CMPS 101: Graph Theory/Recurrence

by Kameron Gill

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- THM: Let T be a tree on n vertices, then T has n-1 edges

  I. Base case:P(1) says: if T tree only has 1 vertice, then T has 0 edges.

  There is only one tree on 1 vertice.

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  It has no edges.

  II. Induction Step:

  For all n≥1: P(n)....P(n-1)⇒P(n).

  Let, n>1 be abritary, asumme for all k in the range 1≤k<n that: If T' is a tree on k vertices, then T' has k-1 edges. We must show: if T is a tree with n vertices, then T has n-1 edges.

  1. Let T be a tree with n vertices. Pick any edge e in T and remove it

     The resulting graph results of two trees: T_1 and T_2 each with fewer than n vertices.

  2. Let T_i have k_i vertices (i=1,2).

  3.T_1 has k_1-1 edges (By Induction hyp)

  4.T_2 has $k_2 - 1$ edges (By Induction hyp.

  5. Note $n = k_1 + k_2$ since no vertices were remove. Therefore # of edges in T is

     $|E(T)| = |E(T_1)| + |E(T_2)| + 1$

     $=(k_1 - 1) + (k_2 - 1) + 1$

     $=(k_1 + k_2) - 1$

     =n-1

  6. Therefore T has n-1 edges.

- Recurrance Relations

  EX: $T(n) = \left\{ \begin{matrix} 0 \\ T(\frac{n}{2})+1 \end{matrix} \right\} \begin{matrix} n=1 \\ n\geq 2 \end{matrix}$

  Iteration method:

  $T(x) = 1 = T\left(\frac{x}{2}\right)$

  $T(n) = 1 + T\left(\frac{n}{2}\right) = 1 + 1 + T\left(\frac{\frac{n}{2}}{2}\right)$

  $=2 + T\left(\frac{n}{4}\right)$