Supplementary Material

1 SUPPLEMENTARY MATHEMATICS

Here we prove the equation

$$|Q_L| = 2^{L-2}(2^{L-1}+1) \tag{S1}$$

Proof: For $L \geq 2$, there are 4^L IBD (identical by descent) probabilities $Q(i_1, i_2, \ldots i_L)$ since $i_l = 0, 1, 2$ or 3 and furthermore they add up to 1. A number of these probabilities are equal because of two symmetries: (1) the two homologous chromosomes in each individual play identical roles, and (2) the siblings play identical roles (assuming no sex-dependence of meiosis, so that the recombination rates $r_{l,l'}$ are sex-independent). It is thus appropriate to use only one representative of each symmetry equivalence class. A way to do this is to impose that this representative have its first index, i_1, equal to zero. In fact we can identify exactly one element in each class by imposing that the indices of the representative Q's have either

1. $i_l \in \{0, 1\}\ \forall l \in \{2, .., L\}$, *or*
2. $i_l \in \{0, 1\}\ \forall l \in \{2, .., K-1\}$, $i_K = 2$ and $i_l \in \{0, 1, 2, 3\}\ \forall l \in \{K+1, .., L\}$

The number of equivalence classes and thus of Q's to consider is then

$$|Q_L| = 2^{L-1} + \sum_{l=2}^{L} 2^{l-2}4^{L-l} = 2^{L-1} + 2^{2L-2}\sum_{l=2}^{L} 2^{-l} \tag{S2}$$

Given that $\sum_{l=2}^{L} 2^{-l}$ is a geometric progression of common ratio 2^{-1} from 2 to L, the sum of its terms can be expressed as:

$$\sum_{l=2}^{L} 2^{-l} = \frac{2^{-2} - 2^{-(L-1)}}{1 - 2^{-1}} = 2^{-1} - 2^{-L} \tag{S3}$$

Substituting S3 in S2, we get

$$|Q_L| = 2^{L-1} + 2^{2L-2}(2^{-1} - 2^{-L}) = 2^{L-1} + 2^{2L-3} - 2^{L-2} \tag{S4}$$

Factorizing with respect to 2^{L-2} and after simplification, this gives

$$|Q_L| = 2^{L-2}(1 + 2^{L-1}). \tag{S5}$$

2 THE SELF-CONSISTENT EQUATIONS FOR THREE LOCI

Here we provide the coefficients entering each of the $|Q_L| = 10$ self-consistent equations for $L = 3$.

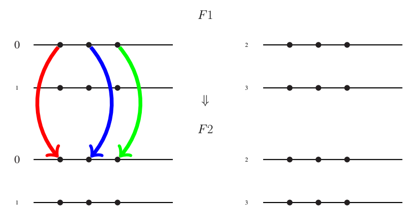
2.1 The self consistent equation for $Q(0,0,0)$

Figure S1 displays the 8 factors in the self-consistent equation for $Q(0,0,0)$:

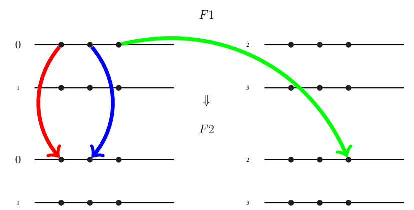
$$Q(0,0,0) = \tfrac{1}{2}(1-r_{12})(1-r_{23})[Q(0,0,0)+Q(2,2,2)] + \tfrac{1}{4}(1-r_{12})[Q(0,0,2)+Q(2,2,0)] + \tfrac{1}{4}(1-r_{13})[Q(0,2,0)+Q(2,0,2)] + \tfrac{1}{4}(1-r_{23})[Q(0,2,2)+Q(2,0,0)] \tag{S6}$$

After use of symmetry to keep only non-equivalent Qs, this leads to

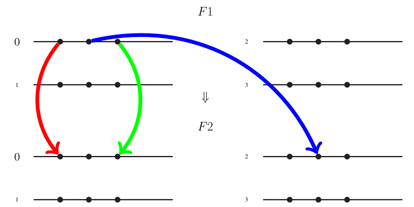
$$Q(0,0,0) = (1-r_{12})(1-r_{23})Q(0,0,0) + \tfrac{1}{2}(1-r_{12})Q(0,0,2) + \tfrac{1}{2}(1-r_{13})Q(0,2,0) + \tfrac{1}{2}(1-r_{23})Q(0,2,2)$$



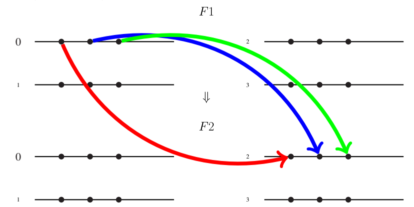
(a) $Q(0,0,0)$: $\frac{1}{2} \times (1-r_{12}) \times (1-r_{23}) \times Q(0,0,0)$



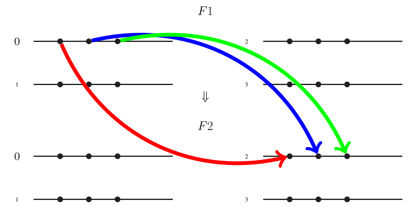
(b) $Q(0,0,0)$: $\frac{1}{2} \times (1-r_{12}) \times \frac{1}{2} \times Q(0,0,2)$



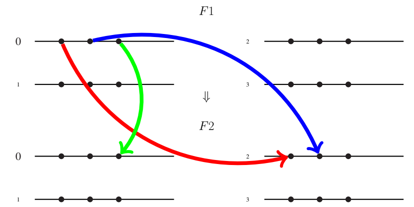
(c) $Q(0,0,0)$: $\frac{1}{2} \times \frac{1}{2} \times (1-r_{13}) \times Q(0,2,0)$



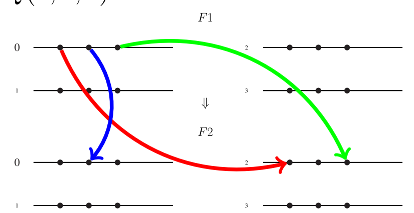
(d) $Q(0,0,0)$: $\frac{1}{2} \times \frac{1}{2} \times (1-r_{23}) \times Q(0,2,2)$



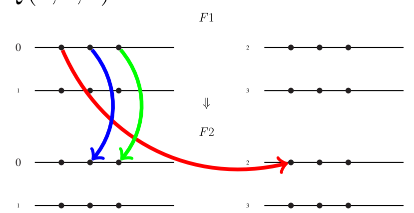
(e) $Q(0,0,0)$: $\frac{1}{2} \times (1-r_{12})(1-r_{23}) \times Q(2,2,2)$



(f) $Q(0,0,0)$: $\frac{1}{2} \times \frac{1}{2} \times (1-r_{12}) \times Q(2,2,0)$



(g) $Q(0,0,0)$: $\frac{1}{2} \times \frac{1}{2} \times (1-r_{13}) \times Q(2,0,2)$



(h) $Q(0,0,0)$: $\frac{1}{2} \times \frac{1}{2} \times (1-r_{23}) \times Q(2,0,0)$

Figure S1: The graphical representation of the factors multiplying each Q on the right-hand side of Eq. S6 for $Q(0,0,0)$.

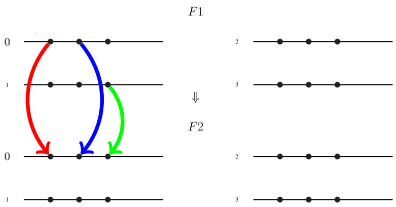
2.2 The self consistent equation for $Q(0,0,1)$

Figure S2 displays the 8 factors in the self-consistent equation for $Q(0,0,1)$:

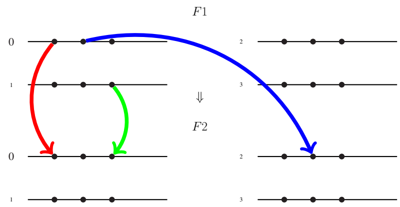
$$Q(0,0,1)=\tfrac{1}{2}(1-r_{12})r_{23}[Q(0,0,0)+Q(2,2,2)]+\tfrac{1}{4}(1-r_{12})[Q(0,0,2)+Q(2,2,0)]+\tfrac{1}{4}r_{13}[Q(0,2,0)Q(2,0,2)]+\tfrac{1}{4}r_{23}[Q(0,2,2)+Q(2,0,0)] \quad \text{(S7)}$$

After use of symmetry to keep only non-equivalent Qs, this leads to

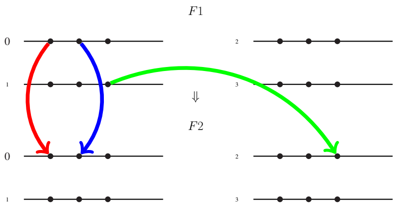
$$Q(0,0,1)=(1-r_{12})r_{23}Q(0,0,0)+\tfrac{1}{2}(1-r_{12})Q(0,0,2)+\tfrac{1}{2}r_{13}Q(0,2,0)+\tfrac{1}{2}r_{23}Q(0,2,2)$$



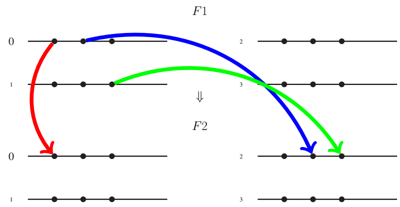
(a) $Q(0,0,1):\frac{1}{2}\times(1-r_{12})\times r_{23}\times Q(0,0,0)$



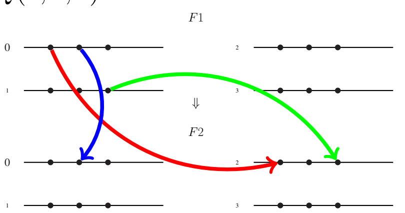
(b) $Q(0,0,1):\frac{1}{2}\times\frac{1}{2}\times r_{13}\times Q(0,2,0)$



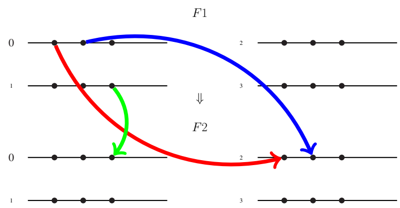
(c) $Q(0,0,1):\frac{1}{2}\times\frac{1}{2}\times(1-r_{12})\times Q(0,0,2)$



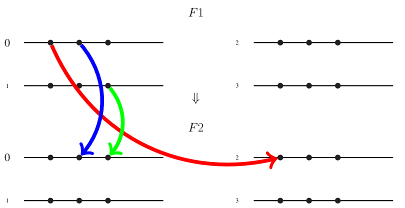
(d) $Q(0,0,1):\frac{1}{2}\times\frac{1}{2}\times r_{23}\times Q(0,2,2)$



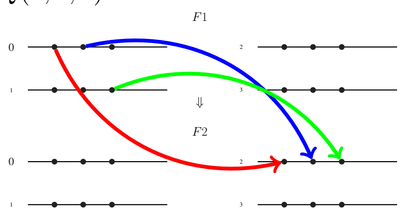
(e) $Q(0,0,1):\frac{1}{2}\times\frac{1}{2}\times r_{13}\times Q(2,0,2)$



(f) $Q(0,0,1):\frac{1}{2}\times\frac{1}{2}\times(1-r_{12})\times Q(2,2,0)$



(g) $Q(0,0,1):\frac{1}{2}\times\frac{1}{2}\times r_{23}\times Q(2,0,0)$



(h) $Q(0,0,1):\frac{1}{2}\times(1-r_{12})\times r_{23}\times Q(2,2,2)$

Figure S2: The graphical representation of the factors multiplying each Q on the right-hand side of Eq. S7 for $Q(0,0,1)$.

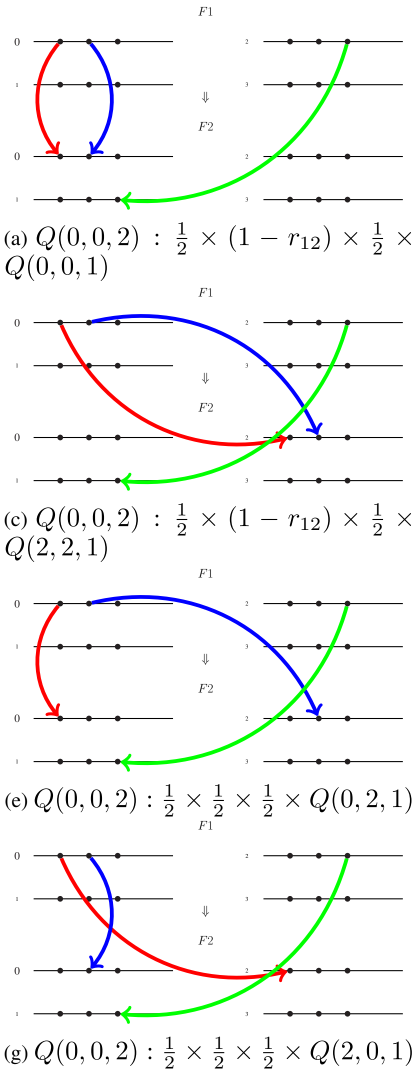
2.3 The self consistent equation for $Q(0,0,2)$

Figure S3 displays the 8 factors in the self-consistent equation for $Q(0,0,2)$:

$$Q(0,0,2) = \tfrac{1}{4}(1-r_{12})[Q(0,0,1)+Q(2,2,3)] + \tfrac{1}{4}(1-r_{12})[Q(0,0,3)+Q(2,2,1)] + \tfrac{1}{8}[Q(0,2,1)+Q(2,0,3)] + \tfrac{1}{8}[Q(0,2,3)+Q(2,0,1)] \tag{S8}$$

After use of symmetry to keep only non-equivalent Qs, this leads to

$$Q(0,0,2) = \tfrac{1}{2}(1-r_{12})Q(0,0,1) + \tfrac{1}{2}(1-r_{12})Q(0,0,2) + \tfrac{1}{4}Q(0,2,1) + \tfrac{1}{4}Q(0,2,3)$$



(a) $Q(0,0,2)\ :\ \frac{1}{2}\times(1-r_{12})\times\frac{1}{2}\times Q(0,0,1)$

(b) $Q(0,0,2)\ :\ \frac{1}{2}\times(1-r_{12})\times\frac{1}{2}\times Q(0,0,3)$

(c) $Q(0,0,2)\ :\ \frac{1}{2}\times(1-r_{12})\times\frac{1}{2}\times Q(2,2,1)$

(d) $Q(0,0,2)\ :\ \frac{1}{2}\times(1-r_{12})\times\frac{1}{2}\times Q(2,2,3)$

(e) $Q(0,0,2) : \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(0,2,1)$

(f) $Q(0,0,2) : \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(0,2,3)$

(g) $Q(0,0,2) : \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(2,0,1)$

(h) $Q(0,0,2) : \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(2,0,3)$

Figure S3: The graphical representation of the factors multiplying each Q on the right-hand side of Eq. S8 for $Q(0,0,2)$.

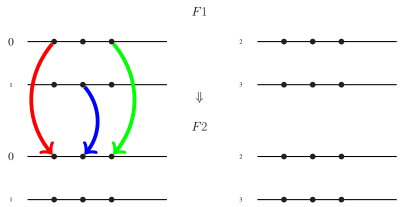
2.4 The self consistent equation for $Q(0,1,0)$

Figure S4 displays the 8 factors in the self-consistent equation for $Q(0,1,0)$:

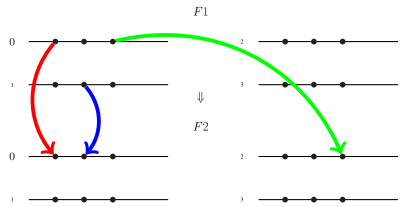
$$Q(0,1,0) = \tfrac{1}{2}r_{12}r_{23}[Q(0,0,0)+Q(2,2,2)] + \tfrac{1}{4}r_{12}[Q(0,0,2)+Q(2,2,0)] + \tfrac{1}{4}(1-r_{13})[Q(0,2,0)+Q(2,0,2)] + \tfrac{1}{4}r_{23}[Q(0,2,2)+Q(2,0,0)] \tag{S9}$$

After use of symmetry to keep only non-equivalent Qs, this leads to

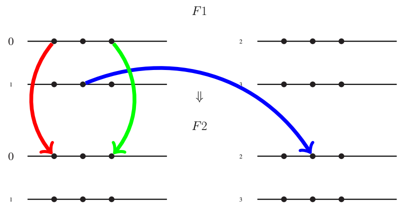
$$Q(0,1,0) = r_{12}r_{23}Q(0,0,0) + \tfrac{1}{2}r_{12}Q(0,0,2) + \tfrac{1}{2}(1-r_{13})Q(0,2,0) + \tfrac{1}{2}r_{23}Q(0,2,2)$$



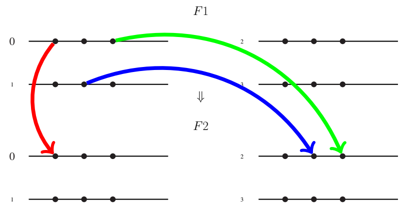
(a) $Q(0,1,0) : \frac{1}{2} \times r_{12} \times (1-r_{23}) \times Q(0,0,0)$



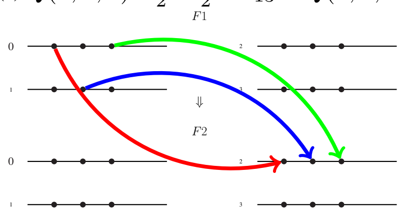
(b) $Q(0,1,0) : \frac{1}{2} \times r_{12} \times \frac{1}{2} \times Q(0,0,2)$



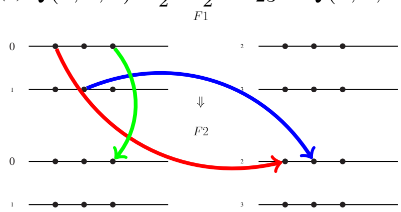
(c) $Q(0,1,0) : \frac{1}{2} \times \frac{1}{2} \times r_{13} \times Q(0,2,0)$



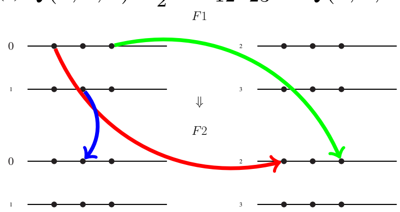
(d) $Q(0,1,0) : \frac{1}{2} \times \frac{1}{2} \times r_{23} \times Q(0,2,2)$



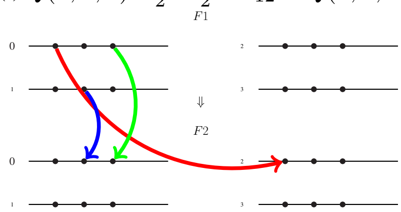
(e) $Q(0,1,0) : \frac{1}{2} \times r_{12}r_{23} \times Q(2,2,2)$



(f) $Q(0,1,0) : \frac{1}{2} \times \frac{1}{2} \times r_{12} \times Q(2,2,0)$



(g) $Q(0,1,0) : \frac{1}{2} \times \frac{1}{2} \times (1-r_{13}) \times Q(2,0,2)$



(h) $Q(0,1,0) : \frac{1}{2} \times r_{23} \times \frac{1}{2} \times Q(2,0,0)$

Figure S4: The graphical representation of the factors multiplying each Q on the right-hand side of Eq. S9 for $Q(0,1,0)$.

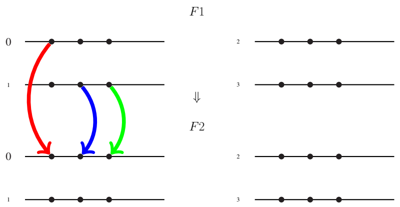
2.5 The self consistent equation for $Q(0,1,1)$

Figure S5 displays the 8 factors in the self-consistent equation for $Q(0,1,1)$:

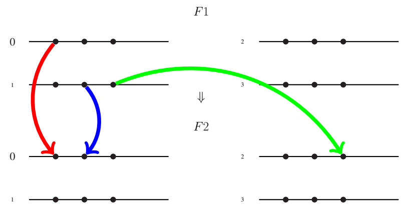
$$Q(0,1,1) = \frac{1}{2}r_{12}(1-r_{23})[Q(0,0,0)+Q(2,2,2)] + \frac{1}{4}r_{12}[Q(0,0,2)+Q(2,2,0)] + \frac{1}{4}r_{13}[Q(0,2,0)+Q(2,0,2)] + \frac{1}{4}(1-r_{23})[Q(0,2,2)+Q(2,0,0)] \tag{S10}$$

After use of symmetry to keep only non-equivalent Qs, this leads to

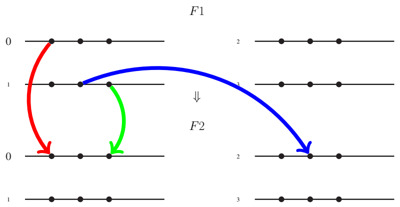
$$Q(0,1,1) = r_{12}(1-r_{23})Q(0,0,0) + \frac{1}{2}r_{12}Q(0,0,2) + \frac{1}{2}r_{13}Q(0,2,0) + \frac{1}{2}(1-r_{23})Q(0,2,2)$$



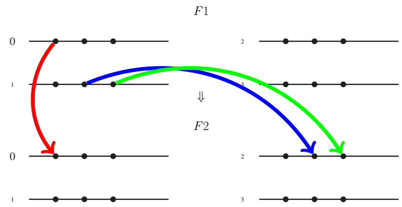
(a) $Q(0,1,1)$: $\frac{1}{2} \times r_{12} \times (1-r_{23}) \times Q(0,0,0)$



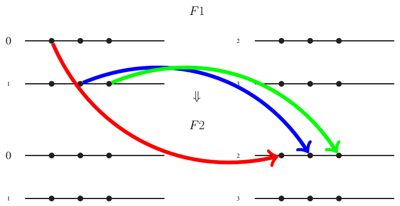
(b) $Q(0,1,1)$: $\frac{1}{2} \times \frac{1}{2} \times r_{12} \times Q(0,0,2)$



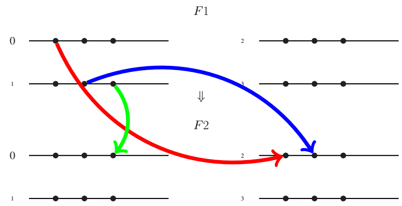
(c) $Q(0,1,1)$: $\frac{1}{2} \times \frac{1}{2} \times r_{13} \times Q(0,2,0)$



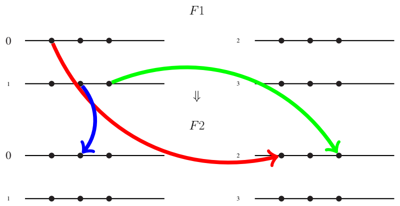
(d) $Q(0,1,1)$: $\frac{1}{2} \times \frac{1}{2} \times (1-r_{23}) \times Q(0,2,2)$



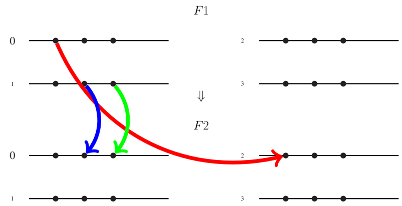
(e) $Q(0,1,1)$: $\frac{1}{2} \times r_{12} \times (1-r_{23}) \times Q(2,2,2)$



(f) $Q(0,1,1)$: $\frac{1}{2} \times r_{12} \times \frac{1}{2} \times Q(2,2,0)$



(g) $Q(0,1,1)$: $\frac{1}{2} \times r_{13} \times \frac{1}{2} \times Q(2,0,2)$



(h) $Q(0,1,1)$: $\frac{1}{2} \times \frac{1}{2} \times (1-r_{23}) \times Q(2,0,0)$

Figure S5: The graphical representation of the factors multiplying each Q on the right-hand side of Eq. S10 for $Q(0,1,1)$.

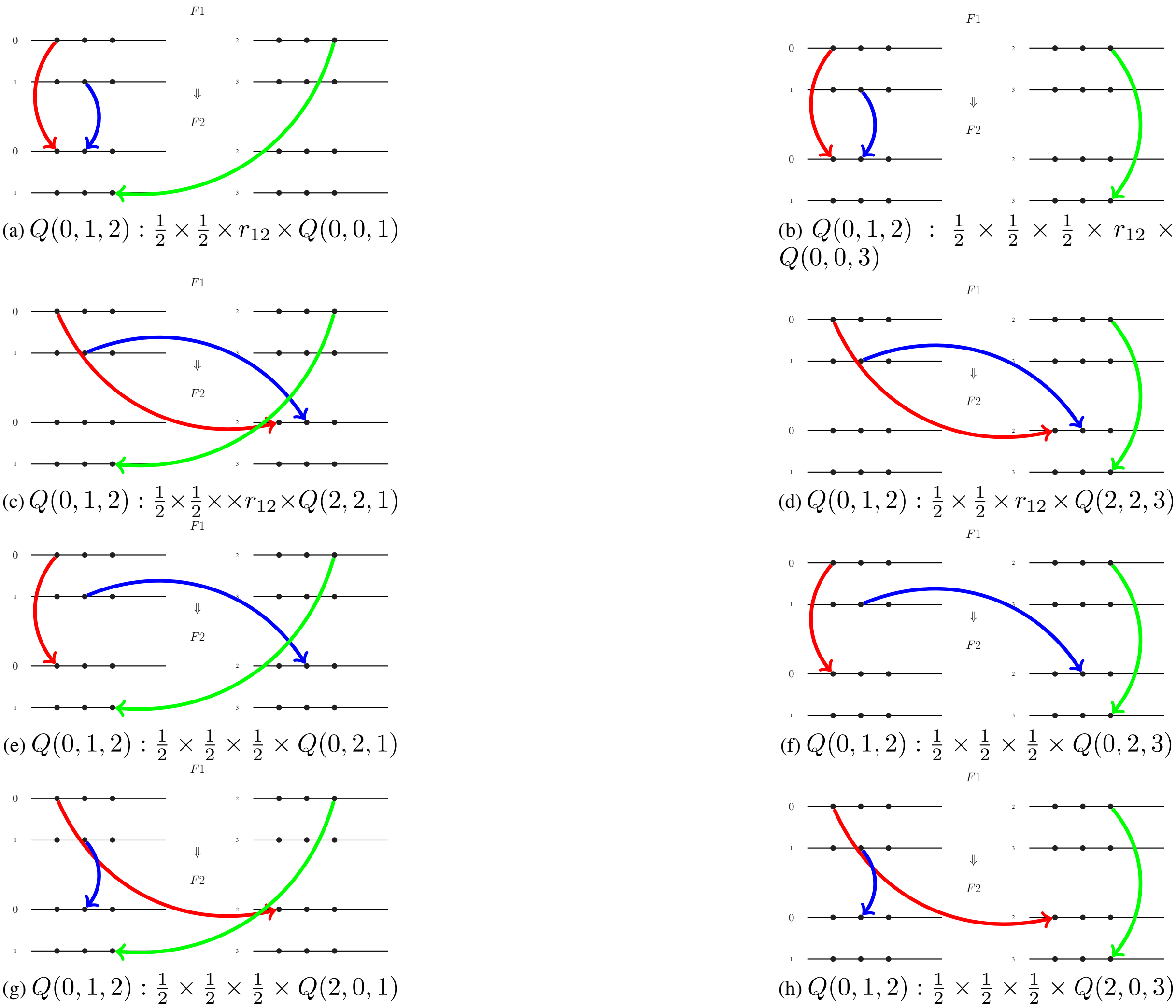
2.6 The self consistent equation for $Q(0,1,2)$

Figure S6 displays the 8 factors in the self-consistent equation for $Q(0,1,2)$:

$$Q(0,1,2) = \frac{1}{4}r_{12}[Q(0,0,1)+Q(2,2,3)] + \frac{1}{4}r_{12}[Q(0,0,3)+Q(2,2,1)] + \frac{1}{8}[Q(0,2,1)+Q(2,0,3)] + \frac{1}{8}[Q(0,2,3)+Q(2,0,1)] \quad \text{(S11)}$$

After use of symmetry to keep only non-equivalent Qs, this leads to

$$Q(0,1,2) = \frac{1}{2}r_{12}Q(0,0,1) + \frac{1}{2}r_{12}Q(0,0,2) + \frac{1}{4}Q(0,2,1) + \frac{1}{4}Q(0,2,3)$$



(a) $Q(0,1,2) : \frac{1}{2} \times \frac{1}{2} \times r_{12} \times Q(0,0,1)$

(b) $Q(0,1,2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times r_{12} \times Q(0,0,3)$

(c) $Q(0,1,2) : \frac{1}{2} \times \frac{1}{2} \times \times r_{12} \times Q(2,2,1)$

(d) $Q(0,1,2) : \frac{1}{2} \times \frac{1}{2} \times r_{12} \times Q(2,2,3)$

(e) $Q(0,1,2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0,2,1)$

(f) $Q(0,1,2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0,2,3)$

(g) $Q(0,1,2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2,0,1)$

(h) $Q(0,1,2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2,0,3)$

Figure S6: The graphical representation of the factors multiplying each Q on the right-hand side of Eq. S11 for $Q(0,1,2)$.

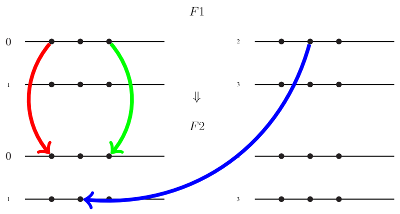
2.7 The self consistent equation for $Q(0,2,0)$

Figure S7 displays the 8 factors in the self-consistent equation for $Q(0,2,0)$:

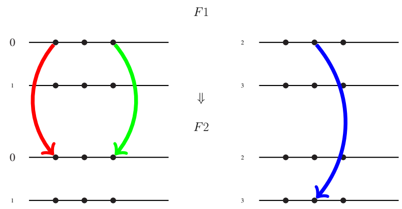
$$Q(0,2,0) = \tfrac{1}{4}(1-r_{13})[Q(0,1,0)+Q(2,3,2)] + \tfrac{1}{8}[Q(0,1,2)+Q(2,3,0)] + \tfrac{1}{4}(1-r_{13})[Q(0,3,0)+Q(2,1,2)] + \tfrac{1}{8}[Q(0,1,3)+Q(2,1,0)] \tag{S12}$$

After use of symmetry to keep only non-equivalent Qs, this leads to

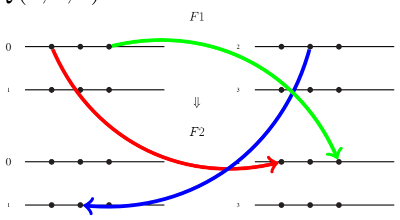
$$Q(0,2,0) = \tfrac{1}{2}(1-r_{13})Q(0,1,0) + \tfrac{1}{4}Q(0,1,2) + \tfrac{1}{2}(1-r_{13})Q(0,2,0) + \tfrac{1}{4}Q(0,1,2)$$



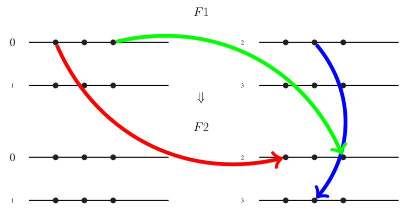
(a) $Q(0,2,0)\ :\ \frac{1}{2}\times(1-r_{13})\times\frac{1}{2}\times Q(0,1,0)$



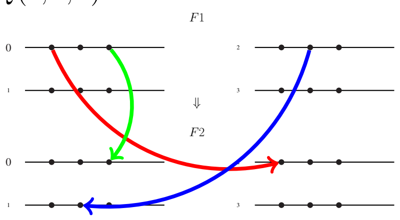
(b) $Q(0,2,0)\ :\ \frac{1}{2}\times(1-r_{13})\times\frac{1}{2}\times Q(0,3,0)$



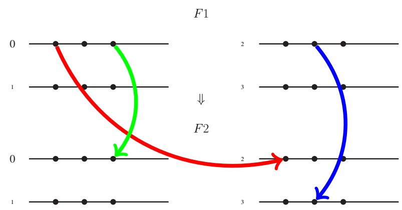
(c) $Q(0,2,0)\ :\ \frac{1}{2}\times(1-r_{13})\times\frac{1}{2}\times Q(2,1,2)$



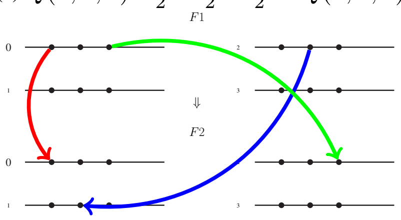
(d) $Q(0,2,0)\ :\ \frac{1}{2}\times(1-r_{13})\times\frac{1}{2}\times Q(2,3,2)$



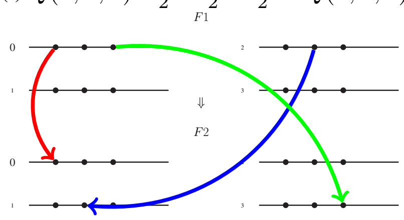
(e) $Q(0,2,0) : \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(2,1,0)$



(f) $Q(0,2,0) : \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(2,3,0)$



(g) $Q(0,2,0) : \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(0,1,2)$



(h) $Q(0,2,0) : \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(0,1,3)$

Figure S7: The graphical representation of the factors multiplying each Q on the right-hand side of Eq. S12 for $Q(0,2,0)$.

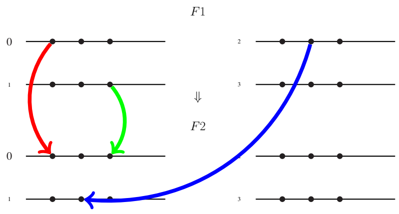
2.8 The self consistent equation for $Q(0,2,1)$

Figure S8 displays the 8 factors in the self-consistent equation for $Q(0,2,1)$:

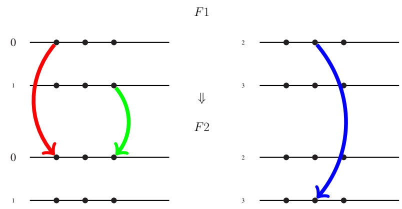
$$Q(0,2,1) = \tfrac{1}{4}r_{13}[Q(0,1,0)+Q(2,3,2)] + \tfrac{1}{8}[Q(0,1,2)+Q(2,3,0)] + \tfrac{1}{4}r_{13}[Q(0,3,0)+Q(2,1,2)] + \tfrac{1}{8}[Q(0,3,2)+Q(2,1,0)] \tag{S13}$$

After use of symmetry to keep only non-equivalent Qs, this leads to

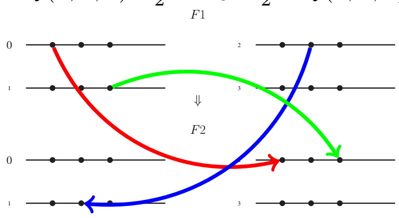
$$Q(0,2,1) = \tfrac{1}{2}r_{13}Q(0,1,0) + \tfrac{1}{4}Q(0,1,2) + \tfrac{1}{2}r_{13}Q(0,2,0) + \tfrac{1}{4}Q(0,2,3)$$



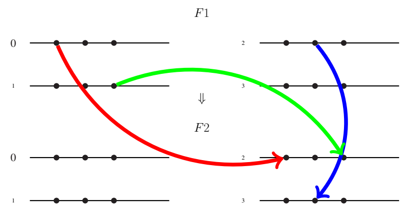
(a) $Q(0,2,1) : \frac{1}{2} \times r_{13} \times \frac{1}{2} \times Q(0,1,0)$



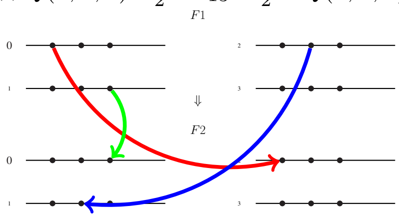
(b) $Q(0,2,1) : \frac{1}{2} \times r_{13} \times \frac{1}{2} \times Q(0,3,0)$



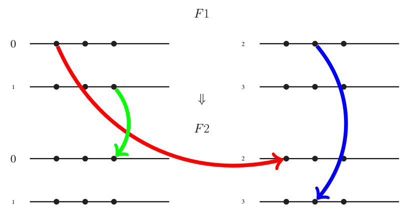
(c) $Q(0,2,1) : \frac{1}{2} \times r_{13} \times \frac{1}{2} \times Q(2,1,2)$



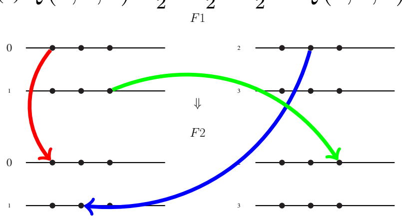
(d) $Q(0,2,1) : \frac{1}{2} \times r_{13} \times \frac{1}{2} \times Q(2,3,2)$



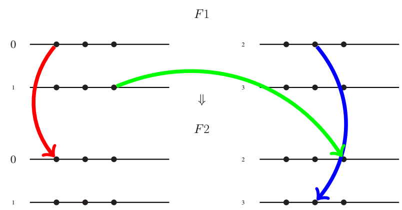
(e) $Q(0,2,1) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2,1,0)$



(f) $Q(0,2,1) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2,3,0)$



(g) $Q(0,2,1) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0,1,2)$



(h) $Q(0,2,1) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0,3,2)$

Figure S8: The graphical representation of the factors multiplying each Q on the right-hand side of Eq. S13 for $Q(0,2,1)$.

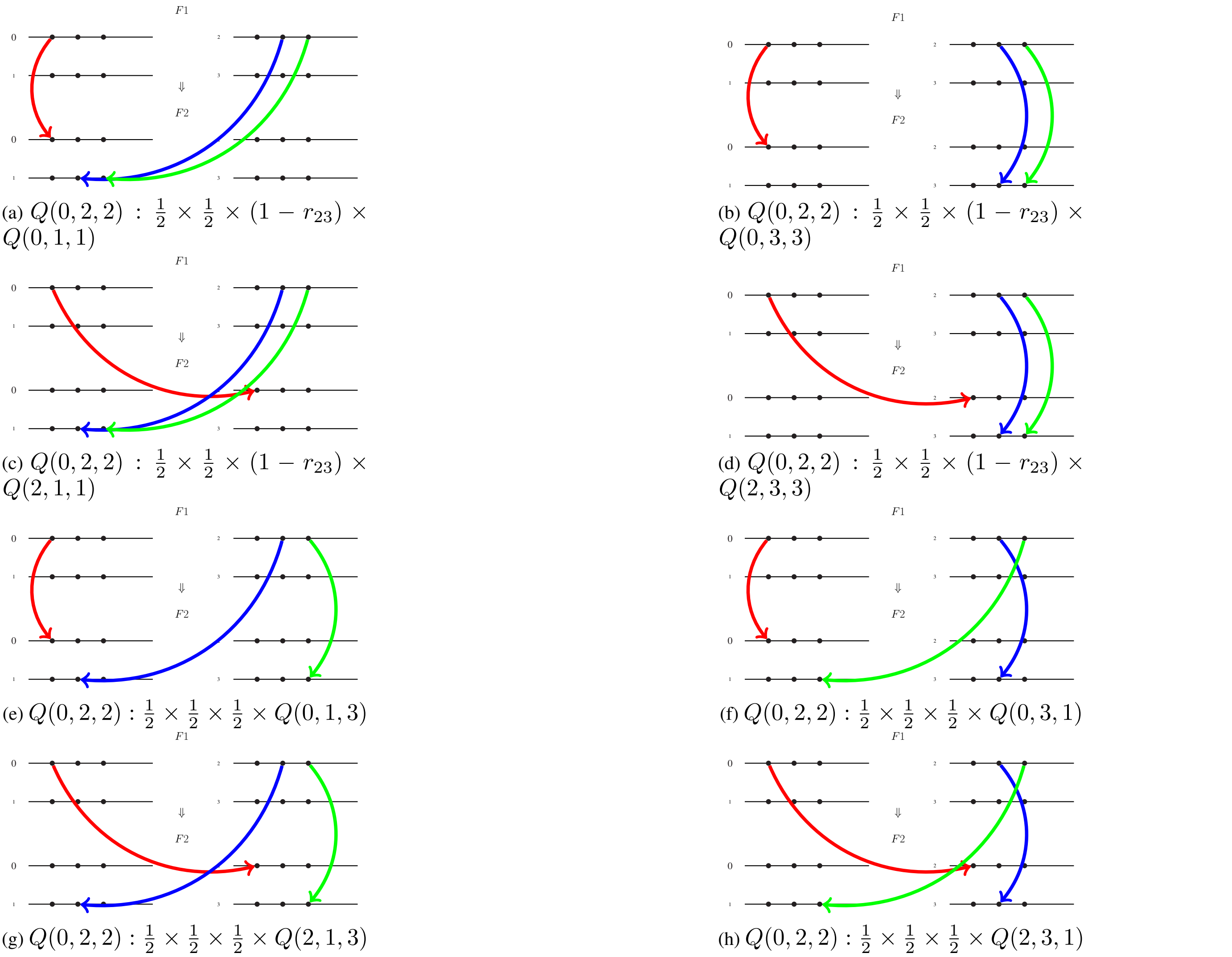
2.9 The self consistent equation for $Q(0, 2, 2)$

Figure S9 displays the 8 factors in the self-consistent equation for $Q(0, 2, 2)$:

$$Q(0,2,2) = \tfrac{1}{4}(1-r_{23})[Q(0,1,1)+Q(2,3,3)] + \tfrac{1}{8}[Q(0,1,3)+Q(2,3,1)] + \tfrac{1}{8}[Q(0,3,1)+Q(2,1,3)] + \tfrac{1}{4}(1-r_{23})[Q(0,3,3)+Q(2,1,1)] \quad \text{(S14)}$$

After use of symmetry to keep only non-equivalent Qs, this leads to

$$Q(0,2,2) = \tfrac{1}{2}(1-r_{23})Q(0,1,1) + \tfrac{1}{4}Q(0,1,2) + \tfrac{1}{4}Q(0,2,1) + \tfrac{1}{2}(1-r_{23})Q(0,2,2)$$



(a) $Q(0,2,2)\ :\ \frac{1}{2} \times \frac{1}{2} \times (1-r_{23}) \times Q(0,1,1)$

(b) $Q(0,2,2)\ :\ \frac{1}{2} \times \frac{1}{2} \times (1-r_{23}) \times Q(0,3,3)$

(c) $Q(0,2,2)\ :\ \frac{1}{2} \times \frac{1}{2} \times (1-r_{23}) \times Q(2,1,1)$

(d) $Q(0,2,2)\ :\ \frac{1}{2} \times \frac{1}{2} \times (1-r_{23}) \times Q(2,3,3)$

(e) $Q(0,2,2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0,1,3)$

(f) $Q(0,2,2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0,3,1)$

(g) $Q(0,2,2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2,1,3)$

(h) $Q(0,2,2) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2,3,1)$

Figure S9: The graphical representation of the factors multiplying each Q on the right-hand side of Eq. S14 for $Q(0, 2, 2)$.

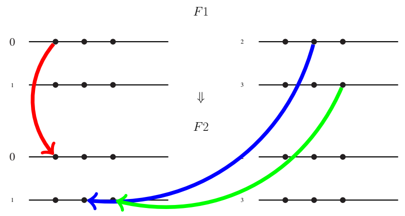
2.10 The self consistent equation for $Q(0,2,3)$

Figure S10 displays the 8 factors in the self-consistent equation for $Q(0,2,3)$:

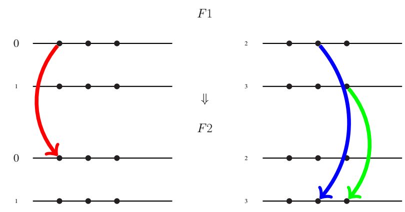
$$Q(0,2,3) = \tfrac{1}{4}r_{23}[Q(0,1,1)+Q(2,3,3)] + \tfrac{1}{8}[Q(0,1,3)+Q(2,3,1)] + \tfrac{1}{8}[Q(0,2,1)+Q(2,1,3)] + \tfrac{1}{4}r_{23}[Q(0,3,3)+Q(2,1,1)] \tag{S15}$$

After use of symmetry to keep only non-equivalent Qs, this leads to

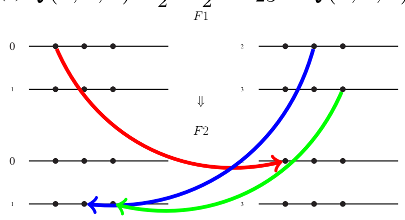
$$Q(0,2,3) = \tfrac{1}{2}r_{23}Q(0,1,1) + \tfrac{1}{4}Q(0,1,2) + \tfrac{1}{4}Q(0,2,1) + \tfrac{1}{2}r_{23}Q(0,2,2)$$



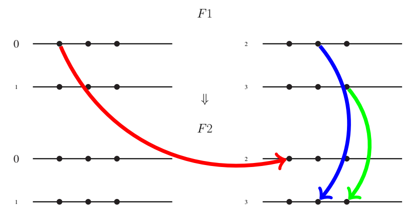
(a) $Q(0,2,3) : \frac{1}{2} \times \frac{1}{2} \times r_{23} \times Q(0,1,1)$



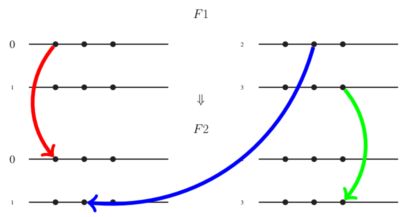
(b) $Q(0,2,3) : \frac{1}{2} \times \frac{1}{2} \times r_{23} \times Q(0,3,3)$



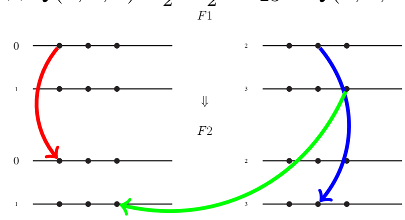
(c) $Q(0,2,3) : \frac{1}{2} \times \frac{1}{2} \times r_{23} \times Q(2,1,1)$



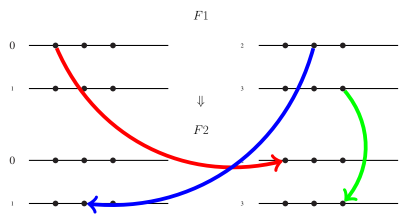
(d) $Q(0,2,3) : \frac{1}{2} \times \frac{1}{2} \times r_{23} \times Q(2,3,3)$



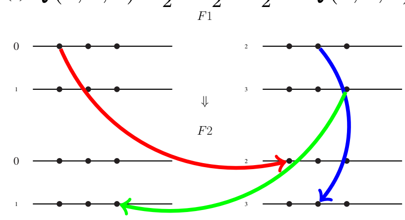
(e) $Q(0,2,3) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0,1,3)$



(f) $Q(0,2,3) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0,3,1)$



(g) $Q(0,2,3) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2,1,3)$



(h) $Q(0,2,3) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2,3,1)$

Figure S10: The graphical representation of the factors multiplying each Q on the right-hand side of Eq. S15 for $Q(0,2,3)$.