AMS 20

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I.

- $\frac{\mathrm{dx}}{\mathrm{dt}} = \mathrm{ax} + b \quad a \,\mathrm{and}\, b \,\mathrm{are\,constants}$

  $x(t) = \mathrm{ce}^{\mathrm{at}} - \frac{b}{a}$

- $\frac{\mathrm{dx}}{\mathrm{dt}} = a(t)x$

  $x(t) = \mathrm{ce}^{\int a(t)}$

- $\frac{\mathrm{dx}}{\mathrm{dt}} = a(t)x + b(t)$

  $-a(t)x + \frac{\mathrm{dx}}{\mathrm{dt}} = b(t)$

  $\mu(t)a(t)x + \mu(t)\frac{\mathrm{dx}}{\mathrm{dt}} = \frac{d(\mu, x)}{\mathrm{dt}}$

  $\frac{d\mu}{\mathrm{dt}} = -a(t)\mu$

  $\frac{d(\mu(t), x(t))}{\mathrm{dt}} = \mu(t)b(t)$

- $\frac{\mathrm{dx}}{\mathrm{dt}} = \frac{-2}{t-1}x + 4t \quad x(0) = 1$

  $\frac{2}{t-1}x + \frac{\mathrm{dx}}{\mathrm{dt}} = 4t$

  $\mu(t)\frac{2}{t-1}x + \mu(t)\frac{\mathrm{dx}}{\mathrm{dt}} = \mu(t)4t$

  $\frac{d(\mu x)}{\mathrm{dt}} = \frac{d\mu}{\mathrm{dt}}x + \mu\frac{\mathrm{dx}}{\mathrm{dt}}$

  $\frac{d\mu}{\mathrm{dt}} = \frac{2}{t-1}\mu(t) \Rightarrow \frac{1}{\mu}\frac{d\mu}{\mathrm{dt}} = \frac{2}{t-1}$

  $\int \frac{1}{\mu}d\mu = \int \frac{2}{t-1}\mathrm{dt} = 2\ln|t-1| + c$

  $\ln|\mu| = 2\mathrm{lm}|t-1| + c$

  $|\mu| = e^{2\mathrm{lm}|t-1+c|} \Rightarrow \mu(t) = (t-1)^2$

  $\frac{d(\mu(t)x)}{\mathrm{dt}} = \mu(t)4t$

  $(t-1)^2 x = \int (t-1)^2 4t\mathrm{dt} + c$

  $x(t) = \frac{t^4 - \frac{8}{3}t^3 + 2t^2 + c}{(t-1)^2} \Rightarrow x(0) = c \Rightarrow c = 1$

II. Interval of definition

- the maxium inteval of independnt variable t, that the solution to IVP is defined

- Consider IVP for $\frac{\mathrm{dx}}{\mathrm{dt}} = a(t)x + b(t)$, where $x(t_0) = x_a$. Where (t_0, x_0) is a given intial condition. if $a(t)$ and $b(t)$ are continous on an open interval $d < t < p$ and $t_0 \epsilon (d, p)$

- EXAMPLE:

  $\frac{\mathrm{dx}}{\mathrm{dt}} = \frac{1}{\cos(t)} x, x(0 = x_0)$

  i. $a(t) = \frac{1}{\cos(t)} \Rightarrow \text{where} \cos(t) = 0 \Rightarrow \frac{\pi}{2}, \frac{3\pi}{2}$

  ii. Interval would be $(\frac{-\pi}{2}, \frac{\pi}{2})$

III. Non-linear 1st Order ODEs

- $\frac{\mathrm{dx}}{\mathrm{dt}} = f(x, t)$

- Seperable equations- Any equations that can be written as $N(x)\frac{\mathrm{dx}}{\mathrm{dt}} = M(t)$

  $\int N(x)\frac{\mathrm{dx}}{\mathrm{dt}}\mathrm{dt} = \int M(t)\mathrm{dt} \Rightarrow \int N(x)\mathrm{dx} = \int M(t)\mathrm{dt}$

  $H_1(x) = H_2(t) + c$

- EXAMPLE:$\frac{\mathrm{dx}}{\mathrm{dt}} = \frac{2t}{1+x} \quad \text{where } x(0) = 0$

  i. $\frac{\mathrm{dx}}{\mathrm{dt}}(1+x) = 2t \Rightarrow \int (1+x)\mathrm{dx} = \int 2t\mathrm{dt}$

  ii. $x + \frac{x^2}{2} = t^2 + c \Rightarrow \text{where } x(0) = 0 \Rightarrow 0 + \frac{0^2}{0} = 0^2 + c \qquad c = 0$

  iii. $\frac{x^2}{2} + x = t^2 \Rightarrow \frac{x^2}{2} + x - t^2 = 0$

  iv. $x = \frac{-1 \pm \sqrt{1 - 4\frac{1}{2}(-t^2)}}{2\frac{1}{2}} = \frac{1 \pm \sqrt{1 + 2t^2}}{1}$

- EXAMPLE: $\frac{\mathrm{dx}}{\mathrm{dt}} = \frac{\cos(t)}{2x}$ where x(0)=$\frac{1/}{\sqrt{2}}$

  i. $\int 2\mathrm{x}\mathrm{dx} = \int \cos(t)\mathrm{dt} \Rightarrow x^2 = \sin(t) + c$

  ii. $\left(\frac{1}{\sqrt{2}}\right)^2 = \sin(0) + c \Rightarrow c = \frac{1}{2}$

  iii. $x(t) = \pm\sqrt{\sin(t) + \frac{1}{2}} \qquad t \text{ must be } \frac{-\pi}{6} < t < \frac{7\pi}{6}$