AM5600: Computational Methods in Mechanics (July-Nov. 2019)

Assignment #2

Due: At the beginning of class on Aug 23, 2019

1. Does $f(x) = x^{1/3}$ have a Taylor series expansion about $x_0 = 0$ & $x_0 = 1$? Justify your answers.
2. Let $L_0(x), L_1(x), \ldots, L_N(x)$ be the Lagrange coefficients for the Nth order polynomial based on the $N+1$ nodes $x_0, \ldots, x_N$. Show that $\sum_{k=0}^{N} L_k(x) = 1$ for any real number x.
3. Prove that the r^{th} divided difference (Δ^r) of $(1/x)$ can be expressed in the following manner assuming that $x_0, \ldots, x_N$ are equally spaced points at an interval of h:
$$\Delta^r\left(\frac{1}{x}\right) = \frac{(-1)^r r!\, h^r}{x(x+h)\ldots(x+rh)}$$
where, $\Delta\big(f(x)\big) = f(x+h) - f(x)$ and $\Delta^2\big(f(x)\big) = \Delta f(x+h) - \Delta f(x)$ and so on.
4. For the given data:

x	1	2	2.5	3	4	5
$f(x)$	0	5	7	6.5	2	0

   Calculate $f(3.4)$ using Newton's interpolation for polynomials of the order 1 through 3. Choose your base points to attain best accuracy $\{f_{best}(3.4) = 4.824832\}$. What is the order of polynomial could have been used to generate this data?

5. For the given data:

x	4.0	5.0	6.0	7.0	8.0
$f(x)$	1.58740105	1.709976	1.81712059	1.912931	2.0000

   a. Compute the divided difference table for the Newton's interpolation.
   b. Write down the Newton polynomials $P_1(x), P_2(x), P_3(x)$ *and* $P_4(x)$.
   c. Using the 4^{th} order polynomial, approximate the value of $f(4.5)$.
   d. Compare the findings in (b) with $f(x) = x^{1/3}$

AM5801/AM5810: Computational Lab (optional for students crediting AM5600)

Due: At the end of lab on Aug 28, 2019

I. Write a MATLAB code to find the Lagrange polynomial for problem 5 from the previous section. Plot the graph of each of the polynomials $P_1(x), P_2(x), P_3(x)$ *and* $P_4(x)$ and compare with the actual function. Furthermore, compute the total number of mathematical operations required for each of the polynomials.

II. Repeat problem I using the Newton's polynomial interpolation. Furthermore, compute the total number of mathematical operations required for each of the polynomials.

Note: *The MATLAB codes should be general enough to perform any order of interpolation.*