GI/G/1
Model and Simulator

A general case of single server system with general independent arrivals and general service times

Distributions of interarrival and service times

- M = exponential distribution
- D = deterministic case (constant value)
- U = uniform distribution (from A to B)
- GI = general independent arrivals (this case includes uniform distribution)

GI/G/1 – a general single server model

$S =$ mean service time with coefficient of variation v_s

$$v_s = \sigma_s / S = \begin{cases} 1, & \text{for exponential distribution} \\ \dfrac{B-A}{\sqrt{3}(A+B)}, & \text{for uniform distribution [A,B]} \end{cases}$$

$a =$ mean interarrival time with coefficient of variation v_a

$v_a = \sigma_a / a =$ coefficient of variation of interarrival time

$X = 1/a =$ throughput (input arrival and output departure rate)

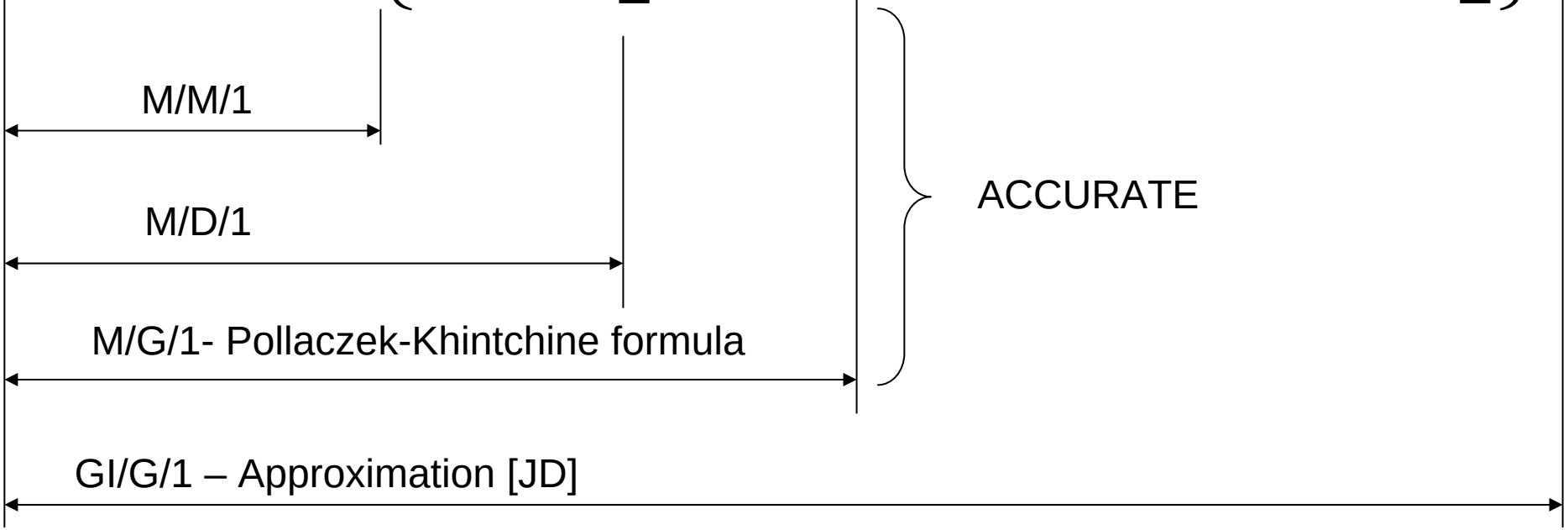
$U = SX = S/a =$ server utilization

$R =$ response time

$W = R - S =$ queue weight time

GI/G/1 formula

$$R \cong \frac{S}{1-U}\left\{1-\frac{U}{2}\left[1-v_s^2-\frac{(v_s^2+1)(v_a^2-1)}{U^2 v_s^2+1}\right]\right\}$$



G/G/1 Calculator

```
#include<iostream.h>
#include<math.h>

int main(void)  // Jozo Dujmovic, Fall 2010
{
    char adist, sdist;
    double S, U, vs,va, Tmin, Tmax, a, X, R;
    cout << "G/G/1 MODEL ANALYZER";
    while(1)
    {
        cout << "\n\n-----------------------------------------------------"
             << "\n\nThe available interarrival time distributions are:"
             << "\n   (1) Constant"
             << "\n   (2) Exponential"
             << "\n   (3) Uniform"
             << "\nEnter your choice: ";
        cin >> adist;
        if(adist=='1') {va=0.; cout << "Constant interarrival time = "; cin >> a; X=1./a;}
        else if(adist=='2') {va=1.; cout << "Mean interarrival time = "; cin >> a; X=1./a;}
        else if(adist=='3')
        {
            cout << "Enter min and max interarrival time = ";
            cin >> Tmin >> Tmax;
            a = (Tmax+Tmin)/2.;
            X=1./a;
            va = (Tmax-Tmin)/((Tmax+Tmin)*sqrt(3.));
        }
        else return 1;

        cout << "\n\nThe available service time distributions are:"
             << "\n   (1) Constant"
             << "\n   (2) Exponential"
             << "\n   (3) Uniform"
             << "\nEnter your choice: ";
        cin >> sdist;
        if(sdist=='1') {vs=0.; cout << "Constant service time = "; cin >> S;}
        else if(sdist=='2') {vs=1.; cout << "Mean service time = "; cin >> S;}
        else if(sdist=='3')
        {
            cout << "Enter min and max service time = ";
            cin >> Tmin >> Tmax;
            S = (Tmax+Tmin)/2.;
            vs = (Tmax-Tmin)/((Tmax+Tmin)*sqrt(3.));
        }
        else return 1;

        U = S*X;
        R = (S/(1.-U))*(1 - 0.5*U*(1. - vs*vs - ((vs*vs + 1.)*(va*va-1.)/(U*U*vs*vs+1.))));
        cout << "\nServer utilization U = " << U
             << "\nMean response time R = " << R
             << "\nMean queue length  Q = " << ((adist=='1' && sdist == '1') ? U : U/(1-U))
             << "\nTime spent waiting W = " << R-S ;

    }

}
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G/G/1 MODEL ANALYZER

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The available interarrival time distributions are:
   (1) Constant
   (2) Exponential
   (3) Uniform
Enter your choice: 3
Enter min and max interarrival time = 1  3

The available service time distributions are:
   (1) Constant
   (2) Exponential
   (3) Uniform
Enter your choice: 3
Enter min and max service time = 1  2

Server utilization U = 0.75
Mean response time R = 1.7381
Mean queue length  Q = 3
Time spent waiting W = 0.238095

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The available interarrival time distributions are:
   (1) Constant
   (2) Exponential
   (3) Uniform
Enter your choice: 3
Enter min and max interarrival time = 1  3


The available service time distributions are:
   (1) Constant
   (2) Exponential
   (3) Uniform
Enter your choice: 2
Mean service time = 1

Server utilization U = 0.5
Mean response time R = 1.26667
Mean queue length  Q = 1
Time spent waiting W = 0.266667

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The available interarrival time distributions are:
   (1) Constant
   (2) Exponential
   (3) Uniform
Enter your choice: 3
Enter min and max interarrival time = 1  3


The available service time distributions are:
   (1) Constant
   (2) Exponential
   (3) Uniform
Enter your choice: 1
Constant service time = 1

Server utilization U = 0.5
Mean response time R = 1.04167
Mean queue length  Q = 1
Time spent waiting W = 0.0416667

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The available interarrival time distributions are:
   (1) Constant
   (2) Exponential
   (3) Uniform
Enter your choice: 2
Mean interarrival time = 2


The available service time distributions are:
   (1) Constant
   (2) Exponential
   (3) Uniform
Enter your choice: 3
Enter min and max service time = 1  2

Server utilization U = 0.75
Mean response time R = 3.83333
Mean queue length  Q = 3
Time spent waiting W = 2.33333

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```

Results of Markus Neubrand simulator

a dist	s dist	Uan	Usim	E[%]	Qan	Qsim	E[%]
D(2.0)	D(1.0)	0.500	0.500	0.000	0.500	0.500	0.000
D(2.0)	M(1.0)	0.500	0.500	-0.033	0.600	0.627	4.564
D(2.0)	G(U 1.0-2.0)	0.750	0.750	0.014	0.774	0.750	-3.074
M(0.5)	D(1.0)	0.500	0.500	0.020	0.750	0.750	0.034
M(0.5)	M(1.0)	0.500	0.500	-0.055	1.000	1.001	0.107
M(0.5)	G(U 1.0-2.0)	0.750	0.750	0.033	1.917	1.919	0.099
G(U 1.0-3.0)	D(1.0)	0.500	0.500	-0.021	0.521	0.500	-3.977
G(U 1.0-3.0)	M(1.0)	0.500	0.500	0.023	0.633	0.658	3.927
G(U 1.0-3.0)	G(U 1.0-2.0)	0.750	0.750	0.009	0.869	0.825	-5.105

a dist	s dist	Ran	Rsim	E[%]	Wan	Wsim	E[%]
D(2.0)	D(1.0)	1.000	1.000	0.000	0.000	0.000	0.000
D(2.0)	M(1.0)	1.200	1.255	4.570	0.200	0.255	27.315
D(2.0)	G(U 1.0-2.0)	1.548	1.500	-3.063	0.048	0.000	-100.000
M(0.5)	D(1.0)	1.500	1.501	0.036	0.500	0.501	0.113
M(0.5)	M(1.0)	2.000	1.997	-0.136	1.000	1.001	0.138
M(0.5)	G(U 1.0-2.0)	3.833	3.840	0.163	2.333	2.336	0.135
G(U 1.0-3.0)	D(1.0)	1.042	1.000	-4.000	0.042	0.000	-100.000
G(U 1.0-3.0)	M(1.0)	1.267	1.317	3.999	0.267	0.317	18.868
G(U 1.0-3.0)	G(U 1.0-2.0)	1.738	1.649	-5.115	0.238	0.149	-37.279

Exponential Distribution (M/M/1)

Simulation controls

Arrival distribution

Type: Exponential (M) Lambda (λ): 1

Service distribution

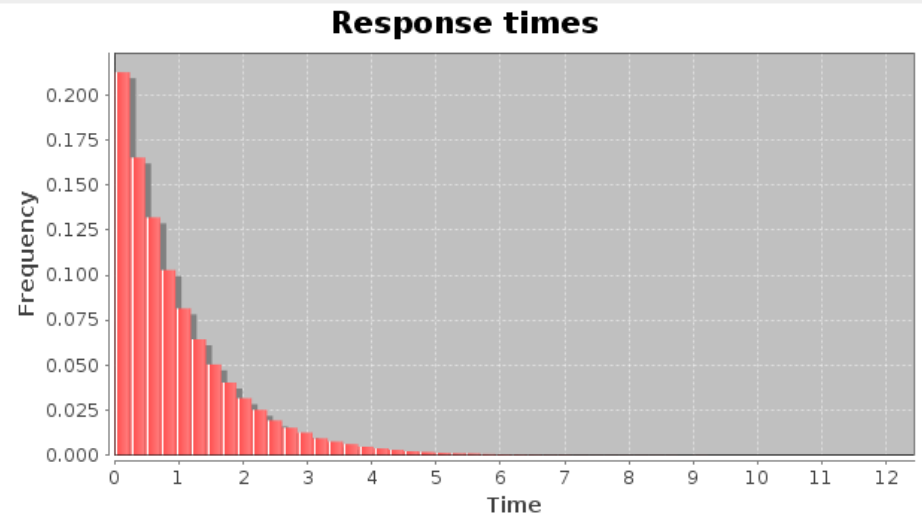
Type: Exponential (M) Lambda (λ): 2

Simulation parameter

Simulation steps (leave empty for infinite simulation): 200000

Finished | Start simulation | Stop simulation

Response times distribution

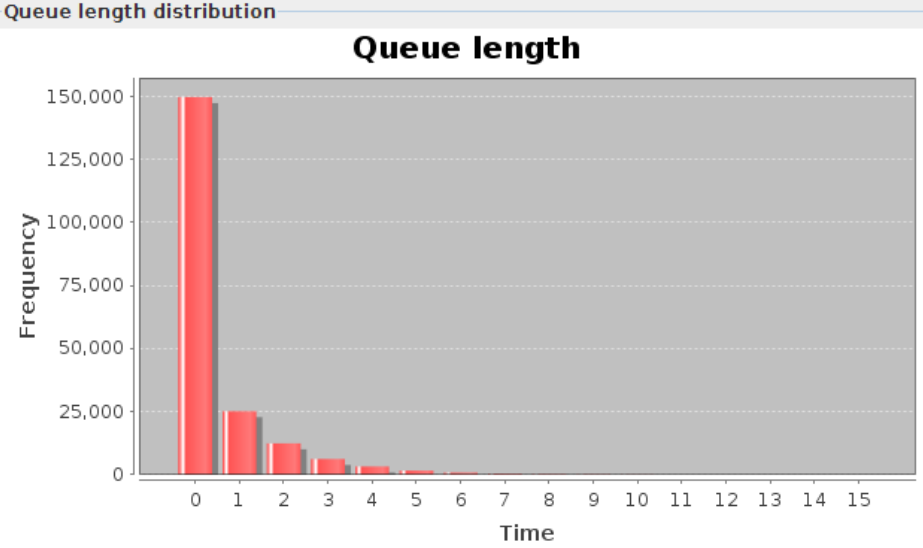


Simulation results

Total number of processed jobs: 200000

Interarrival time	Average: 0.998	Std. dev.: 0.994
Service time	Average: 0.499	Std. dev.: 0.5
Queue length	Average: 0.499	Std. dev.:1.115
Jobs in System	Average: 0.999	Std. dev.:1.411
Server utilization	0.5	
Response time	Average: 0.997	Std. dev.: 0.996

Queue length distribution



Uniform[0,10] / Uniform[1,5] (GI/G/1)

Simulation controls

Arrival distribution

Type: Uniform (G) Lower bound (a): 0 Upper bound (b): 10

Service distribution

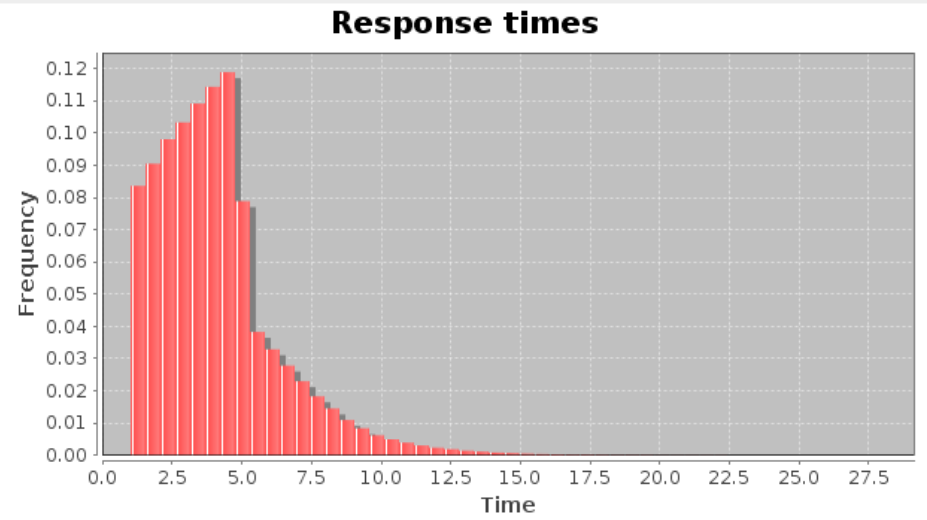
Type: Uniform (G) Lower bound (a): 1 Upper bound (b): 5

Simulation parameter

Simulation steps (leave empty for infinite simulation): 1000000

Finished | Start simulation | Stop simulation

Response times distribution

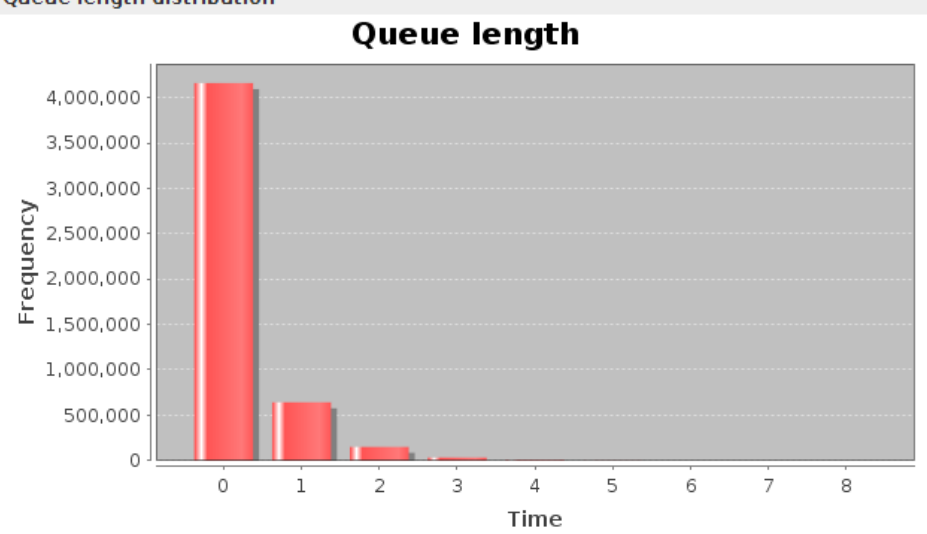


Simulation results

Total number of processed jobs: 1000000

Interarrival time	Average: 4.994	Std. dev.: 2.89
Service time	Average: 2.998	Std. dev.: 1.155
Queue length	Average: 0.216	Std. dev.:0.543
Jobs in System	Average: 0.817	Std. dev.:0.841
Server utilization	0.6	
Response time	Average: 4.078	Std. dev.: 2.261

Queue length distribution



Uniform[0,6.667] / Uniform[1,5] (GI/G/1)

Simulation controls

Arrival distribution

Type: Uniform (G) Lower bound (a): 0 Upper bound (b): 6.667

Service distribution

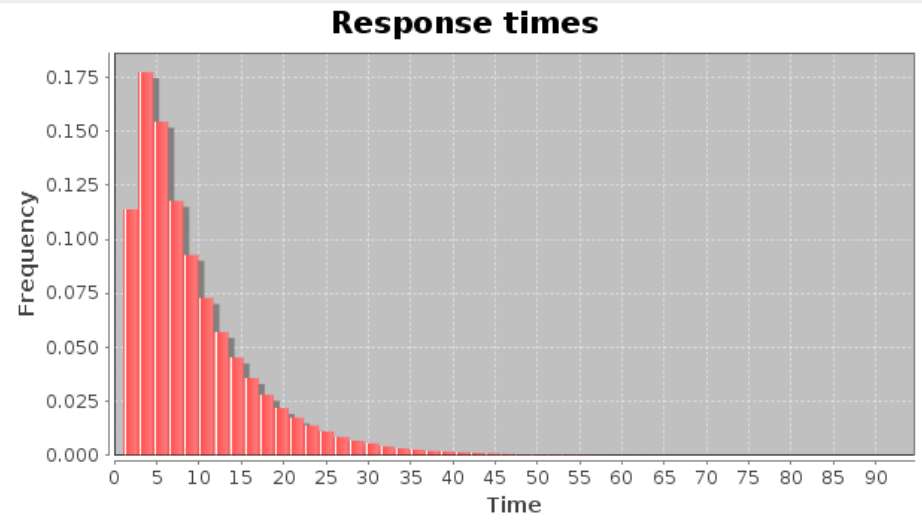
Type: Uniform (G) Lower bound (a): 1 Upper bound (b): 5

Simulation parameter

Simulation steps (leave empty for infinite simulation): 1000000

Finished | Start simulation | Stop simulation

Response times distribution



Simulation results

Total number of processed jobs: 1000000

Interarrival time	Average: 3.332	Std. dev.: 1.924
Service time	Average: 3.001	Std. dev.: 1.154
Queue length	Average: 1.914	Std. dev.:2.41
Jobs in System	Average: 2.815	Std. dev.:2.505
Server utilization	0.901	
Response time	Average: 9.38	Std. dev.: 7.54

Queue length distribution