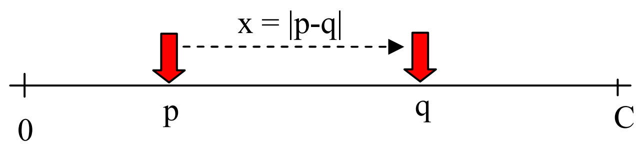
Elementary Disk Performance Models

Dr. Jozo Dujmović

1. Movement of disk head

Suppose that the disk head mechanism can travel the maximum distance of *C* cylinders. The head moves from a random position *p* to a random position *q* as follows:



The distance traveled is x=|p-q|. Assuming the uniform distribution of p and q, the mean distance traveled is

$$\bar{D} = \frac{1}{c^2}\int_0^C dp \int_0^C |\, p-q \,|\, dq = \frac{1}{C^2}\int_0^C dp\left[\int_0^p (p-q)dq + \int_p^C (q-p)dq\right] =$$

$$= \frac{1}{C^2}\int_0^C dp\left[\left(p^2 - \frac{p^2}{2}\right) + \left(\frac{C^2}{2} - \frac{p^2}{2} - Cp + p^2\right)\right] =$$

$$= \frac{1}{C^2}\int_0^C \left(\frac{C^2}{2} - Cp + p^2\right)dp = \frac{1}{C^2}\left(\frac{C^3}{2} - \frac{C^3}{2} + \frac{C^3}{3}\right) = \frac{C}{3}$$

So, the head mechanism travels on the average one third of existing cylinders.

2. The simplest linear model

The minimum number of cylinders the head mechanism can travel is 1, and the corresponding seek time is $t_{\min}$. Let *C* be the maximum number of cylinders the head can travel, and let $t_{\max}$ be the maximum travel time. The simplest model of access motion (seek) time is the linear model:

$$T_{seek}(x) = t_{\min} + \frac{t_{\max} - t_{\min}}{C-1}(x-1), \quad 1 \le x \le C$$

The mean seek time is now obtained for *x*=*C*/3 and can be approximated (taking into account that $C >> 1$ and $C/3 >> 1$) as follows:

$$\bar{T}_{seek} = T_{seek}\left(\frac{C}{3}\right) = t_{\min} + \frac{t_{\max} - t_{\min}}{C-1}\left(\frac{C}{3} - 1\right)$$
$$\approx t_{\min} + \frac{t_{\max} - t_{\min}}{3} = \frac{2}{3}t_{\min} + \frac{1}{3}t_{\max}$$

So, if a disk manufacturer specifies $t_{\min} = 1.5$ ms and $t_{\max} = 9$ ms then $\bar{T}_{seek} = 4$ ms .

3. The simplest nonlinear model (square root model)

Suppose that the head mechanism has the mass *m* and that we apply a constant force *F* to move the head over *x* cylinders (from its current position to a destination position). According to Newton's law, the acceleration *a*, the speed *v*, and the position *x* are as follows:

$$a = \frac{F}{m} = \frac{dv}{dt}$$
$$v = \int a\,dt = at = \frac{dx}{dt}$$
$$x = \int v\,dt = \int at\,dt = a\frac{t^2}{2}$$

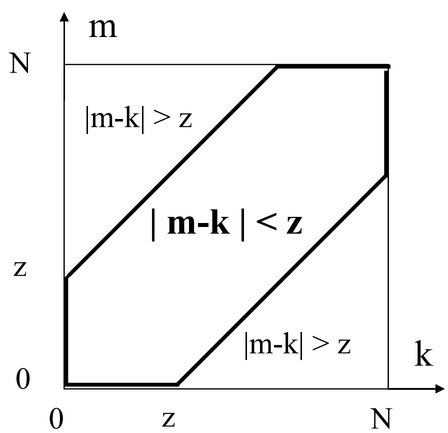
The movement of head over *x* cylinders takes the time T_{seek} . Ideally, a half of that time is spent in acceleration of mechanism and a half of that time is spent in deceleration. If the maximum distance traveled is $x_{\max} = C$ the seek time can be computed from $2x = at^2$ as follows:

$$2\frac{x}{2} = a\left(\frac{T_{seek}}{2}\right)^2$$
$$T_{seek}(x) = 2\sqrt{\frac{x}{a}}$$
$$t_{\max} = 2\sqrt{\frac{C}{a}}$$
$$\frac{T_{seek}(x)}{t_{\max}} = \sqrt{\frac{x}{C}} \quad \Rightarrow \quad T_{seek}(x) = t_{\max}\sqrt{\frac{x}{C}}$$

The time to travel the mean distance of *C*/3 cylinders is $T_{seek}(C/3) = t_{\max}/\sqrt{3} = 0.577t_{\max}$ but this is not the mean seek time. Computation of the mean seek time must be based on averaging all possible seek operations.

4. Computing the mean seek time

Suppose that a program processes a disk file that is located on N adjacent cylinders of disk and that all cylinders are accessed with equal probability. The distance s for traveling from cylinder m to cylinder k inside the file is $s=|m-k|$. The probability that the distance s is less than a given value z can be computed using the following geometric interpretation:



Since $0 \le m \le N$ and $0 \le k \le N$ it follows that the square with the area N^2 has three regions shown in the above figure, and the central region which has the area $N^2-(N-z)^2$ is the region where $s \le z$. In the case of uniform distribution of disk accesses we can compute the probability distribution for s and the corresponding probability density as follows:

$$\text{Probability}[s \le z] = P(z) = \frac{N^2-(N-z)^2}{N^2} = \frac{2NZ-z^2}{N^2}$$

$$p(z) = \frac{dP}{dz} = \frac{2}{N^2}(N-z)$$

Consequently, the mean seek time can be computed as the following mathematical expectation:

$$\overline{T}_{seek}(N) = \int_0^N T(z)\,p(z)\,dz = \frac{2}{N^2}\int_0^N T(z)\,(N-z)\,dz$$

For example, if $T_{seek}(z) = t_{\max}\sqrt{z/C}$ then the mean seek time as a function of file size N can be computed as follows:

$$\overline{T}_{seek}(N) = \frac{2}{N^2}\int_0^N T(z)\,(N-z)\,dz = \frac{2t_{\max}}{N^2\sqrt{C}}\int_0^N z^{1/2}(N-z)\,dz$$

$$= \frac{2t_{\max}}{N^2\sqrt{C}}\left[N\frac{2}{3}z^{3/2} - \frac{2}{5}z^{5/2}\right]_0^N = \frac{2t_{\max}\sqrt{N}}{\sqrt{C}}\left(\frac{2}{3}-\frac{2}{5}\right)$$

$$= \frac{8t_{\max}}{15}\sqrt{\frac{N}{C}} \quad , \quad 0 \le N \le C$$

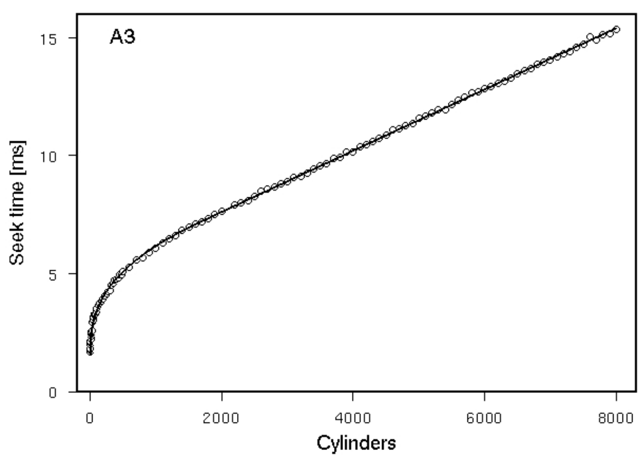
In the extreme case the file size can be equal to the disk size (N=C) and the corresponding mean disk seek time is

$$\overline{T}_{seek} = \frac{8t_{\max}}{15} = 0.533t_{\max}$$

It is interesting to note that the difference between the mean seek time and the seek time for the mean distance ($0.577t_{\max}$) is only 8.26%, i.e. in is not very significant.

5. An accurate exponential seek time model

The seek time characteristic of real disk units is exemplified in the following figure (for Quantum Atlas III):



The actual seek time characteristic consists of a nonlinear part (for small number of cylinders traveled) followed by a linear part (for large number of cylinders traveled, where the disk head moves at the constant coasting speed). Therefore, the disk seek time cannot be modeled using the square root characteristic, not and combination of square root and other functions because they cannot accurately model neither the nonlinear part nor the linear part of the disk seek time characteristic. We introduced an exponential model that uses the power function to accurately model the nonlinear segment of the seek time characteristic followed by the linear model, a follows

$$T_{seek}(x) = \begin{cases} t + c(x-1)^r & , \quad 1 \le x \le x^* \\ \\ \dfrac{cr(x-x^*)}{(x^*-1)^{1-r}} + t + c(x^*-1)^r & , \quad x \ge x^* \end{cases}$$

This model has four parameters: *t, c, r,* and *x**. The parameter *r* is the exponent that is usually less than 0.5 (i.e. different from the square root model). The meaning of *t* and c is the following: $t = T_{seek}(1) = t_{\min}$, and $T_{seek}(2) = t + c$. The parameter x^* denotes the critical distance of head movement which is the end of the nonlinear segment of the seek time characteristic and the beginning of the linear segment. These four parameters have to be adjusted so that they model the actual measured disk characteristic in an optimum way, as shown in the above figure for the Quantum Atlas III disk (where the average error is only 1.3%). For this disk we have the following characteristic values:

Number of cylinders $C = 8057$
Number of revolutions per minute $n = 7200$ rpm
Disk capacity = 9.1 MB
Minimum seek time $t = t_{\min} = 1.5455$ ms
Second cylinder increment time c=0.3197 ms
Critical number of cylinders traveled to attain the maximum speed x^*=1686
Exponent r=0.3868
Fraction of cylinders modeled using nonlinear characteristic = 21%

Therefore, for this disk, if the file size is less than 1.9 GB, there will be no head traveling at the constant speed, but only acceleration and deceleration. In such a case the head speed obtained at the end of acceleration period is less than the maximum coasting speed for the disk unit. Consequently, the seek time characteristic can be reduced to the simple nonlinear segment $T_{seek}(x) = t + c(x-1)^r$.

In a general case of a file occupying N adjacent cylinders we can compute the mean seek time for the whole characteristic (nonlinear and linear segment). Let us use the following exponential model:

$$T_{seek}(x) = \begin{cases} 0, & x = 0 \\ t + c(x-1)^r, & 1 \le x \le x^* \\ ax + b, & x \ge x^* \end{cases}$$

The corresponding mean seek time for an N-cylinder file is the following:

$$\overline{T}_{seek}(N) = \begin{cases} \dfrac{2(N-1)^2}{N^2}\left[\dfrac{t}{2}+\dfrac{c(N-1)^r}{(r+1)(r+2)}\right], & N \le x^* \\[2ex] \dfrac{2}{N^2}\left[\begin{array}{l} \dfrac{t(x^*-1)(2N-x^*-1)}{2}+ \\ \dfrac{c(x^*-1)^{r+1}[N(r+2)-x^*(r+1)-1]}{(r+1)(r+2)}+ \\ \dfrac{(N-x^*)^2(aN+2cx^*+3b)}{6} \end{array}\right], & N \ge x^* \end{cases}$$

In the case of Quantum Atlas III, the result is $\overline{T}_{seek}(C) = 8.31\,ms$. In this case, if we take approximately $t_{\min} = 1.5ms$ and $t_{\max} = 15ms$ then the approximate mean time based on formula $\overline{T}_{seek}(C) = 2t_{\min}/3 + t_{\max}/3$ gives $\overline{T}_{seek}(C) = 6ms$ which is significantly less (28%) than the accurate value. This result is expected because the actual nonlinear seek time characteristic is located significantly above its linear approximation. On the other hand, as a rule of thumb, if we compute $2t_{\min}/3 + t_{\max}/3$ and then add 30%, i.e. if we use the approximation $1.3(2t_{\min}/3 + t_{\max}/3)$ the result (7.8 ms for Quantum Atlas III) might be useful as a rough approximation of the mean disk seek time $\overline{T}_{seek}(C)$.

6. Mean access and transfer time

If we have the mean seek time for a disk unit $\overline{T}_{seek}$ and the number of revolutions per minute N_{rpm}, then it is easy to compute all other disk performance indicators:

Full revolution time: $T_{rev} = 60 / N_{rpm}$ [sec]

Rotational delay (latency) time: $T_{rd} = 30 / N_{rpm}$ [sec] (half revolution time)

Mean disk access time: $T_a = \overline{T}_{seek} + T_{rd} = \overline{T}_{seek} + 30 / N_{rpm}$ [sec]

Mean disk access and transfer time: $\overline{T}_{at}(N) = \overline{T}_{seek}(N) + \dfrac{60}{N_{rpm}}\left(\dfrac{1}{2} + \dfrac{N_{\sec}}{\overline{N}_{spt}}\right)$, where

$\overline{N}_{spt}$ = mean number of sectors per track

$N_{\sec}$ = number of sectors written/read per disk access