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MAT 512 Prof. Hurlbert

Daily HW 8/29/09

5 **Problem 1** *Use three of the IMPORTANT IDENTITIES to prove that*

$$\sum_{k=0}^{38204629939869} \frac{\binom{38204629939869}{k}}{\binom{76409259879737}{k}} = 2$$

Proof. Since numbers are too large, we try to generalize it. Both two numbers are odd numbers with a relation 76409259879737 = 2 $\times$ 38204629939869 -1. So the generalized version of this problem is to prove that $\sum_{k=0}^{2m+1} \frac{\binom{2m+1}{k}}{\binom{4m+1}{k}} = 2$.

To prove generalied version of this problem, we use this identity

$$\sum_{i=0}^{k} \binom{n+i}{i} = \binom{n+k+i}{k} \cdots (*)$$

Let's start

$$\sum_{k=0}^{2m+1} \frac{\binom{2m+1}{k}}{\binom{4m+1}{k}} = 1 + \sum_{k=1}^{2m+1} \frac{\binom{2m+1}{k}}{\binom{4m+1}{k}}$$

$$= 1 + \sum_{k=1}^{2m+1} \frac{(4m+1-k)!(2m+1)!}{(2m+1-k)!(4m+1)!}$$

$$= 1 + \frac{(2m+1)!}{(4m+1)!} \sum_{k=1}^{2m+1} \frac{(4m+1-k)!}{(2m+1-k)!}$$

$$= 1 + \frac{(2m+1)!}{(4m+1)!} \sum_{k=1}^{2m+1} \frac{(4m+1-k)!(2m)!}{(2m+1-k)!(2m)!}$$

$$= 1 + \frac{(2m+1)!}{(4m+1)!} \sum_{k=1}^{2m+1} \binom{4m+1-k}{2m} (2m)!$$

$$= 1 + \frac{(2m+1)!(2m)!}{(4m+1)!} \sum_{k=1}^{2m+1} \binom{4m+1-k}{2m}$$

$$
\begin{aligned}
&= 1 + \frac{1}{\binom{4m+1}{2m}} \sum_{k=1}^{2m+1} \binom{4m+1-k}{2m} \\
&= 1 + \frac{1}{\binom{4m+1}{2m}} \sum_{i=0}^{2m} \binom{2m+i}{i} (by\ inverse\ order) \\
&= 1 + \frac{1}{\binom{4m+1}{2m}} \binom{4m+1}{2m} (\because (*)) \\
&= 2
\end{aligned}
$$

$\square$