AMS 20

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Date April 3, 2017

1. Differential Equations

   - unknown is a function (such as velocity over time)
   - Equations involve unknown function and its derivatives

   In a pendelum, we have...

   $F = \text{ma} \quad F = \text{mgsin}\Theta \Rightarrow \text{ma} = -\text{mL}\frac{d^2\Theta}{\text{dt}^2}$

   $f(x) = L\frac{d^2\Theta}{\text{dt}^2} + \text{gsin}\Theta = 0$

   $F = m\frac{\text{dv}}{\text{dt}} \Rightarrow m\frac{d^2\Theta}{\text{dt}^2}$

   $R - C$ transistor

   - $Q(t)$ charge on c
   - $\text{iR} + \frac{Q}{c} = 0 \; i = \frac{\text{dQ}}{\text{dt}}$
   - $R\frac{\text{dQ}}{\text{dt}} + \frac{Q}{c} = 0$

2. Independent Variable

   In previous problems, t or time, was the independent variable

3. Classification of diffierntial equation

   i. ODE/ PDE (Ordinary Differential Equation or Partial Differential Equation)

      ODE- unknown x(t)

      PDE- $c\frac{d^2u(\text{xt})}{\text{dx}^2} = \frac{d^2u(\text{xt})}{\text{dt}^2}$

   ii. Order: highest derivative of unknown function involved in the ODE

   iii. Time varying/Time invariant

      Time varying- independent variable t appears in ODE explicity

      $\frac{\text{dx}}{\text{dt}} + (\sin^2(t) + 1)x = 0 \quad \text{when } t = 0 \text{ then } \frac{\text{dx}}{\text{dt}} + x = 0 \quad t = 2 \quad \frac{\text{dx}}{\text{dt}} + 2x$

      As time varies, then equation changes

      In time invariance, then independent variable does not change equation

   iv. Linear ODE/nonlinear ODE

      $f(x_1, x_2) = a_1x_1 + a_2x_2 + b$

$$f\left(x(t), \frac{\mathrm{dx}}{\mathrm{dt}}, \frac{d^2x}{\mathrm{dt}^2}......t\right) = 0$$

f is a linear function of $x(t), \frac{\mathrm{dx}}{\mathrm{dt}}, \frac{d^2x}{\mathrm{dt}^2}\cdots\frac{d^nx}{\mathrm{dt}^n}$

Linear form: $a_{n(t)}\frac{d^nx}{\mathrm{dt}^n} + a_{n-1(t)}\frac{d^{n-1}x}{\mathrm{dt}^{n-1}} + ...a_{1(t)}\frac{\mathrm{dx}}{\mathrm{dt}} + a_0(t)x(t) = g(t)$

4. Example

$\frac{\mathrm{du}(x,t)}{\mathrm{dt}} + \frac{u\,\mathrm{du}(x,t)}{\mathrm{dx}} = 1 + \frac{d^2u(x,t)}{\mathrm{dx}^2}$ PDE

$\frac{d^3y}{\mathrm{dt}^3} - 3\frac{d^2y}{\mathrm{dt}^2} + 2\frac{\mathrm{dy}}{\mathrm{dt}} = 0$ ODE, linear

$\frac{\mathrm{dx}}{\mathrm{dt}} - 2t^2\,e^{\mathrm{sint}}\,x(t) = 1$ ODE, Linear

$\frac{d^2y}{\mathrm{dt}^2} + (o^2t)y = t^3$ ODE, linear