AMS 20

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Date April 5, 2017

1. First Order linear time invariant ODE

   - $a_n(t)\frac{d^n x}{\mathrm{dt}^n} + a_{n-1}(t)\frac{d^{n-1}x}{\mathrm{dt}^{n-1}}....... + a_1(t)\frac{\mathrm{dx}}{\mathrm{dt}} + x(t) = y(t)$

     $a_1, a_0....y\,\text{are constants}$

     $a_1\frac{\mathrm{dx}}{\mathrm{dt}} + a_0 x(t) = y \Rightarrow \frac{\mathrm{dx}}{\mathrm{dt}} = -\frac{-a_0}{a_1}x(t) + \frac{y}{a_1}$

   - $\frac{\mathrm{dx}}{\mathrm{dt}} = \mathrm{ax} + b$

     $\frac{\mathrm{dx}}{\mathrm{dt}} = x \quad \text{therefore}\, x(t) = e^t\,\text{since derivative of}\, e^t\,\text{is}\, e^t$

   - $\frac{\mathrm{dx}}{\mathrm{dt}} = \mathrm{ax} \Rightarrow \frac{1}{x}\frac{\mathrm{dx}}{\mathrm{dt}} = a$

     $\int \frac{1}{x}\frac{\mathrm{dx}}{\mathrm{dt}}\,\mathrm{dt} = \int a\,\mathrm{dt} \Rightarrow \mathrm{at} + c_1$

     $\int \frac{1}{x}\mathrm{dx} = \mathrm{at} + c_1$

     $\ln|x| + c_2 = \mathrm{at} + c_1 \Rightarrow \ln|x| = \mathrm{at} + c_1 - c_2$

     $|x| = e^{\mathrm{at} + c_1 - c_2} = e^{c_1 - c_2}\,e^{\mathrm{at}}$

     $x(t) = e^{c_1 - c_2}\,e^{\mathrm{at}} \Rightarrow \mathrm{ce}^{\mathrm{at}}$

   - $\frac{\mathrm{dx}}{\mathrm{dt}} = \mathrm{ax} + b \Rightarrow \frac{1}{\mathrm{ax} + b}\frac{\mathrm{dx}}{\mathrm{dt}} = 1$

     $\int \frac{1}{\mathrm{ax} + b}\frac{\mathrm{dx}}{\mathrm{dt}}\mathrm{dt} = \int 1\mathrm{dt} \Rightarrow \int \frac{1}{\mathrm{ax} + b}\mathrm{dx} = t + c_1$

     $\int \text{sumsteps here lol}$

     $\frac{1}{a}\ln|s| = \frac{1}{a}\ln|\mathrm{ax} + b| + c_2 = t + c_1$

     $\ln|\mathrm{ax} + b| = a(t_1 + c_1 - c_2)$

     $\mathrm{ax} + b = \mathrm{ce}^{\mathrm{at}}$

     $x(t) = \frac{\mathrm{ce}^{\mathrm{at}}}{a} - \frac{b}{a}$

   - $d = -\mathrm{kv}$

     $\mathrm{mg} - \mathrm{kv} = m\frac{\mathrm{dv}}{\mathrm{dt}} \Rightarrow \frac{\mathrm{dv}}{\mathrm{dt}} = \frac{-k}{m}v + g$

     $v(t) = \frac{c}{\frac{-k}{m}}e^{\frac{-k}{m}}t - \frac{g}{\frac{-k}{m}}$

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