AM5600: Computational Methods in Mechanics (July-Nov. 2019)

Assignment #5

Due: At the beginning of class on Oct. 4, 2019

1. Use the Taylor series / polynomial interpolation expansions for equally spaced points along x with step size h. Derive the backward difference formula that is $O(h^2)$, truncation error and optimal step size, assuming ε as the maximum round off error.

   $f^{(2)}(x_0) = [2f_0 - 5f_{-1} + 4f_{-2} - f_{-3}]/(h^2)$, where $f_{-1} = f(x_0 - h)$ and so on.

2. Find $f'(x)$ for $f(x) = e^x$ @ $x = 2$ for a step size of $h = 0.1$ using central difference approximation of $O(h^2)$. Repeat calculation by multiplying the step size by 1/5 till $h = 0.001$. Comment on the optimal step size for approximating $f'(x)$ @ $x = 2$ by finding the absolute error during numerical differentiation.

3. Compute the first-order central difference approximation of $O(h^4)$ for the following function at $x = 0.4$ for a step size of $h = 0.1$:

   $$y = x^2 \cos(x)$$

   Compare your findings with the analytical solution.

4. Evaluate the integral for the following data using composite trapezoidal and Simpson's 1/3rd rule:

x	0	0.1	0.2	0.3	0.4
$f(x)$	1	8	4	3.5	5

   Compare and comment on the findings.

5. Determine the number of intervals (M) and width (h) such that: composite trapezoidal method; and b) composite Simpson's 1/3 rule $(2M\ subintervals)$ can be used to compute the integral below with an accuracy of 5×10^{-9}.

   $$I = \int_{-\pi/6}^{\pi/6} \cos(x)dx$$

6. Evaluate the integral below using Gauss 2- and 3-point quadrature methods

   $$I = \int_{-3}^{3} \frac{1}{1 + x^2} dx$$

   Compare the results with exact values.

AM5801/AM5810: Computational Lab (optional for students crediting AM5600)

Due: At the end of lab on Oct. 9, 2019

I. Write the MATLAB code for finding the numerical derivate of any general function $f(x)$ using central difference approximation of $O(h^2)$. The implementation should incorporate a numerical scheme to find the optimum step size (h) (*Hint: Carryout numerical differentiation with decreasing step size till absolute error reaches the minima*). Compare with forward and backward difference approximation of $O(h)$. Note: $f'(x)$ should be returned as a vector for analysis.

II. Write the MATLAB codes for carrying out numerical integration using composite trapezoidal and Simpson's 1/3 methods, 2- *and* 3- point Gauss quadrature (including the change of integration limits). Comment on number of function evaluations, error and optimum step size (h).