# Supplementary Material

## 1 SUPPLEMENTARY MATHEMATICS

### 1.1 The derivation of equation

$$|Q_L| = 2^{L-2}(2^{L-1}+1) \tag{S1}$$

**Proof:** For $L \geq 2$, there are $4^L$ IBD (identical by descent) probabilities $Q(i_1, i_2, \ldots i_L)$ since $i_l = 1, 2, 3$ or 4 and furthermore they add up to 1. A number of these probabilities are equal because of two symmetries: (1) the two homologous chromosomes in each individual play identical roles, and (2) the siblings play identical roles (assuming no sex-dependence of meiosis, so that for instance the recombination rates $r_{l,l'}$ are sex-independent. It is thus appropriate to use only one representative of each symmetry equivalence class, so that for instance one may impose this representative to have its first index, $i_1$, equal to zero. In fact one can identify exactly one element in each class by imposing that the indices of the representative $Q$'s have either

1. $i_l \in \{0,1\}\ \forall l \in \{2,..,L\}$, *or*
2. $i_l \in \{0,1\}\ \forall l \in \{2,..,K-1\}$, $i_K = 2$ and $i_l \in \{0,1,2,3\}\ \forall l \in \{K+1,..,L\}$

The number of equivalence classes and thus of $Q$'s to consider is then

$$|Q_L| = 2^{L-1} + \sum_{l=2}^{L} 2^{l-2}4^{L-l} = 2^{L-1} + 2^{2L-2}\sum_{l=2}^{L} 2^{-l} \tag{S2}$$

Given that $\sum_{l=2}^{L} 2^{-l}$ is a geometric progression of common ratio $2^{-1}$ from 2 to $L$, the sum of its terms can be expressed as:

$$\sum_{l=2}^{L} 2^{-l} = \frac{2^{-2} - 2^{-(L-1)}}{1 - 2^{-1}} = 2^{-1} - 2^{-L} \tag{S3}$$

Substituting S3 in S2, we get

$$|Q_L| = 2^{L-1} + 2^{2L-2}(2^{-1} - 2^{-L}) = 2^{L-1} + 2^{2L-3} - 2^{L-2} \tag{S4}$$

Factorizing with respect to $2^{L-2}$ and after simplification, this gives

$$|Q_L| = 2^{L-2}(1 + 2^{L-1}) \tag{S5}$$

### 1.2 The derivation of equation

$$|marginals| = 2^{L-2}(L+1) + 2^{2L-3} - 3^{L-1} \tag{S6}$$

**Proof:** By exploiting symmetries, the number of independent marginal equations is given by

$$|marginals| = \underbrace{2^{L-3} \cdot (2^{L-2}+1)}_{1} + \underbrace{(L-1) \cdot 2^{L-2}}_{2} + \underbrace{3 \cdot 2^{L-3} \cdot (2^{L-2}+1) - 3^{L-1}}_{3} \tag{S7}$$

This result can be explained as follows. The marginals are constructed by marginalizing over the last index $i_L$ first. Such an operation considers the $Q$'s for the first $L-1$ loci, and since they are independent variables, there are exactly $num_{Q_{L-1}}$ marginals, leading to the first term. The second term is corresponds to marginalizing on $i_l$ where $l \in \{2, .., L-1\}$ and in which the other $L-2$ indices can take the values zero and one. The third set of independent marginal equations is then given by marginalizing on $i_l$ as before, but for those in which an index has value 2 for a first time in a position prior to $l$, let's say $k$, so that the indices $i_h$ for $h \in \{k+1, .., l-1\}$ take values in $\{0, 1, 2, 3\}$ while the indices $i_{h'}$ for $h' \in \{l+1, .., L\}$ take values in $\{0, 1, 2\}$. At fixed $l$ and $k$, there are $2^{k-2} \cdot 4^{l-k-1} \cdot 3^{L-l}$ such possibilities. Summing over all the possible values of $k$ and $l$ leads to two geometric sums and then to the third term of (S7). Finally, we can easily check that

$$|marginals| = 2^{L-2} \cdot (L+1) + 2^{2L-3} - 3^{L-1} < |Q_L|$$

## 2 THE INHOMOGENEOUS EQUATION

$$4Q(0,0,0) + 4Q(0,0,1) + 8Q(0,0,2) + 4Q(0,1,0) + 4Q(0,1,1) + \\ 8Q(1,1,2) + 8Q(0,2,0) + 8Q(0,2,1) + 8Q(0,2,2) + 8Q(0,2,3) = 1 \tag{S8}$$

From equation S8 and Figures S9, S18, S27, S36, S45, S54, S54, S63, S72, S81, and S90 we get this system

$$\begin{bmatrix} 4 & 4 & 8 & 4 & 4 & 8 & 8 & 8 & 8 & 8 \\ \bar{r}_{12}\bar{r}_{23} - 1 & 0 & \frac{\bar{r}_{12}}{2} & 0 & 0 & 0 & \frac{\bar{r}_{13}}{2} & 0 & \frac{\bar{r}_{23}}{2} & 0 \\ r_{23}\bar{r}_{12} & -1 & \frac{\bar{r}_{12}}{2} & 0 & 0 & 0 & \frac{r_{13}}{2} & 0 & \frac{r_{23}}{2} & 0 \\ 0 & \frac{\bar{r}_{12}}{2} & \frac{\bar{r}_{12}-2}{2} & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ r_{12}r_{23} & 0 & \frac{r_{12}}{2} & -1 & 0 & 0 & \frac{\bar{r}_{13}}{2} & 0 & \frac{r_{23}}{2} & 0 \\ r_{12}\bar{r}_{23} & 0 & \frac{r_{12}}{2} & 0 & -1 & 0 & \frac{r_{13}}{2} & 0 & \frac{\bar{r}_{23}}{2} & 0 \\ 0 & \frac{r_{12}}{2} & \frac{r_{12}}{2} & 0 & 0 & -1 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{\bar{r}_{13}}{2} & 0 & \frac{1}{4} & \frac{\bar{r}_{13}-2}{2} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{r_{13}}{2} & 0 & \frac{1}{4} & \frac{r_{13}}{2} & -1 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{\bar{r}_{23}}{2} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{\bar{r}_{23}-2}{2} & 0 \end{bmatrix} \begin{bmatrix} Q(0,0,0) \\ Q(0,0,1) \\ Q(0,0,2) \\ Q(0,1,0) \\ Q(0,1,1) \\ Q(0,1,2) \\ Q(0,2,0) \\ Q(0,2,1) \\ Q(0,2,2) \\ Q(0,2,3) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{S9}$$

where $\bar{r} = 1 - r$; the complement value of the recombination rate.

## 3 THE SCHP EQUATIONS

### 3.1

See Figure S9

$$((1-r_{12})(1-r_{23})-1)Q(0,0,0)+\frac{1}{2}(1-r_{12})Q(0,0,2)+\frac{1}{2}(1-r_{13})Q(0,2,0)+\frac{1}{2}(1-r_{23})Q(0,2,2)=0 \tag{S10}$$

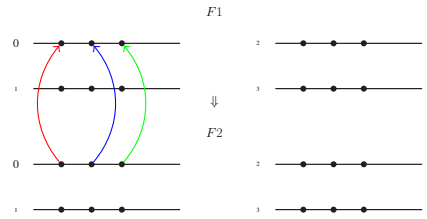


**Figure S1.** $Q(0,0,0) : \frac{1}{2} \times (1-r_{12}) \times (1-r_{23}) \times Q(0,0,0)$

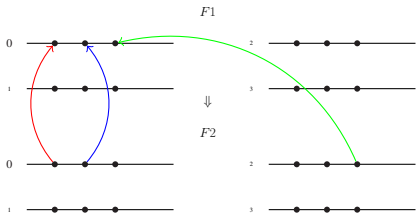


**Figure S2.** $Q(0,0,0) : \frac{1}{2} \times (1-r_{12}) \times \frac{1}{2} \times Q(0,0,2)$

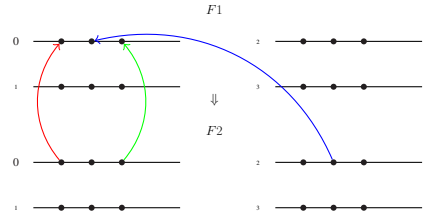


**Figure S3.** $Q(0,0,0) : \frac{1}{2} \times \frac{1}{2} \times (1-r_{13}) \times Q(0,2,0)$

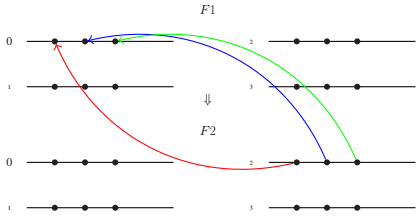


**Figure S4.** $Q(0,0,0) : \frac{1}{2} \times \frac{1}{2} \times (1-r_{23}) \times Q(0,2,2)$

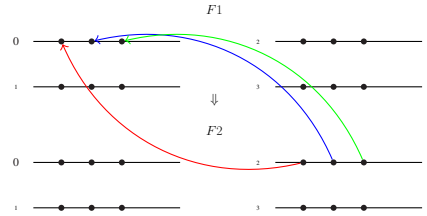


**Figure S5.** $Q(0,0,0) : \frac{1}{2} \times (1-r_{12})(1-r_{23}) \times Q(2,2,2)$

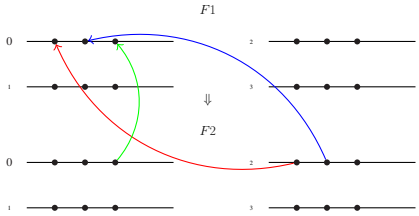


**Figure S6.** $Q(0,0,0) : \frac{1}{2} \times \frac{1}{2} \times (1-r_{12}) \times Q(2,2,0)$

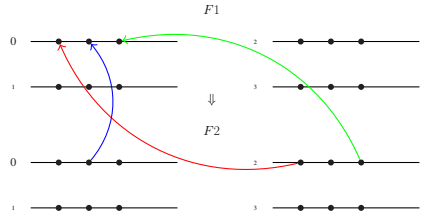


**Figure S7.** $Q(0,0,0) : \frac{1}{2} \times \frac{1}{2} \times (1-r_{13}) \times Q(2,0,2)$

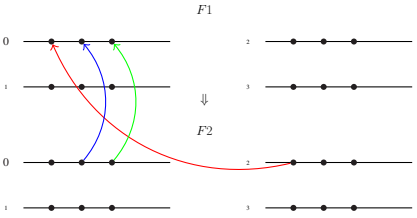


**Figure S8.** $Q(0,0,0) : \frac{1}{2} \times \frac{1}{2} \times (1-r_{23}) \times Q(2,0,0)$

**Figure S9.** $Q(0,0,0)$

### 3.2

See Figure S18

$$(1-r_{12})r_{23}Q(0,0,0) - Q(0,0,1) + \frac{1}{2}(1-r_{12})Q(0,0,2) + \frac{1}{2}r_{13}Q(0,2,0) + \frac{1}{2}r_{23}Q(0,2,2) = 0 \tag{S11}$$

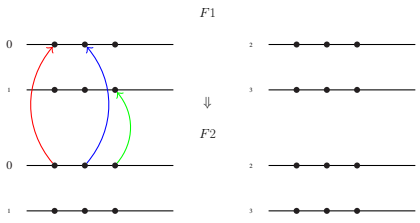


**Figure S10.** $Q(0,0,1) : \frac{1}{2} \times (1-r_{12}) \times r_{23} \times Q(0,0,0)$

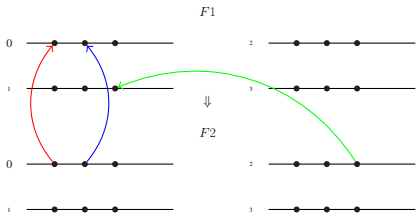


**Figure S12.** $Q(0,0,1) : \frac{1}{2} \times \frac{1}{2} \times (1-r_{12}) \times Q(0,0,2)$

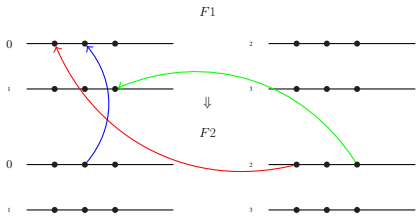


**Figure S14.** $Q(0,0,1) : \frac{1}{2} \times \frac{1}{2} \times r_{13} \times Q(2,0,2)$

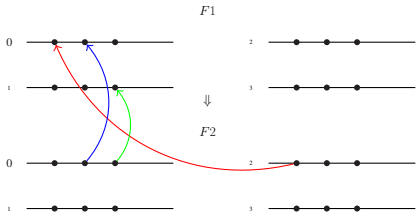


**Figure S16.** $Q(0,0,1) : \frac{1}{2} \times \frac{1}{2} \times r_{23} \times Q(2,0,0)$

**Figure S18.** $Q(0,0,1)$

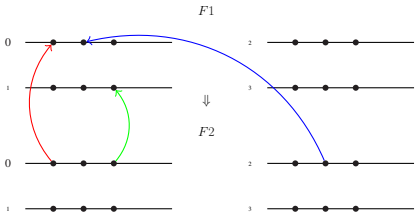


**Figure S11.** $Q(0,0,1) : \frac{1}{2} \times \frac{1}{2} \times r_{13} \times Q(0,2,0)$

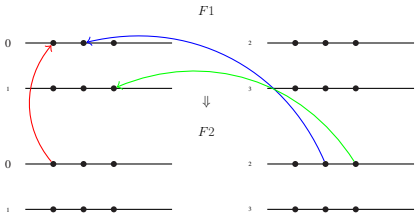


**Figure S13.** $Q(0,0,1) : \frac{1}{2} \times \frac{1}{2} \times r_{23} \times Q(0,2,2)$

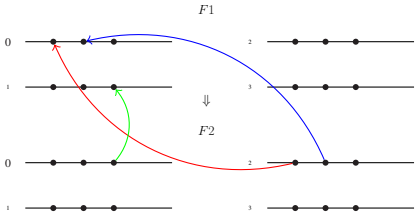


**Figure S15.** $Q(0,0,1) : \frac{1}{2} \times \frac{1}{2} \times (1-r_{12}) \times Q(2,2,0)$

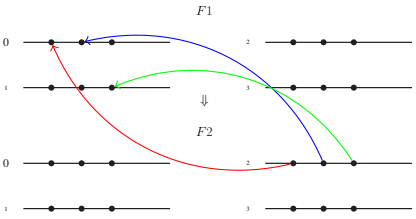


**Figure S17.** $Q(0,0,1) : \frac{1}{2} \times (1-r_{12}) \times r_{23} \times Q(2,2,2)$

### 3.3

See Figure S27.

$$\frac{1}{2}(1-r_{12})Q(0,0,1)+(\frac{1}{2}(1-r_{12})-1)Q(0,0,2)+\frac{1}{4}Q(0,2,1)+\frac{1}{4}Q(0,2,3)=0 \tag{S12}$$

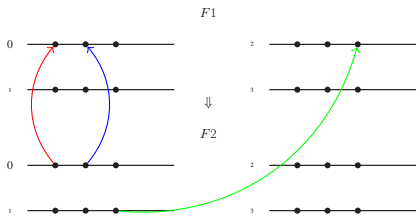


**Figure S19.** $Q(0,0,2):\frac{1}{2}\times(1-r_{12})\times\frac{1}{2}\times Q(0,0,1)$

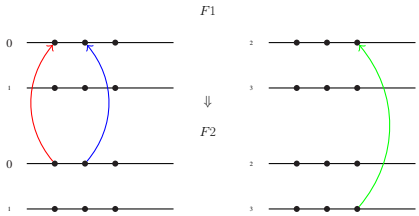


**Figure S20.** $Q(0,0,2):\frac{1}{2}\times(1-r_{12})\times\frac{1}{2}\times Q(0,0,3)$

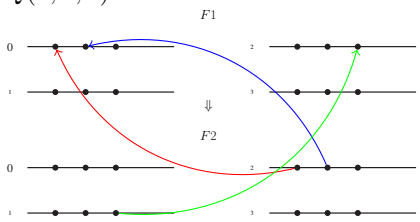


**Figure S21.** $Q(0,0,2):\frac{1}{2}\times(1-r_{12})\times\frac{1}{2}\times Q(2,2,1)$

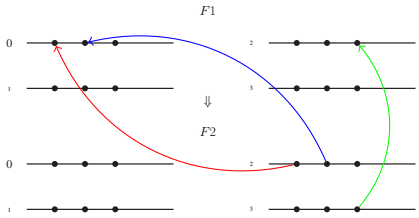


**Figure S22.** $Q(0,0,2):\frac{1}{2}\times(1-r_{12})\times\frac{1}{2}\times Q(2,2,3)$

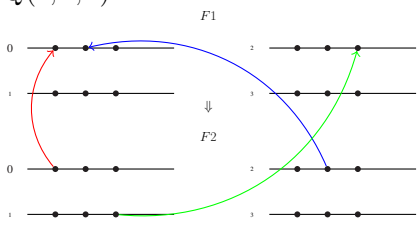


**Figure S23.** $Q(0,0,2):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(0,2,1)$

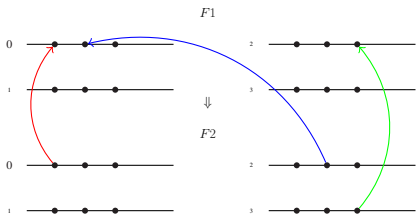


**Figure S24.** $Q(0,0,2):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(0,2,3)$

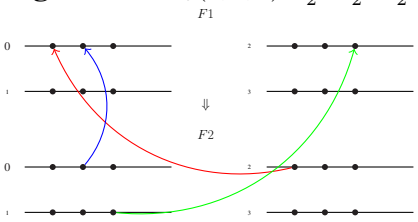


**Figure S25.** $Q(0,0,2):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(2,0,1)$

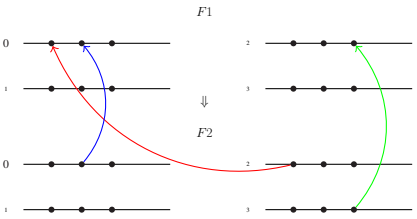


**Figure S26.** $Q(0,0,2):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(2,0,3)$

**Figure S27.** $Q(0,0,2)$

### 3.4

See Figure S36.

$$r_{12}r_{23}Q(0,0,0) + \frac{1}{2}r_{12}Q(0,0,2) - Q(0,1,0) + \frac{1}{2}(1-r_{13})Q(0,2,0) + \frac{1}{2}r_{23}Q(0,2,2) \tag{S13}$$

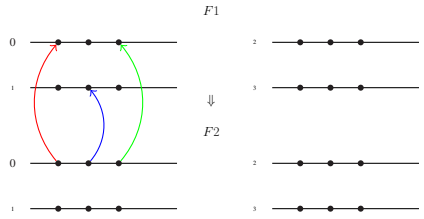


**Figure S28.** $Q(0,1,0) : \frac{1}{2} \times r_{12} \times (1-r_{23}) \times Q(0,0,0)$

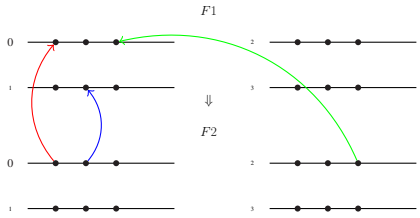


**Figure S29.** $Q(0,1,0) : \frac{1}{2} \times r_{12} \times \frac{1}{2} \times Q(0,0,2)$

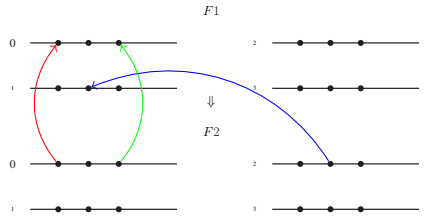


**Figure S30.** $Q(0,1,0) : \frac{1}{2} \times \frac{1}{2} \times r_{13} \times Q(0,2,0)$

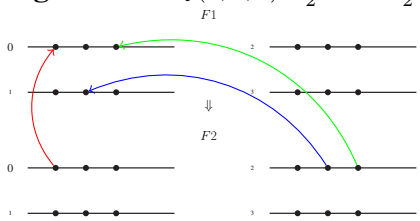


**Figure S31.** $Q(0,1,0) : \frac{1}{2} \times \frac{1}{2} \times r_{23} \times Q(0,2,2)$

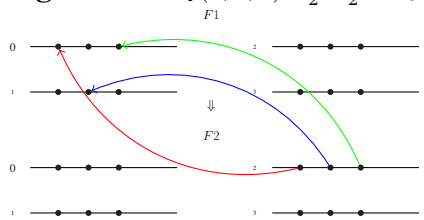


**Figure S32.** $Q(0,1,0) : \frac{1}{2} \times r_{12}r_{23} \times Q(2,2,2)$

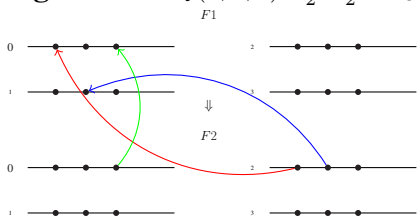


**Figure S33.** $Q(0,1,0) : \frac{1}{2} \times \frac{1}{2} \times r_{12} \times Q(2,2,0)$

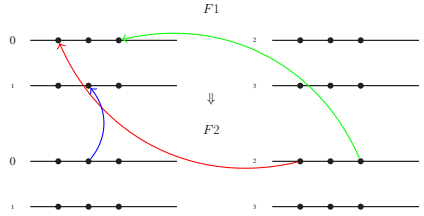


**Figure S34.** $Q(0,1,0) : \frac{1}{2} \times \frac{1}{2} \times (1-r_{13}) \times Q(2,0,2)$

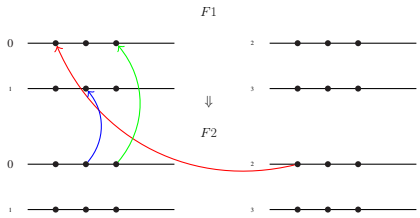


**Figure S35.** $Q(0,1,0) : \frac{1}{2} \times r_{23} \times \frac{1}{2} \times Q(2,0,0)$

**Figure S36.** $Q(0,1,0)$

### 3.5

See Figure S45

$$r_{12}(1-r_{23})Q(0,0,0)+\frac{1}{2}r_{12}Q(0,0,2)-Q(0,1,1)+\frac{1}{2}r_{13}Q(0,2,0)+\frac{1}{2}(1-r_{23})Q(0,2,2) \quad \text{(S14)}$$

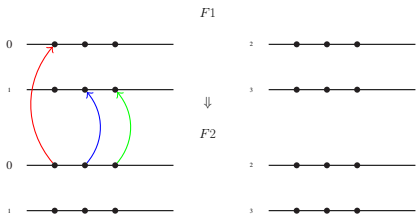


**Figure S37.** $Q(0,1,1):\frac{1}{2}\times r_{12}\times(1-r_{23})\times Q(0,0,0)$

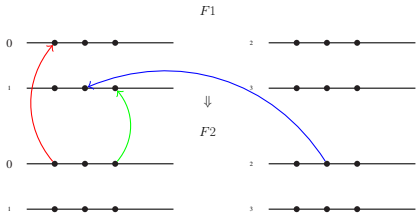


**Figure S39.** $Q(0,1,1):\frac{1}{2}\times\frac{1}{2}\times r_{13}\times Q(0,2,0)$

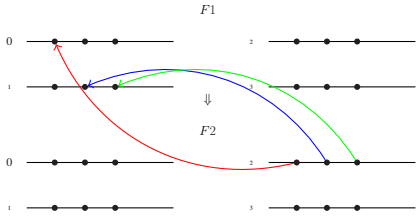


**Figure S41.** $Q(0,1,1):\frac{1}{2}\times r_{12}\times(1-r_{23})\times Q(2,2,2)$

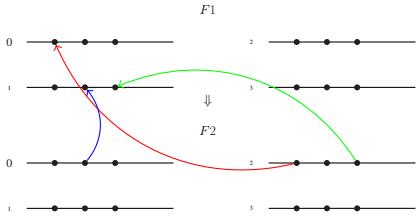


**Figure S43.** $Q(0,1,1):\frac{1}{2}\times r_{13}\times\frac{1}{2}\times Q(2,0,2)$

**Figure S45.** $Q(0,1,1)$

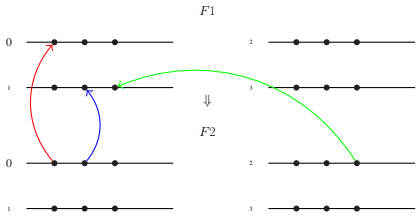


**Figure S38.** $Q(0,1,1):\frac{1}{2}\times\frac{1}{2}\times r_{12}\times Q(0,0,2)$

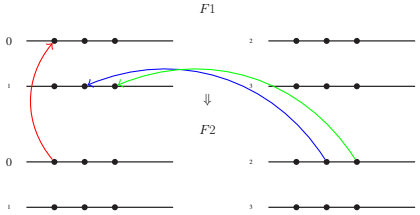


**Figure S40.** $Q(0,1,1):\frac{1}{2}\times\frac{1}{2}\times(1-r_{23})\times Q(0,2,2)$

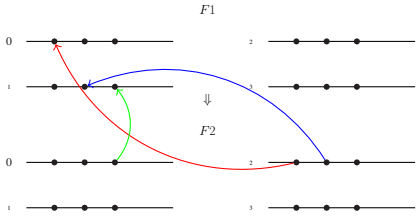


**Figure S42.** $Q(0,1,1):\frac{1}{2}\times r_{12}\times\frac{1}{2}\times Q(2,2,0)$

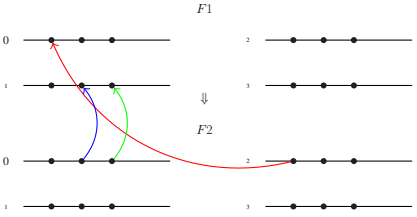


**Figure S44.** $Q(0,1,1):\frac{1}{2}\times\frac{1}{2}\times(1-r_{23})\times Q(2,0,0)$

### 3.6

See Figure S54.

$$\frac{1}{2}r_{12}Q(0,0,1)+\frac{1}{2}r_{12}Q(0,0,2)-Q(0,1,2)+\frac{1}{4}Q(0,2,1)+\frac{1}{4}Q(0,2,3) \tag{S15}$$

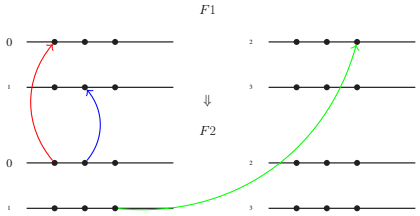


**Figure S46.** $Q(0,1,2):\frac{1}{2}\times\frac{1}{2}\times r_{12}\times Q(0,0,1)$

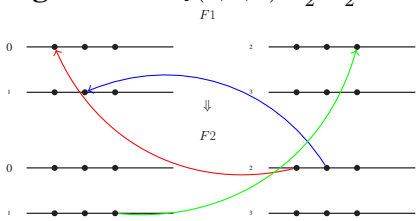


**Figure S48.** $Q(0,1,2):\frac{1}{2}\times\frac{1}{2}\times\times r_{12}\times Q(2,2,1)$

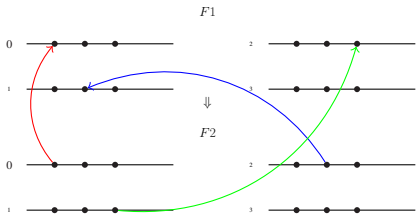


**Figure S50.** $Q(0,1,2):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(0,2,1)$

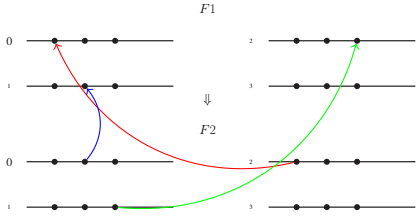


**Figure S52.** $Q(0,1,2):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(2,0,1)$

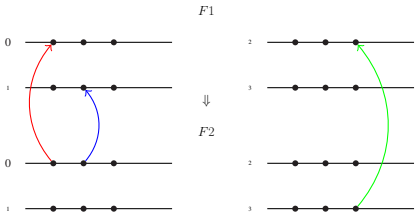


**Figure S47.** $Q(0,1,2):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times r_{12}\times Q(0,0,3)$

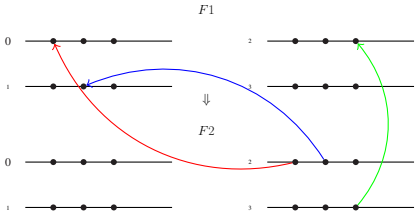


**Figure S49.** $Q(0,1,2):\frac{1}{2}\times\frac{1}{2}\times r_{12}\times Q(2,2,3)$

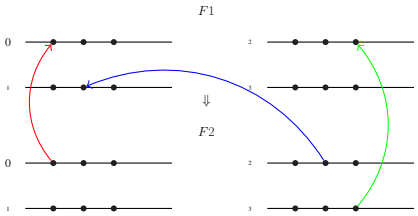


**Figure S51.** $Q(0,1,2):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(0,2,3)$

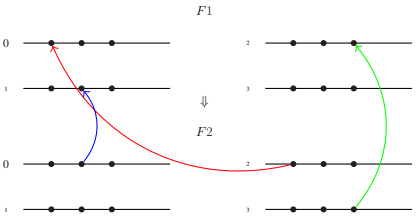


**Figure S53.** $Q(0,1,2):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(2,0,3)$

**Figure S54.** $Q(0,1,2)$

### 3.7

See Figure S63.

$$\frac{1}{2}(1-r_{13})Q(0,1,0)+\frac{1}{4}Q(0,1,2)+(\frac{1}{2}(1-r_{13})-1)Q(0,2,0)+\frac{1}{4}Q(0,1,2) \tag{S16}$$

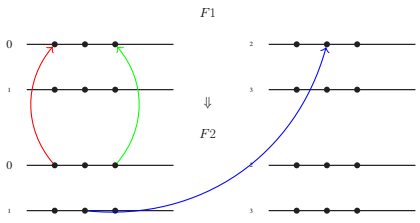


**Figure S55.** $Q(0,2,0):\frac{1}{2}\times(1-r_{13})\times\frac{1}{2}\times Q(0,1,0)$

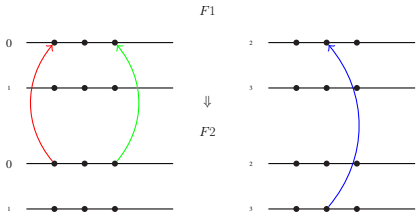


**Figure S56.** $Q(0,2,0):\frac{1}{2}\times(1-r_{13})\times\frac{1}{2}\times Q(0,3,0)$

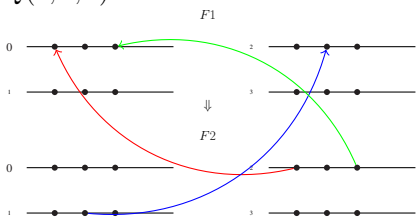


**Figure S57.** $Q(0,2,0):\frac{1}{2}\times(1-r_{13})\times\frac{1}{2}\times Q(2,1,2)$

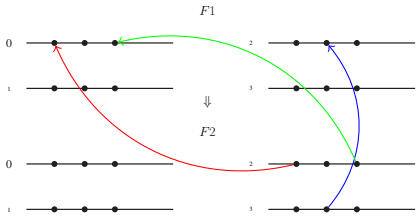


**Figure S58.** $Q(0,2,0):\frac{1}{2}\times(1-r_{13})\times\frac{1}{2}\times Q(2,3,2)$

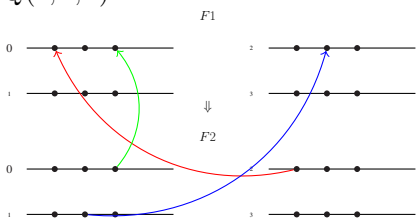


**Figure S59.** $Q(0,2,0):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(2,1,0)$

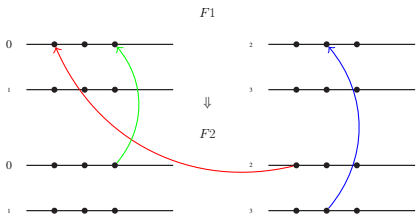


**Figure S60.** $Q(0,2,0):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(2,3,0)$

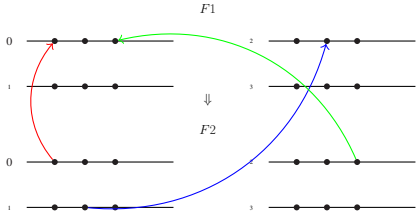


**Figure S61.** $Q(0,2,0):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(0,1,2)$

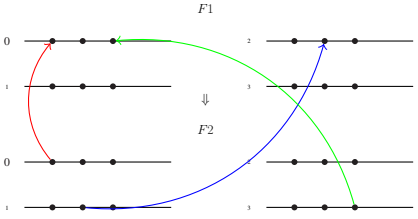


**Figure S62.** $Q(0,2,0):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(0,1,3)$

**Figure S63.** $Q(0,2,0)$

### 3.8

See Figure S72.

$$\frac{1}{2}r_{13}Q(0,1,0) + \frac{1}{4}Q(0,1,2) + \frac{1}{2}r_{13}Q(0,2,0) - Q(0,2,1) + \frac{1}{4}Q(0,2,3) \tag{S17}$$

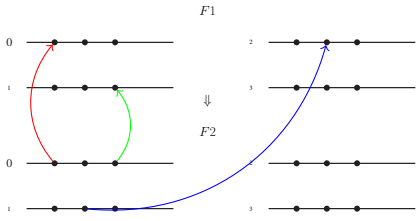


**Figure S64.** $Q(0,2,1) : \frac{1}{2} \times r_{13} \times \frac{1}{2} \times Q(0,1,0)$

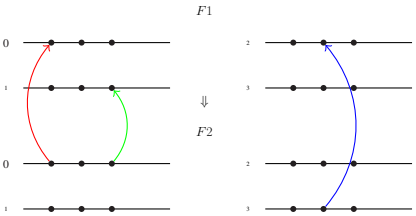


**Figure S65.** $Q(0,2,1) : \frac{1}{2} \times r_{13} \times \frac{1}{2} \times Q(0,3,0)$

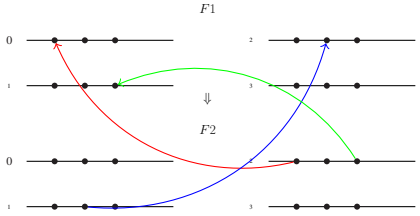


**Figure S66.** $Q(0,2,1) : \frac{1}{2} \times r_{13} \times \frac{1}{2} \times Q(2,1,2)$

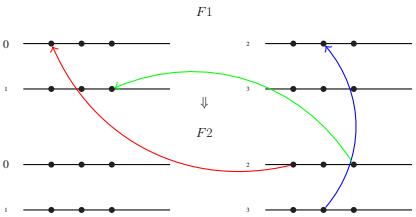


**Figure S67.** $Q(0,2,1) : \frac{1}{2} \times r_{13} \times \frac{1}{2} \times Q(2,3,2)$

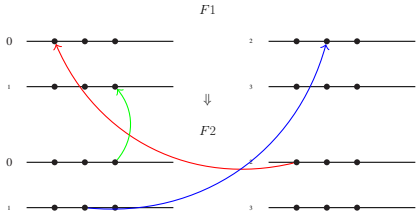


**Figure S68.** $Q(0,2,1) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2,1,0)$

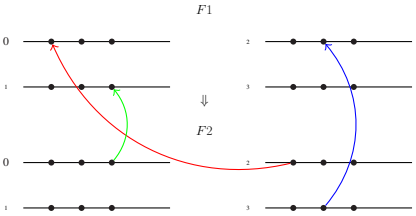


**Figure S69.** $Q(0,2,1) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(2,3,0)$

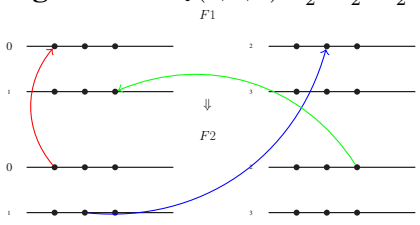


**Figure S70.** $Q(0,2,1) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0,1,2)$

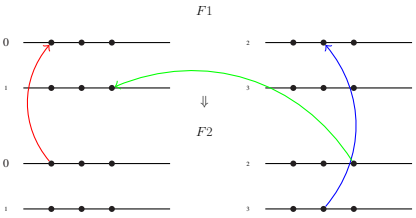


**Figure S71.** $Q(0,2,1) : \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times Q(0,3,2)$

**Figure S72.** $Q(0,2,1)$

### 3.9

See Figure S81.

$$\frac{1}{2}(1-r_{23})Q(0,1,1)+\frac{1}{4}Q(0,1,2)+\frac{1}{4}Q(0,2,1)+(\frac{1}{2}(1-r_{23})-1)Q(0,2,2) \tag{S18}$$

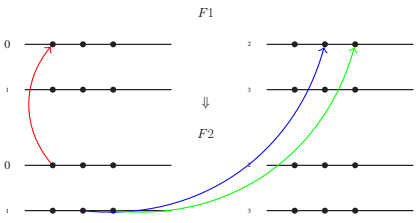


**Figure S73.** $Q(0,2,2):\frac{1}{2}\times\frac{1}{2}\times(1-r_{23})\times Q(0,1,1)$

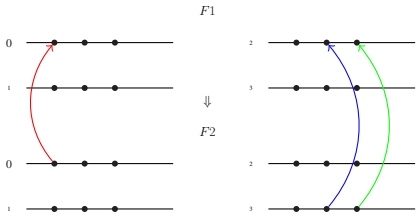


**Figure S74.** $Q(0,2,2):\frac{1}{2}\times\frac{1}{2}\times(1-r_{23})\times Q(0,3,3)$

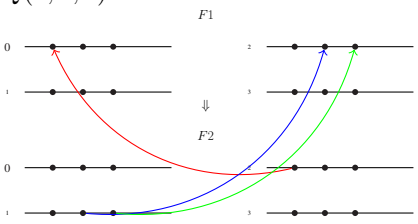


**Figure S75.** $Q(0,2,2):\frac{1}{2}\times\frac{1}{2}\times(1-r_{23})\times Q(2,1,1)$

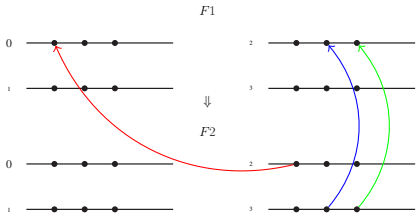


**Figure S76.** $Q(0,2,2):\frac{1}{2}\times\frac{1}{2}\times(1-r_{23})\times Q(2,3,3)$

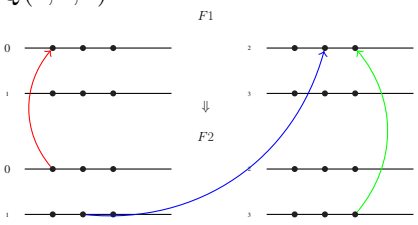


**Figure S77.** $Q(0,2,2):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(0,1,3)$

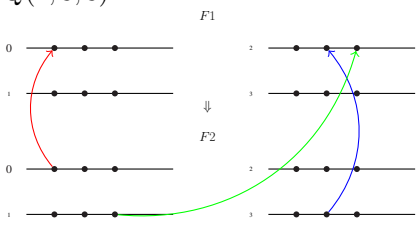


**Figure S78.** $Q(0,2,2):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(031)$

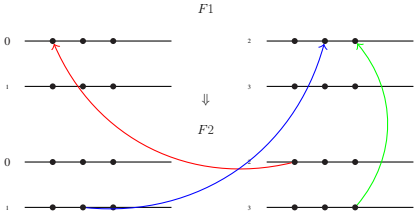


**Figure S79.** $Q(0,2,2):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(2,1,3)$

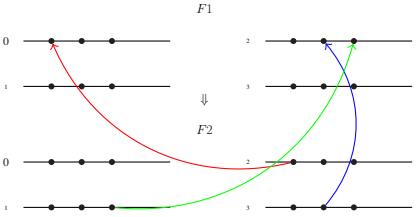


**Figure S80.** $Q(0,2,2):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(2,3,1)$

**Figure S81.** $Q(0,2,2)$

### 3.10

See Figure S90.

$$\frac{1}{2}r_{23}Q(0,1,1)+\frac{1}{4}Q(0,1,2)+\frac{1}{4}Q(0,2,1)+\frac{1}{2}r_{23}Q(0,2,2)-Q(0,2,3)$$

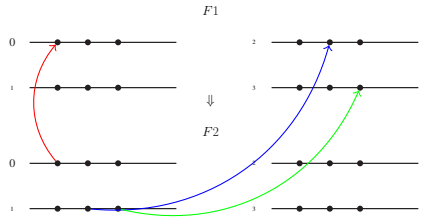


**Figure S82.** $Q(0,2,3):\frac{1}{2}\times\frac{1}{2}\times r_{23}\times Q(0,1,1)$

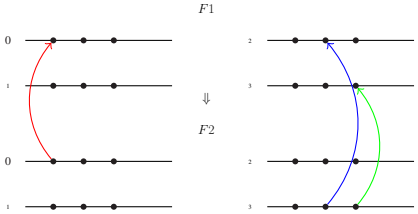


**Figure S83.** $Q(0,2,3):\frac{1}{2}\times\frac{1}{2}\times r_{23}\times Q(0,3,3)$

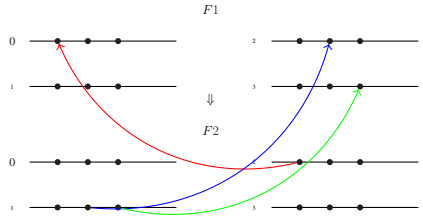


**Figure S84.** $Q(0,2,3):\frac{1}{2}\times\frac{1}{2}\times r_{23}\times Q(2,1,1)$

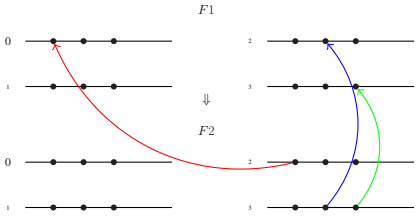


**Figure S85.** $Q(0,2,3):\frac{1}{2}\times\frac{1}{2}\times r_{23}\times Q(2,3,3)$

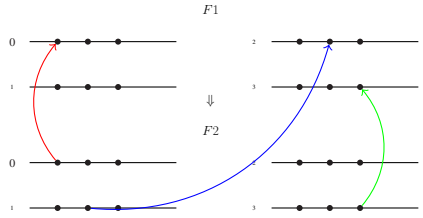


**Figure S86.** $Q(0,2,3):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(0,1,3)$

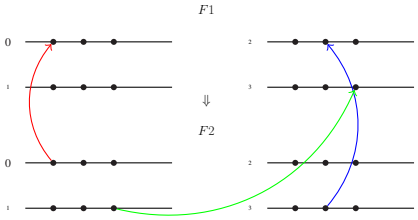


**Figure S87.** $Q(0,2,3):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(031)$

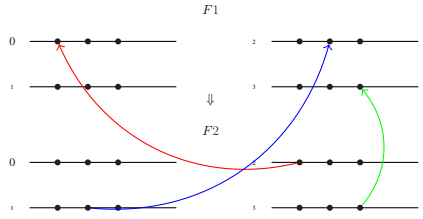


**Figure S88.** $Q(0,2,3):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(2,1,3)$

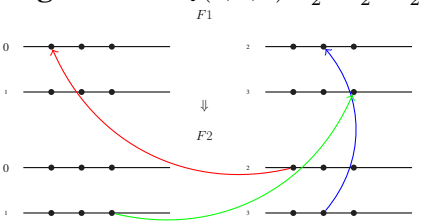


**Figure S89.** $Q(0,2,3):\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times Q(2,3,1)$

**Figure S90.** $Q(0,2,3)$