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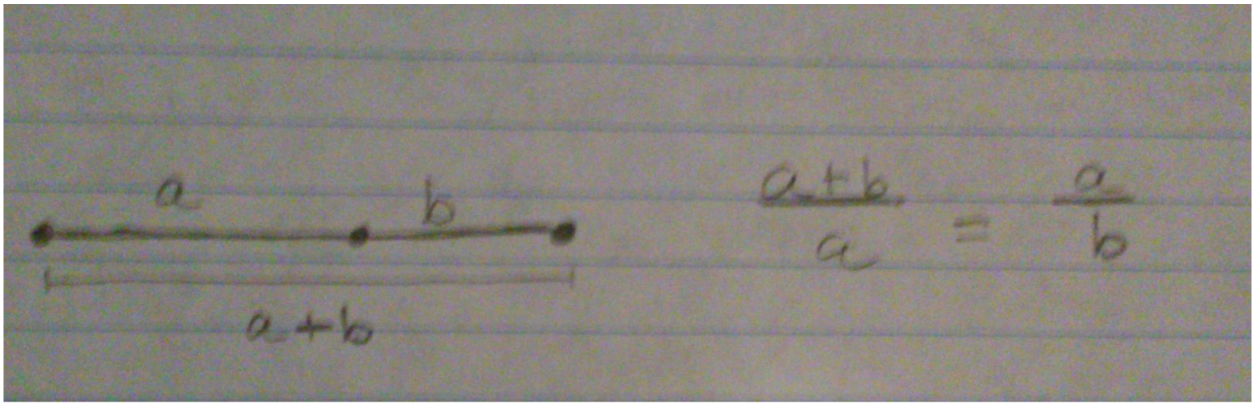
The Golden Ratio

Mathematicians often talk about the beauty of math. Where scientist are concerned with math as a tool to solve problems, mathematicians like to explore math for math's own sake. Luckily the golden ratio fulfills both these sets of views. It is mysterious and beautiful, but also has many practical applications. The world has been using the golden ratio for more than two thousand years, and it is still puzzling mathematicians and inducing many existential crises.

Every summer thousands of tourist flock to see the wonders of ancient Greece. The crowds climb a small hill to catch a glimpse of the famous Parthenon that has stood overlooking Athens since 432 BC (Connelly). Although it was built before the first definition of the golden mean was given by Euclid, many think that the temple's construction uses the golden number (Meisner 1). If you head a little south, you might be able to catch a glimpse of the pyramids at Giza. These too are thought to have been constructed using knowledge of the golden ratio (Posamentier 4). These two wonders and others tell us that ancient peoples also saw a deep connection between the natural world and this peculiar number.

The golden mean was first defined by Euclid in 325 BC (Livio 3). In his book the *Elements* he states, "A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the lesser" (Heath). In other words, when a line is cut into two segments one of which is larger, the larger of the two segments ratio with the whole line, is the same ratio that the smaller piece has with the larger.

Consider the following picture:



This shows the ratio that Euclid was referring to. It's easy to take this ratio and begin to manipulate it to really see why this is special.

Start with the ratio above, we'll call phi:

$$\varphi = \frac{a+b}{a} = \frac{a}{b}$$

Then:

$\varphi = 1 + \frac{b}{a}$ and $\frac{b}{a} = \frac{1}{\varphi}$ because $\frac{a}{b} = \varphi \Rightarrow \frac{1}{a} = \frac{1}{\varphi}$

We have the following definition for phi, or the golden ratio:

$$\varphi = 1 + \frac{1}{\varphi}$$

Let's square this equation and try to find what phi is:

$\varphi(1 + \frac{1}{\varphi}) = \varphi(\varphi)$

$\varphi + 1 = \varphi^2$

Set this equal to 0

$\varphi^2 - \varphi + 1 = 0$

Apply quadratic formula

$\varphi = \frac{1 \pm \sqrt{5}}{2} \approx 1.6180339887...$ The golden ratio. (We're interested in the positive portion).

Why is this number so special? It's certainly irrational, but not transcendental like pi. Consider a few things about the derivation above. One interesting thing is that phi is defined by itself. The equation $\varphi = 1 + \frac{1}{\varphi}$ is a recursive definition. This means that the golden mean is contained within the golden mean itself.

For example:

$$\varphi = 1 + \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\dots}}}}$$

This helps to explain why the golden ratio pops up so much in nature. If you construct a rectangle whose proportions follow the golden ratio (called the golden rectangle) the recursive properties above become apparent. One can create another golden rectangle inside of the first. Repeating this over and over we end up with the special golden spiral or the logarithmic spiral (Posanmentier 4). This is the design seen so often in nature on the shells of animals (Posanmentier 4). The fascinating thing is that no matter how many times you find the golden mean inside the spiral there is always more left. It will continually repeat and give more golden means.

This might help to explain why the golden ratio is thought to be aesthetically pleasing. Although some might chalk this up to a matter of opinion, one has to consider that many famous artist have used the golden ratio in their work (Meisner 1). As discussed above Phidias used the ratio in the famous Parthenon as the proportions for the columns and structure (Meisner). Leonardo Da Vinci’s *The Last Supper* used phi in much of the painting (Meisner). At the time it was even referred to as the divine proportion (Meisner). It seems there is an inherent beauty to the golden ratio that is pleasing to the human eye.

If that isn't enough mystery, there is also a relationship between the golden mean and the fibonacci numbers. In the 1500s Johannes Kepler proved that the golden ratio is the limit of the ratio of consecutive Fibonacci numbers (Posamentier 4). This relationship is truly surprising. Kepler himself said, "Geometry has two great treasures: one is the theorem of Pythagoras; the other, the division of a line into extreme and mean ratio. The first we may compare to a measure of gold; the second we may name a precious jewel", (Meisner).

Kepler is astounded by this discovery because the fibonacci numbers also seem to have a deep relationship with nature. The number of petals a flower has seems to follow the fibonacci sequence [8]. Parental trees can be modeled after the fibonacci numbers [8]. Even your own body seems to follow the sequence: you have eight fingers, five digits on each hand, three bones per finger, two bones per thumb, two hands, one thumb per hand. Once again the golden number tends to pop up in places that seem unrelated more than two millennia after its discovery.

This is just a sneak peek into why the golden number is so mysterious. For two thousand years it has been studied by mathematicians for its beauty and peculiar properties. It keeps popping up in mysterious places it has no business being. From the great Parthenon at Athens to the design on a snail's shell. It gives scientist and artist a useful tool to model the natural world. For more than two millennia this little number has inspired mathematicians scientist, and artist; and it doesn't look like it's going to stop any time soon.

Works Cited

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