Robust Subspace Tracking

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1 Introduction

It is often desired to reduce redundancy in data or remove dimensions that don't provide any additional information. PCA is used in such scenarios, which can solved using SVD. But the problem becomes more challenging when the data is by corrupted outliers. PCA in the presence of outliers is called Robust PCA.

One way to approach the Robust PCA problem is to decompose the data matrix(M) into low-rank matrix(L) and sparse matrix(S). Here the low-rank matrix represents the estimate of the outlier-free data and the sparse matrix is the outlier matrix.

$$M = L + S + W,$$

where W is noise.

When the data sequence is too long and the subspace it lies in is changing, we might not be able to do dimensionality reduction. As the whole data collectively might be high dimensional. It can alternatively be modelled as data sequence with sparse outliers, lying in a slowly changing low-dimensional subspace. The problem of tracking this slowly changing subspace is called Robust Subspace Tracking(RST) or Dynamic Robust PCA.

2 Problem Statement

The objective of this assignment is Foreground Background separation in videos. We formulate this as a RST problem.

$$m_t = l_t + s_t + w_t,$$

where t is the frame number, m_t is the image, l_t is the background, s_t is the foreground and w_t is noise.

Things to be estimated:

1. l_t background at each frame.
2. s_t foreground at each frame.

3. P_t subspace estimate at each frame given an initial estimate.

The initial estimate of subspace can be obtained using RPCA on first few frames.
Input : Video
Output : Foreground video and Background video
Link to Example

3 Evaluation

The following evaluation metrics can be used to evaluate the algorithm.

1. Subspace Error : Distance between two subspaces spanned by columns of matrices A and B is as follows.

$$SE(A, B) = ||(I - AA')B||_2$$

2. Relative error in the recovered true data or the low-rank estimate.

$$RE(l_t, \hat{l_t}) = \frac{||\hat{l_t} - l_t||_2^2}{||l_t||_2^2}$$

Evaluate your code on a synthetic dataset:

1. Generate an orthonormal matrix P of size nxr, where n is the length of data vector and r is the desired subspace rank.

2. Generate data vectors by sampling from the subspace and adding sparse outliers.

3. Plot subspace error between estimated subspace and the actual subspace, and relative error in the low-rank estimate at each frame.

Link to Real datasets

4 References

1. P. Narayanamurthy and N. Vaswani, "A Fast and Memory-Efficient Algorithm for Robust PCA (MEROP)," 2018 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Calgary, AB, 2018, pp. 4684-4688.)

2. N. Vaswani, T. Bouwmans, S. Javed and P. Narayanamurthy, "Robust Subspace Learning: Robust PCA, Robust Subspace Tracking, and Robust Subspace Recovery," in IEEE Signal Processing Magazine, vol. 35, no. 4, pp. 32-55, July 2018.