CMPS 101

by Kameron Gill

Date 2/01/2017

- Example:

  $T(n) = \begin{cases} 1 & 1 \leqslant n \leqslant 2 \\ 4T\left(\frac{n}{3}\right)+n & n \geqslant 3 \end{cases}$

  Show: For all n≥1: T(n) $\leq n^2$ $\leftarrow P(n)$

  Hence T(n) $= O(n^2)$

  1. Base case:

     P(1): T(1)$\leq 1^2$ I.E $1 \leq 1^2$ TRUE

     P(2): T(2)$\leq 2^2$ I.E $1 \leq 2^2$ TRUE

  2. let n>2, asssume for all k in the range 1≤k<n that

     $T(k) \leq k^2$

     In particular k $= \frac{n}{3}$ we have $T(\frac{n}{3}) \leq \left(\frac{n}{3}\right)^2$

     We must show: $T(n) \leq n^2$

  3. $T(n) = 4T\left(\frac{n}{3}\right) + n$ By the rec. for T(n)

     $\leq 4\left(\frac{n}{3}\right)^2 + n$ By the Induction Hypothesis

     $\leqslant 4\left(\frac{n}{3}\right)^2 + n$ Since $x \leq x$

     $= \frac{4}{9}n^2 + n \leqslant n^2$ True!