CS 101:

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I. Lemma 5:

   Let G be connected with n vertices and m edges. Suppose m=n-1 then G is also acylic, hence a tree.

   Proof:

   i. Assume to get a x, that G counties a cycle. Pick any edge e on that cycle and remove it, call the resulting graph G-e

   ii. Observe that G-E has m-1 edges, n vertices, and connected

   iii. By Lemma 3: $m-1 \geq n-1$, so $m \geq n$, But $m = n-1$ by hypothesis, so $n-1 \geq n$. This X shows no such exists because G is acycic.

II. Lemma 6:

   Let G be acyclic with m edges, n vertice, and suppose m=n-1. Then G is also connected, hence a tree.

   Proof:

   i. Let k be the # of connected componenets in G. must show k=1

   ii. by Lemma 2: m=n-k Also m=n-1 by hypthosis so n-1=n-k

   iii. -1=-k

   iv. k=1

III. Consider

   1. G is conncedted
   2. G is acyclic.
   3. m=n-1

   - Lemm1: 1 & 2 =>3
   - Lemma 5: 1 & 3 =>2
   - Lemma 6: 2 & 3 =>1

IV. Theorem1: Tree Theorem

   Let G=(V,E) be a graph. Then the following are eqauivalent:

   a) G is a tree

   b) G contains a unique xy path for all x,yεV.

   c) G is connected, but if any edge is removed, the resultung graph is disconnected

d) G is connected and $|E| = |V - 1|$

e) G is acyclic and E=V-1

f) G is acyclic, but if any edge is added (joining to non-adj vertices), then G+E contains a unique cycle

By Lemma 1, 5, 6:

a<=>d<=>e

V. Directed Graph

Defintion G=(V,E) where V≠0, E⊆VxV

- adjacency, incidence
- walk, trail, path, cycle
- subgraph
- isomorphism

A directed graph is called strongly connected iff for all x,yεV

X is reachable from y

Y is reachable from y

A subset S$\subseteq V$ is called strongly connected iff for ll $x.y\varepsilon V: x$ is reachable from y and y from x

A subsect S$\subseteq V$ is called a strongly connected component of G iff

- S is strongly connected
- S is maximal wrt

VI. Representing Graphs

- Incidence Matrix

  Requires V=$\{x_1........., x_n\}$

  E=$\{e_{1.................,e_n}\}$

  I(G) is an n x m matrix. Row i $(i \leqslant j \leqslant m)$ represents e_j

  Undirected case: $I_{\mathrm{ij}} = \left\{ {}^{1\, x_i \text{incident with } e_j}_{0 \quad \text{otherwise}} \right\}$

  Directed: $I_{\mathrm{ij}} \left\{ -1\, x_i \text{is origin of } e_j {}^{+1\, x_i \text{ is termination of } e_j}_{0 \text{ othwerise}} \right\}$

- Adjacency Matrix

  row i: x_i

  col j: x_j

  Undirected: $A_{\mathrm{ij}} = \left\{ {}^{1 \quad x_i \text{ adj to } x_j}_{0 \quad \text{otherwise}} \right\}$

  Directed: $A_{\mathrm{ij}} = \left\{ {}^{1 \quad \text{If } x_i \rightarrow x_j}_{0 \quad \text{otherwise}} \right\}$