AM5600: Computational Methods in Mechanics (July-Nov. 2019)

Assignment #7

Due: At the beginning of class on Nov. 11, 2019

1. For the wave equation $u_{tt}(x,t) = 9u_{xx}(x,t)$, what relationship between h and k must occur to produce the following finite difference equation:
$$u_{i,j+1} = u_{i+1,j} + u_{i-1,j} - u_{i,j-1}$$
2. Solve $u_{xx} + u_{yy} = -9u$ over $\mathcal{R} = \{(x,y): 0 \le x \le 1, 0 \le y \le 1\}$ with the boundary values
$$u(x,y) = \sin(2x) + \cos(2y)$$
3. For the wave equation $u_{tt}(x,t) = 4u_{xx}(x,t)$, can it be solved numerically using finite difference for $h = 0.03$ and $k = 0.02$?
4. Consider a cross-section of a long rectangular electrical conductor which generates internal heat energy due to electrical resistance. The 2D system is governed by the Poisson's equation:
$$T_{xx} + T_{yy} = \frac{-\dot{Q}}{k}$$
The conductor has a width of 1 cm and height of 1.5 cm. The bottom and top sides of the conductor are held at 0 °C, while the left and right side has the derivate boundary condition $T_x(0,y) = 0$ and $T_x(1,y) = 0$. The conductor is made of a copper alloy ($K = 0.4$ J/cm-s-°C). The rate of heat energy generation within the conductor, due to the electrical resistance $\dot{Q}$, is equal to 100 J/cm^3-s. Utilize the second-order centered difference scheme to find the temperature distribution ($T(x,y)$) for a 5 x 7 grid along the width and height of the conductor.
5. Assume a solid plate of thickness, $L = 1\ cm$ and thermal diffusivity, $\alpha = 0.01\ cm^2/s$ and the heat transfer is governed by $T_t(x,t) = \alpha T_{xx}(x,t)$. The plate is heated to an initial temperature distribution, $T(x,0)$ (refer below) after which the source was turned off.
$$T(x,0) = 100x,\ 0 \le x \le 0.5$$
$$T(x,0) = 100(1-x), 0.5 \le x \le 1$$
where, T is in ℃. The temperature of the two faces of the plate is held at 0℃ at all times. Find $T(x,t)$ at $t = 3s$ for $h = 0.1$ and $k = [0.5, 1]$ using the forward time centered-space (FTCS) method. Comment on your findings by comparing the solution between the two different step sizes.

AM5801/AM5810: Computational Lab (optional for students crediting AM5600)

Due: At the end of lab on Nov. 6, 2019

I. Consider steady heat diffusion in the unit square $0 \leq x \leq 1, 0 \leq y \leq 1$. Develop a numerical scheme for 5- and 9- point finite difference schemes for the boundary conditions given below. Solve the problem for several different step sizes. Use *surf* and *contour* for visualization of results.
$T(0, y) = 100;\ 0 \leq y \leq 1$
$T(x, 0) = 50;\ 0 < x \leq 1$
$T(x, 1) = 50;\ 0 < x \leq 1$
$T_x(1, y) = 0;\ 0 < y < 1$

II. Solve the heat equation $T_t(x, t) = \alpha T_{xx}(x, t)$, for $0 \leq x \leq 1, 0 \leq t \leq 0.1$, with initial conditions $T(x, 0) = 3 - |3x - 1| - |3x - 2|$ for $0 \leq x \leq 1$ and $t = 0$ and the boundary conditions are:

$$T(0, t) = 0\ 0 \leq t \leq 0.1$$

$$T(1, t) = 0\ 0 \leq t \leq 0.1$$

utilize several different values for h and k and $\alpha k/h^2 = 0.25\ and\ 1$. Develop the Crank-Nicholson and FTCS method.

III. Solve the wave equation $u_{tt}(x, t) = 4u_{xx}(x, t)$, for $0 \leq x \leq 1, 0 \leq t \leq 1$ with the following initial and boundary conditions:

$$u(0, t) = 0 \text{ and } u(1, t) = 0 \text{ for } 0 \leq t \leq 1$$

$$u(x, 0) = \sin(2\pi x) + \sin(4\pi x) \text{ and } u_t(x, 0) = 0 \text{ for } 0 \leq x \leq 1$$

Choose different combinations of h and k and plot the solutions using *surf* and *contour*.