Problem 4.

$$(1+x)^n + (1-x)^n = \sum_{k=0}^{n}\binom{n}{k}x^k + \sum_{k=0}^{n}(-1)^k\binom{n}{k}$$

There are two cases.

First, n is even.

Let, $n=2k$

$$(1+x)^n + (1-x)^n = 2[\binom{n}{0} + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n]$$

$$\therefore \frac{(1+x)^n + (1-x)^n}{2} = \binom{n}{0} + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n$$

Second, n is odd.

Let, $n=2k+1$

$$(1+x)^n + (1-x)^n = 2[\binom{n}{0} + \binom{n}{2}x^2 + \cdots + \binom{n}{n-1}x^{n-1}]$$

$$\therefore \frac{(1+x)^n + (1-x)^n}{2} = \binom{n}{0} + \binom{n}{2}x^2 + \cdots + \binom{n}{n-1}x^{n-1}$$

The arithmetic mean of $(1+x)^n$ and $(1-x)^n$ is a positive integer.