Calibration and Comparison of Disk Unit Models

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Contents

Seek Time

- Seek time is the time necessary to move disk R/W mechanism from a current position at cylinder *c* to a destination cylinder *d* .
- Traveled distance: $x = | c - d |$.
- Seek time is important because it is the primary component of the disk access time. Seek time is *not* productive.

The Basic Square Root Model

a = acceleration of the R/W mechanism

x = distance traveled by the R/W mechanism

T = total seek time (T/2 for acceleration and
T/2 for deceleration)

$$x = at^2 / 2$$

$$x/2 = a(T/2)^2/2,$$

acceleration or deceleration (half distance yields half seek time)

$$T(x) = 2\sqrt{x/a}$$

The Basic Square Root Model: T(x)

$$T(x) = 2\sqrt{x/a}$$

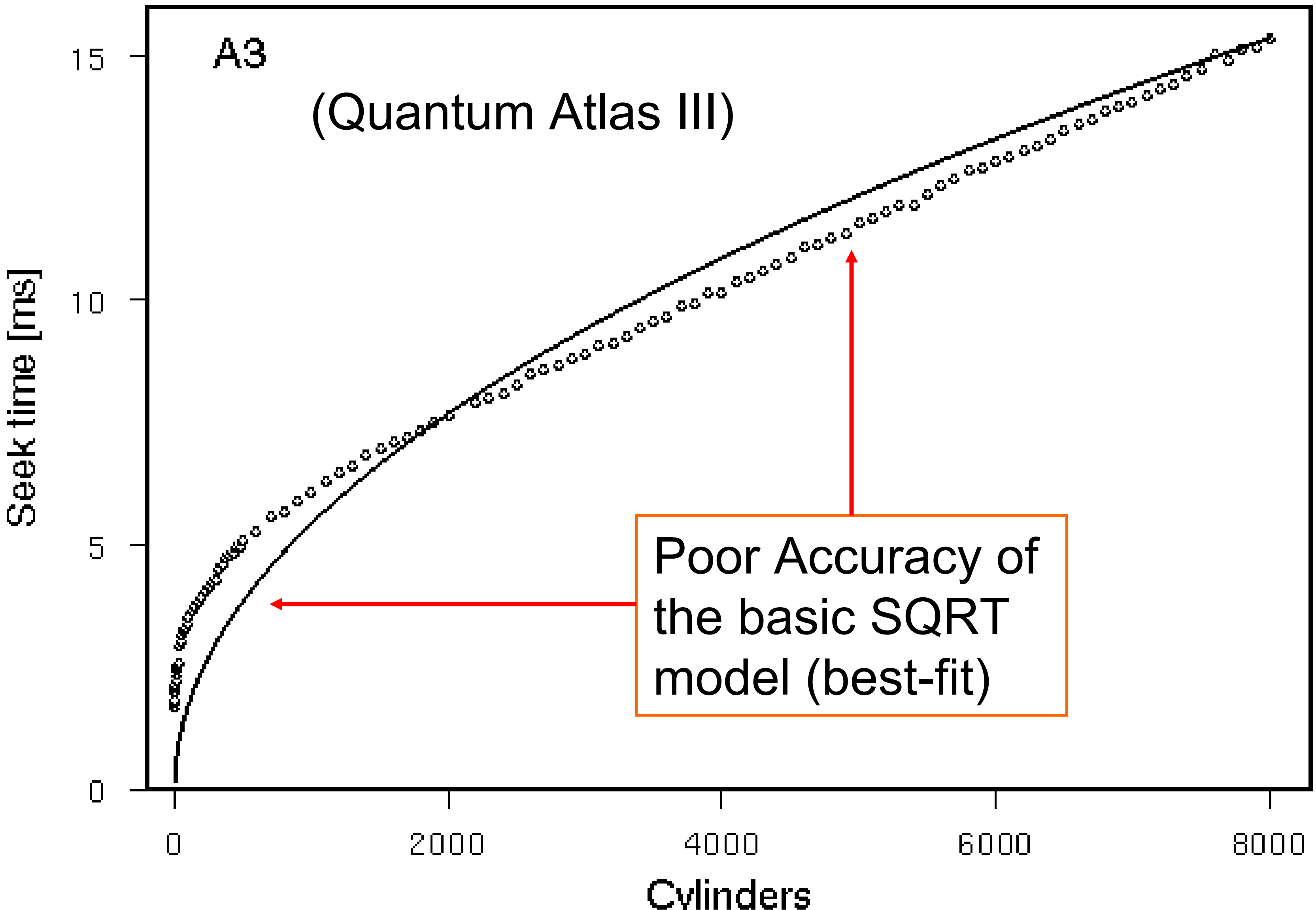
x=distance

T=seek time

$$T_{\max} = 2\sqrt{x_{\max}/a}$$

$$T(x) = T_{\max}\sqrt{x/x_{\max}}$$

The Basic Square Root Model and a Measured Real Characteristic



Reasons for Poor Accuracy of the Basic Square Root Model

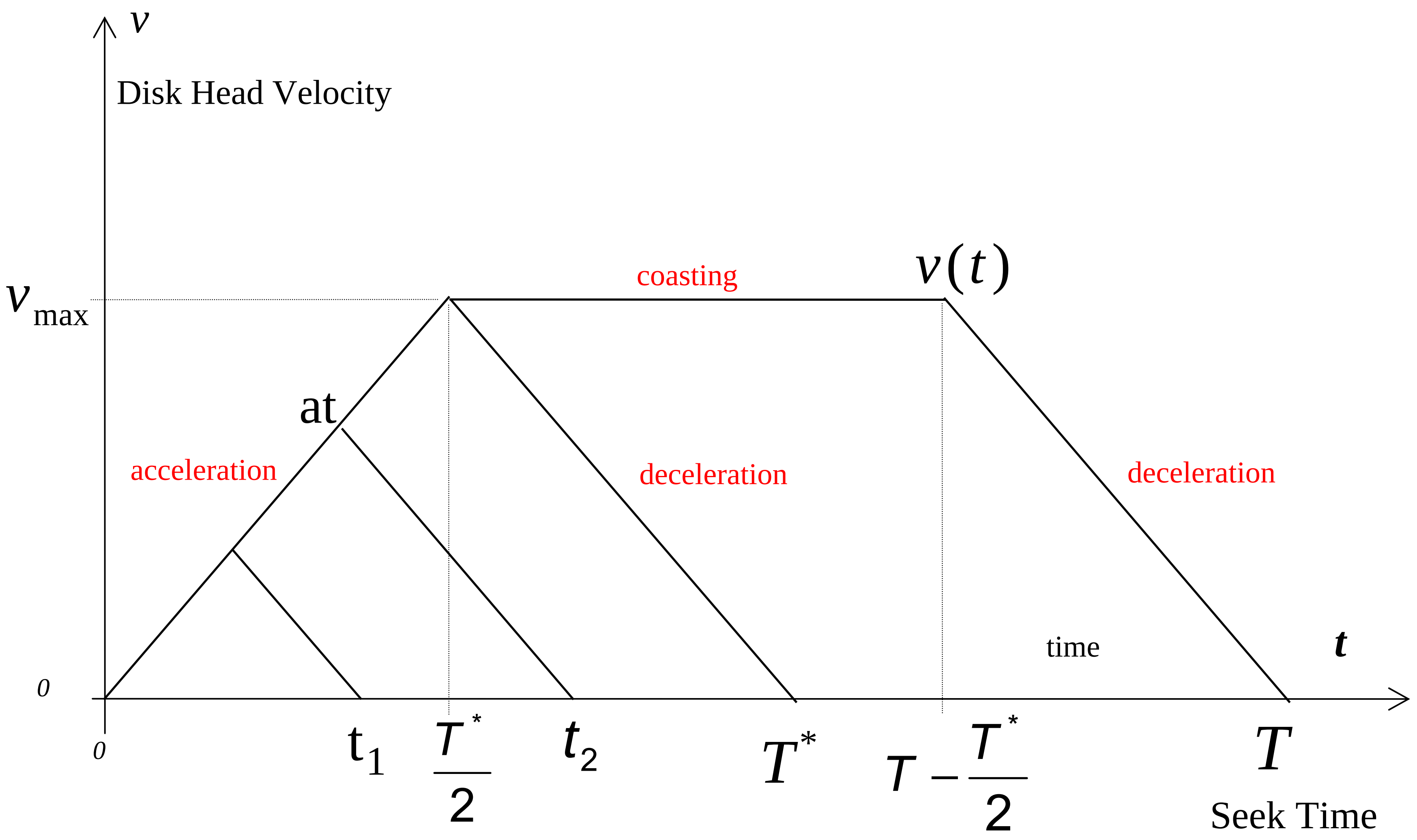
Real seek time characteristic consists of two parts:

- Initial nonlinear segment that reflects acceleration and deceleration for small distances
- Linear segment that is caused by moving at constant velocity for large distances

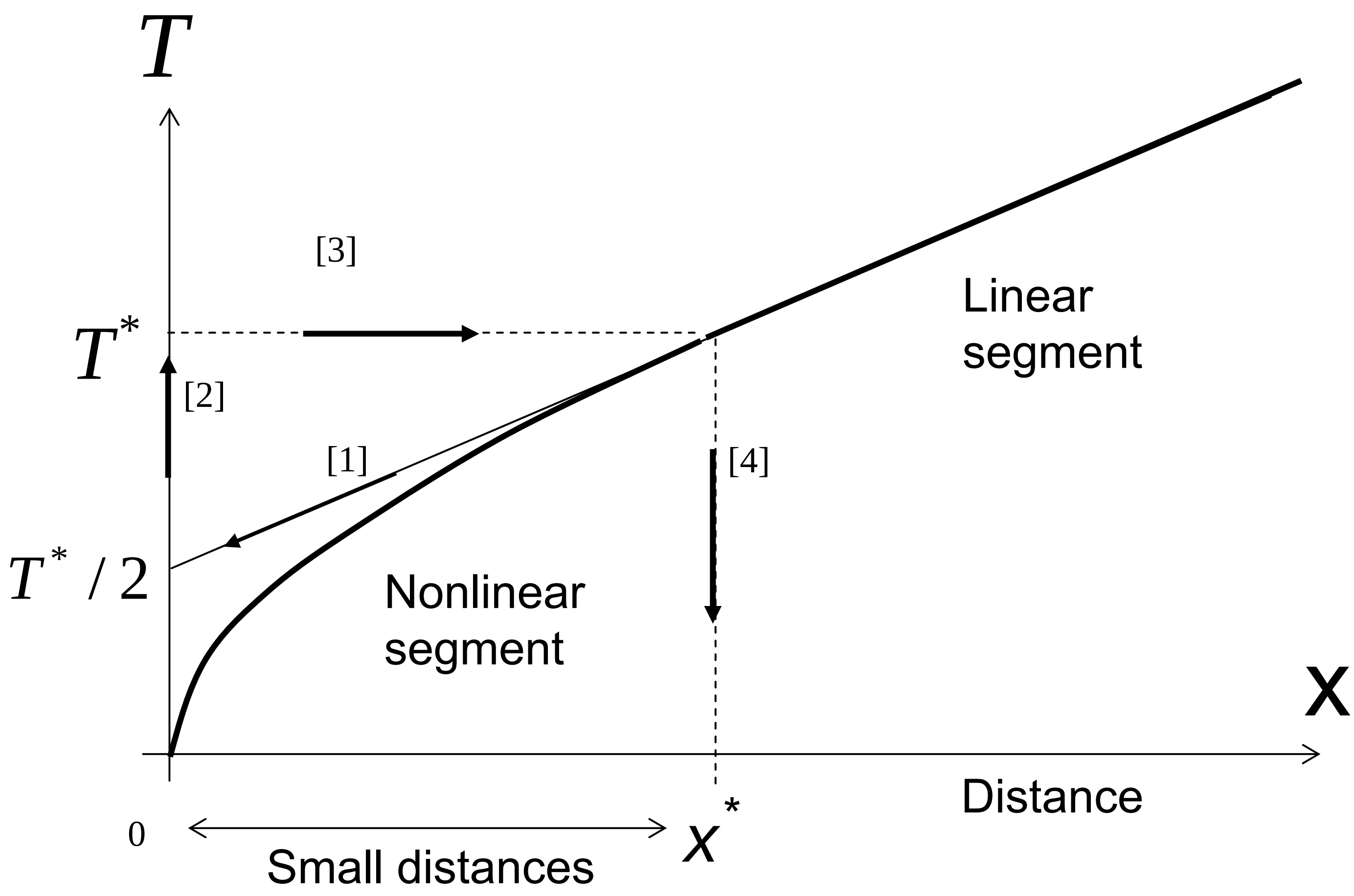
As opposed to that the basic square root model is non-linear and cannot accurately model real characteristics.

∴ **A more precise model is needed.**

A Limited Maximum Velocity Model of Seek Time (SQRT)



Computation of X* and T* From the Seek Time Characteristic



Traditional SQRT Model and the Boundary Point Slope Error

$$T(x) = \begin{cases} a + b\sqrt{x}, \quad \text{for short seeks}, \quad x \le x^* \quad (a, b = const.) \\ Ax + B, \quad \text{for long seeks}, \quad x > x^* \quad (A, B = const.) \end{cases}$$

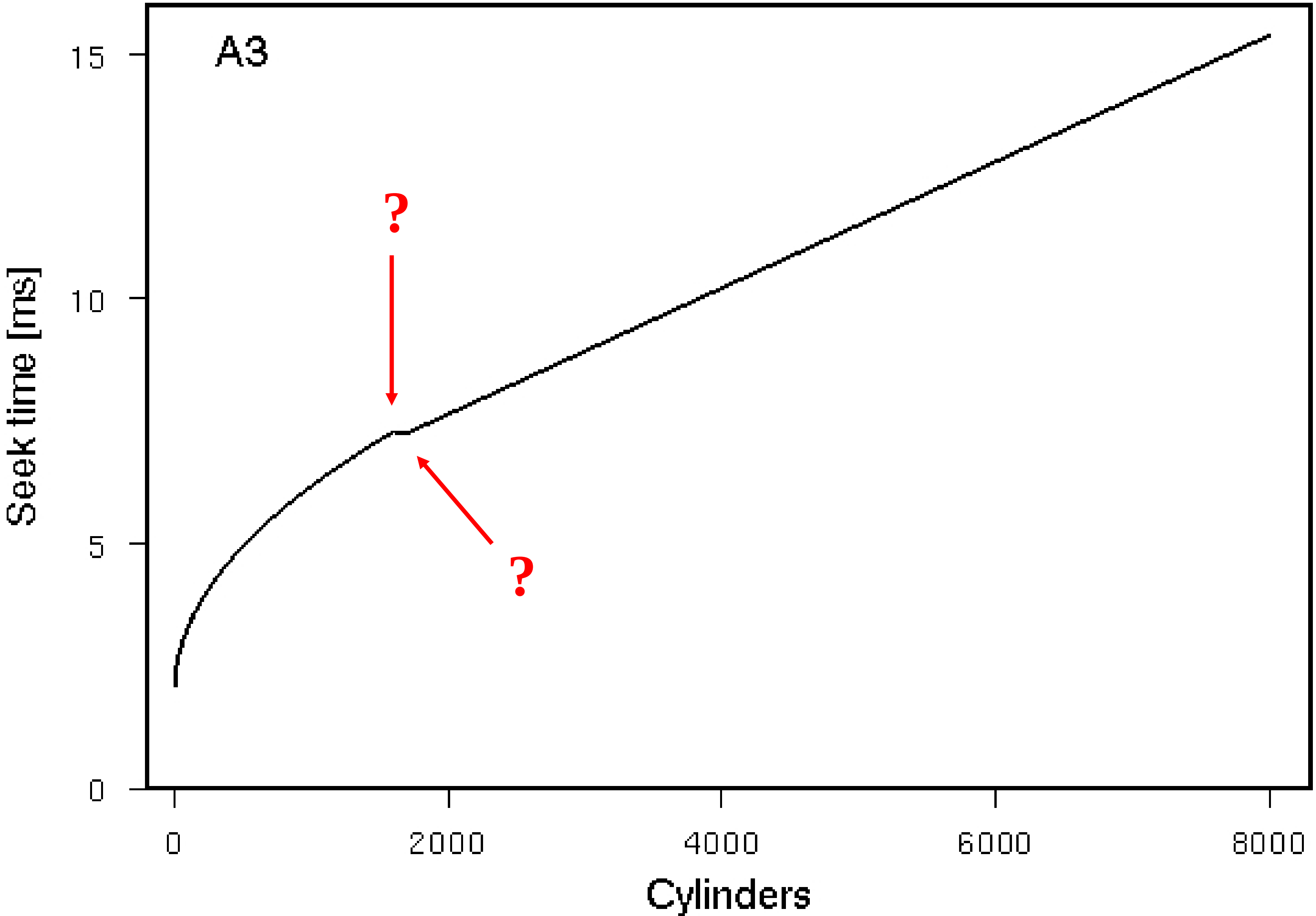
$$T(x) = \begin{cases} 3.24 + 0.4\sqrt{x} \quad , \quad x \le 383 \\ 8.2 + 0.0075x \quad , \quad x > 383 \end{cases}$$

Ruemmler and Wilkes [Computer'94]

$$\frac{dT}{dx} = \begin{cases} 0.2/\sqrt{x} = 0.01, \quad x = 383 \\ 0.0075 \quad , \quad x = 384 \end{cases}$$

R&W left/right slope difference = 25%

SQRT Model: Sample Slope Error



A Smooth SQRT Model

$$T(x^*) = a + b\sqrt{x^*} = Ax^* + B$$

Boundary point x*

$$\left.\frac{dT(x)}{dx}\right|_{x = x^*} = \frac{b}{2\sqrt{x^*}} = A$$

Smooth connection at x*

$$T(x) = \begin{cases} B - Ax^* + 2A\sqrt{xx^*}, & x \le x^* \\ Ax + B \quad , & x > x^* \end{cases}$$

Smooth SQRT model (uses 3 parameters: A, B, and x*)

SQRT Model: Variations

Ng [1998]: d=recording density

$$T(x) = a + b\sqrt{x} + c\log(d), \quad (a, b, c = const.)$$

Shriver [1997]:

$$T(x) = \begin{cases} 0, & x = 0 \\ a + b\sqrt{x}, & 0 < x \le x_1 \\ c + d\sqrt{x}, & x_1 < x \le x_2 \\ Ax + B, & x > x_2 \end{cases} \quad (a, b, c, d, A, B = const)$$

SQRT Model: Problems

- The nonlinear part is not adjustable and the real characteristics differ from the square root.
- The 4-parameter SQRT model usually has a boundary point slope error.
- The smooth square root model eliminates the slope error, but it has only 3 parameters and relatively large maximum and average errors.

POLY Model (3 Parameters)

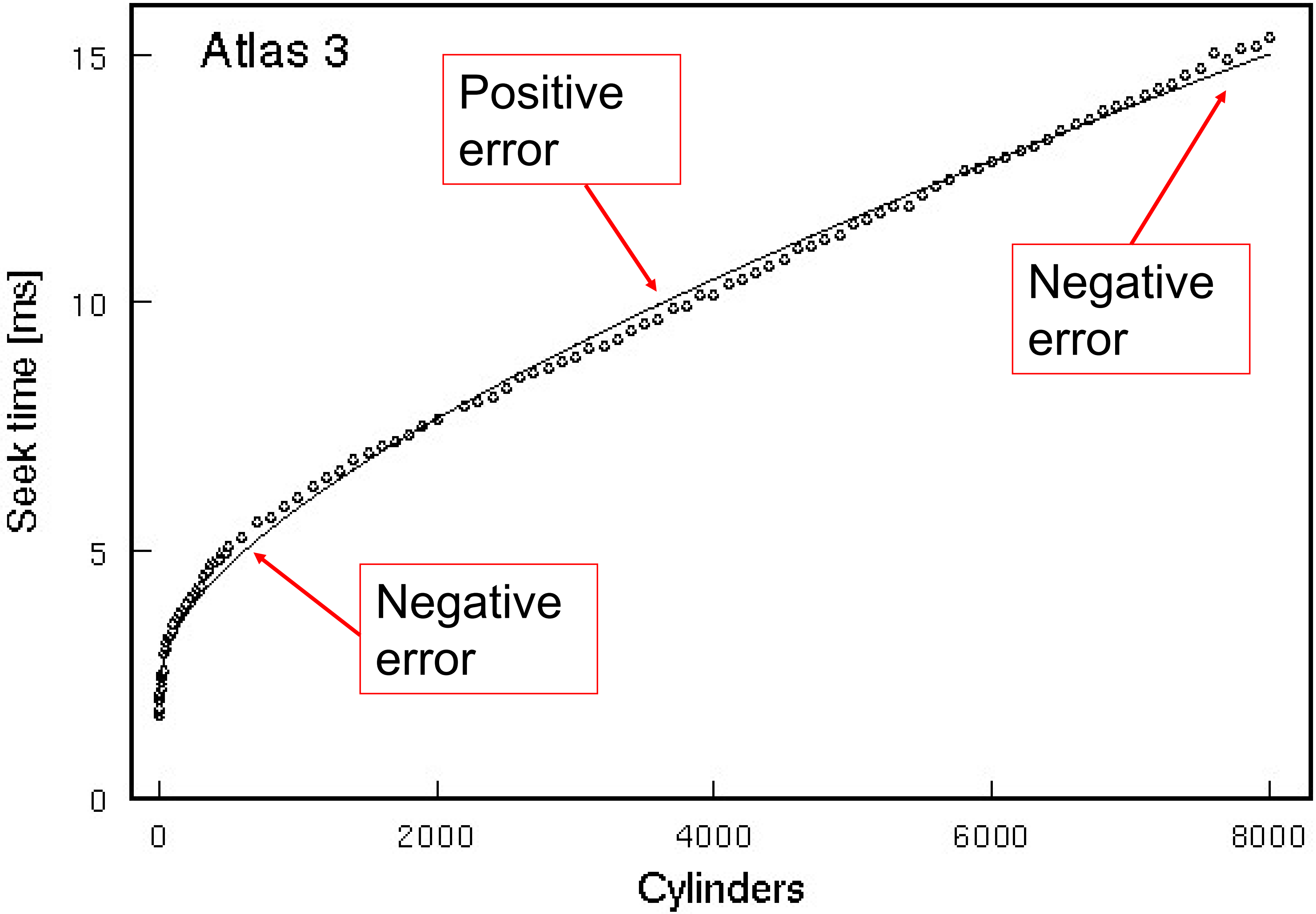
Lee and Katz [1993]:

$$T(x) = \begin{cases} 0 \quad , & x = 0 \\ A\sqrt{x-1} + B(x-1) + t \, , & x > 0 \end{cases}$$

t=T(1) (single cylinder seek time)

Problem: How to model the linear segment?

The Best-Fit POLY Model



Quality of Modeling of Existing SQRT and POLY Models

- The SQRT model (initial square root plus a linear segment) has breakpoint errors.
- Both the basic square root model and the POLY model are insufficiently precise and cannot model the linear segment of real disk characteristics.
- The smooth best-fit SQRT model has insufficient accuracy in the initial nonlinear segment of the seek time characteristic.

∴ **There is a need to improve the quality of modeling of the nonlinear segment.**

EXPO Model of Seek Time

$$T(x) = \begin{cases} 0, & x = 0 \\ t + c(x-1)^r, & 1 \le x \le x^* \\ ax + b, & x \ge x^* \end{cases}$$

Parameters: t=T(1) , c=T(2)-T(1) , r , a , b , x*

(These 6 parameters are not independent!)

4-Parameter EXPO Model

$$t + c(x^* - 1)^r = ax^* + b$$ Connectivity condition

$$\left.\frac{dT}{dx}\right|_{x = x^*} = a = cr(x^* - 1)^{r-1}$$ Equal slope condition

$$b = t + c(x^* - 1)^r - cr(x^* - 1)^{r-1} x^*$$

$$T(x) = \begin{cases} t + c(x-1)^r & , \quad 1 \le x \le x^* \\ \dfrac{cr(x - x^*)}{(x^* - 1)^{1-r}} + t + c(x^* - 1)^r & , \quad x \ge x^* \end{cases}$$

Adjustable parameters: t=T(1) , c=T(2)-T(1) , r , x*

EXPO Model Calibration

Model calibration: adjustment of parameters of the model so that it yields the best approximation of measured data.

EXPO Model Calibration Criteria

$$E_1(t,c,r,x^*) = \frac{1}{n}\sum_{i=1}^{i=n}(T(x_i) - T_i)^2$$

$$E_2(t,c,r,x^*) = \frac{1}{n}\sum_{i=1}^{i=n}|T(x_i) - T_i|$$

$$E_3(t,c,r,x^*) = \max_{1\le i\le n}|T(x_i) - T_i|$$

A Survey of 10 Analyzed Disks

Name	Brand / Model	Cyl	GB	RPM
A10	QUANTUM TORNADO	10042	9.1	10000
A3	QUANTUM QM39100TD	8057	9.1	7200
Bar	SEAGATE ST32171W	5172	2.16	7200
Ch4	SEAGATE ST34501N	6581	4.5	10000
Ch9	SEAGATE ST39102LW	6962	9.1	10000
DEC	DEC RZ26	2570	1.03	5400
HP2	HP C2490A	2582	2.13	6400
HP3	HP C3323A	2910	1.05	5400
IBM	IBM DNES-309170W	11474	9	7200
Sea	Seagate ST41601N	2098	1.37	5400

Calibrated EXPO Models

Disk	t [ms]	c [ms]	r	x^* [cyl]
A10	1.0752	0.193	0.3848	1813
A3	1.5455	0.3197	0.3868	1686
Bar	1.6057	0.3898	0.4051	1357
Ch4	0.7078	0.3183	0.4058	1322
Ch9	0.98	0.1395	0.4499	1613
DEC	1.2002	0.7405	0.3799	734
HP2	1.1166	0.8504	0.328	452
HP3	1.2998	0.4287	0.4328	794
IBM	0.9586	0.4033	0.3374	2864
Sea	1.5115	0.6632	0.4103	462

Acceleration/Deceleration Distance

Disk	Xmax	X*	X*/Xmax
A10	10042	1813	0.18
A3	8057	1686	0.21
Bar	5172	1357	0.26
Ch4	6581	1322	0.20
Ch9	6962	1613	0.23
DEC	2570	734	0.29
HP2	2582	452	0.18
HP3	2910	794	0.27
IBM	11474	2864	0.25
Sea	2098	462	0.22

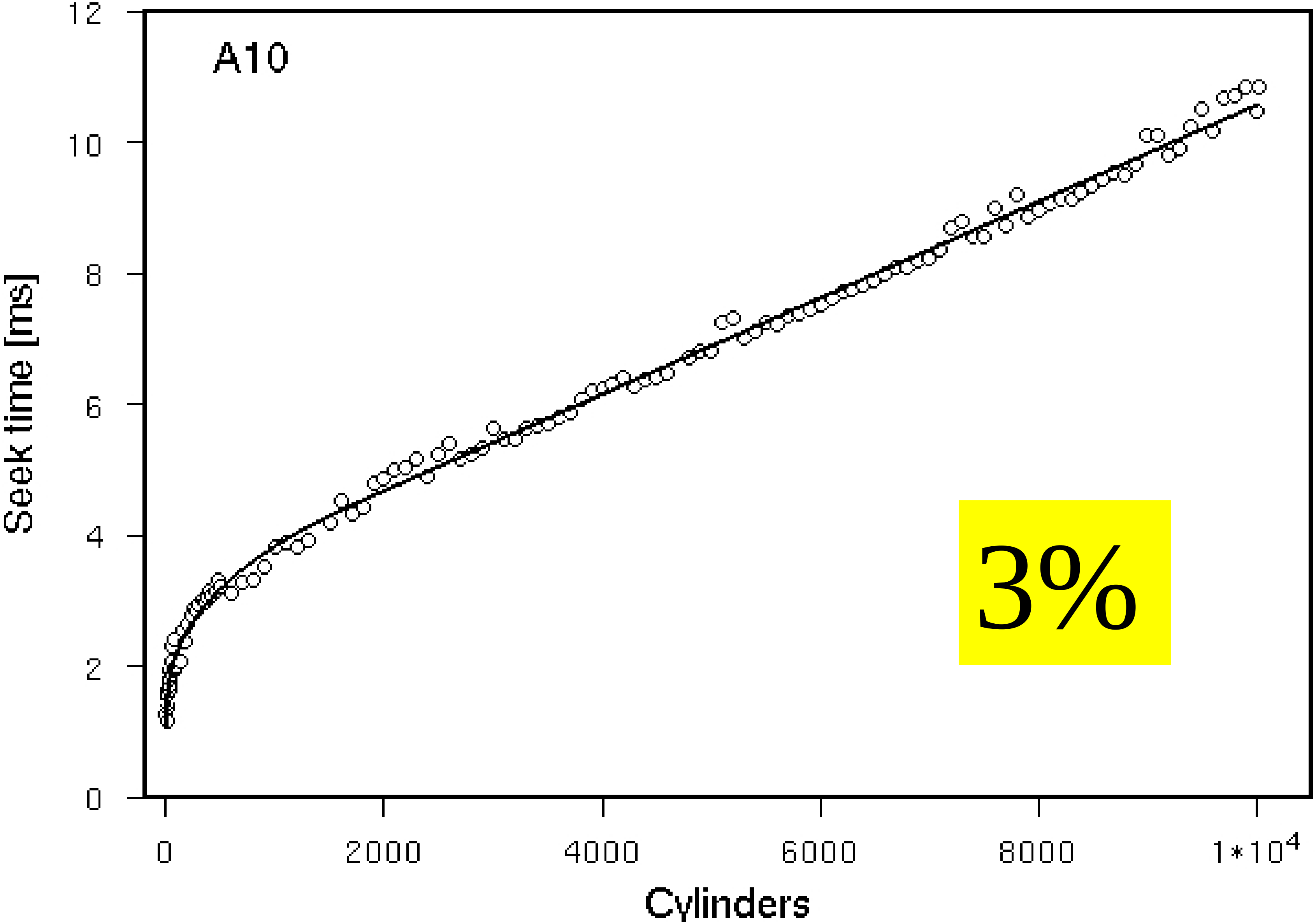
Comparison of the SQRT, POLY, and EXPO Models (Nonlinear Segment Only)

	Max. relative error [%]			Average rel. error [%]		
Disk	SQRT	POLY	EXPO	SQRT	POLY	EXPO
A10	30.37	40.35	14.12	7.41	10.13	4.98
A3	30.25	33.78	11.54	6	8.45	2.36
Bar	25.27	27.35	18.08	8.61	9.27	6
Ch4	98.67	113.5	13.63	7.56	11.87	4.08
Ch9	43.4	51.9	17.93	3.74	6.35	2.64
DEC	132.7	107.7	18.31	7.04	6.25	3.06
HP2	88.91	84.19	33.84	9.57	10.3	5.09
HP3	60.28	55.26	21.13	6.38	7.26	3.75
IBM	104.2	104.1	76.94	10.39	10.52	5.14
Sea	46.65	41.04	10.46	7.09	8.37	3.81
AVG	66.1	65.9	23.6	7.4	8.9	4.1

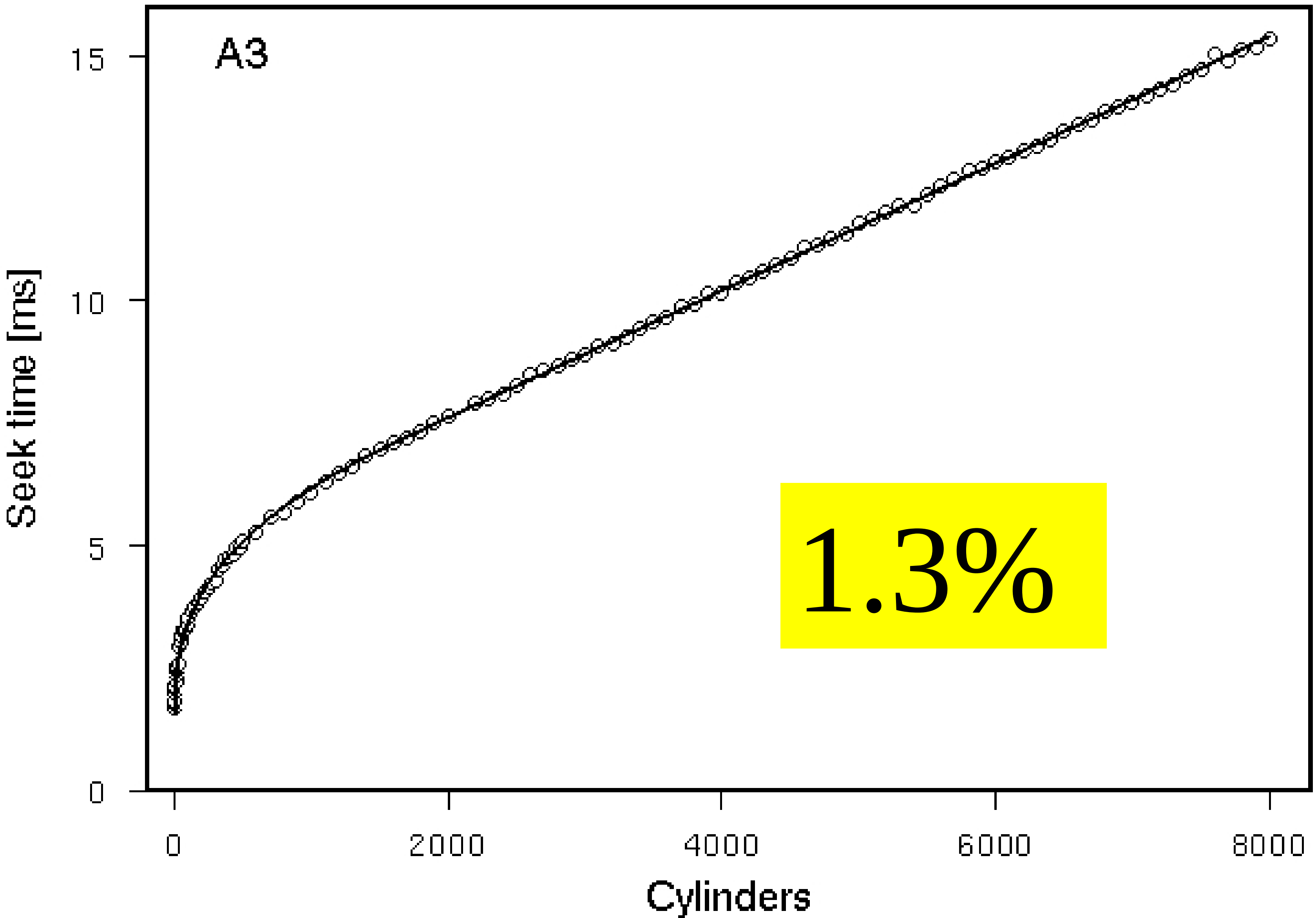
Comparison of Models (Full Range)

Disk	Max. relative error [%]			Average rel. error [%]		
	SQRT	POLY	EXPO	SQRT	POLY	EXPO
A10	30.37	40.35	14.12	4.06	5.55	3.03
A3	30.25	33.78	11.54	3.08	4.78	1.3
Bar	25.28	27.35	18.08	5.29	6.13	3.73
Ch4	98.67	113.5	13.63	3.96	6.71	2.22
Ch9	43.4	51.9	17.93	2.15	3.98	1.6
DEC	132.7	107.7	18.31	5.25	5.07	2.32
HP2	88.91	84.19	33.84	6.51	7.56	3.37
HP3	60.28	55.26	21.13	4.39	5.47	2.63
IBM	104.2	104.1	76.95	5.13	5.59	2.79
Sea	46.65	41.04	10.46	5.28	6.7	2.8
AVG	66.1	65.9	23.6	4.51	5.76	2.58

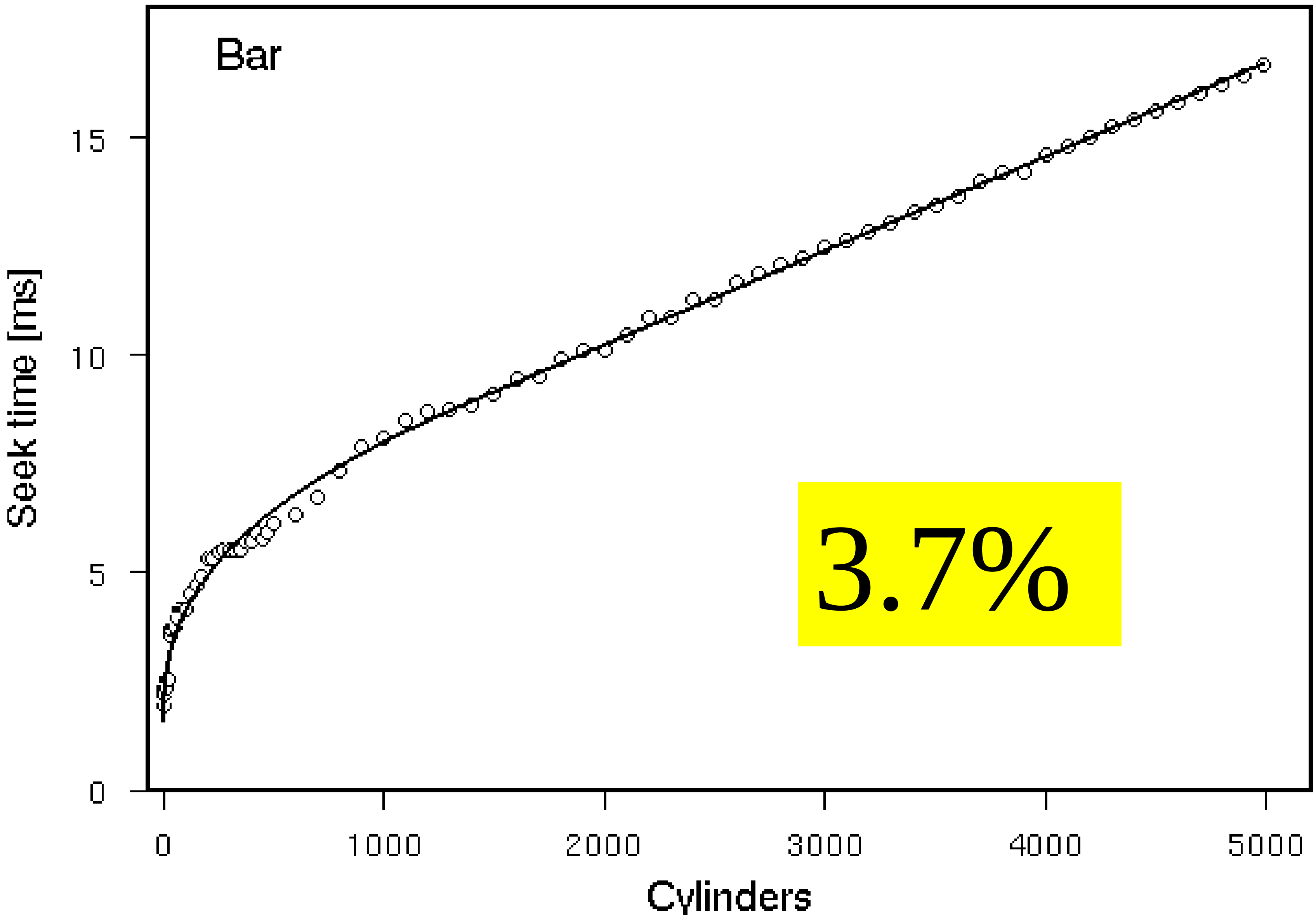
EXPO Model: Quantum Atlas 10



EXPO Model: Quantum Atlas III

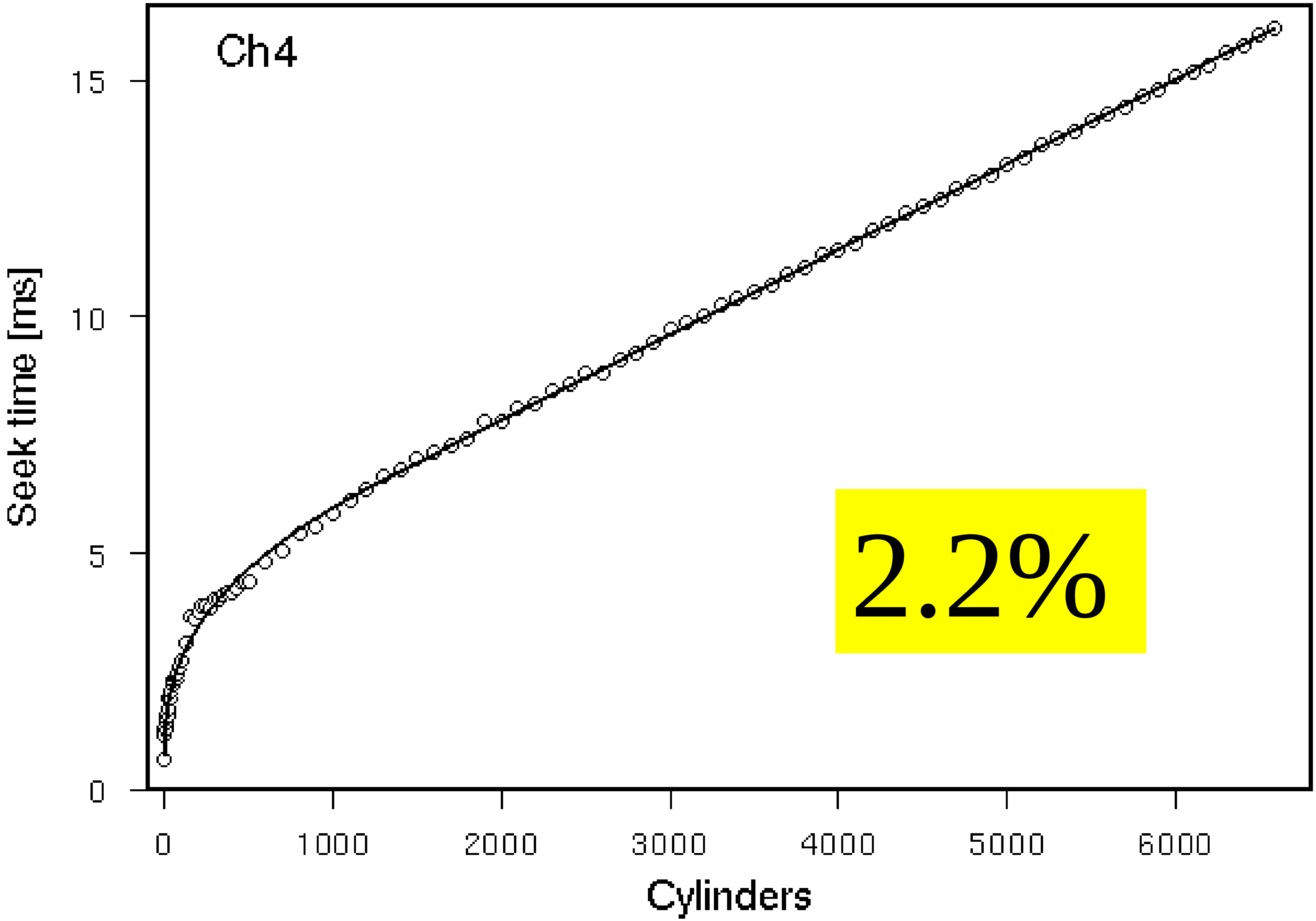


EXPO Model: Seagate Barracuda

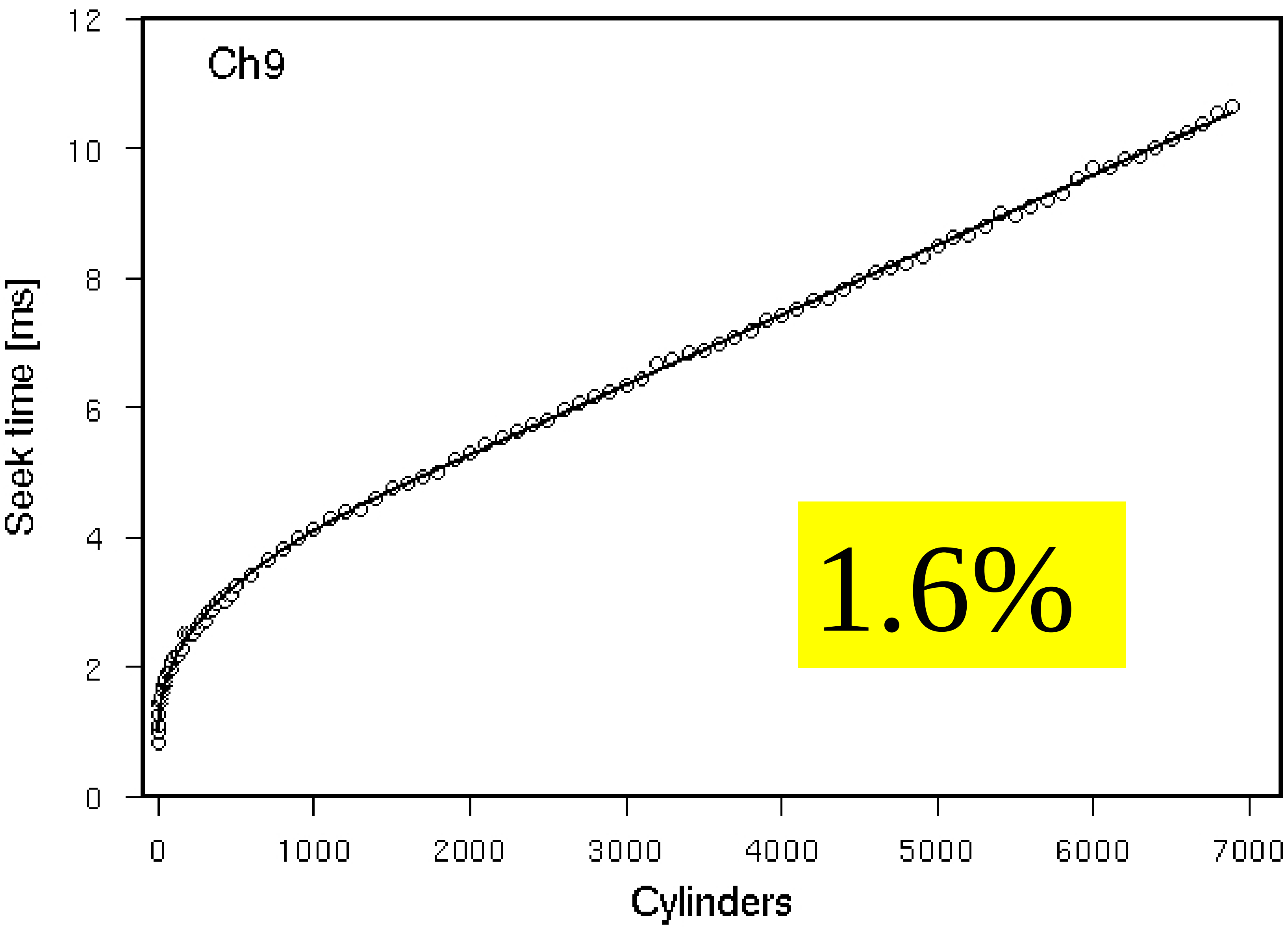


3.7%

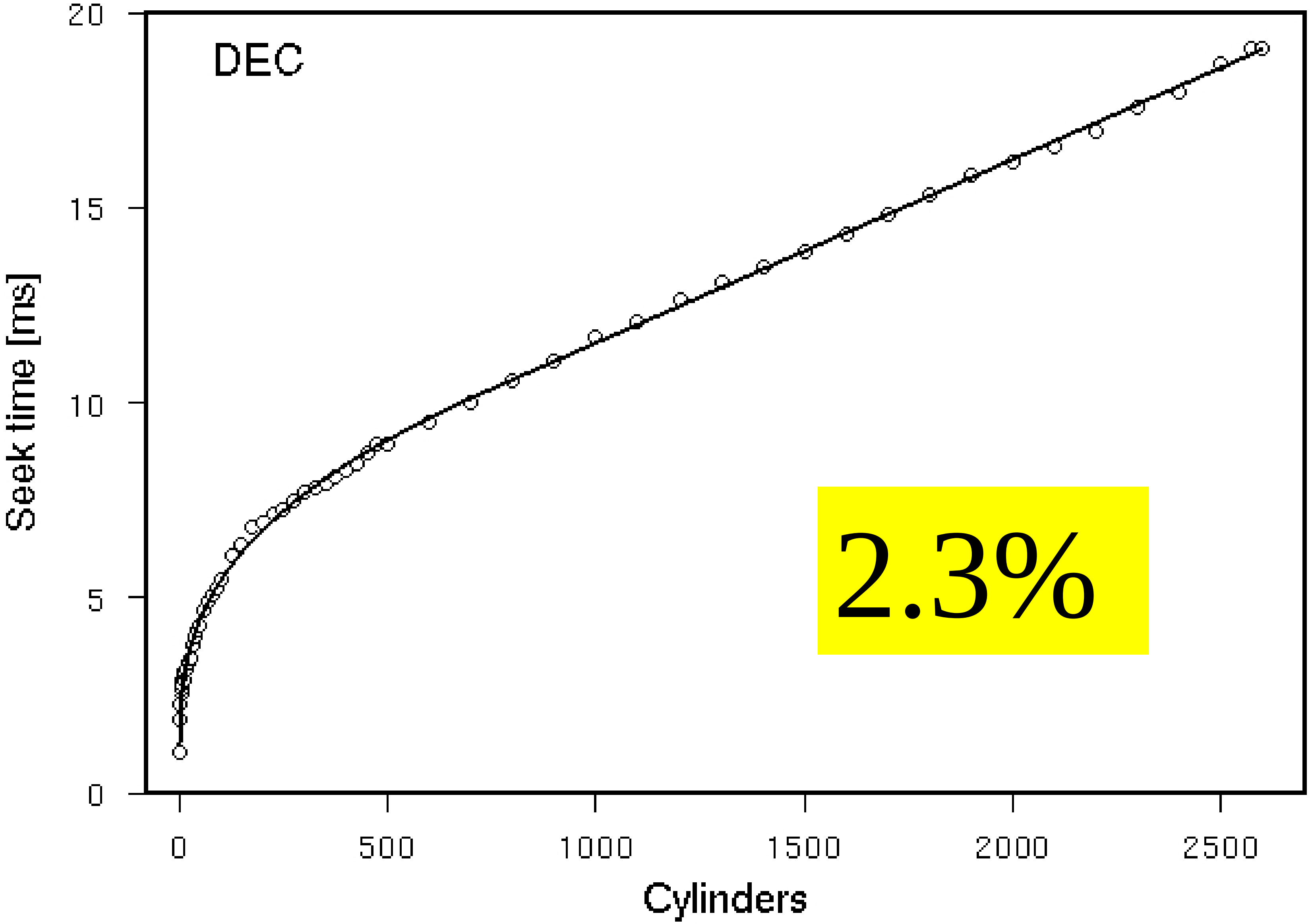
EXPO Model: Seagate Cheetah 4



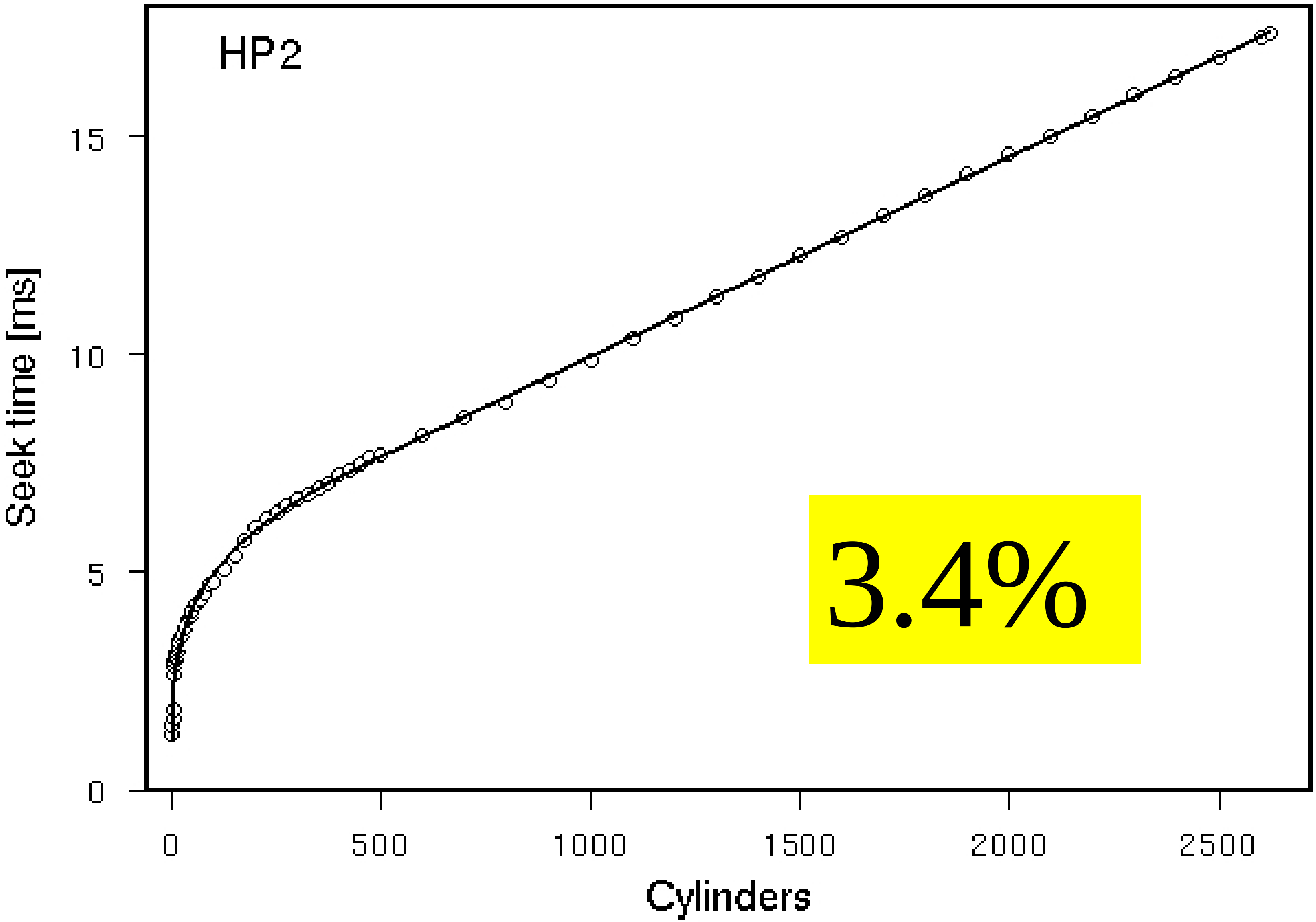
EXPO Model: Seagate Cheetah 9



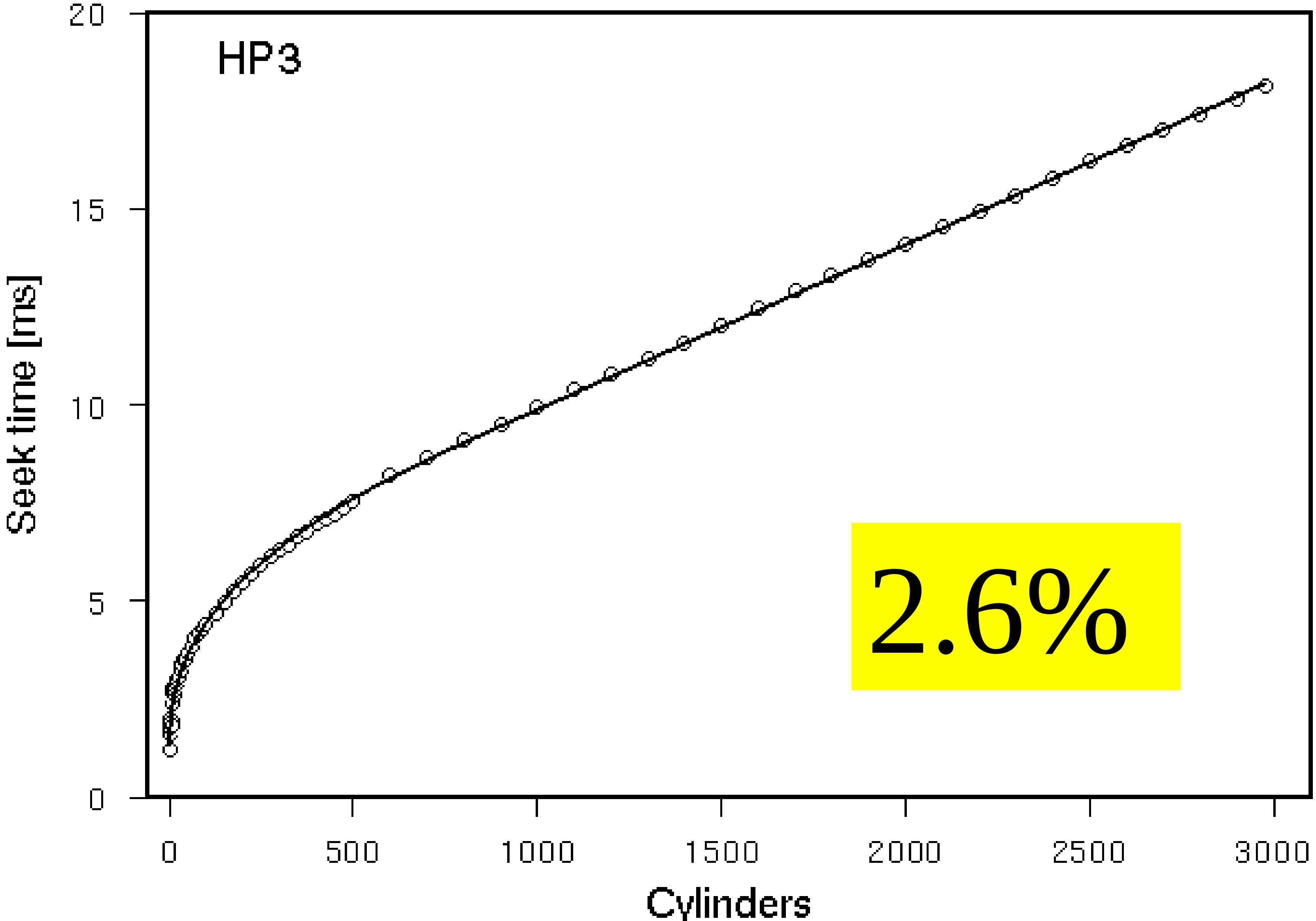
EXPO Model: DEC RZ 26



EXPO Model: HP C2490A

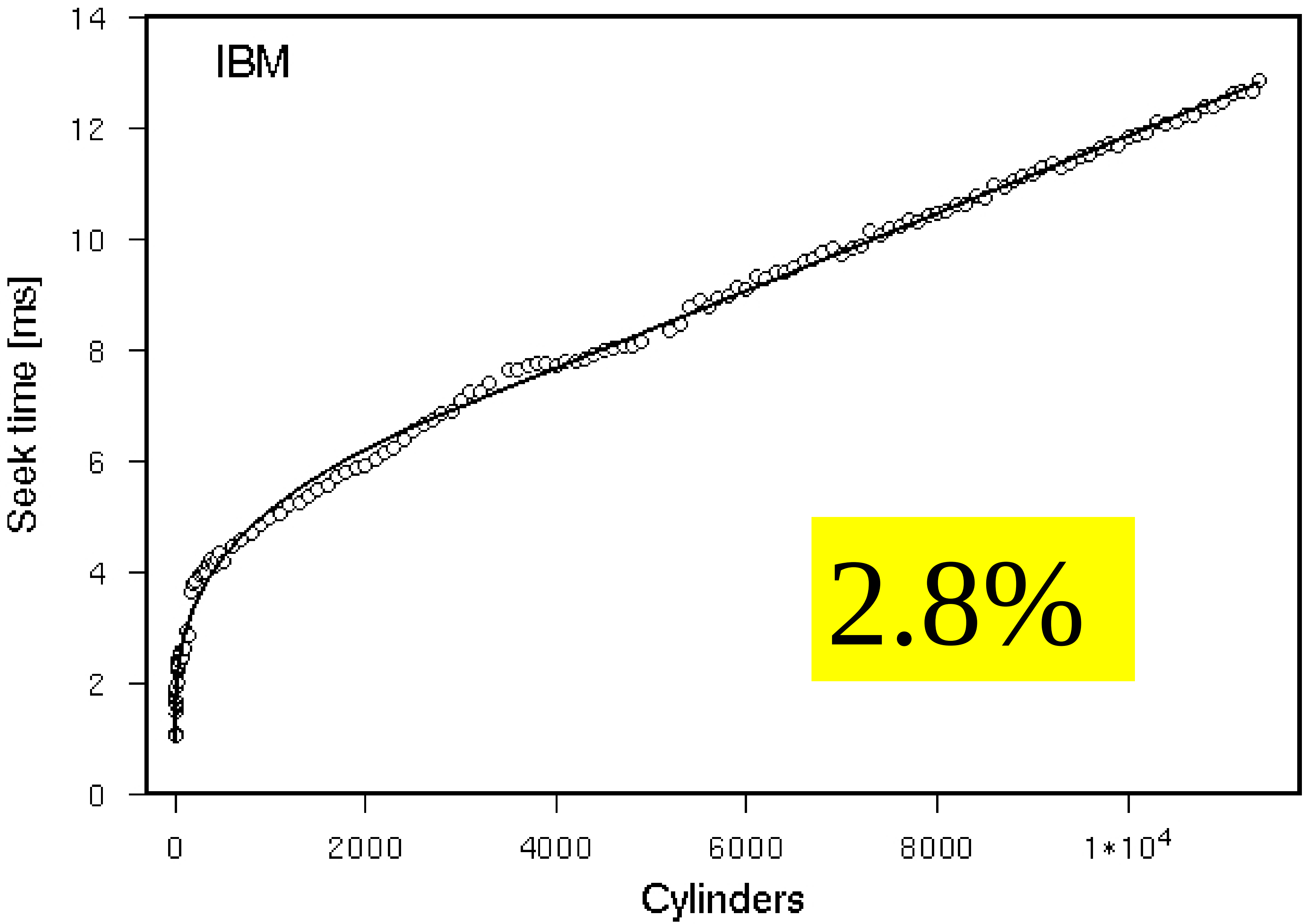


EXPO Model: HP C3323A

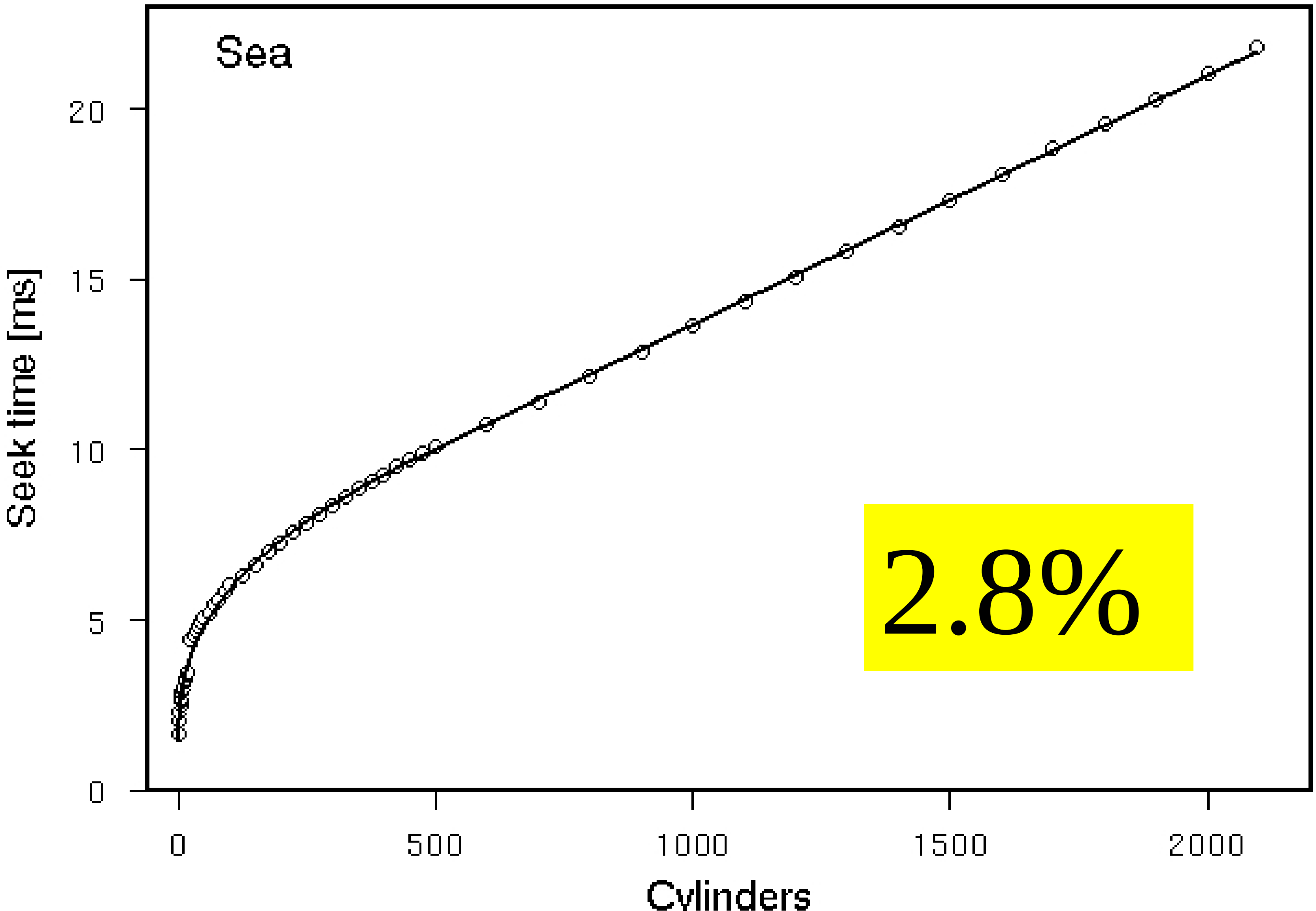


2.6%

EXPO Model: IBM 309170W



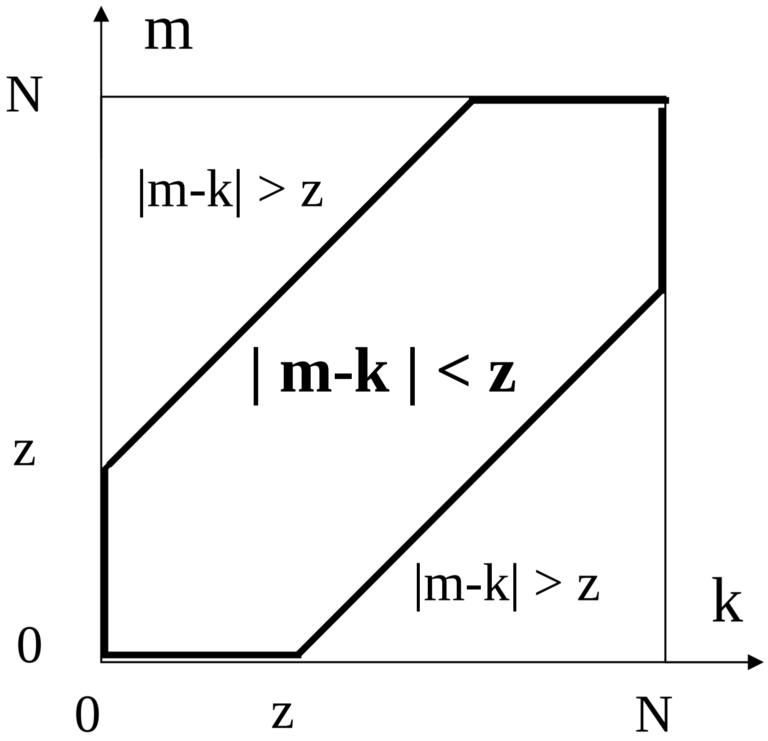
EXPO Model: Seagate ST4160N



Features of the EXPO Model

1. Accurate modeling of the nonlinear segment
2. Accurate modeling of the linear segment
3. Smoothness

Computing the Mean Seek Time



$$P_x(z) = \text{probability}[x \le z]$$

$$= \frac{N^2 - (N-z)^2}{N^2} = \frac{2Nz - z^2}{N^2}$$

$$p_x(z) = \frac{dP}{dz} = \frac{2}{N^2}(N - z)$$

$$\overline{T}_{seek}(N) = \int_0^N T(z)p_x(z)dz = \frac{2}{N^2}\int_0^N T(z)(N-z)dz$$

Mean Seek Time (Example)

$$T(x) = T_{max}\sqrt{\frac{x}{x_{max}}}$$

Basic SQRT model

$$\overline{T}_{seek}(N) = \frac{2}{N^2}\int_0^N T(z)(N-z)dz = \frac{2T_{max}}{N^2\sqrt{x_{max}}}\int_0^N z^{1/2}(N-z)dz$$

$$= \frac{2T_{max}}{N^2\sqrt{x_{max}}}\left[N\frac{2}{3}z^{3/2} - \frac{2}{5}z^{5/2}\right]_0^N = \frac{2T_{max}\sqrt{N}}{\sqrt{x_{max}}}\left(\frac{2}{3} - \frac{2}{5}\right)$$

$$= \frac{8T_{max}}{15}\sqrt{\frac{N}{x_{max}}} \quad , \quad 0 \le N \le x_{max} \quad , \quad 0 \le \overline{T}_{seek} \le \frac{8T_{max}}{15}$$

Mean Seek Time for EXPO

$$\overline{T}_{seek}(N) = \begin{cases} \dfrac{2}{N^2}\displaystyle\int_0^N T(z)(N-z)dz\,, & N \le x^* \\[2ex] \dfrac{2}{N^2}\left[\displaystyle\int_0^{x^*} T(z)(N-z)dz + \int_{x^*}^{N} T(z)(N-z)dz\right], & N \ge x^* \end{cases}$$

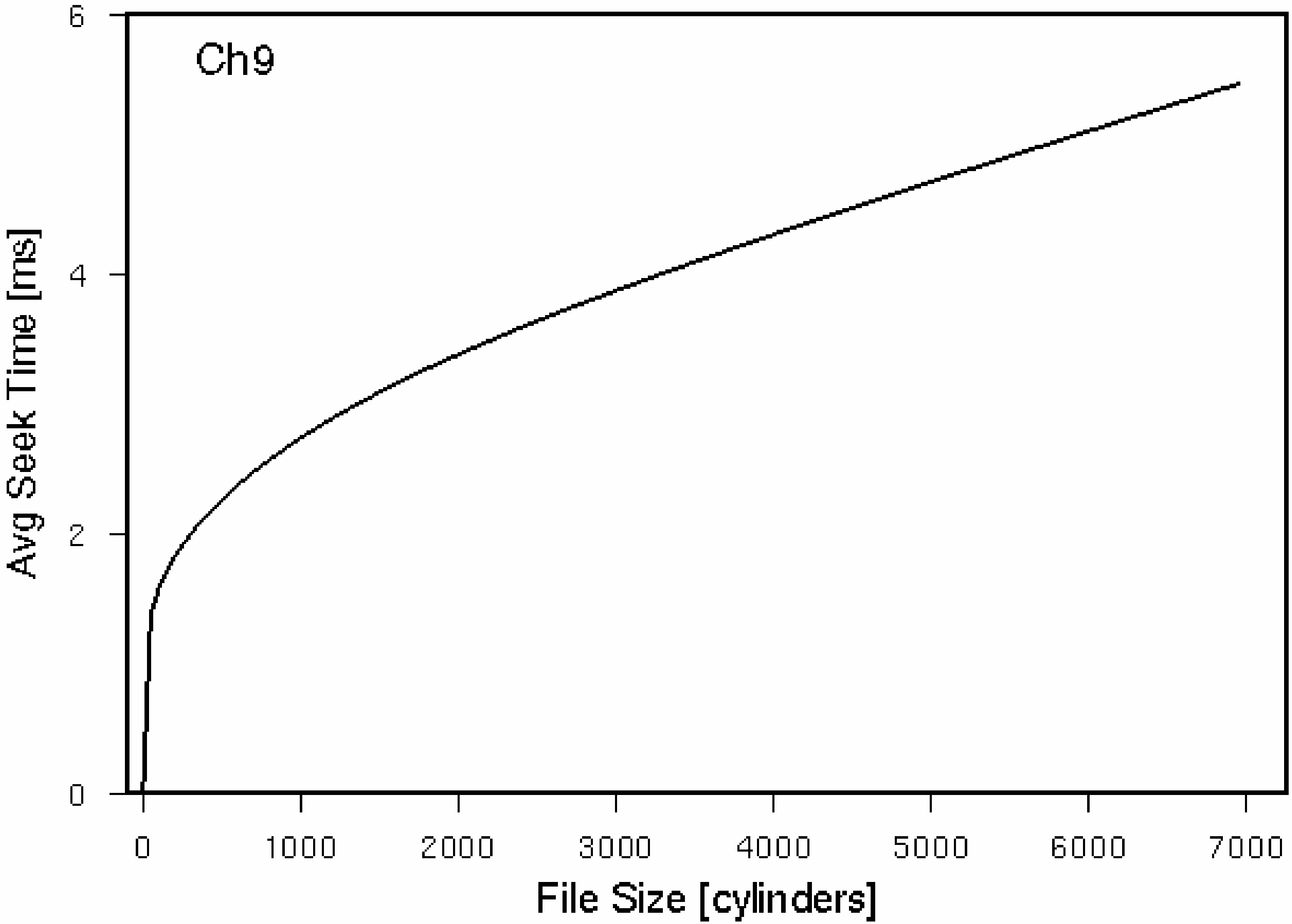
SQRT & EXPO

$$T(x) = \begin{cases} 0\,, & x = 0 \\ t + c(x-1)^r, & 1 \le x \le x^* \\ ax + b, & x \ge x^* \end{cases}$$

EXPO

$$\overline{T}_{seek}(N) = \begin{cases} \dfrac{2(N-1)^2}{N^2}\left[\dfrac{t}{2} + \dfrac{c(N-1)^r}{(r+1)(r+2)}\right], & N \le x^* \\[2ex] \dfrac{2}{N^2}\left[\begin{array}{l} \dfrac{t(x^*-1)(2N-x^*-1)}{2} + \\ \dfrac{c(x^*-1)^{r+1}[N(r+2)-x^*(r+1)-1]}{(r+1)(r+2)} + \\ \dfrac{(N-x^*)^2(aN+2cx^*+3b)}{6} \end{array}\right], \\ N \ge x^* \end{cases}$$

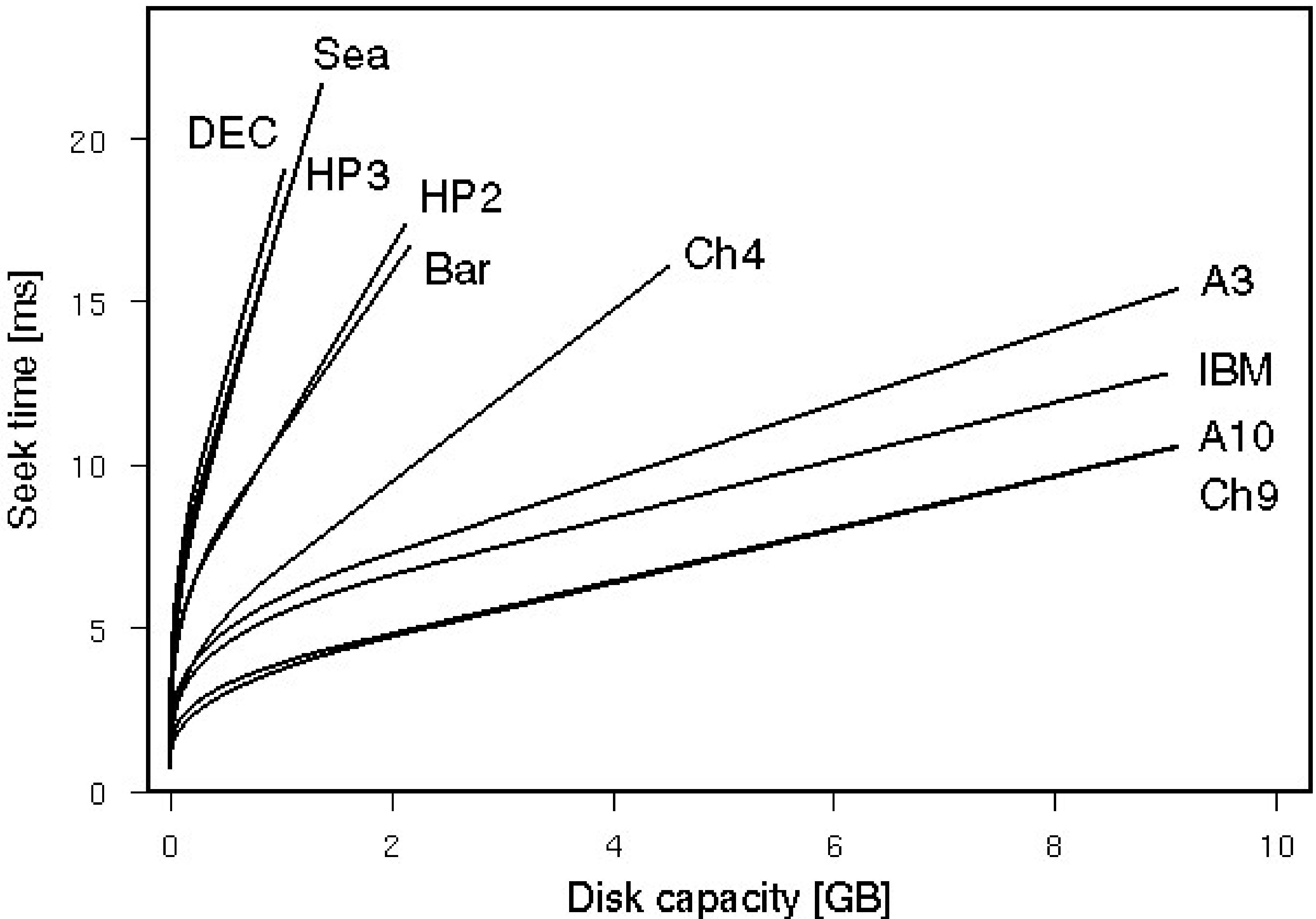
Mean Seek Time As a Function of File Size (EXPO Model)



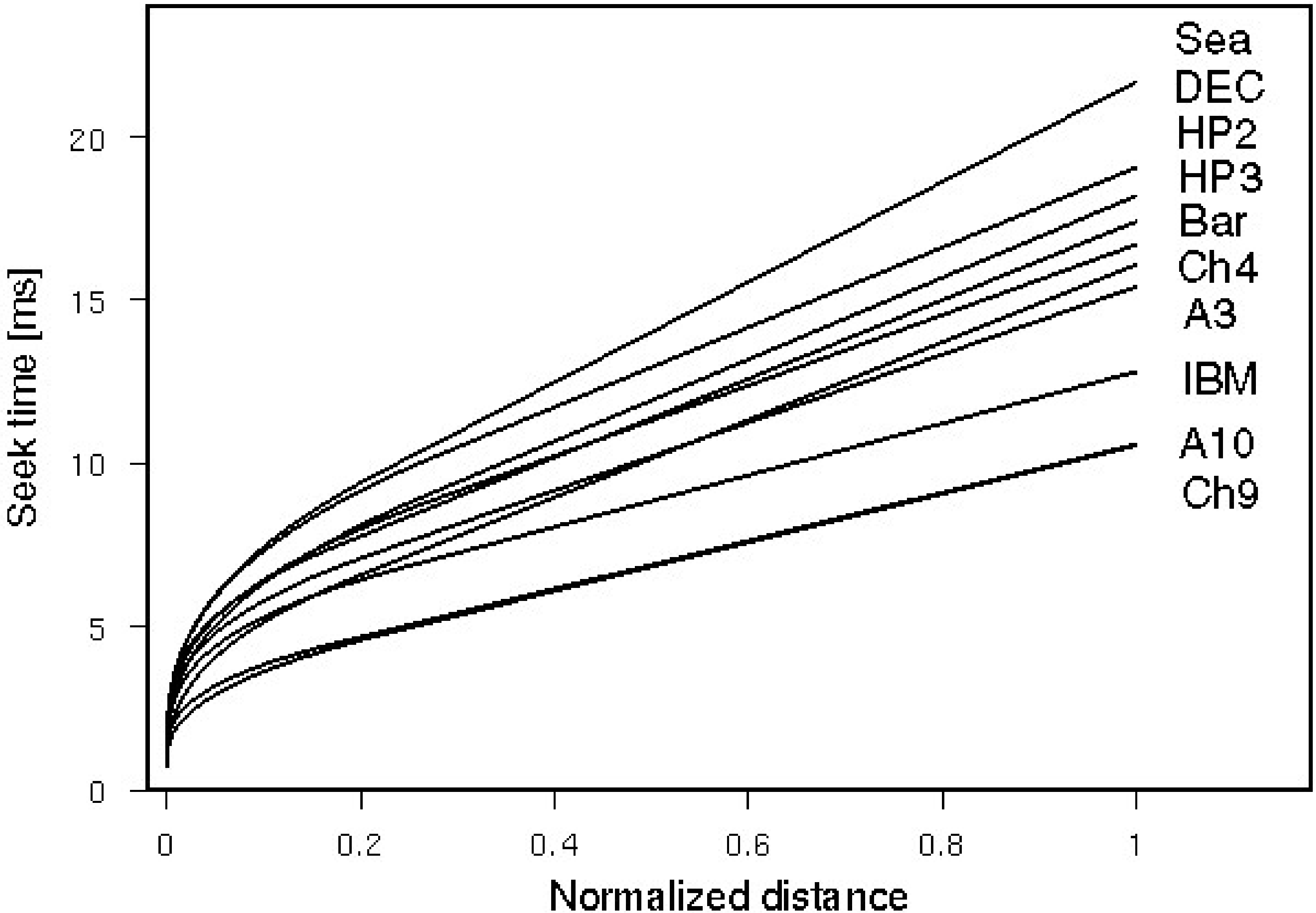
Mean Disk Seek Times (EXPO)

Table 5. Mean seek times [milliseconds]			
Disk	Manufacturer's data	Tseek(Nmax) **[EXPO model]**	Difference %
A10	5	5.56	11.2
A3	7.8	8.31	6.5
Bar	9.4	9.33	-0.7
Ch4	7.7	7.97	3.5
Ch9	5.4	5.47	1.3
IBM	7	7.3	4.3
Sea	11.5	11.16	-3

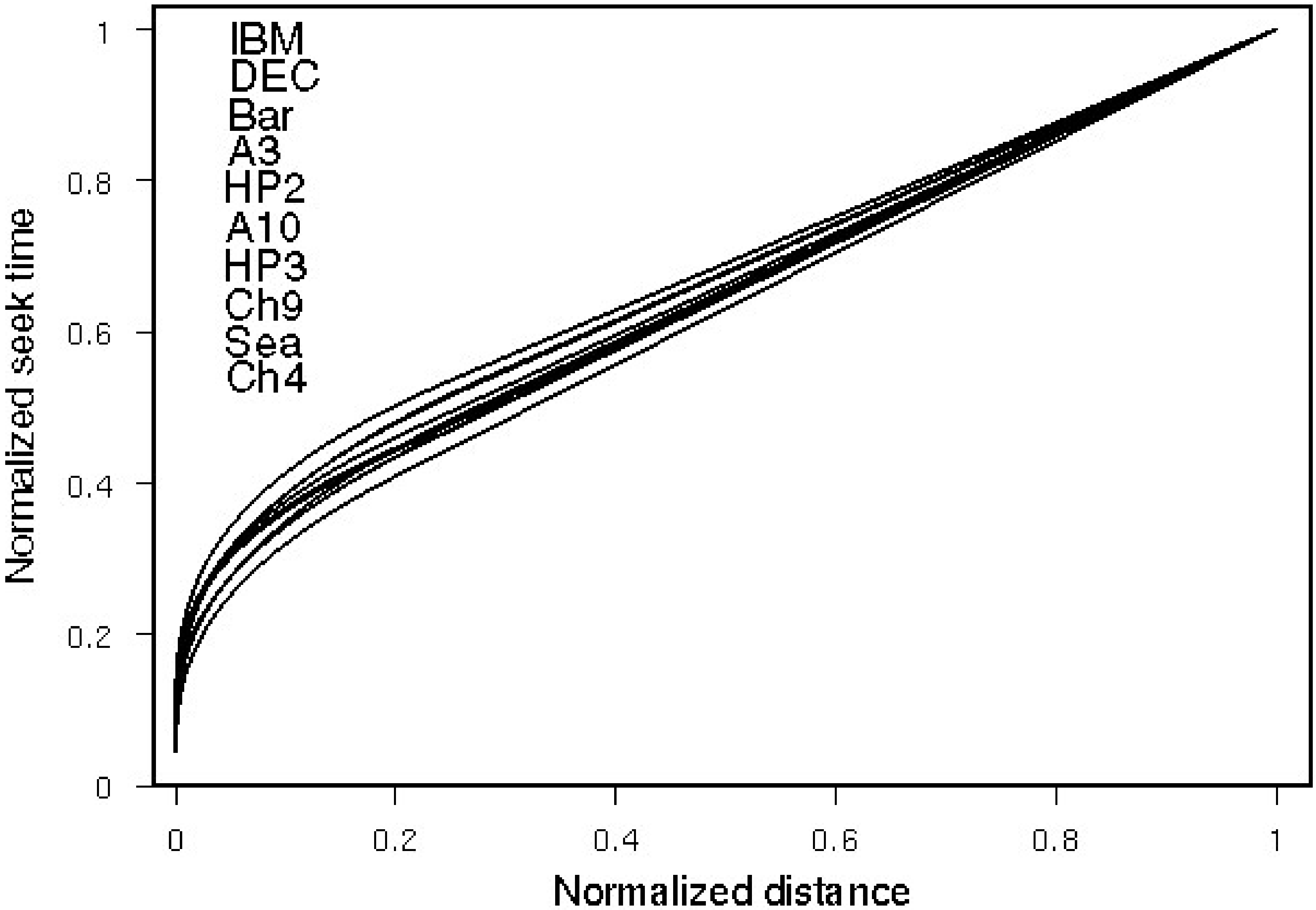
EXPO Model: Disk Comparison



EXPO Model: Normalized Distance



EXPO Model: Similarities of Normalized Seek Times



Mean Disk Access Time

$$\overline{T}_{disk}(N) = \overline{T}_{seek}(N) + \frac{60}{N_{rpm}}\left(\frac{1}{2} + \frac{N_{sect}}{\overline{N}_{spt}}\right)$$

- No caching.
- No disk access optimization
- N_{rpm} = revolutions per minute
- $\overline{N}_{spt}$ = mean number of sectors per track
- N_{sect} = sectors written/read per access

Disk Access Optimization (SSTF) and Disk Service Time

$$S(Q) = \overline{T}_{seek}(N)e^{-\alpha(Q-1)} + \frac{60}{N_{rpm}}\left(\frac{1}{2} + \frac{N_{sect}}{\overline{N}_{spt}}\right)$$

- Q = disk queue length (no optimization for Q=1 , the case of single process)
- α = quality of optimization
- S(Q) = load-dependent disk service time

Cached Disk Mean Access Time

$$\overline{T}_{access}(F,Q) = \begin{cases} t_{cache}, & F \le C \\ t_{cache} + \dfrac{F-C}{F} S(Q), & F \ge C \end{cases}$$

- F = file size
- C = cache size
- Assumption: uniform access distribution

Model Refinements

- zoning (typically 10 zones for modern disks)
- settle time (1-3ms [RW])
- head switch delay (0.5-1.5ms [RW])
- track and cylinder skewing
- sparing (bad sectors/tracks)
- controller caching (64KB-1MB [RW])
- controller firmware overhead (up to 1ms)
- recalibrations

Conclusions

- Square root and polynomial models of seek time are not sufficiently accurate.
- EXPO model provides very good numerical accuracy for disk modeling and significantly outperforms SQRT and POLY models .
- Disk models must be load-dependent because of disk access optimization.
- EXPO model can be integrated with models of caching and access optimization.