1. Найти $\frac{dz}{dt}$, если $z = z(x; y), x = x(t), y = y(t)$:

1. $z = x^2 + y^2 + xy, x = a\sin t, y = a\cos t$;

$$\frac{dz}{dt} = \frac{dz}{dx} * \frac{dx}{dt} + \frac{dz}{dy} * \frac{dy}{dt}$$

$$\frac{dz}{dx} = (x^2 + y^2 + xy)'_x = 2x + y$$

$$\frac{dz}{dy} = (x^2 + y^2 + xy)'_y = 2y + x$$

$$\frac{dx}{dt} = (a\sin t)' = a\cos t$$

$$\frac{dy}{dt} = (a\cos t)' = -a\sin t$$

$$\frac{dz}{dt} = (2x + y) * (a\cos t) + (2y + x) *$$

$$* (-a\sin t) = a((2x + y)\cos t -$$

$$- (2y + x)\sin t)$$

2. Для данных $z = f(x;y), x = x(u;v), y = y(u;v)$ найти $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}$ и dz:

1. $z = x^3 + y^3$, где $x = uv, y = \frac{u}{v}$;

$$z = u^3v^3 + \frac{u^3}{v^3}$$

$$\frac{dz}{du} = \left(u^3v^3 + \frac{u^3}{v^3}\right)'_u = 3u^2v^3 +$$

$$+ \frac{3u^2}{v^3} = \frac{3u^2v^6 + 3u^2}{u^3} = \frac{3u^2}{u^3}(v^6+1)$$

$$\frac{dz}{du} = \left(u^3v^3 + \frac{u^3}{v^3}\right)'_v = 3v^2u^3 - \frac{3u^3v^2}{v^6} =$$

$$= \frac{3v^6u^3 - 3u^3}{v^4} = \frac{3u^3}{v^4}(v^6 - 1)$$

$$\boxed{dz = \frac{dz}{du}du + \frac{dz}{du}du} \Rightarrow$$

$$\Rightarrow dz = \frac{3u^2}{v^3}(v^6+1)du + \frac{3u^3}{v^4} *$$

$$* (v^6 - 1)du$$

3. Найти производные $y'(x)$ неявных функций, заданных уравнениями:

$xe^{2y} - y\ln x = 8.$

$$xe^{2y} - y * \ln(x) - 8 = 0$$

$$\boxed{\frac{dy}{dx} = -\frac{\frac{dF(x,y)}{dx}}{\frac{dF(x,y)}{dy}}}$$

$$\frac{dy}{dx} = -\frac{e^{2y} - \frac{y}{x}}{2xe^{2y} - \ln(x)}$$

4. Составить уравнение касательной прямой и нормали к кривой $y = y(x)$, заданной уравнением $F(x; y) = 0$ в точке $M_0(x_0; y_0)$:

$x^3y - y^3x = 6, M_0(2; 1).$

$$y_k = y_0 + y'(x_0)(x - x_0)$$

$$\boxed{\frac{dy}{dx} = -\frac{\frac{dF(x,y)}{dx}}{\frac{dF(x,y)}{dy}}}$$

$$\Rightarrow 3x^2y - y^3$$

$$\Rightarrow x^3 - 3xy^2$$

$$\frac{dy}{dx} = -\frac{3x^2y - y^3}{x^3 - 3xy^2} \Rightarrow F_x'(2;1) =$$

$$= \frac{-3*2^2*1 + 1^3}{2^3 - 3*2*1^2} = -\frac{11}{2} \Rightarrow$$

$$\Rightarrow y_k = 1 - \frac{11}{2}(x-2)$$

$$y_n = y_0 - \frac{1}{y'(x_0)}(x - x_0) \Rightarrow$$

$$\Rightarrow y_n = 1 - \frac{1}{-11/2}(x-2)$$

5. Для данных функций найти требуемую частную производную или дифференциал:

1. $z = \sin x \sin y, d^2 z;$

$$\left.\begin{aligned} \frac{dz}{dx} &= \sin(y) * \cos(x) \\ \frac{dz}{dy} &= \sin(x) * \cos(y) \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \frac{d^2 z}{dx\,dy} = \left(\sin(y) * \cos(x)\right)'_y =$$

$$= \cos(x) * \cos(y)$$

2. $z = xy + \sin(x+y), \frac{\partial^2 z}{\partial x^2};$

$$\frac{dz}{dx} = y + \cos(x+y)$$

$$\frac{dz}{dy} = x + \cos(x+y)$$

$$\boxed{dz = \frac{dz}{dx}dx + \frac{dz}{dy}dy}$$ полный диф.

$$dz = (y + \cos(x+y))dx +$$

$$+ (x + \cos(x+y))dy$$

$$\frac{d^2z}{dxdy} = (y + \cos(x+y))'_y = 1 - \sin(x+y)$$