

# CMPS 101:

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- Lemma 1:

- I. Let  $x \in V$ , and suppose Initialize ( $G, s$ ) is executed. Suppose some sequence of calls to Relax( $u, v$ ) results in  $d[x] < \infty$ . Then  $G$  contains an  $s$ - $x$  Path of weight  $d[x]$
- II. Let  $n = \#$  of calls to Relax()
- III.  $n=0$ , then only limit  $d$ -value is 0, i.e must have  $x=s$ . There is an  $s$ - $s$  path of weight  $d(s)=0$ , namely trivial path.
- IV. Let  $n>0$ , Assume for any  $u \in V$  that if  $d(u)$  becomes finite after fewer than  $n$  Relaxations, then  $G$  contains sum path of weight  $d(u)$ .
- V. We must show that if  $x \in V$ , and  $d(x)$  becomes finite after  $n$  relaxations, then  $G$  contains an  $s$ - $x$  Path of weight  $d(x)$
- VI. Let  $x \in V$ , and suppose some relaxation sequence causes  $d(x) < \infty$
- VII. then some edge of form  $y$ - $x$  must have been relaxed in the sequence.
- VIII. Let  $y$  be the origin of that edge on that call to Relax( $y, x$ ),  $d(x)$  was set to
$$d(x) = d(y) + w(y, x)$$
Since we suppose this number to be finite,  $d(y)$  must have been finite, before the call to Relax()
- IX. Therefore  $d(y)$  becomes finite after fewer than  $n$  relaxations.
- X. By the induction hypothesis,  $G$  contain an  $s$ - $y$  Path of weight  $d(y)$ !
- XI. That Path, followed by the edge  $(y, x)$  constitute an  $s$ - $x$  Path of weight
$$d(x) = d(y) + w(y, x)$$
 as required

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