CS101:

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Date 3/8/17

- The height of a rooted tree is its maximum node depth (I.E depth at deepest lfear)

  Notation: height(T)

- Given a Node x in a rooted Tree T, the subtree rooted at x concides of all nodes decensing from x, with x as root.

- The height of a Node x is the height of the subtree rooted at x

- A Binary Tree is a rooted tree in which every node has at most 2 children, identified as left child and right child

- Recursive Definition of height(T) for Binary Tree:

  $h(T) = \{$

  - -∞ n=0
  - 0 n=1
  - 1+max(h(L),h(R) n≥2

- Exercise

  Lets T be a binary Tree on n nodes show $h(t) = \lfloor \text{lgn} \rfloor$

  Hint1: use strong induction on n starting at n=1

  Hint2: use and prove the follwing $\forall$xε$\mathbb{Z}^{\perp}$: $\lfloor \lg(2k+1) \rfloor = \lfloor \lg(2k) \rfloor$

- A complete Binary Tree is a binary tree in which every iternal node had 2 children all leave at same depth

  If T has height h and n nodes then

  $n = \sum_{d=0}^{h} 2^d = \frac{2^{h+1}-1}{2-1} = 2^{h+1} - 1$

- An Almost Complete Binary Tree is a Binary Tree that is filled at all levels, except possibly bottom, and bottom level is filled left to right

  Observce the number of nodes n in an Almost Complete Binary tree of height h must satisfy:

  $2^h - 1 < n \leq 2^{h+1} - 1$

  $2^h \leq n < 2^{h+1}$

  $h \leq \text{lgn} < h + 1$

  $h = \lfloor \text{lgn} \rfloor$

- Heaps:

  A Heap is a data structure stored in an array, based on an Almost Complete Binary Tree

  Two kinds of heaps:

  Max-heap propety: $A[\text{parent}(i) \geq A(i)$

  Min heap property: $A[\text{parent}(i)] \leq A[i]$

  We Assume always a max heap