CMPS 101:

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I. Topological sort

- Definition: a digraph is acyclic iff it contains no directed cycles

- A Directed Acyclic Graph is called a DAG

- LEMMA:

  A digraph is acyclic iff DFS yields no back edges

  Negation: a digram is acyclic iff DFS produces a back edge.

  PROOF:

    i. <=
       (easy) suppose DFS produces a back edge then obviously there is a directed cycle!

    ii. =>(harder) Use White path theorem

        Assume G contains a directed cycle. Must show DFS(G) produces a back edge

        Call the directed cycle C

        Let x be the vertex that prrecdes y along C

        Since at time d[y] all vertices along C are white, there exists a white path from y to x(I.E travel along the cycle C)

        By the White path theorem, x is a decendent of y

- Defintion: Let G be a DAG. A topological sort of of V(G) is a linear ordering of V(G) such that if $(x,y)\varepsilon E(G)$ then x comes before y in the ordering.

    i. Theorem:

       To peform a topological sort on a DAG G:

    ii. Run DFS(G)

    iii. as vertices finish, push onto stack

    iv. When DFS is complete, the stack (top to bottom) is a topological sort

        Equivatnly: Run DFS(G), then sort by decreasing finish times.