Assignment 4

General Instructions

- Solutions due by Submission due by 1^{st} October 2019 for Batch I (Dr. Tiwari's class); 4^{th} October 2019 for Batch II (Dr. Pallab's class).
- Hand calculations can be done as homework and should be submitted as a report.

Hand calculations

1. The equation $lnx = x^2 - 1$ has exactly two real roots, $\alpha_1 = 0.45$ and $\alpha_2 = 1$. Determine for which initial approximation x_0, the iteration $x_{n+1} = \sqrt{1 + lnx_n}$ converges to α_1 and α_2.

2. Find the interval in which the smallest positive root of the following equations lies,

   (a) $tanx + tanhx = 0$. Take the initial guess as $(0, \pi/2)$.

   (b) $x^3 - x - 4 = 0$. Take the initial guess as $(1, 2)$.

   Determine the roots correct to two decimal places using bisection method.

3. For the following equations,

   (a) $x^4 - x - 10 = 0$, take the initial guess as $(1, 2)$.

   (b) $x - e^{-x} = 0$, take the initial guess as $(0, 1)$.

   Determine the smallest positive root correct to three decimal places using Regula-Falsi and Secant methods.

4. Consider a function $f(x)$ defined in $[0, 2\pi]$ as

$$f(x) = \begin{cases} -1 & x \in [-4, 0] \\ x - 1 & x \in [0, 2] \\ 1 & x \in [2, 4] \end{cases}$$

   Sketch the function f vs x. What is the appropriate method to be used to solve for the root of $f(x) = 0$ and why? Perform five iterations using that method and report the solution at the end of each iteration.

5. A good approximation for $n!$ is given by the function

$$f(x) = (2\pi)^{1/2} x^{x+1/2} e^{-x} \left(1 + \frac{1}{12x} + \frac{1}{288x^2}\right)$$

   Though $n!$ is defined for integer values of n, for large n, the approximation is very close. For this $f(x)$, if $f(x) = 1000$, calculate the value of x accurate up to 4 decimal places.

6. Determine the order of convergence of the iterative method

$$x_{k+1} = \frac{x_0 f(x_k) - x_k f(x_0)}{f(x_k) - f(x_0)}$$

   for finding a simple root of the equation $f(x) = 0$.

7. Calculate all the solutions of the system,

$$x^2 + y^2 = 1.12 \tag{1}$$

$$xy = 0.23 \tag{2}$$

   correct to three decimal places. Start with initial guess of $x_0 = 1$ and $y_0 = 0.23$.

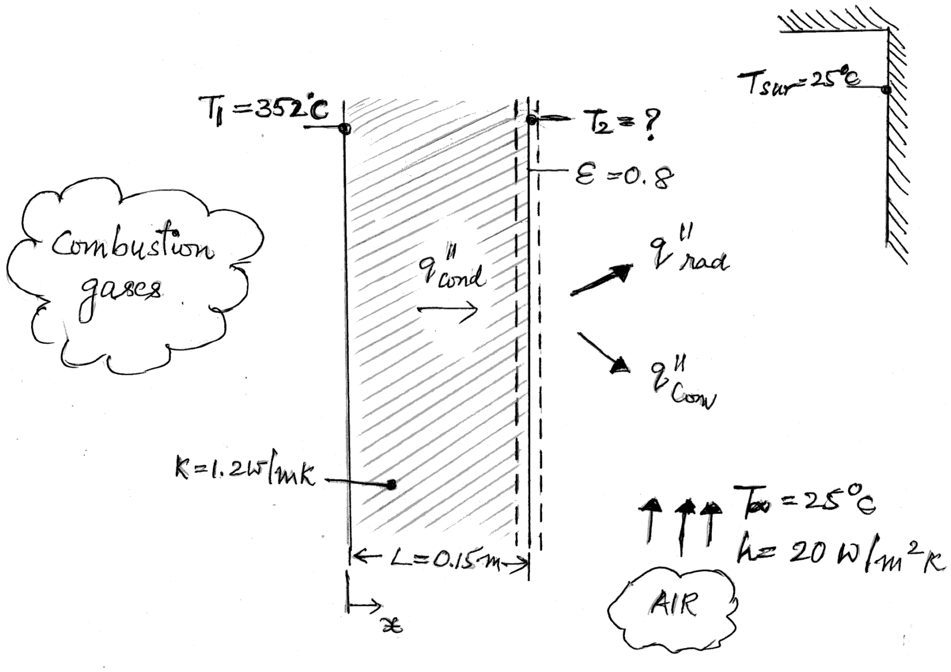

Figure 1: A schematic of the furnace indicating several heat transfer processes.

8. The hot combustion gases of a furnace are separated from the ambient air and its surroundings, as shown in figure 1, which are at 25^oC, by a brick wall 0.15 m thick. The brick has a thermal conductivity of 1.2 W/m.K and a surface emissivity of 0.8. Under steady-state conditions an inner surface temperature of 352^oC is measured. Free convection heat transfer to the air adjoining the surface is characterised by a convection coefficient of $h = 20$ W/m^2. K. Calculate the brick outer surface temperature T_2 using Newoton's method by performing 5 iterations with an initial guess of 1000 K. An energy balance on the outer surface of the furnace yields the following equation:

$$k\frac{T_1 - T_2}{L} = h(T_2 - T_\infty) + \epsilon\sigma(T_2^4 - T_{sur}^4) \tag{3}$$

where $\sigma = 5.67 \times 10^{-8}$ W/m^2 K^4 is the Stefan-Boltzmann constant.

Programming

1. Show that the equation

$$f(x) = cos\left(\frac{\pi(x+1)}{8}\right) + 0.148x - 0.9062 = 0 \tag{4}$$

has one root at the interval $(-1, 0)$ and one in $(0, 1)$. Calculate the negative root correct to six decimals after 10 iterations.

2. The equation

$$2e^{-x} = \frac{1}{x+2} + \frac{1}{x+1} \tag{5}$$

has two roots greater than -1. Calculate these roots correct to five decimal places.

3. Solve the system of non-linear equations given below with two starting guesses - $x = [0.1,\ 1.2,\ 2.5]^T$ and $x = [1,\ 0,\ 1]^T$. Do the two solutions converge to the same root? If not, why?

$$x + y + z = 3 \tag{6}$$
$$x^2 + y^2 + z^2 = 5 \tag{7}$$
$$e^x + xy - xz = 1 \tag{8}$$