$

$$17.\ \int_0^{+\infty} e^{-4x}\,dx \Rightarrow$$

несобств. $\int$ $\lim\limits_{a\to+\infty}\left(\int_0^a e^{-4x}\,dx\right) =$

$$= \lim_{a\to+\infty}\left(-\frac{1}{4e^{4a}}+\frac{1}{4}\right) = \frac{1}{4} = 0,25$$

$$18.\ \int_0^1 \ln x\,dx \Rightarrow$$

несобств. $\int$ $\lim\limits_{a\to 0^+}\left(\int_a^1 \ln(x)\,dx\right) =$

$$= \lim_{a\to 0^+}\left(-1-\ln(a)\cdot a+a\right) = -1$$