## AM5600: Computational Methods in Mechanics (July-Nov. 2019)

## Assignment #3

## Due: At the beginning of class on Sep 2, 2019

1. Show that the triangular factorization is unique. In other words, If **A** is non-singular then **L** and **U** are unique upper and lower triangular matrices.
2. Find the triangular factorization $\boldsymbol{A} = \boldsymbol{LU}$ for the matrix below:

$$A = \begin{bmatrix} -5 & 2 & -1 \\ 1 & 0 & 3 \\ 3 & 1 & 6 \end{bmatrix}$$

3. Determine if $\boldsymbol{A}$ is a strictly diagonally dominant matrix (perform pivoting if necessary):

$$A = \begin{bmatrix} 3.02 & -1.05 & 2.53 \\ 4.33 & 0.56 & -1.78 \\ -0.83 & -0.54 & 1.47 \end{bmatrix}$$

   Similarly, find the condition number of $\boldsymbol{A}$ ($Cond[\boldsymbol{A}] = \|\boldsymbol{A}\|.\|\boldsymbol{A}^{-1}\|$, where $\|\boldsymbol{A}\| = \max_{1 \le i \le n} \sum_{i=1}^{n} |a_{ij}|$ quantify the condition number for $\boldsymbol{A}$. Can we solve $\boldsymbol{AX} = \boldsymbol{b}$ to obtain unique solutions by using any of the iterative schemes?
4. a) Prove that $Cond[\boldsymbol{A}] \geq 1$ for any square matrix. Are there any exceptions to this? b) Find a $2 \times 2$ matrix $\boldsymbol{A}$ which has $Cond[\boldsymbol{A}] = 169$?
5. Find the solution to the following system of equations:

$$4x_1 + x_2 - x_3 = 13$$
$$x_1 - 5x_2 - x_3 = -8$$
$$2x_1 - x_2 - 6x_3 = -2$$

6. $\boldsymbol{A} = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$ is singular and almost diagonally dominant matrix for $\boldsymbol{AX} = \boldsymbol{0}$
   a.) Perform 5 iterations of Jacobi method with these initial guesses [1,1], [1,-1] and [-1,1].
   b.) Repeat part (a) with Gauss Seidel method.
   c.) Now change $a_{12} = a_{21} = -2.99$ and repeat part (a) and comment on your findings.

**AM5801/AM5810: Computational Lab (optional for students crediting AM5600)**

**Due: At the end of lab on Sep 18, 2019**

I. Write a MATLAB code using the Jacobi iterative method for solving the following system of equations:

$$3x_1 - 0.1x_2 - 0.2x_3 = 7.85$$
$$0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$
$$0.3x_1 - 0.2x_2 + 10x_3 = 71$$

*Convergence Criteria:* $tolerance > \sum_{i=1}^{n} \left| \frac{x_i^{(k)} - x_i^{(k-1)}}{x_i^{(k)}} \right|$ *& tolerance = 1e-12 (k = iteration counter).*

II. Solve the following equation using Gauss Seidel iterative method:

$$x_1 + (1/2)x_2 + (1/3)x_3 + (1/4)x_4 = (25/12)$$

$$(1/2)x_1 + (1/3)x_2 + (1/4)x_3 + (1/5)x_4 = (77/60)$$

$$(1/3)x_1 + (1/4)x_2 + (1/5)x_3 + (1/6)x_4 = (57/60)$$

$$(1/4)x_1 + (1/5)x_2 + (1/6)x_3 + (1/7)x_4 = (319/420)$$

Use only three significant digits in your arithmetic operations to find the solution. Next, find the solution using 6 significant digits and compare with earlier results.

*Convergence Criteria:* $tolerance > \sum_{i=1}^{n} \left| \frac{x_i^{(k)} - x_i^{(k-1)}}{x_i^{(k)}} \right|$ *& tolerance = 1e-12 (k = iteration counter).*

III. Write a program for triangular factorization ($A = LU$) algorithm of the matrix below:

$$A = \begin{bmatrix} 2 & -1 & 3 & 2 \\ 2 & 2 & 0 & 4 \\ 1 & 1 & -2 & 2 \\ 1 & 3 & 4 & -1 \end{bmatrix}$$