Physics 5B: Waves II

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Wave Motion **-** the wave transports energy and momentum from left to right, but the individual parts of the rope are just moving up and down.

- Average Energy per unit time (power) passing through some area perpendicular to propagation is equal to

  Mechanical Wave Equation:

  $$P = 2\pi^2 \mathrm{pv}_{\mathrm{wave}} f^2 A^2 S$$

- Intensity Equation:

  $$\frac{P}{S} = 2\pi^2\, p\, v\, f^2\, A^2$$

  Remeber: ω=2πf

  $$I = \frac{1}{2} v\, p\, \omega^2\, A^2$$

- the Power transported by a wave is proportional to the square of the amplitude and the square of the fregeuency

- For a wave moving through a wolume, intensity is power per units area

- $I = \frac{P}{S} = \frac{P}{4\pi r^2}$ For sound source r distance away in an open area (space)

- Wave Equation for a String:

  The magnitude of the tension, F_T, is the same everywhere in the string!

  But, due to the curvature of the string, the Vertial componenet of F_T is less at x+Δx than it is at x.

  This provides a net resotring force to pull this point on the string back to equlibrium.

  μ = Mass per unit Length

  Mass = μΔx

  Vertial acceleration: $\frac{d^2 y}{\mathrm{dt}^2}$

  $F_T \sin(\theta_2) - F_T \sin(\theta_1) = \mu \Delta x \frac{d^2 y}{\mathrm{dt}^2}$

  For, ϑ_1 and θ_2 are small, sin(ϑ)-tan(ϑ) = slope!

  $F_T[(\frac{\mathrm{dy}}{\mathrm{dx}} - \frac{\mathrm{dy}}{\mathrm{dx}}) = \mu \frac{d^2 y}{\mathrm{dt}^2}$

  $$\frac{d^2 y}{\mathrm{dx}^2} = \frac{\mu}{F_T} \frac{d^2 y}{\mathrm{dt}^2} \Rightarrow \frac{d^2 y}{\mathrm{dx}^2} = \frac{1}{v^2} \frac{d^2 y}{\mathrm{dt}^2}$$

  Take any function F, $y = F(x - \mathrm{vt})$

$\frac{\mathrm{dy}}{\mathrm{dt}} = F'x(-v)$

$\frac{d^2y}{\mathrm{dt}^2} = F''x(-v)(-v) \Rightarrow v^2F''$

$\frac{\mathrm{dy}}{\mathrm{dx}} = F'$

$\frac{d^2y}{\mathrm{dt}^2} = F''$

- Wave Equation (One-Dimension)

  $v = \sqrt{\dfrac{F_T}{\mu}} \qquad \frac{d^2D}{\mathrm{dx}^2} = \dfrac{1}{v^2}\dfrac{d^2y}{\mathrm{dt}^2}$

- Wave equation is a linear P.D.E

- Then, $\quad y(x,t) = \mathrm{ay}_1(x,t) + \mathrm{by}_2(x,t)$