AMS 20

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I. $\frac{\mathrm{dx}}{\mathrm{dt}} = a(t)x + b(t)$
   $x(t_0) = x_0$

   - $\frac{\mathrm{dx}}{\mathrm{dt}} = \frac{2t}{4-t^2}x + 3t^2 \quad x(0) = 2$

     i. $4 - t^2 = 0 \Rightarrow t \pm 2$: means that there are three intevrals $(-\infty, -2), (-2, 2), (2, \infty)$

     ii. $N(x)\frac{\mathrm{dx}}{\mathrm{dt}} = M(t)$

     iii. $\int N(x)\mathrm{dx} = \int M(t)\mathrm{dt} \Rightarrow H_1(x) = H_2(t) + c$

II. $\frac{\mathrm{dx}}{\mathrm{dt}} = -\frac{2t}{1+2x} \quad x(2) = 0$

   i. $(1+2x)\frac{\mathrm{dx}}{\mathrm{dt}} = 2t \Rightarrow \int 1 + 2\mathrm{xdx} = \int 2\mathrm{tdt}$

   ii. $x + x^2 = t^2 + c \Rightarrow 0 + 0^2 = 2^2 + c \Rightarrow c = -4$

   iii. $x^2 + x - (t^2 - 4)$=0 Use quadratic

   iv. $x(t) = \frac{-1 \pm \sqrt{1 + 4(t^2-4)}}{2} = \frac{-1 \pm \sqrt{t^2 - 15}}{2} \Rightarrow t^2 \gg \frac{15}{4} \Rightarrow t \ll -\frac{\sqrt{15}}{2}$ OR $t \gg \frac{\sqrt{15}}{2}$

   v. $t > \frac{\sqrt{15}}{2}$

III. $\frac{\mathrm{dx}}{\mathrm{dt}} = \frac{4t - t^3}{4 + x^2}$ x(0)=1

   i. $(4 + x^2)\frac{\mathrm{dx}}{\mathrm{dt}} = 4t - t^3$

   ii. $\int 4 + x^2\mathrm{dx} = \int 4t - t^3\mathrm{dt} \Rightarrow 4x + \frac{x^4}{4} = 2t^2 - \frac{t^4}{4} + c$=>0+0=2$-\frac{1}{4} = c \Rightarrow c = \frac{17}{4}$

   iii. $x^4 + 16x - (8t^2 - t^4 + 17) = 0$

   iv. $4 + x^3 \neq 0 \Rightarrow x \neq -4^{\frac{1}{3}} \approx -1.59$
   t=3.34

IV. $\frac{\mathrm{dx}}{\mathrm{dt}} = f(t, x) \quad x(t_0) = x_0$

1. If f(t,y) is continous in some rectangle d<t<S. r<x<S continuity (t_0, x_0) then IVP has at least one solution defined in some interval $(t_0 - h, t_0 + h)$

2. If $f(t,\ x)$ and $\frac{\mathrm{dt}}{\mathrm{dx}}$ are continous in $d < t < S$, $r < x < S$ contains (t_0, x_0). Then the solution to IVP is uniquely define on $(t_0 - h, t_0 + h)$

$\frac{\mathrm{dx}}{\mathrm{dt}} = x^{1/3} \Rightarrow x^{-1/3}\frac{\mathrm{dx}}{\mathrm{dt}} = 1 \Rightarrow \int x^{-1/3}\mathrm{dx}$=$t + c \Rightarrow \frac{3}{2}x^{2/3} = t + c \Rightarrow c = 0$

$\frac{3}{2}x^{2/3} = t \Rightarrow x = \sqrt{\left(\frac{2t}{3}\right)^3}$

1. 2n Order Linear ODE
   - $y'' + P(t)y' + q(t)y = g(t)$