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Kepler's Laws

Humanity has always looked skyward for meaning. For tens of thousands of years humans have looked to the heavens and wondered at the mysteries they hold. Before the advent of television, the internet, or even books, man craned his neck to the skies. Almost 200,000 years ago ancient humans gazed up at stars very similar to the ones we see at night today. They watched, and remembered, and tried to understand. It wasn't until very recently however, that man has understood how the stars and planets dance across the great expanse of space outside our little refuge of earth.

Although there have been many mystical explanations of the heavens over human history, the first scientific ones started appearing around 330 B.C (Linton). The Babylonians and Greeks both recorded observations of astronomical phenomena. Heraclides used these records and observations to construct a model of the solar system. This model was geocentric but Heraclides also played with a heliocentric model which he abandoned because he could not reconcile this model with perfectly circular orbits of the planets; the Greeks believing that things in the heavens must be perfect, and therefore orbits must be perfectly circular (Katz 443). It would be over a thousand years until Kepler proved this to be a huge misstep in understanding how the planets and stars moved.

In ~200 A.D. another famous Greek astronomer/mathematician Ptolemy revisited the geocentric model of Heraclides. Using observational data Ptolemy was able to explain the observed motion of orbits in almost full completeness [4]. The problem was that Ptolemy's model was extremely complex. In order to explain retrograde motion (where an observed body seems to move backwards in the sky before continuing in its original direction) Ptolemy developed the idea of epicycles and deferents. In his model there was a main orbit called the deferent centered on the earth and the smaller epicycle orbit whose center moved along the deferent . The sun, moon, and planets moved along the circumference of their respected epicycles [4]. Because of the need for these epicycles to explain the observed motion of the planets, Ptolemy's theory was very complicated with as many as 28 epicycles needed to account for observed planetary motion [4].

In the 14th century another famous astronomer, Copernicus, revisited the heliocentric model of the solar system. The Catholic church had long since accepted the geocentric model into dogma, and Copernicus's heliocentric model went against both religion and popular culture.. Although we see it as a major breakthrough today, Copernicus's model did little to simplify the problems of Ptolemy. He too used circular orbits and was forced to use even more epicycles than Ptolemy to account for the observed motion of the planets.

The next person critical to the formulation of Kepler's three laws was Tycho Brahe. Brahe was not much of a theorist but he was fanatical about observation. He was able to acquire funds to build one of the first true observatories. A very impressive feat considering the telescope had not yet been invented [5]. He too believed in a geocentric model like so many astronomers before him. Along with his assistants and students he was able to collect a huge amount of observational data. It was one of these students, Johannes Kepler, who would truly make waves in the world of astronomy.

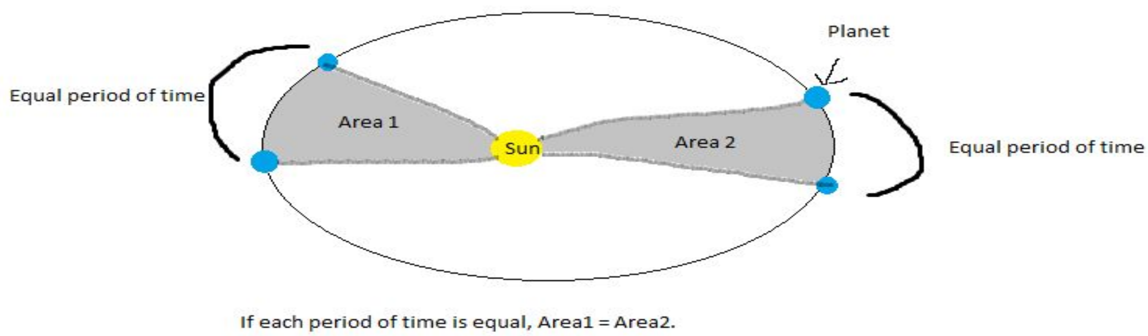
Unlike Brahe, Kepler did not believe in the geocentric model. Using the large amount of data collected by Brahe, and by studying the orbit of mars, Kepler was able to formulate his three laws of planetary motion below:

Kepler's First Law - Law of Ellipses
The orbit of a planet is an ellipse where one focus of the ellipse is the sun.

Most elementary or middle school students in modern times are aware that the planets' orbits are not perfect circles and instead are elliptical (even though a circle is an ellipse whose foci are in the same location). This was a truly revolutionary idea when Kepler published his laws. By using ellipses for orbits he was able to get rid of the need for epicycles, thus simplifying his model greatly.

Kepler's Second Law - Law of Equal Areas

A line from the planet to the sun sweeps out equal area in equal amounts of time.



Kepler's second law says that if a line is drawn from a planet to the sun, then as that planet moves around the sun over a given interval of time the area it "sweeps" out will always be equal given equal time intervals. The implications of this law are better seen in the picture above. As the planet moves closer and closer to the sun, the planet must move faster in order to sweep out the same area. This might seem intuitive to us today but it is extremely important. It would be almost another 200 years before Isaac Newton developed his theory of gravity and an explanation for why this would occur.

Kepler's Third Law - Law of Harmonies

Kepler's constant is given by the equation:

$k = \frac{T^2}{R^3}$ *where T is the orbital period and R is the semi-major axis.*

The ratio of the square of the revolutionary periods for two planets is equal to the ratio of the cubes of their semimajor axes. Given by the equation:

$$\frac{T_1^{\ 2}}{T_2^{\ 2}} = \frac{R_1^{\ 3}}{R_2^{\ 3}}$$

This comes from the fact that Kepler's constant is a constant and therefore we can relate any two bodies by this ratio.

Where T is the orbital period of the first body, R_1 is the semi-major axis of the elliptical orbit , and the same for the second body with the given subscripts. Kepler's third law is perhaps the most difficult to understand. Its importance is easiest to see when dealing with a planet and the sun. Take earth to be the first body and the sun to be the second body. Then the period of the earth around the sun is one year, and its radius is one astronomical unit, then the formula above for any other planet is:

$$T^2 = R^3$$

Where the orbital period T, is in years, and the distance of the semi-major axis R, is in astronomical units. Take the planet Mars, which has an orbital period of 1.88 years, and a distance from the sun of 1.5 au (Stern 1). $1.88^2 \approx 3.5\ and\ 1.5^3 \approx 3.5.$ Take the planet Jupiter, with an orbital period of 11.9 years, and a distance of 5.203 au (Stern 1). Then T^2 and R^3 are within experimental error of each other. By looking at the data above we begin to see another clear trend. Kepler's 3rd law implies that planets closer to the sun must have shorter orbital periods and planets farther away must have longer ones. Above we used the sun and planets, but Kepler's 3rd law holds for any two bodies, therefore you can center these orbits around a different body. We can use Kepler's 3rd law to see how Newton derived an even more general form of this law that is much more powerful.

Start with Kepler's law and add a constant of proportionality k so that we can write it as an equality. T is still the orbital period but R has been replaced with a for the semi-major axis.

$$T^2 = ka^3$$

Newton's law of gravity states for any two masses M and m:

$$F = \frac{GMm}{r^2}$$

Where G is the gravitational constant and r is the distance between the two bodies (Note: this equation assumes circular orbits). Newton also tells us the equation for centripetal force:

$F = \frac{mv^2}{r}$

Where m is the mass of the moving body, v is the velocity and r is the radius of the circular orbit. This force is caused by gravity so we can relate the two equations:

$\frac{GMm}{r^2} = \frac{mv^2}{r}$

Canceling we get

$\frac{GM}{r} = v^2$

We know that velocity is distance over time. In this case the distance is the circumference of the orbit and the time is the orbital period. Therefore:

$v = \frac{2\pi}{T}$

Putting this together we get:

$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$

or

$T^2 = \frac{4\pi^2}{GM} a^3$

This is Kepler's third law with the constant of proportionality equal to $\frac{4\pi^2}{GM}$ if k is in astronomical units and T in years then it becomes 1. We can generalize this equation even more using two bodies with masses of M_1, M_2:

$$T^2 = \frac{4\pi^2}{G(M_1+M_2)} a^3$$

Let us use this formula to calculate the mass of the Sun. The Earth is on average 1.52×10^8 km from the Sun and has an average orbital period of about 365 days (Wikipedia). Then:

$$(3.15 \times 10^7 sec) = \frac{4(3.14159)^2}{(6.673 \times 10^{-11} \frac{m^3}{kg \cdot s^2})(M_1 + M_2)} (1.52 \times 10^{11} meters)^3$$

$$\frac{(1.52 \times 10^{11} m)^3}{(3.15 \times 10^7)^2} \times \frac{4(3.14159)^2}{(6.673 \times 10^{-11} \frac{m^3}{kg \cdot s^2})} = (M_1 + M_2)$$

$$(M_1 + M_2) \approx 2.0 \times 10^{30} kg$$

If we know the mass of the earth we can use it to solve for the mass of the Sun. Otherwise if one of the masses is relatively much smaller than the other (as would be the case for the Sun and Earth) we can get a very good estimate with just the above. If we want to do better, the mass of the earth is $5.972 \times 10^{24} kg$ (Wikipedia). $2.0 \times 10^{30} - 5.972 \times 10^{24} \approx 1.99 \times 10^{30} kg$ which is a very good estimate for the mass of the sun given these relatively simple calculations.

As evident in the example above, Kepler's laws truly revolutionized how we view the universe. With his laws it was possible to calculate how far away the heavens were. They paved the way for Newton to develop his theory of gravity and push astronomy even farther. Without Kepler it might have taken another thousand years before someone decided that orbits are elliptical and that the earth isn't the center of the solar system, let alone the universe.

Works Cited

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