Supplementary Material

1 SUPPLEMENTARY MATHEMATICS

Here we prove the equation

$$|Q_L| = 2^{L-2}(2^{L-1} + 1) \tag{S1}$$

Proof: For $L \geq 2$, there are 4^L IBD (identical by descent) probabilities $Q(i_1, i_2, \ldots i_L)$ since $i_l = 0, 1, 2$ or 3 and furthermore they add up to 1. A number of these probabilities are equal because of two symmetries: (1) the two homologous chromosomes in each individual play identical roles, and (2) the siblings play identical roles (assuming no sex-dependence of meiosis, so that for instance the recombination rates $r_{l,l'}$ are sex-independent. It is thus appropriate to use only one representative of each symmetry equivalence class, so that for instance one may impose this representative to have its first index, i_1, equal to zero. In fact one can identify exactly one element in each class by imposing that the indices of the representative Q's have either

1. $i_l \in \{0, 1\}$ $\forall l \in \{2, .., L\}$, *or*
2. $i_l \in \{0, 1\}$ $\forall l \in \{2, .., K-1\}$, $i_K = 2$ and $i_l \in \{0, 1, 2, 3\}$ $\forall l \in \{K+1, .., L\}$

The number of equivalence classes and thus of Q's to consider is then

$$|Q_L| = 2^{L-1} + \sum_{l=2}^{L} 2^{l-2} 4^{L-l} = 2^{L-1} + 2^{2L-2} \sum_{l=2}^{L} 2^{-l} \tag{S2}$$

Given that $\sum_{l=2}^{L} 2^{-l}$ is a geometric progression of common ratio 2^{-1} from 2 to L, the sum of its terms can be expressed as:

$$\sum_{l=2}^{L} 2^{-l} = \frac{2^{-2} - 2^{-(L-1)}}{1 - 2^{-1}} = 2^{-1} - 2^{-L} \tag{S3}$$

Substituting S3 in S2, we get

$$|Q_L| = 2^{L-1} + 2^{2L-2}(2^{-1} - 2^{-L}) = 2^{L-1} + 2^{2L-3} - 2^{L-2} \tag{S4}$$

Factorizing with respect to 2^{L-2} and after simplification, this gives

$$|Q_L| = 2^{L-2}(1 + 2^{L-1}). \tag{S5}$$

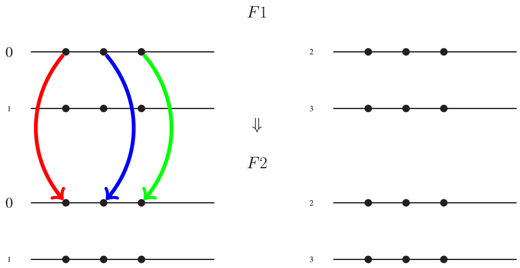
2 THE SELF-CONSISTENT EQUATIONS FOR THREE LOCI

Here we provide the coefficients entering each of the self-consistent equations.

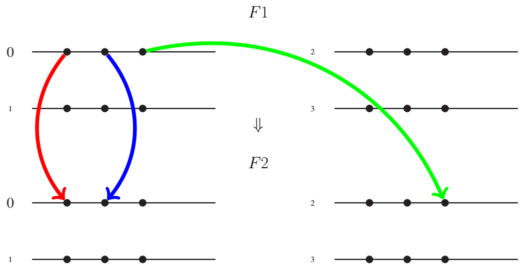
2.1 The self consistent equation for $Q(0,0,0)$

In Figure S1 we show the graphical representation for each term entering this self-consistent equation.

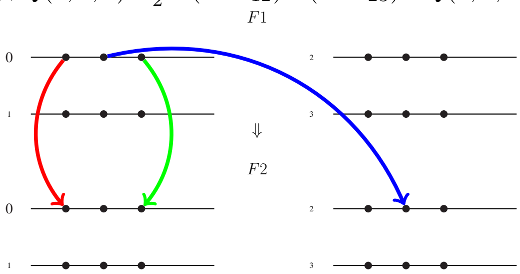
$$((1-r_{12})(1-r_{23})-1)Q(0,0,0)+\frac{1}{2}(1-r_{12})Q(0,0,2)+\frac{1}{2}(1-r_{13})Q(0,2,0)+\frac{1}{2}(1-r_{23})Q(0,2,2)=0 \tag{S6}$$



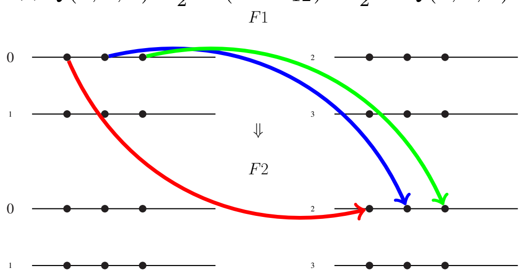
(a) $Q(0,0,0):\frac{1}{2}\times(1-r_{12})\times(1-r_{23})\times Q(0,0,0)$



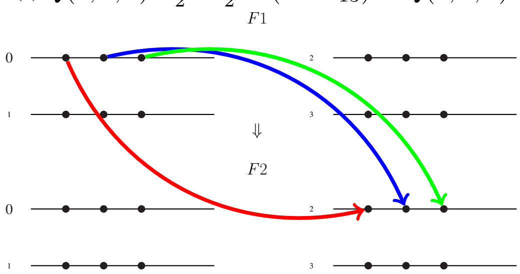
(b) $Q(0,0,0):\frac{1}{2}\times(1-r_{12})\times\frac{1}{2}\times Q(0,0,2)$



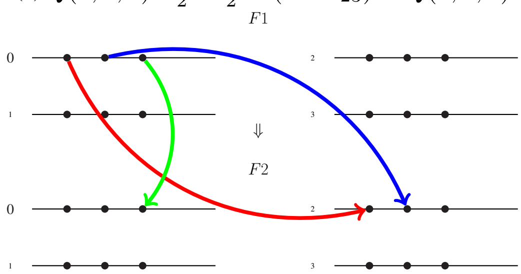
(c) $Q(0,0,0):\frac{1}{2}\times\frac{1}{2}\times(1-r_{13})\times Q(0,2,0)$



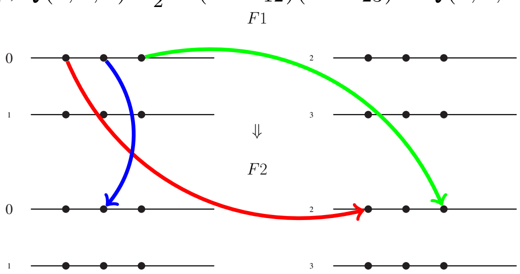
(d) $Q(0,0,0):\frac{1}{2}\times\frac{1}{2}\times(1-r_{23})\times Q(0,2,2)$



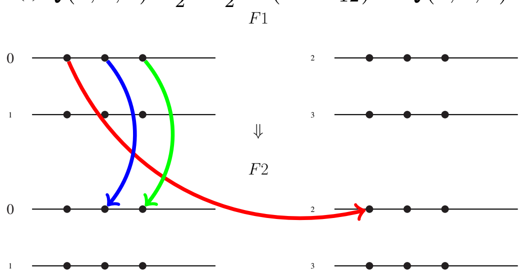
(e) $Q(0,0,0):\frac{1}{2}\times(1-r_{12})(1-r_{23})\times Q(2,2,2)$



(f) $Q(0,0,0):\frac{1}{2}\times\frac{1}{2}\times(1-r_{12})\times Q(2,2,0)$



(g) $Q(0,0,0):\frac{1}{2}\times\frac{1}{2}\times(1-r_{13})\times Q(2,0,2)$



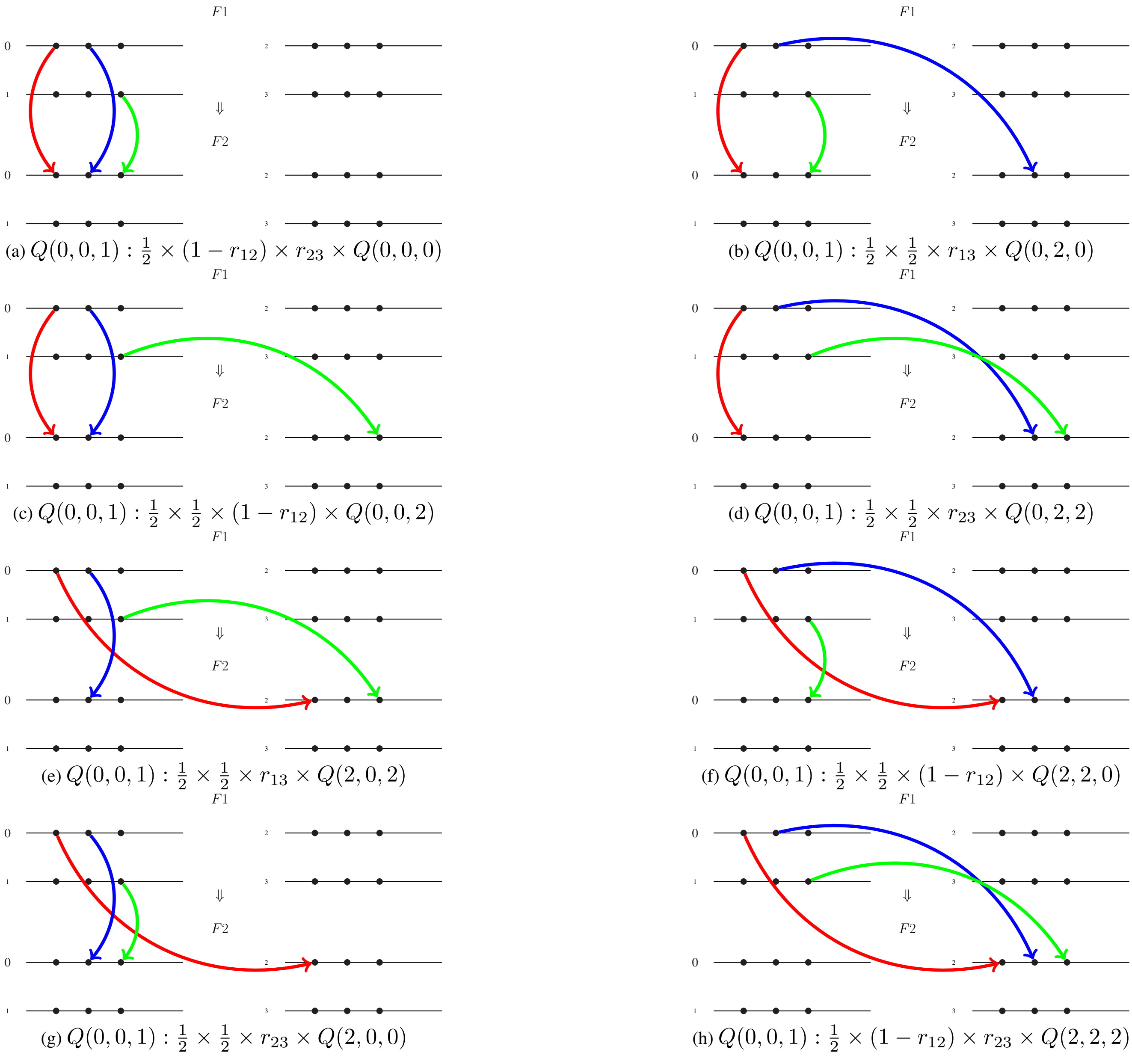
(h) $Q(0,0,0):\frac{1}{2}\times\frac{1}{2}\times(1-r_{23})\times Q(2,0,0)$

Figure S1: $Q(0,0,0)$

2.2 The self consistent equation for $Q(0,0,1)$

In Figure S2 we show the graphical representation for each term entering this self-consistent equation.

$$(1-r_{12})r_{23}Q(0,0,0) - Q(0,0,1) + \frac{1}{2}(1-r_{12})Q(0,0,2) + \frac{1}{2}r_{13}Q(0,2,0) + \frac{1}{2}r_{23}Q(0,2,2) = 0 \tag{S7}$$



(a) $Q(0,0,1) : \frac{1}{2} \times (1-r_{12}) \times r_{23} \times Q(0,0,0)$

(b) $Q(0,0,1) : \frac{1}{2} \times \frac{1}{2} \times r_{13} \times Q(0,2,0)$

(c) $Q(0,0,1) : \frac{1}{2} \times \frac{1}{2} \times (1-r_{12}) \times Q(0,0,2)$

(d) $Q(0,0,1) : \frac{1}{2} \times \frac{1}{2} \times r_{23} \times Q(0,2,2)$

(e) $Q(0,0,1) : \frac{1}{2} \times \frac{1}{2} \times r_{13} \times Q(2,0,2)$

(f) $Q(0,0,1) : \frac{1}{2} \times \frac{1}{2} \times (1-r_{12}) \times Q(2,2,0)$

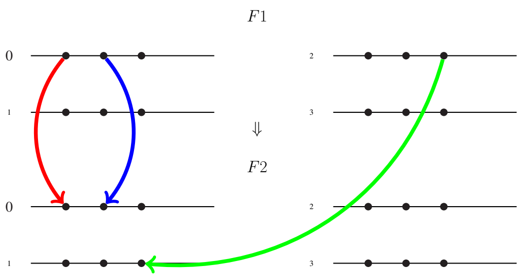
(g) $Q(0,0,1) : \frac{1}{2} \times \frac{1}{2} \times r_{23} \times Q(2,0,0)$

(h) $Q(0,0,1) : \frac{1}{2} \times (1-r_{12}) \times r_{23} \times Q(2,2,2)$

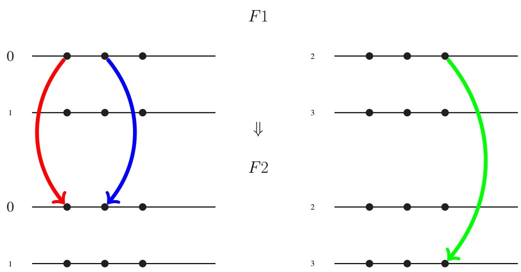
Figure S2: $Q(0,0,1)$

2.3 The self consistent equation for $Q(0,0,2)$

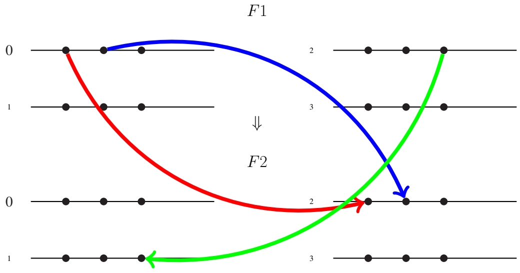
In Figure S3 we show the graphical representation for each term entering this self-consistent equation.



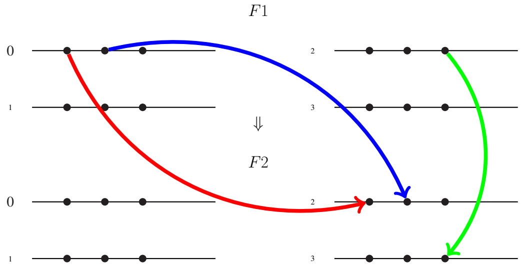
(a) $Q(0,0,2) : \frac{1}{2} \times (1 - r_{12}) \times \frac{1}{2} \times Q(0,0,1)$



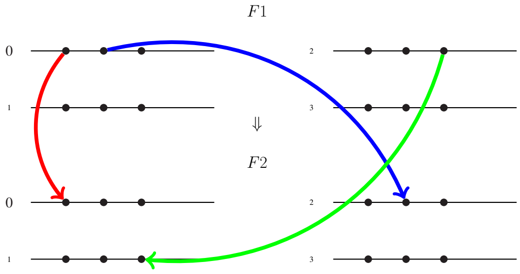
(b) $Q(0,0,2) : \frac{1}{2} \times (1 - r_{12}) \times \frac{1}{2} \times Q(0,0,3)$



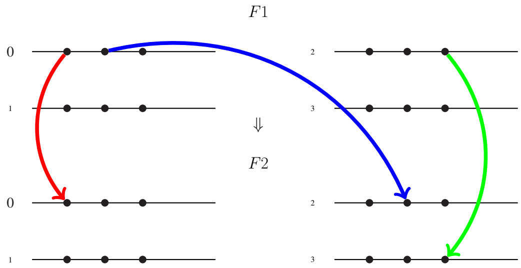
(c) $Q(0,0,2) : \frac{1}{2} \times (1 - r_{12}) \times \frac{1}{2} \times Q(2,2,1)$



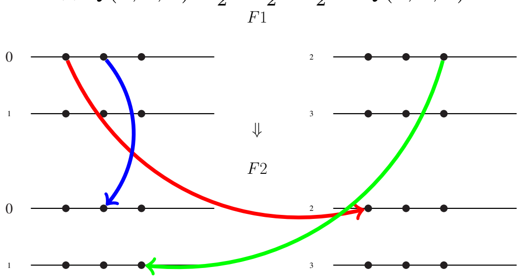
(d) $Q(0,0,2) : \frac{1}{2} \times (1 - r_{12}) \times \frac{1}{2} \times Q(2,2,3)$



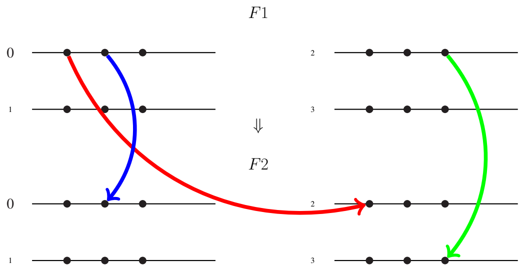
(e) $Q(0,0,2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0,2,1)$



(f) $Q(0,0,2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0,2,3)$



(g) $Q(0,0,2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2,0,1)$

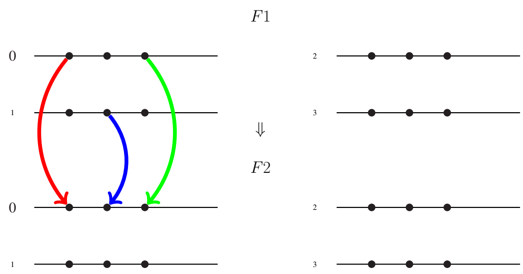


(h) $Q(0,0,2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2,0,3)$

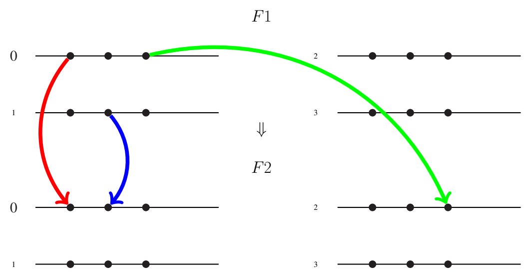
Figure S3: $Q(0,0,2)$

2.4 The self consistent equation for $Q(0,1,0)$

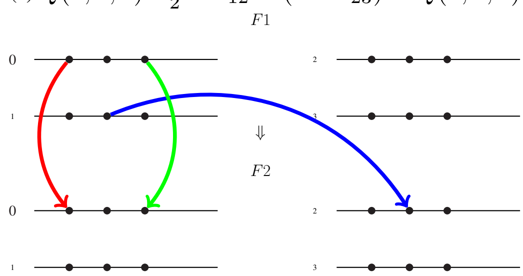
In Figure S4 we show the graphical representation for each term entering this self-consistent equation.



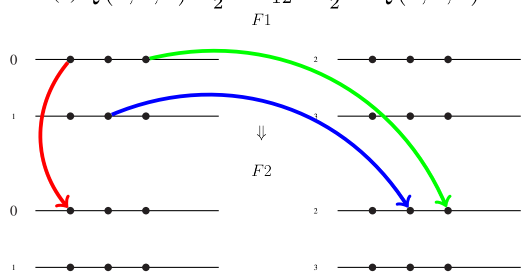
(a) $Q(0,1,0) : \frac{1}{2} \times r_{12} \times (1 - r_{23}) \times Q(0,0,0)$



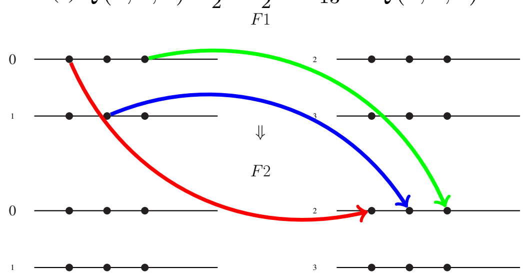
(b) $Q(0,1,0) : \frac{1}{2} \times r_{12} \times \frac{1}{2} \times Q(0,0,2)$



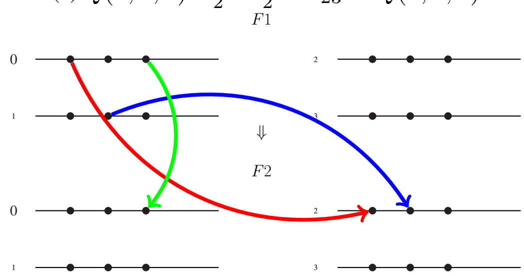
(c) $Q(0,1,0) : \frac{1}{2} \times \frac{1}{2} \times r_{13} \times Q(0,2,0)$



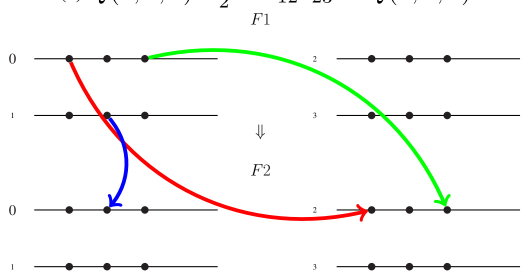
(d) $Q(0,1,0) : \frac{1}{2} \times \frac{1}{2} \times r_{23} \times Q(0,2,2)$



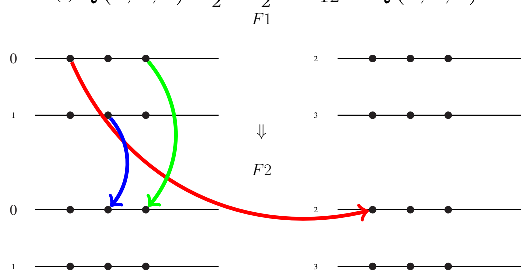
(e) $Q(0,1,0) : \frac{1}{2} \times r_{12} r_{23} \times Q(2,2,2)$



(f) $Q(0,1,0) : \frac{1}{2} \times \frac{1}{2} \times r_{12} \times Q(2,2,0)$



(g) $Q(0,1,0) : \frac{1}{2} \times \frac{1}{2} \times (1 - r_{13}) \times Q(2,0,2)$

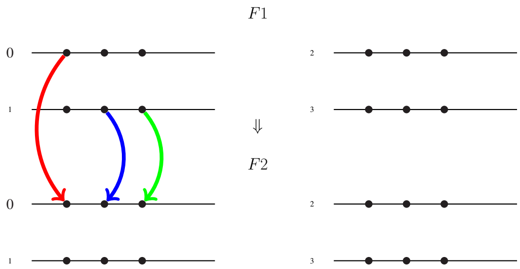


(h) $Q(0,1,0) : \frac{1}{2} \times r_{23} \times \frac{1}{2} \times Q(2,0,0)$

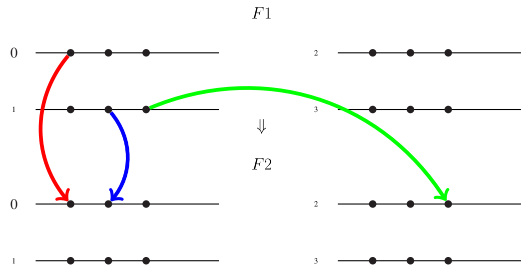
Figure S4: $Q(0,1,0)$

2.5 The self consistent equation for $Q(0,1,1)$

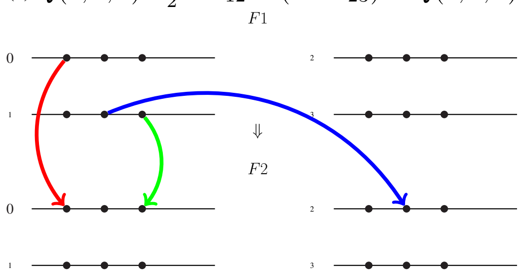
In Figure S5 we show the graphical representation for each term entering this self-consistent equation.



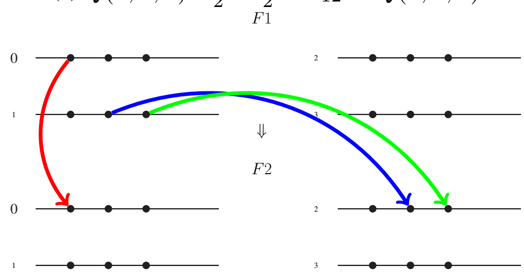
(a) $Q(0,1,1) : \frac{1}{2} \times r_{12} \times (1 - r_{23}) \times Q(0,0,0)$



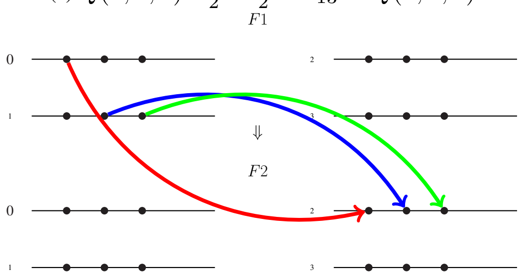
(b) $Q(0,1,1) : \frac{1}{2} \times \frac{1}{2} \times r_{12} \times Q(0,0,2)$



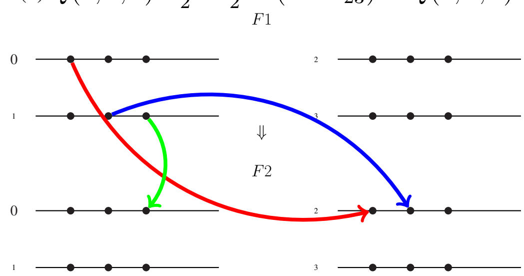
(c) $Q(0,1,1) : \frac{1}{2} \times \frac{1}{2} \times r_{13} \times Q(0,2,0)$



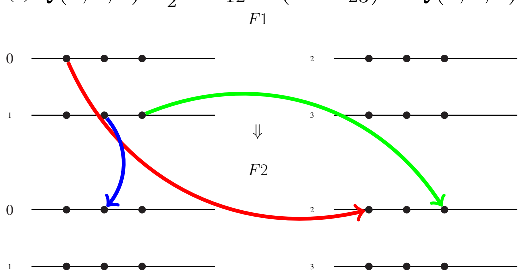
(d) $Q(0,1,1) : \frac{1}{2} \times \frac{1}{2} \times (1 - r_{23}) \times Q(0,2,2)$



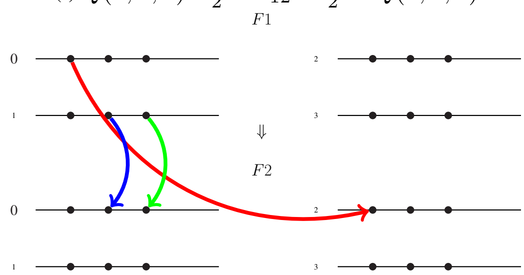
(e) $Q(0,1,1) : \frac{1}{2} \times r_{12} \times (1 - r_{23}) \times Q(2,2,2)$



(f) $Q(0,1,1) : \frac{1}{2} \times r_{12} \times \frac{1}{2} \times Q(2,2,0)$



(g) $Q(0,1,1) : \frac{1}{2} \times r_{13} \times \frac{1}{2} \times Q(2,0,2)$

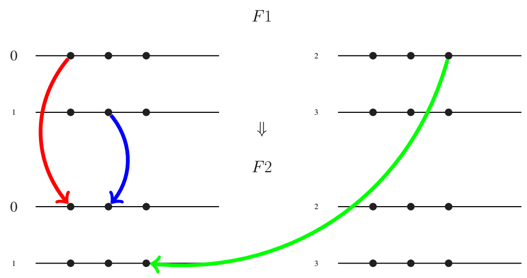


(h) $Q(0,1,1) : \frac{1}{2} \times \frac{1}{2} \times (1 - r_{23}) \times Q(2,0,0)$

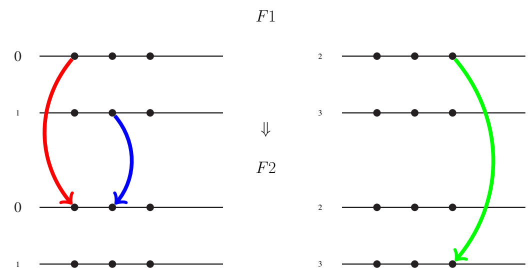
Figure S5: $Q(0,1,1)$

2.6 The self consistent equation for $Q(0, 1, 2)$

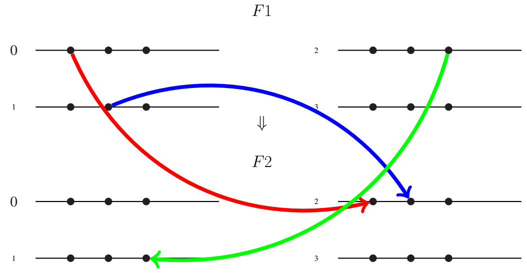
In Figure S6 we show the graphical representation for each term entering this self-consistent equation.



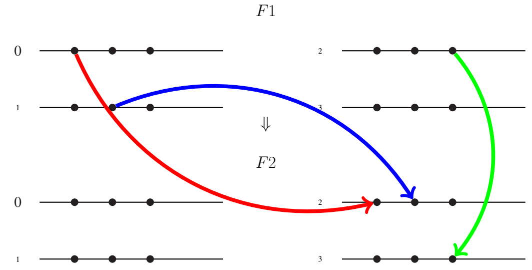
(a) $Q(0, 1, 2) : \frac{1}{2} \times \frac{1}{2} \times r_{12} \times Q(0, 0, 1)$



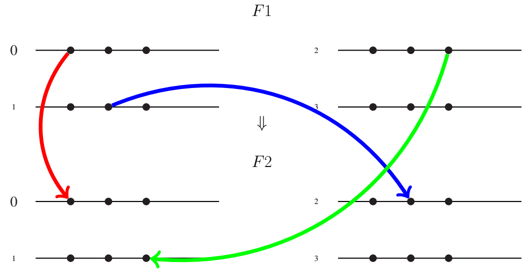
(b) $Q(0, 1, 2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times r_{12} \times Q(0, 0, 3)$



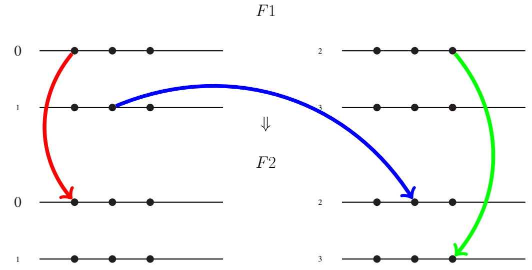
(c) $Q(0, 1, 2) : \frac{1}{2} \times \frac{1}{2} \times \times r_{12} \times Q(2, 2, 1)$



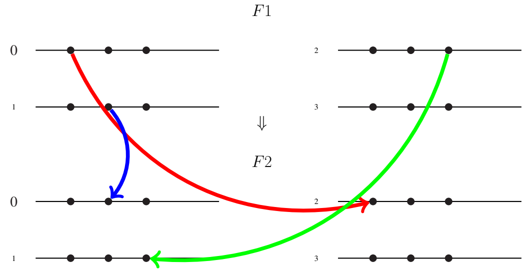
(d) $Q(0, 1, 2) : \frac{1}{2} \times \frac{1}{2} \times r_{12} \times Q(2, 2, 3)$



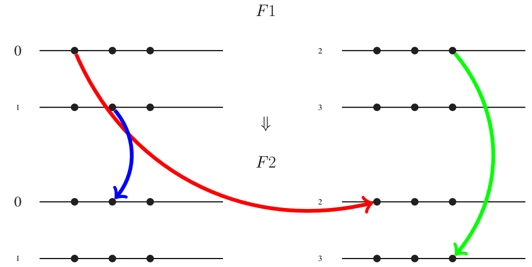
(e) $Q(0, 1, 2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0, 2, 1)$



(f) $Q(0, 1, 2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0, 2, 3)$



(g) $Q(0, 1, 2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2, 0, 1)$

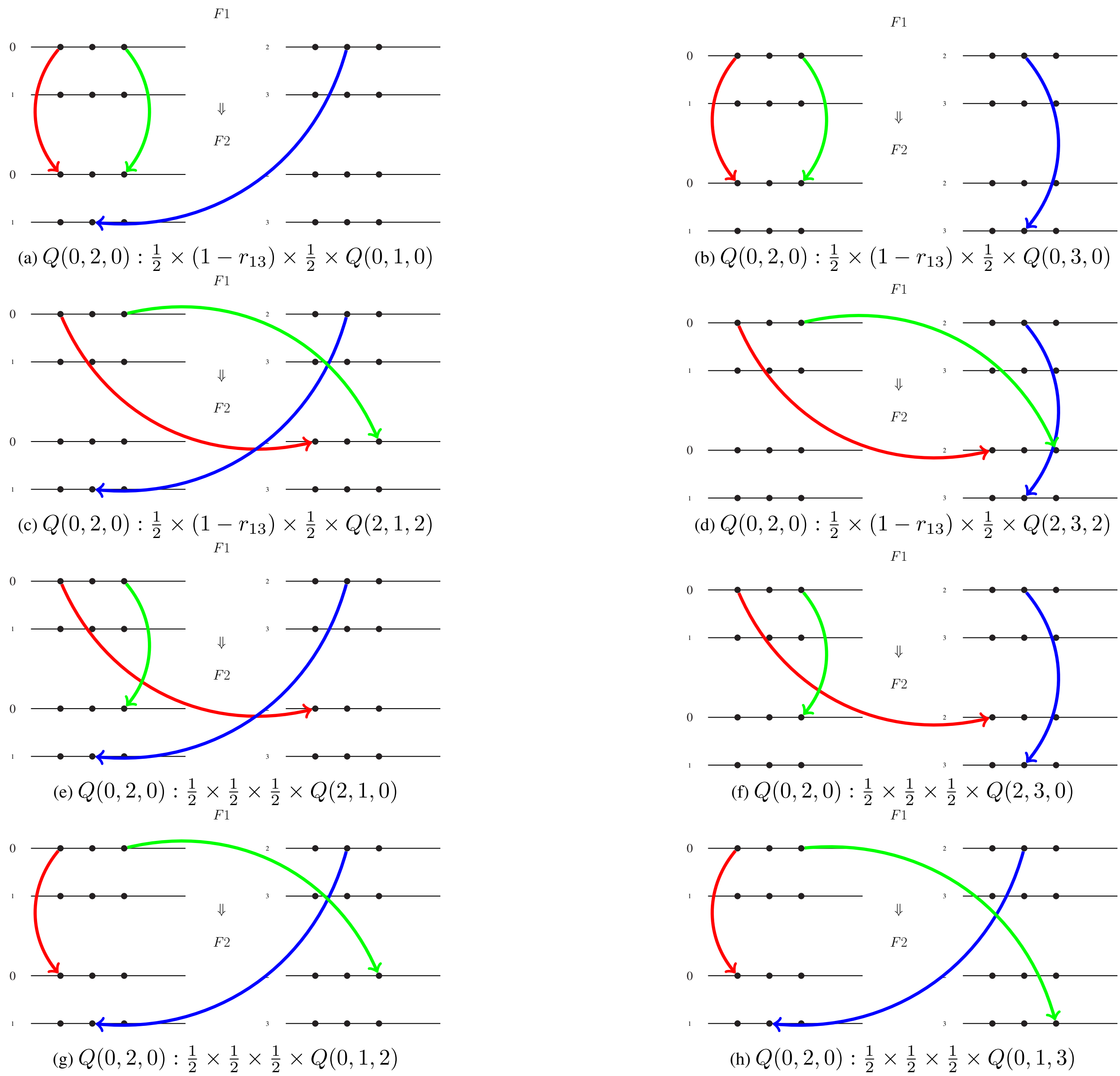


(h) $Q(0, 1, 2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2, 0, 3)$

Figure S6: $Q(0, 1, 2)$

2.7 The self consistent equation for $Q(0,2,0)$

In Figure S7 we show the graphical representation for each term entering this self-consistent equation.



(a) $Q(0,2,0) : \frac{1}{2} \times (1 - r_{13}) \times \frac{1}{2} \times Q(0,1,0)$

(b) $Q(0,2,0) : \frac{1}{2} \times (1 - r_{13}) \times \frac{1}{2} \times Q(0,3,0)$

(c) $Q(0,2,0) : \frac{1}{2} \times (1 - r_{13}) \times \frac{1}{2} \times Q(2,1,2)$

(d) $Q(0,2,0) : \frac{1}{2} \times (1 - r_{13}) \times \frac{1}{2} \times Q(2,3,2)$

(e) $Q(0,2,0) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2,1,0)$

(f) $Q(0,2,0) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2,3,0)$

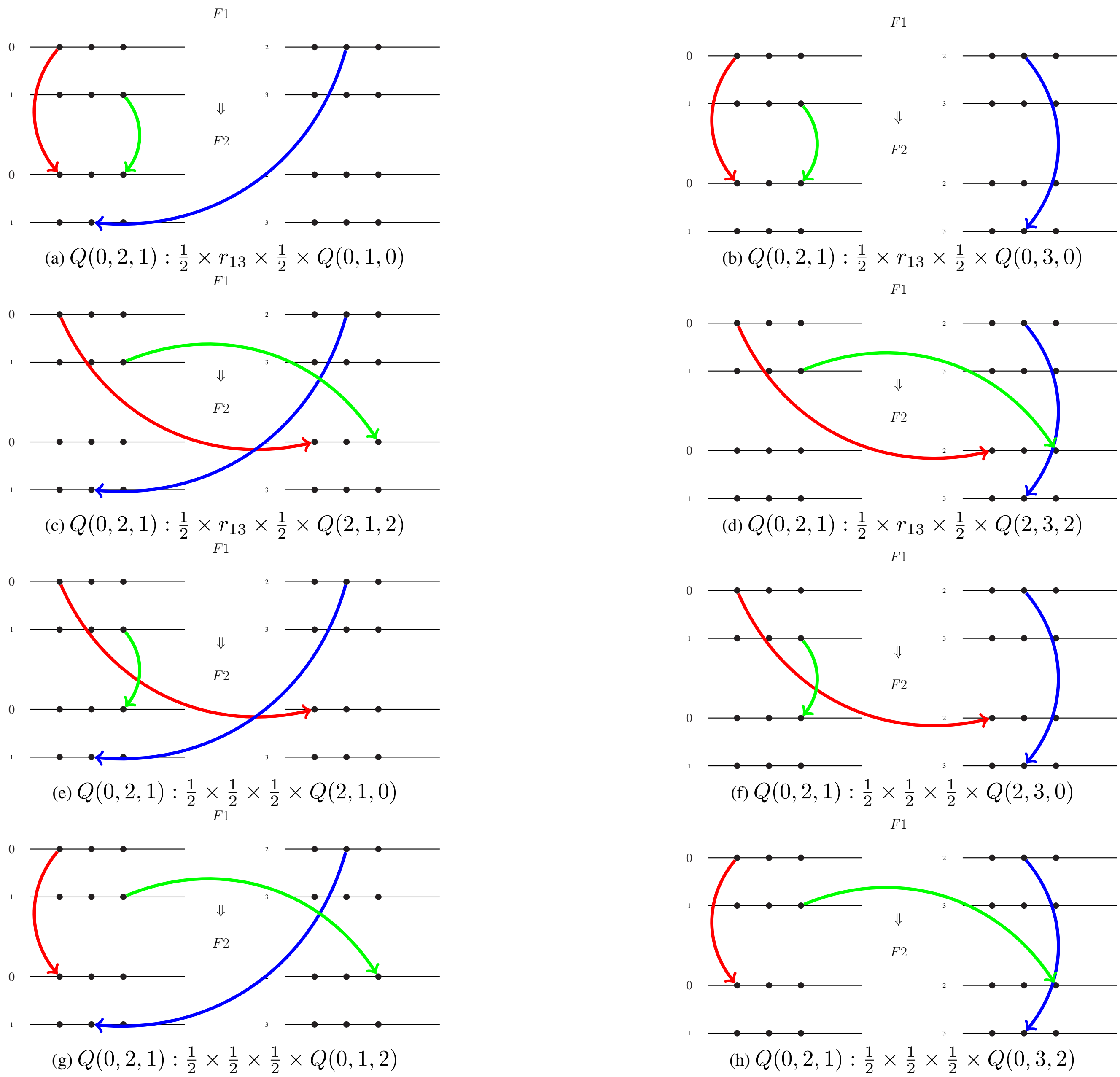
(g) $Q(0,2,0) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0,1,2)$

(h) $Q(0,2,0) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0,1,3)$

Figure S7: $Q(0,2,0)$

2.8 The self consistent equation for $Q(0, 2, 1)$

In Figure S8 we show the graphical representation for each term entering this self-consistent equation.



(a) $Q(0,2,1) : \frac{1}{2} \times r_{13} \times \frac{1}{2} \times Q(0,1,0)$

(b) $Q(0,2,1) : \frac{1}{2} \times r_{13} \times \frac{1}{2} \times Q(0,3,0)$

(c) $Q(0,2,1) : \frac{1}{2} \times r_{13} \times \frac{1}{2} \times Q(2,1,2)$

(d) $Q(0,2,1) : \frac{1}{2} \times r_{13} \times \frac{1}{2} \times Q(2,3,2)$

(e) $Q(0,2,1) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2,1,0)$

(f) $Q(0,2,1) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2,3,0)$

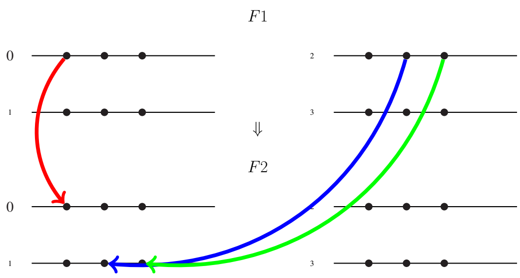
(g) $Q(0,2,1) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0,1,2)$

(h) $Q(0,2,1) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0,3,2)$

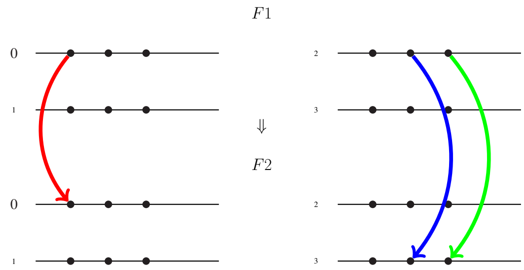
Figure S8: $Q(0, 2, 1)$

2.9 The self consistent equation for $Q(0, 2, 2)$

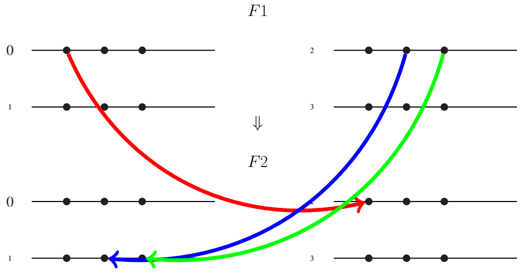
In Figure S9 we show the graphical representation for each term entering this self-consistent equation.



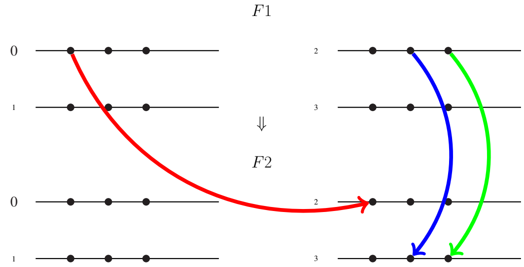
(a) $Q(0, 2, 2) : \frac{1}{2} \times \frac{1}{2} \times (1 - r_{23}) \times Q(0, 1, 1)$



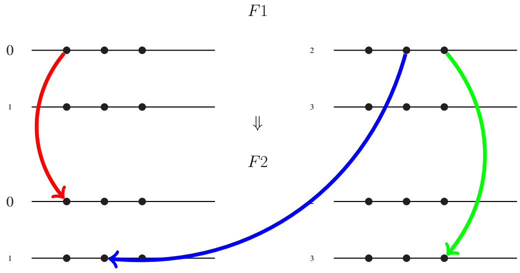
(b) $Q(0, 2, 2) : \frac{1}{2} \times \frac{1}{2} \times (1 - r_{23}) \times Q(0, 3, 3)$



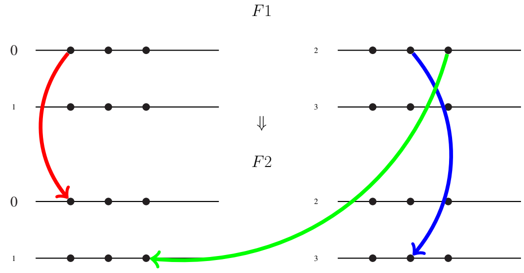
(c) $Q(0, 2, 2) : \frac{1}{2} \times \frac{1}{2} \times (1 - r_{23}) \times Q(2, 1, 1)$



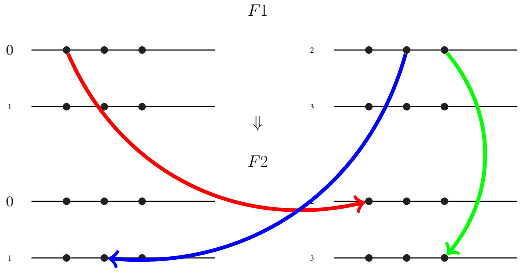
(d) $Q(0, 2, 2) : \frac{1}{2} \times \frac{1}{2} \times (1 - r_{23}) \times Q(2, 3, 3)$



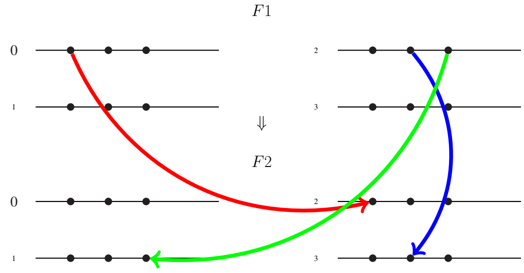
(e) $Q(0, 2, 2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0, 1, 3)$



(f) $Q(0, 2, 2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0, 3, 1)$



(g) $Q(0, 2, 2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2, 1, 3)$

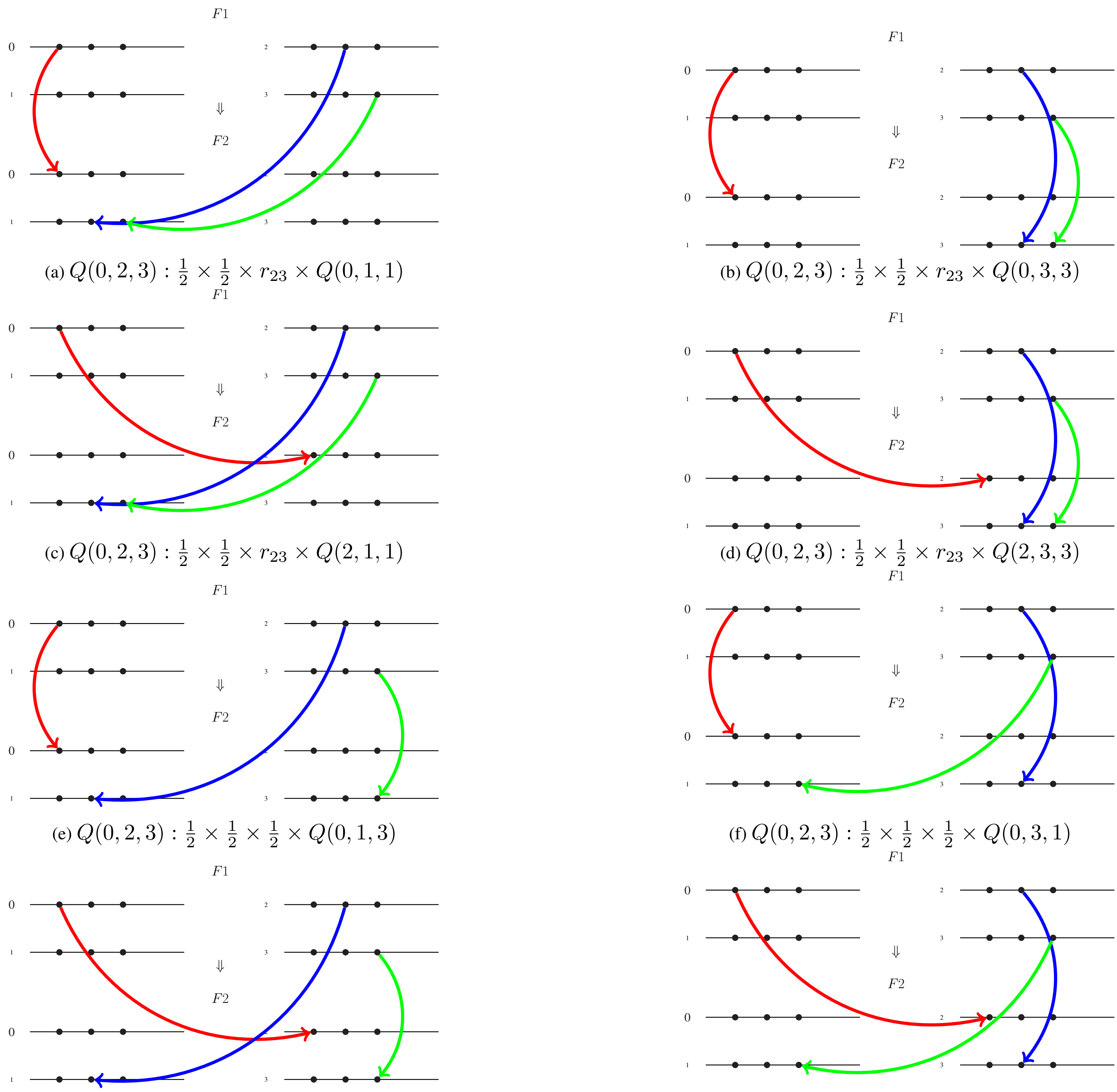


(h) $Q(0, 2, 2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2, 3, 1)$

Figure S9: $Q(0, 2, 2)$

2.10 The self consistent equation for $Q(0, 2, 3)$

In Figure S10 we show the graphical representation for each term entering this self-consistent equation.



(a) $Q(0,2,3) : \frac{1}{2} \times \frac{1}{2} \times r_{23} \times Q(0,1,1)$

(b) $Q(0,2,3) : \frac{1}{2} \times \frac{1}{2} \times r_{23} \times Q(0,3,3)$

(c) $Q(0,2,3) : \frac{1}{2} \times \frac{1}{2} \times r_{23} \times Q(2,1,1)$

(d) $Q(0,2,3) : \frac{1}{2} \times \frac{1}{2} \times r_{23} \times Q(2,3,3)$

(e) $Q(0,2,3) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0,1,3)$

(f) $Q(0,2,3) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0,3,1)$

(g) $Q(0,2,3) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2,1,3)$

(h) $Q(0,2,3) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2,3,1)$

Figure S10: $Q(0, 2, 3)$