AMS 20

by Kameron Gill

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I. FIrst Order Linear Time ODE

- General form $\frac{\mathrm{dx}}{\mathrm{dt}}$
- $\frac{\mathrm{dx}}{\mathrm{dt}} = x + t$
  - i. $\frac{1}{x+t}\frac{\mathrm{dx}}{\mathrm{dt}} = 1$
  - ii. $\int \frac{1}{x+t}\mathrm{dx} = t + c_1$
  - iii. $\ln|x+t| = t + c_1$
  - iv. $x + t = \mathrm{ce}^t$
  - v. $x = \mathrm{ce}^t - t$
  - vi. VERIFY
  - vii. $\frac{d(\mathrm{ce}^t - t)}{\mathrm{dx}} = x + t \Rightarrow \mathrm{ce}^t - t - t$
  - viii. x+t=ce^t
  - ix. WRONG!!!!(step 2)
- $\int F(x(t), t)\frac{\mathrm{dx}}{\mathrm{dt}}\mathrm{dt} \neq \int F(x, t)\mathrm{dx}$
- $\frac{\mathrm{dx}}{\mathrm{dt}} = a(t)\, x$
  - i. $\frac{1}{x}\frac{\mathrm{dx}}{\mathrm{dt}} = a(t)$
  - ii. $\int \frac{1}{x}\mathrm{dx} = \int a(t)\mathrm{dt}$
  - iii. $\ln|x| + c = \int a(t)\mathrm{dt} + c$
  - iv. $x(t) = \mathrm{ce}^{\int a(t)\mathrm{dt}}$
- $\frac{\mathrm{dx}}{\mathrm{dt}} = -x + e^{2t}$
  - i. $\frac{\mathrm{dx}}{\mathrm{dt}} + x = e^{-2t}$
  - ii. $e^t x + e^t\frac{\mathrm{dx}}{\mathrm{dt}} = e^t e^{-2t} = e^{-t}$
  - iii. $\frac{\mathrm{de}^t}{\mathrm{dt}}x + e^t\frac{\mathrm{dx}}{\mathrm{dt}} = \mathrm{LHS}$

iv. $\frac{d(e^t x)}{\mathrm{dt}} = e^{\mathrm{tt}}x \Rightarrow \int \frac{d(e^t x)}{\mathrm{dt}}\mathrm{dt} = \int e^{-t} = -e^{-t} + c$

v. $x = \mathrm{ce}^{-t} - e^{-2t}$

- $\frac{\mathrm{dx}}{\mathrm{dt}} = a(t)x + b(t)$

  i. $\frac{\mathrm{dx}}{\mathrm{dt}} - a(t)x = b(t)$

  ii. $-\mu(t)a(t)x + \mu(t)\frac{\mathrm{dx}}{\mathrm{dt}} = \mu(t)b(t)$ determine $\mu(t)$ for later

  iii. $\frac{d(\mu(t)\,x)}{\mathrm{dt}} = \mathrm{LHS} = \frac{d\mu}{\mathrm{dt}}x + \mu(t)\frac{\mathrm{dx}}{\mathrm{dt}}$

  if$\frac{d\mu}{\mathrm{dt}} = -a(t)\mu(t)$ requirement for μ

  iv. $\frac{d\mu(t)x}{\mathrm{dt}} = \mu(t)\,b(t)$

  v. $\mu(t)x = \int \mu(t)b(t)\mathrm{dt} + c$

  vi. $x = \frac{c}{\mu(t)} + \frac{1}{\mu(t)}\int \mu(t)b(t)\mathrm{dt}$

  vii. $-a(t)\mu(t) \Rightarrow \mu(t) = \mathrm{ce}^{-\int a(t)\mathrm{dt}}$

- $\frac{\mathrm{dx}}{\mathrm{dt}} = \mathrm{ax} + b(t) \quad a = \mathrm{constant}$

  i. $\frac{\mathrm{dx}}{\mathrm{dt}} - \mathrm{ax} = b(t)$

  ii. $\mu(\mathrm{t})\frac{\mathrm{dx}}{\mathrm{dt}} - \mu(t)\mathrm{ax} = \mu(t)b(t)$

  iii. $-\frac{d\mu}{\mathrm{dt}}x + \mu(t)\frac{\mathrm{dx}}{\mathrm{dt}} = \frac{d(\mu(t)x(t))}{\mathrm{dt}}$

  $\frac{d\mu}{\mathrm{dt}} = a\mu \Rightarrow \mu(t) = \mathrm{ce}^{-\mathrm{at}}$

  iv. $\frac{d(e^{-\mathrm{at}}x)}{\mathrm{dt}} = e^{-\mathrm{at}}b(t)$

  v. $e^{-\mathrm{at}}x = \int e^{-\mathrm{at}}b(t)\mathrm{dt} + c$

  vi. $\mathrm{x(t)} = \mathrm{ce}^{\mathrm{at}} + e^{\mathrm{at}}\int e^{-\mathrm{at}}b(t)\mathrm{dt}$

- $\frac{\mathrm{dx}}{\mathrm{dt}} = -x + \sin(t)$

  i. $\frac{\mathrm{dx}}{\mathrm{dt}} + x = \sin(t)$

  ii. $\mu(\mathrm{t})\frac{\mathrm{dx}}{\mathrm{dt}} + \mu(t)x = \mu(t)\sin(t)$

  iii. $\frac{d(\mu(t)x)}{\mathrm{dt}} = \frac{d\mu}{\mathrm{dt}} + \mu(t)\frac{\mathrm{dx}}{\mathrm{dt}}$

  $\frac{d\mu}{\mathrm{dt}} = \mu \Rightarrow \mu(t) = e^t$

  iv. $\frac{d(e^t x)}{\mathrm{dt}} = e^t\sin(t) \Rightarrow e^t x = \int e^t\sin(t)\mathrm{dt} + c$

  v. $x = \mathrm{ce}^{-t} + e^{-t}\int e^t\sin(t)\mathrm{dt}$

  vi. $x(t) = \mathrm{ce}^{-t} + e^{-t}(\mathrm{insert\ formula\ here})$