

Problem 1.

To simplify this problem, let's consider a party of 3 people, denote A, B, C. The number of pairs of people shake hands in this party is $\binom{3}{2} = 3$. In other words, there are three pairs, AB, AC and BC. Now we can classify distinct parties according to the number of handshakes.

$$\text{i) } \#handshakes = 0 \Rightarrow \binom{3}{0}$$

$$\text{ii) } \#handshakes = 1 \Rightarrow \binom{3}{1}$$

$$\text{iii) } \#handshakes = 2 \Rightarrow \binom{3}{2}$$

$$\text{iv) } \#handshakes = 3 \Rightarrow \binom{3}{3}$$

Since $\sum_{k=0}^n \binom{n}{k}$ is 2^n , $\binom{3}{0} + \binom{3}{1} + \binom{3}{2} + \binom{3}{3} = \sum_{k=0}^3 \binom{3}{k} = 2^3 = 8$.

Hence, there are 8 distinguish parties.

Now, in our problem, The number of pairs of people shake hands in this party is $\binom{10}{2} = 45$.

We can classify distinct parties according to the number of handshakes.

$$1) \#handshakes = 0 \Rightarrow \binom{45}{0}$$

$$2) \#handshakes = 1 \Rightarrow \binom{45}{1}$$

\vdots

$$45) \#handshakes = 45 \Rightarrow \binom{45}{45}$$

Therefore, the number of possible parties is $\sum_{k=0}^{45} \binom{45}{k} = 2^{45}$.