CS 101: Master and Graph Theory

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- Master Method

  Example 1:

  i. $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\text{logn}}$

  ii. compare $\frac{n}{\text{logn}}$ to $n^{\log_2 2}$

  iii. case 1: requires $\frac{n}{\text{logn}} = O(n^{1-\varepsilon})$

  iv. some ε>0 Pick any ε>0 Then

  v. $\frac{\frac{n}{\text{logn}}}{n^{1-2}} = \frac{\frac{n}{\text{logn}}}{\frac{n}{n^2}} = \frac{n^2}{\text{logn}} \Rightarrow \infty$

  vi. Conclusion:$\frac{n}{\text{logn}} = \omega(n^{1-2})$

  vii. Conclusion:$\frac{n}{\text{logn}} = \neq O(n^{1-2})$

  viii. So maser theorem does not apply!

  Execise: find another example illustrating the gap between case 2 and case 3

  Example 2:

  i. $T(n) = T\left(\frac{n}{2}\right) + 1$

  ii. case 1: 1=n^0 to $n^{\log_2 1} = n^0$

  iii. case 2:$T(n) = \Theta(\text{logn})$

  Mergestort Example:

  i. $T(n) = 2T\left(\frac{n}{2}\right) + n$

  ii. compare n to $n^{\log_2 2} = n^1$

  iii. case 2: $T(n) = \Theta(\text{nlogn})$

  Example 4:

  i.

  Inversion(A,p,r)
  if p<r
  q=$\lfloor \frac{p+r}{2} \rfloor$

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        a=Inversion(A,p,q)
        b=Inversion(A,q+1,r)
        c=Compare(A,p,q,r)
        return a+b+c
    else
        return 0

    Compare(A,p,q,r)
    count = 0
    for i =p to q
        for j = q+1 to r
        if A[i]>A[j]
            count++;
    return count
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ii. $T(n)=\left\{\begin{matrix} 0 \\ T\left(\lfloor\frac{n}{2}\rfloor\right)+T\left(\lceil\frac{n}{2}\rceil\right)+\lfloor\frac{n}{2}\rfloor\lceil\frac{n}{2}\rceil \end{matrix}\right\}\begin{matrix} n=1 \\ n\geq 2 \end{matrix}$

iii. Simplify: $T(n)=2T\left(\dfrac{n}{2}\right)+n^2$

iv. Compare n^2 to $n^{\log_2 2}=n^1$

v. Case 3: Let ε=2-1=1>0, Then,

vi. $n^2=n^{1+\varepsilon}=n^{\log_2 2+\varepsilon}=\Omega(n^{\log_2 2+2})$

vii. Regular Condition: find c in (0,1) S.T $2\left(\dfrac{n}{2}\right)^2\leq \mathrm{cn}^2$

viii. $\dfrac{2}{4}n^2\leq \mathrm{cn}^2$

ix. $\frac{1}{2}\leq c<1$ Pick any such C

x. By case 3: $T(n)=\Theta(n^2)$

- Graph Theory:

    - Isomorphism: A function $\phi V(G_1)\Rightarrow V(G_2)$ iff the following holds:

      $\{x,y|\varepsilon E(G_1)\text{iff}\{\phi(x),\phi(y)|\varepsilon E(G_2)|\}$

    - Let x$\varepsilon V(G)$ the degree ox x, denoted by deg(x) is the # of edges incidented with x

      Note: if $\phi\colon V(G_1)\Rightarrow V(G_2)$ is an isomophism, than $\deg(\phi(x))=\deg(x)$ for any $x\varepsilon V(G_1)$

    - The degree Sequence at G is a list of its vertices degrees in increasing order

      Note: isomorphic graphs some same degree sequence

    - LEMMA: Handshake

      $\sum_{x\varepsilon V(G)}\deg(x)=2|E(G)|$

      Proof: Each edge contributes exactly 2 to the sum of the left.