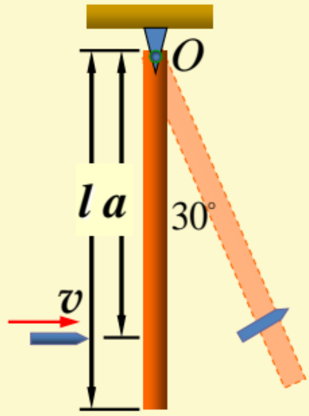
大题

例3-11 一长为l、质量为M的杆可绕支点O自由转动. 一质量为m、速度为v的子弹射入距支点为a的棒内. 若棒偏转角为30°, 问子弹的初速度是多少?



解 碰撞过程角动量守恒

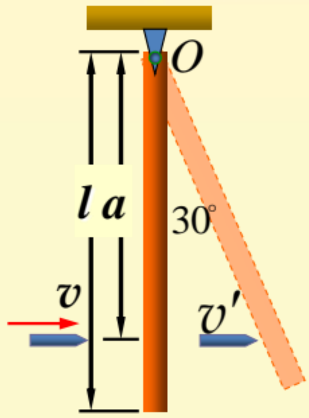
$$mva = \underbrace{\left(\frac{1}{3}Ml^2 + ma^2\right)}_{\text{the new } J}\omega$$

向上偏转过程机械能守恒

$$\frac{1}{2}\left(\frac{1}{3}Ml^2 + ma^2\right)\omega^2 = mga(1-\cos 30°) + Mg\frac{l}{2}(1-\cos 30°)$$

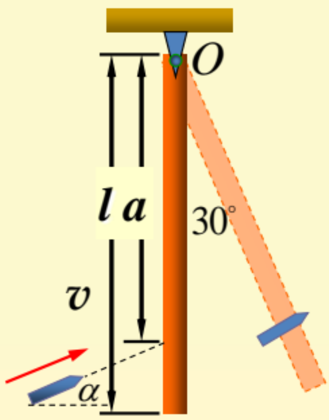
$$v = \frac{1}{ma}\sqrt{\frac{g}{6}(2-\sqrt{3})(Ml+2ma)(Ml^2+3ma^2)}$$

变形问题:



$$mva = \frac{1}{3}M\,l^2\omega + mv'a$$

$$\frac{1}{2}\frac{1}{3}M\,l^2\omega^2 = Mg\frac{l}{2}(1-\cos 30°)$$



$$mva\cos\alpha = \left(\frac{1}{3}M\,l^2 + ma^2\right)\omega$$

$$\frac{1}{2}\left(\frac{1}{3}M\,l^2 + ma^2\right)\omega^2$$

$$= mga(1-\cos 30°) + Mg\frac{l}{2}(1-\cos 30°)$$

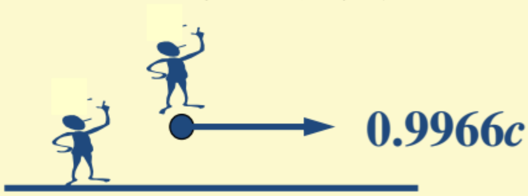
例5-4 静系中μ子的平均寿命为2.2×10^{-6}s. 据报导, 在一组高

例5-4 静系中μ子的平均寿命为2.2×10^{-6}s. 据报导, 在一组高能物理实验中, 当它的速度为u=0.9966c时, 通过的平均距离为8km. 试说明这一现象: (1) 用经典力学计算与上述结果是否一致; (2) 用时间膨胀说明; (3) 用尺缩效应说明.

解 (1) 按经典力学

$$l = u\tau = 3\times10^{8}\times2.2\times10^{-6}\,\mathrm{m} = 660\,\mathrm{m}$$ 不符合事实

(2) 本征寿命: $\tau_0=2.2\times10^{-6}$s



$\boxed{\text{实验室测其寿命:}}$

$$\tau = \frac{\tau_0}{\sqrt{1-u^2/c^2}} = \frac{2.2\times10^{-6}}{\sqrt{1-0.9966^2}}\,\mathrm{s} = 26.7\times10^{-6}\,\mathrm{s}$$

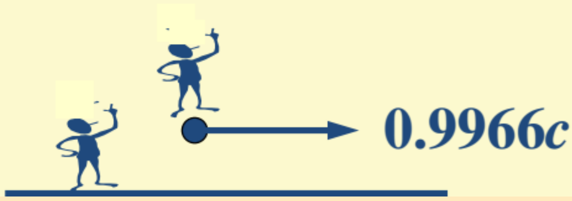
$$l = u\tau = 3\times10^{8}\times26.7\times10^{-6}\,\mathrm{m} \approx 8\times10^{3}\,\mathrm{m}$$

与平均距离一致

(3) $\boxed{\text{μ子参考系测实验室距离:}}$

$$l = l_0\sqrt{1-u^2/c^2} = 8\times10^{3}\times\sqrt{1-0.9966^2}\,\mathrm{m}$$

$$= 0.66\times10^{3}\,\mathrm{m}$$

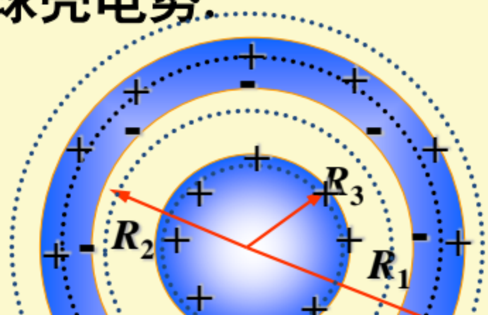


考试题球壳上不带电，只有感应电荷

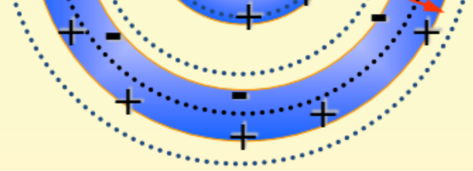
例6-11 有一外半径R_1、内半径R_2的金属球壳, 其中放一半径为R_3的金属球, 球壳和球均带有电量10^{-8}C的正电荷. 求: (1) 两球电荷分布; (2) 球心的电势; (3) 球壳电势.

解: (1) 电荷分布如图所示

球面q, 壳内表面$-q$,壳外表面$2q$



$$\vec{E}_3 = 0 \qquad (r < R_3)$$



$$E_2 = \frac{q}{4\pi\varepsilon_0 r^2} \quad (R_3 < r < R_2)$$

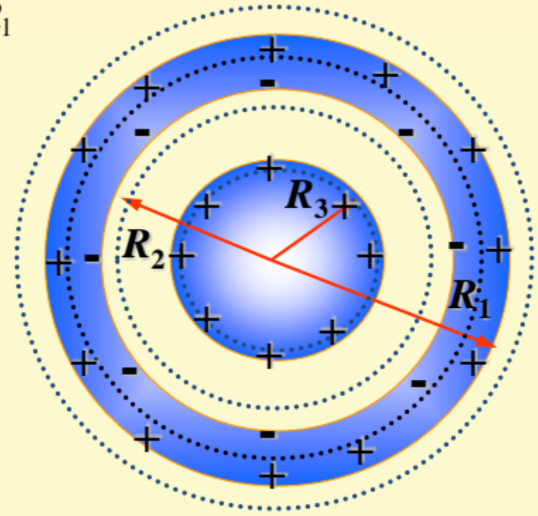
$$E_1 = 0 \qquad (R_2 < r < R_1) \qquad E_0 = \frac{2q}{4\pi\varepsilon_0 r^2} \quad (r > R_1)$$

(2) $$U_o = \int_0^\infty \vec{E}\cdot d\vec{l} = \int_0^{R_3} + \int_{R_3}^{R_2} + \int_{R_2}^{R_1} + \int_{R_1}^{\infty}$$

$$= \int_{R_3}^{R_2} E_2 dr + \int_{R_1}^{\infty} E_0 dr$$

$$= \int_{R_3}^{R_2} \frac{q\,dr}{4\pi\varepsilon_0 r^2} + \int_{R_1}^{\infty} \frac{2q\,dr}{4\pi\varepsilon_0 r^2}$$

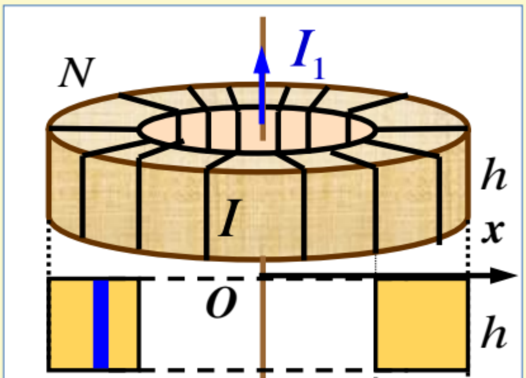
$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R_3} - \frac{1}{R_2} + \frac{2}{R_1}\right)$$



(3) $$U_1 = \int_{R_1}^{\infty} \frac{2q}{4\pi\varepsilon_0 r^2} dr = \frac{2q}{4\pi\varepsilon_0 R_1}$$

考试题求的是L，没有中间的直导线

例8-12 矩形截面螺绕环尺寸如图，密绕N匝线圈，其轴线上置一无限长直导线. 当螺绕环中通有电流 $I = I_0 \cos\omega t$ 时，直导线中的感生电动势为多少？

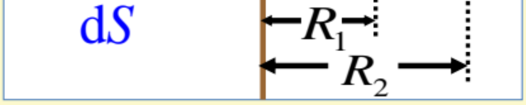


解一 互感问题先求M

建立坐标系Ox

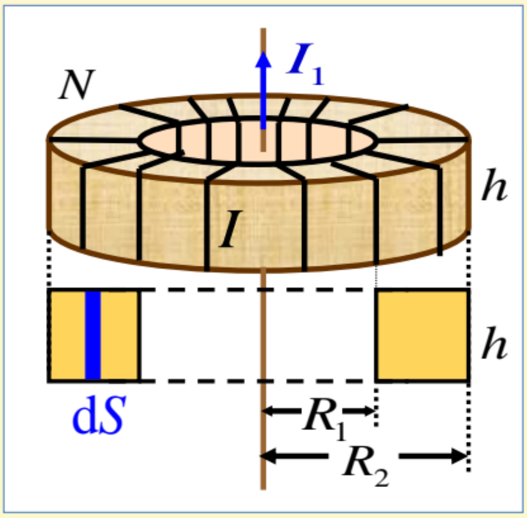
设直导线中通有电流I_1

$$B = \frac{\mu_0 I_1}{}$$



$$B_1 = \frac{\quad}{2\pi x}$$

$$\Psi_{21} = N\Phi_{21} = N\int_{S_2}\vec{B}_1\cdot \mathrm{d}\vec{S} = \frac{N\mu_0 I_1 h}{2\pi}\int_{R_1}^{R_2}\frac{\mathrm{d}x}{x} = \frac{N\mu_0 I_1 h}{2\pi}\ln\frac{R_2}{R_1}$$



$$\Psi_{21} = \frac{N\mu_0 I_1 h}{2\pi}\ln\frac{R_2}{R_1}$$

$$M = \frac{\Psi_{21}}{I_1} = \frac{\mu_0 N h}{2\pi}\ln\frac{R_2}{R_1}$$

$$\varepsilon_1 = -M\frac{\mathrm{d}I_2}{\mathrm{d}t} = -\frac{\mu_0 N h}{2\pi}\ln\frac{R_2}{R_1}\frac{\mathrm{d}}{\mathrm{d}t}\quad (I = I_0\cos\omega t)$$

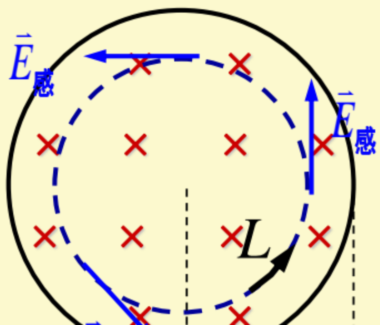
$$= \frac{\mu_0 N h I_0\omega}{2\pi}\ln\frac{R_2}{R_1}\cdot\sin\omega t$$

1. 磁能密度：磁场单位体积内的能量

$$w_{\mathrm{m}} = \frac{W_{\mathrm{m}}}{V} = \frac{B^2}{2\mu} = \frac{1}{2}BH$$

例8-5 已知半径为R的长直螺线管中的电流随时间变化，若管内磁感应强度随时间增大，即 $\frac{\mathrm{d}B}{\mathrm{d}t}$ = 恒量 > 0，求感生电场分布.

解 选择一回路L，逆时针绕行

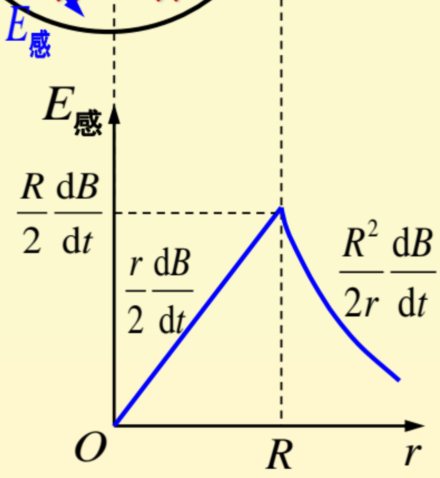


$$\oint_L \vec{E}_{感}\cdot\mathrm{d}\vec{l} = -\iint_S \frac{\mathrm{d}\vec{B}}{\mathrm{d}t}\cdot\mathrm{d}\vec{S}$$

$$E_{感} 2\pi r = \iint_S \frac{dB}{dt} dS$$

$$r < R, E_{感} 2\pi r = \frac{dB}{dt}\pi r^2 \qquad E_{感} = \frac{r}{2}\frac{dB}{dt}$$

$$r > R, E_{感} 2\pi r = \frac{dB}{dt}\pi R^2 \qquad E_{感} = \frac{R^2}{2r}\frac{dB}{dt}$$



8.3.3 感生电动势的计算

1. 定义求解: $\varepsilon_i = \oint_L \vec{E}_{感} \cdot d\vec{l}$

某道填空题

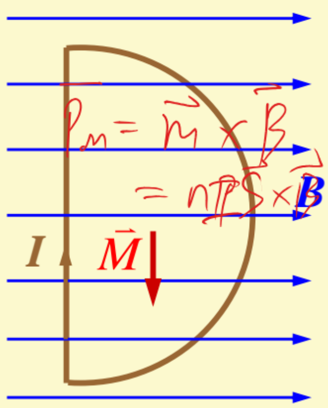
例7-12 一半径为R的闭合载流线圈，载流I，放在磁感应强度为B的均匀磁场中，其方向与线圈平面平行.

(1) 求以直径为转轴、线圈所受磁力矩的大小和方向.

(2) 线圈在力矩作用下转过90°，力矩做了多少功?

解法一 $\vec{M} = \vec{P}_m \times \vec{B} \qquad M = P_m B \sin\frac{\pi}{2}$

$$P_m = I \cdot \frac{\pi R^2}{2} \qquad M = \frac{1}{2}\pi IBR^2$$



线圈转过90°，磁通量增量为

$$\Delta\Phi_m = \frac{\pi R^2}{2}B \Rightarrow A = I\Delta\Phi_m = \frac{\pi R^2}{2}IB$$

某道填空题

例6-14 求半径R的孤立金属球的电容.

解 设其带电量为Q，令$U_\infty = 0$ 则金属球电势：$U = \dfrac{Q}{4\pi\varepsilon_0 R}$

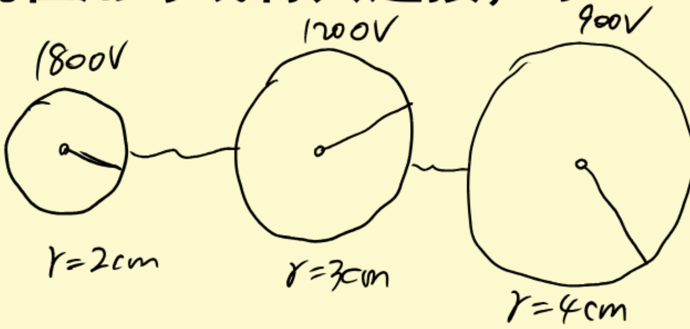
由电容定义： $C = \dfrac{Q}{U} = 4\pi\varepsilon_0 R$

9 三个导体球相距非常远，半径分别为2cm，3cm，4cm；电势分比为1800V，1200V，900V· 现在用导线将其连接，求：

（1）三球的总带电量

（2）连接后的电势

（3）总电容



(1) $Q_1 = CU = 4\pi\varepsilon_0 R_1 \cdot U_1 \quad Q_2 = 4\pi\varepsilon_0 R_2 \cdot U_2 \quad Q_3 = 4\pi\varepsilon_0 R_3 U_3$

$Q_总 = 4\pi\varepsilon_0 \sum_{i=1}^{3} R_i U_i$

(2) $U = \dfrac{Q_总}{C_总} = \dfrac{4\pi\varepsilon_0 \sum_{i=1}^{3} R_i U_i}{4\pi\varepsilon_0 (R_1 + R_2 + R_3)} = \dfrac{R_1 U_1 + R_2 U_2 + R_3 U_3}{R_1 + R_2 + R_3}$

(3) $C_总 = C_1 + C_2 + C_3 = 4\pi\varepsilon_0 (R_1 + R_2 + R_3)$

某道填空题

$F = -\nabla E_p$

11. 已知双原子分子的原子之间相互作用势能函数为

$$E_p = \frac{A}{x^{12}} - \frac{B}{x^6}$$

$(x^{-12})' = -12x^{-11}$

其中A，B都是常量，x为原子间的距离。试求

（1）原子间作用力的函数

（2）原子间的相互作用为0时的距离

$F = -\nabla E_p$

(1) $F = -\dfrac{dE_p}{dx} = -\left(-12x^{-11} \cdot A + (-6)x^{-5} \cdot B\right)$

$F = 12Ax^{-13} + 6Bx^{-7}$

Kami

(2) $F=0$ $\qquad \frac{\cancel{12}^{2}A}{x^{11}} = \frac{\cancel{6}B}{x^5}$ $\qquad 2x^5 A = x^{11} B$

$$x^6 = \frac{2A}{B}$$

$$x = \sqrt[6]{\frac{2A}{B}}$$

某道填空题

$$\oint_S \vec{B} \cdot \mathrm{d}\vec{S} = 0$$

$$\oint_L \vec{H} \cdot \mathrm{d}\vec{l} = \oint_S (\underbrace{\vec{\delta}}_{\vec{j}} + \frac{\partial \vec{D}}{\partial t}) \cdot \mathrm{d}\vec{S}$$

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$$

$$\vec{B} = \mu_0 \mu_r \vec{H}$$