AM5600: Computational Methods in Mechanics (July-Nov. 2019)
Assignment #4
Due: At the beginning of class on Sep. 17, 2019

1. Let $g(x) = x^3\sin(x)$. Solve $x = g(x)$ and find all the fixed points of $g(x)$ for $x \in [-5,5]$. Can fixed-point iteration be used to find all the solutions?

2. Let $f(x) = (x - 2)^4$. Find the roots of $f(x)$ using Newton Raphson with an initial guess of $x_0 = -0.9$ till the true error $\left(\varepsilon_t = \left|\frac{p-x_i}{p}\right|\right)$ falls below 10^{-2} (where, p is the exact root and x_i is the approximation of the root on the ith iteration). Find the order convergence (R). Discuss the observed order of convergence (R)?

3. Use bisection, Secant and Regula-Falsi methods to locate the root of $f(x) = x^{10} - 1$ for initial guess of $x = [0,1.3]$. Perform 5 iterations of each methods (quantify the true error at each iteration). Discuss the observed results.

4. Use Newton Raphson to locate the root of $f(x) = x^{10} - 1$ with an initial guess of $x = 0.5$ for 5 iterations each. Discuss the observed results.

I. Write the MATLAB codes for finding multiple roots using Bisection and Regula-Falsi method for:
$$f(x) = x(16 - 2x)(10 - 2x) - 100, x \in [0,9]$$
First, an incremental search algorithm would be required to locate approximate location of the various roots. In other words, discretize $x \in [0,9]$ with a fine resolution (example: define 100 points in the range) and utilize $f(x_i)f(x_{i-1}) < 0$ root lies between x_{i-1} and x_i. Plot the true error vs iteration for both the methods using a semi-log plot. Finally, find the order of convergence (R) for both the methods (refer Q1 for details).

II. Write the MATLAB codes for finding all the roots using Newton-Raphson and Secant method for:
$$f(x) = x(16 - 2x)(10 - 2x) - 100, x \in [0,9]$$
Plot the true error vs iteration for both the methods using a semi-log plot. Find the order of convergence (R) for both the methods (refer Q1 for details). Comment: do the methods converge for different initial guess (Hint plot $f'(x)$, $x \in [0,9]$). Next, for $f(x) = x^{10} - 1$ find the order of convergence (R) for both Newton-Raphson and Secant method.

Note: *The MATLAB codes should be written in a generalized manner, such that other problems can be solved without major modifications.*