```
^{1} _{\sqcap}
                                                Module Inv
    EXTENDS Implementation, TypeSafety
    SameRoot(t, i, j) \triangleq
                                         t.sigma[i] = t.sigma[j]
     SigmaRespectsShared \triangleq
                                          \forall t \in M : \forall i \in NodeSet :
                                                                               \wedge F[i].bit = 0
                                                                                                      \Rightarrow t.sigma[i] = t.sigma[F[i].pares
                                                                               \wedge F[i].bit = 1
                                                                                                      \Rightarrow t.sigma[i] = i
 7
    SharedRespectsSigma \stackrel{\Delta}{=}
                                          \forall t \in M : \forall i \in NodeSet : \land t.sigma[i] = i \Rightarrow F[i].bit = 1
    InvF2All(p, t) \triangleq
                                \wedge SameRoot(t, c[p], u\_F[p])
                                \wedge F[u\_F[p]].bit = 0
     InvF3All(p, t) \triangleq
15
                                \wedge SameRoot(t, c[p], u\_F[p])
16
    InvF4All(p, t) \triangleq
                                \wedge F[u\_F[p]].bit = 0
18
                                \wedge a F[p].bit = 0
19
                                \land SameRoot(t, c[p], a\_F[p].parent)
20
                                \wedge SameRoot(t, c[p], u\_F[p])
21
    InvF5All(p, t) \triangleq
                                \wedge F[u\_F[p]].bit = 0
23
                                \wedge a F[p].bit = 0
24
                                \land SameRoot(t, c[p], a\_F[p].parent)
25
                                \wedge SameRoot(t, c[p], u\_F[p])
26
                                \land b\_F[p].bit = 0 \Rightarrow SameRoot(t, a\_F[p].parent, b\_F[p].parent)
27
                                \land b_{-}F[p].bit = 1 \Rightarrow t.sigma[a_{-}F[p].parent] = a_{-}F[p].parent
28
    InvF6All(p, t) \triangleq
                                \wedge F[u\_F[p]].bit = 0
30
                                \wedge a F[p].bit = 0
31
                                \wedge F[a\_F[p].parent].bit = 0
32
                                \wedge b F[p].bit = 0
33
                                \land SameRoot(t, c[p], a\_F[p].parent)
34
                                \wedge SameRoot(t, c[p], u\_F[p])
35
                                \land SameRoot(t, a\_F[p].parent, b\_F[p].parent)
36
    InvF7All(p, t) \triangleq
                                \wedge F[u\_F[p]].bit = 0
38
                                \wedge a F[p].bit = 0
39
40
                                \land SameRoot(t, c[p], a\_F[p].parent)
                                \wedge SameRoot(t, c[p], u\_F[p])
41
    InvU2All(p, t) \triangleq
                                \land SameRoot(t, t.arg[p][1], u\_U[p])
43
                                \land SameRoot(t, t.arg[p][2], v_U[p])
44
    InvU5All(p, t) \triangleq
                                \land SameRoot(t, t.arg[p][1], u\_U[p])
47
                                \wedge \ SameRoot(t, \ t.arg[p][2], \ v\_U[p])
48
                                \wedge u_{-}U[p] \neq v_{-}U[p]
49
```

```
\land a\_U[p].bit = 0 \Rightarrow SameRoot(t, a\_U[p].parent, u\_U[p])
50
                                  \land a_{-}U[p].bit = 1 \Rightarrow t.sigma[u_{-}U[p]] = u_{-}U[p]
51
     InvU6All(p, t) \triangleq
                                  \wedge SameRoot(t, t.arg[p][1], u_U[p])
53
                                  \land SameRoot(t, t.arg[p][2], v\_U[p])
54
                                  \wedge u_{-}U[p] \neq v_{-}U[p]
55
                                  \land a\_U[p].bit = 0 \Rightarrow SameRoot(t, a\_U[p].parent, u\_U[p])
56
                                  \land a_-U[p].bit = 1 \Rightarrow t.sigma[u_-U[p]] = u_-U[p]
57
58
                                  \land b_{-}U[p].bit = 0 \Rightarrow SameRoot(t, b_{-}U[p].parent, v_{-}U[p])
                                  \land b_{-}U[p].bit = 1 \Rightarrow t.sigma[v_{-}U[p]] = v_{-}U[p]
59
     InvU7All(p, t) \triangleq
                                  \wedge SameRoot(t, t.arg[p][1], u_U[p])
61
                                  \land SameRoot(t, t.arg[p][2], v_U[p])
62
                                  \land t.ret[p] = ACK \Rightarrow SameRoot(t, u\_U[p], v\_U[p])
63
    InvU8All(p, t) \triangleq
                                  \wedge SameRoot(t, t.arg[p][1], u_U[p])
65
                                  \land SameRoot(t, t.arg[p][2], v_{-}U[p])
66
                                  \land t.ret[p] = ACK \Rightarrow SameRoot(t, u\_U[p], v\_U[p])
67
     InvDecide \stackrel{\triangle}{=}
                                 \forall p \in PROCESSES : \forall t \in M :
69
                                                                    \land \ pc[p] = \text{``0''}
                                                                                             \Rightarrow \land t.ret[p] = BOT
70
71
                                                                                                   \wedge t.op[p] = BOT
                                                                                                   \land t.arg[p] = BOT
72
    InvF1 \triangleq
                                 \forall p \in PROCESSES : \forall t \in M :
73
                                                                    \wedge \ pc[p] = \text{``F1''}
                                                                                               \Rightarrow \land t.ret[p] = BOT
74
                                                                                                    \wedge t.op[p] = \text{``F''}
75
                                                                                                     \land t.arg[p] \in NodeSet
76
                                                                                                    \land SameRoot(t, c[p], t.arg[p])
77
                                                                    \land pc[p] = \text{``F1U1''} \Rightarrow \land t.ret[p] = BOT
78
                                                                                                    \wedge t.op[p] = \text{``U"}
79
                                                                                                    \land t.arg[p] \in NodeSet \times NodeSet
80
                                                                                                    \wedge SameRoot(t, c[p], u_{-}U[p])
81
                                                                    \land pc[p] = \text{``F1U2''} \Rightarrow \land t.ret[p] = BOT
82
                                                                                                    \wedge t.op[p] = \text{``U"}
83
                                                                                                     \land t.arg[p] \in NodeSet \times NodeSet
84
                                                                                                    \wedge InvU2All(p, t)
85
                                                                                                    \wedge SameRoot(t, c[p], v_{-}U[p])
86
                                                                   \land pc[p] = \text{``F1U7''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
87
                                                                                                    \wedge t.op[p] = \text{``U"}
88
                                                                                                     \land \ t.arg[p] \in NodeSet \times NodeSet
89
                                                                                                    \wedge InvU7All(p, t)
90
                                                                                                    \wedge SameRoot(t, c[p], u_{-}U[p])
91
                                                                    \land pc[p] = \text{``F1U8''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
92
                                                                                                     \wedge t.op[p] = \text{``U"}
93
                                                                                                     \land \ t.arg[p] \in NodeSet \times NodeSet
94
                                                                                                     \wedge InvU8All(p, t)
95
```

```
\wedge SameRoot(t, c[p], v_{-}U[p])
 96
     InvF2 \stackrel{\triangle}{=}
                                   \forall p \in PROCESSES : \forall t \in M :
 97
                                                                      \wedge pc[p] = \text{``F2''}
                                                                                                 \Rightarrow \land t.ret[p] = BOT
 98
                                                                                                        \wedge t.op[p] = \text{``F''}
 99
                                                                                                        \land t.arg[p] \in NodeSet
100
                                                                                                       \wedge SameRoot(t, c[p], t.arg[p])
101
                                                                                                       \wedge InvF2All(p, t)
102
                                                                      \land pc[p] = \text{``F2U1''} \Rightarrow \land t.ret[p] = BOT
103
                                                                                                       \wedge t.op[p] = "U"
104
                                                                                                        \land t.arg[p] \in NodeSet \times NodeSet
105
                                                                                                       \wedge SameRoot(t, c[p], u\_U[p])
106
                                                                                                       \wedge InvF2All(p, t)
107
                                                                      \land pc[p] = \text{``F2U2''} \Rightarrow \land t.ret[p] = BOT
108
                                                                                                       \wedge t.op[p] = "U"
109
                                                                                                        \land t.arg[p] \in NodeSet \times NodeSet
110
                                                                                                        \wedge InvU2All(p, t)
111
                                                                                                       \wedge SameRoot(t, c[p], v_U[p])
112
                                                                                                       \wedge InvF2All(p, t)
113
                                                                      \land pc[p] = \text{``F2U7''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
114
                                                                                                       \wedge t.op[p] = "U"
115
                                                                                                        \land \ t.arg[p] \in \mathit{NodeSet} \times \mathit{NodeSet}
116
                                                                                                        \wedge InvU7All(p, t)
117
                                                                                                        \wedge SameRoot(t, c[p], u_U[p])
118
                                                                                                       \wedge InvF2All(p, t)
119
                                                                      \land pc[p] = \text{``F2U8''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
120
                                                                                                       \wedge t.op[p] = \text{``U"}
121
                                                                                                        \land t.arg[p] \in NodeSet \times NodeSet
122
123
                                                                                                        \wedge InvU8All(p, t)
                                                                                                       \wedge SameRoot(t, c[p], v_{-}U[p])
124
                                                                                                        \wedge InvF2All(p, t)
125
     InvF3 \triangleq
                                   \forall p \in PROCESSES : \forall t \in M :
128
                                                                      \wedge pc[p] = \text{``F3''}
                                                                                                 \Rightarrow \land t.ret[p] = BOT
129
                                                                                                       \wedge t.op[p] = \text{``F''}
130
                                                                                                        \land t.arg[p] \in NodeSet
131
132
                                                                                                        \land SameRoot(t, c[p], t.arg[p])
                                                                                                       \wedge InvF3All(p, t)
133
                                                                      \land pc[p] = \text{``F3U1''} \Rightarrow \land t.ret[p] = BOT
134
                                                                                                        \wedge t.op[p] = "U"
135
                                                                                                       \land t.arg[p] \in NodeSet \times NodeSet
136
                                                                                                       \wedge SameRoot(t, c[p], u\_U[p])
137
                                                                                                       \wedge InvF3All(p, t)
138
                                                                      \land \ \ pc[p] = \text{``F3U2''} \ \ \Rightarrow \ \ \land \ t.ret[p] = BOT
139
                                                                                                       \wedge t.op[p] = "U"
```

140

```
\land t.arg[p] \in NodeSet \times NodeSet
141
                                                                                                      \wedge InvU2All(p, t)
142
                                                                                                     \wedge SameRoot(t, c[p], v_{-}U[p])
143
                                                                                                     \wedge InvF3All(p, t)
144
                                                                     \land \ pc[p] = \text{``F3U7''} \ \Rightarrow \ \land t.ret[p] \in \{BOT, ACK\}
145
                                                                                                     \wedge t.op[p] = "U"
146
                                                                                                      \land t.arg[p] \in NodeSet \times NodeSet
147
                                                                                                      \wedge InvU7All(p, t)
148
                                                                                                     \wedge SameRoot(t, c[p], u\_U[p])
149
                                                                                                     \wedge InvF3All(p, t)
150
                                                                     \land pc[p] = \text{``F3U8''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
151
                                                                                                      \wedge t.op[p] = "U"
152
                                                                                                      \land \ t.arg[p] \in NodeSet \times NodeSet
153
154
                                                                                                      \wedge InvU8All(p, t)
                                                                                                      \wedge SameRoot(t, c[p], v_{-}U[p])
155
156
                                                                                                      \wedge InvF3All(p, t)
     InvF4 \stackrel{\triangle}{=}
                                   \forall p \in PROCESSES : \forall t \in M :
158
                                                                     \wedge \ pc[p] = \text{``F4''}
                                                                                                \Rightarrow \land t.ret[p] = BOT
159
                                                                                                      \wedge t.op[p] = \text{``F''}
160
161
                                                                                                      \land t.arg[p] \in NodeSet
                                                                                                     \land SameRoot(t, c[p], t.arg[p])
162
                                                                                                     \wedge InvF4All(p, t)
163
                                                                     \land pc[p] = \text{``F4U1''} \Rightarrow \land t.ret[p] = BOT
164
                                                                                                     \wedge t.op[p] = "U"
165
166
                                                                                                      \land t.arg[p] \in NodeSet \times NodeSet
                                                                                                     \wedge SameRoot(t, c[p], u_U[p])
167
                                                                                                     \wedge InvF4All(p, t)
168
                                                                     \land pc[p] = \text{``F4U2''} \Rightarrow \land t.ret[p] = BOT
169
                                                                                                     \wedge t.op[p] = "U"
170
                                                                                                      \land \ t.arg[p] \in \mathit{NodeSet} \times \mathit{NodeSet}
171
172
                                                                                                     \wedge InvU2All(p, t)
                                                                                                     \wedge SameRoot(t, c[p], v_{-}U[p])
173
                                                                                                     \wedge InvF4All(p, t)
174
                                                                     \land pc[p] = \text{``F4U7''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
175
176
                                                                                                      \wedge t.op[p] = \text{``U"}
                                                                                                      \land t.arg[p] \in NodeSet \times NodeSet
177
178
                                                                                                     \wedge InvU7All(p, t)
                                                                                                     \wedge SameRoot(t, c[p], u\_U[p])
179
                                                                                                     \wedge InvF4All(p, t)
180
                                                                     \land pc[p] = \text{``F4U8''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
181
                                                                                                     \land t.op[p] = \text{``U"}
182
                                                                                                      \land t.arg[p] \in NodeSet \times NodeSet
183
                                                                                                      \wedge InvU8All(p, t)
184
                                                                                                     \wedge SameRoot(t, c[p], v_U[p])
185
```

```
InvF5 \triangleq
                                    \forall p \in PROCESSES : \forall t \in M :
189
                                                                       \land \ \mathit{pc}[\mathit{p}] = \text{``F5''}
190
                                                                                                  \Rightarrow \land t.ret[p] = BOT
                                                                                                         \wedge t.op[p] = \text{``F''}
191
                                                                                                         \land t.arg[p] \in NodeSet
192
                                                                                                        \land SameRoot(t, c[p], t.arg[p])
193
                                                                                                        \wedge InvF5All(p, t)
194
                                                                       \land pc[p] = \text{``F5U1''} \Rightarrow \land t.ret[p] = BOT
195
                                                                                                        \wedge t.op[p] = "U"
196
                                                                                                         \land t.arg[p] \in NodeSet \times NodeSet
197
                                                                                                         \wedge SameRoot(t, c[p], u_{-}U[p])
198
                                                                                                         \wedge InvF5All(p, t)
199
                                                                       \land pc[p] = \text{``F5U2''} \Rightarrow \land t.ret[p] = BOT
200
                                                                                                         \wedge t.op[p] = \text{``U"}
201
                                                                                                         \land t.arg[p] \in NodeSet \times NodeSet
202
                                                                                                         \wedge InvU2All(p, t)
203
                                                                                                        \wedge \ SameRoot(t, \ c[p], \ v\_U[p])
204
                                                                                                        \wedge InvF5All(p, t)
205
                                                                       \land \ pc[p] = \text{``F5U7''} \ \Rightarrow \ \land t.ret[p] \in \{BOT, ACK\}
206
                                                                                                         \land \ t.op[p] = \text{``U"}
207
                                                                                                         \land t.arg[p] \in NodeSet \times NodeSet
208
                                                                                                        \wedge InvU7All(p, t)
209
                                                                                                         \wedge SameRoot(t, c[p], u\_U[p])
210
                                                                                                        \wedge InvF5All(p, t)
211
                                                                       \land pc[p] = \text{``F5U8''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
212
                                                                                                        \wedge t.op[p] = "U"
213
                                                                                                         \land t.arg[p] \in NodeSet \times NodeSet
214
                                                                                                         \wedge InvU8All(p, t)
215
216
                                                                                                         \wedge SameRoot(t, c[p], v_{-}U[p])
217
                                                                                                         \wedge InvF5All(p, t)
      InvF6 \stackrel{\triangle}{=}
                                    \forall p \in PROCESSES : \forall t \in M :
219
                                                                       \wedge \ pc[p] = \text{``F6''}
                                                                                                  \Rightarrow \land t.ret[p] = BOT
220
                                                                                                         \wedge t.op[p] = \text{``F''}
221
                                                                                                         \land t.arg[p] \in NodeSet
222
                                                                                                        \land SameRoot(t, c[p], t.arg[p])
223
                                                                                                         \wedge InvF6All(p, t)
224
                                                                       \land pc[p] = \text{``F6U1''} \Rightarrow \land t.ret[p] = BOT
225
                                                                                                         \wedge t.op[p] = "U"
226
                                                                                                         \land t.arg[p] \in NodeSet \times NodeSet
227
                                                                                                        \wedge SameRoot(t, c[p], u\_U[p])
228
                                                                                                        \wedge InvF6All(p, t)
229
                                                                       \land pc[p] = \text{``F6U2''} \Rightarrow \land t.ret[p] = BOT
```

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230

231

 $\wedge InvF4All(p, t)$ 

 $\wedge t.op[p] = "U"$ 

```
\wedge InvU2All(p, t)
233
                                                                                                     \wedge SameRoot(t, c[p], v_{-}U[p])
234
                                                                                                     \wedge InvF6All(p, t)
235
                                                                     \land \ pc[p] = \text{``F6U7''} \ \Rightarrow \ \land t.ret[p] \in \{BOT, ACK\}
236
                                                                                                     \wedge t.op[p] = "U"
237
                                                                                                     \land t.arg[p] \in NodeSet \times NodeSet
238
                                                                                                     \wedge InvU7All(p, t)
239
                                                                                                     \wedge SameRoot(t, c[p], u\_U[p])
240
                                                                                                     \wedge InvF6All(p, t)
241
                                                                    \land pc[p] = \text{``F6U8''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
242
                                                                                                     \wedge t.op[p] = \text{``U"}
243
                                                                                                     \land \ t.arg[p] \in NodeSet \times NodeSet
244
245
                                                                                                     \wedge InvU8All(p, t)
                                                                                                     \wedge SameRoot(t, c[p], v_{-}U[p])
246
247
                                                                                                     \wedge InvF6All(p, t)
      InvF7 \triangleq
                                  \forall p \in PROCESSES : \forall t \in M :
250
                                                                    \wedge pc[p] = \text{``F7''}
                                                                                               \Rightarrow \land t.ret[p] = BOT
251
                                                                                                     \land t.op[p] = \text{``F''}
252
                                                                                                     \land t.arg[p] \in NodeSet
253
                                                                                                     \wedge SameRoot(t, c[p], t.arg[p])
254
                                                                                                     \wedge InvF7All(p, t)
255
                                                                    \land pc[p] = \text{``F7U1''} \Rightarrow \land t.ret[p] = BOT
256
                                                                                                     \wedge t.op[p] = "U"
257
                                                                                                     \land t.arg[p] \in NodeSet \times NodeSet
258
                                                                                                     \wedge SameRoot(t, c[p], u_U[p])
259
                                                                                                     \wedge InvF7All(p, t)
^{260}
                                                                     \land pc[p] = \text{``F7U2''} \Rightarrow \land t.ret[p] = BOT
261
                                                                                                     \wedge t.op[p] = "U"
262
                                                                                                     \land \ t.arg[p] \in NodeSet \times NodeSet
263
                                                                                                     \wedge InvU2All(p, t)
264
                                                                                                     \wedge SameRoot(t, c[p], v_U[p])
265
                                                                                                     \wedge InvF7All(p, t)
266
                                                                    \land pc[p] = \text{``F7U7''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
267
268
                                                                                                     \wedge t.op[p] = "U"
                                                                                                     \land \ t.arg[p] \in \mathit{NodeSet} \times \mathit{NodeSet}
^{269}
                                                                                                     \wedge InvU7All(p, t)
270
                                                                                                     \wedge SameRoot(t, c[p], u_U[p])
271
                                                                                                     \wedge InvF7All(p, t)
272
                                                                     \land pc[p] = \text{``F7U8''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
273
274
                                                                                                     \wedge t.op[p] = "U"
                                                                                                     \land t.arg[p] \in NodeSet \times NodeSet
275
                                                                                                     \wedge InvU8All(p, t)
276
```

232

 $\land t.arg[p] \in NodeSet \times NodeSet$ 

```
\wedge SameRoot(t, c[p], v_{-}U[p])
277
                                                                                                       \wedge InvF7All(p, t)
278
      InvFR \triangleq
                                   \forall p \in PROCESSES : \forall t \in M :
281
                                                                      \land pc[p] = \text{"FR"}
                                                                                                 \Rightarrow \land t.ret[p] = u_F[p]
282
                                                                                                       \wedge \ t.op[p] = \text{``F''}
283
                                                                                                       \land t.arg[p] \in NodeSet
284
                                                                                                       \land SameRoot(t, t.arg[p], u\_F[p])
285
                                                                                                       \wedge SameRoot(t, c[p], u\_F[p])
286
                                                                      \land pc[p] = \text{``FRU1''} \Rightarrow
                                                                                                       \wedge t.ret[p] = BOT
287
                                                                                                       \wedge t.op[p] = "U"
288
                                                                                                       \land t.arg[p] \in NodeSet \times NodeSet
289
                                                                                                       \wedge SameRoot(t, c[p], u\_U[p])
290
                                                                                                       \wedge SameRoot(t, c[p], u\_F[p])
291
                                                                      \land pc[p] = \text{``FRU2''} \Rightarrow \land t.ret[p] = BOT
292
                                                                                                       \wedge t.op[p] = "U"
293
                                                                                                       \land t.arg[p] \in NodeSet \times NodeSet
294
                                                                                                       \wedge InvU2All(p, t)
295
                                                                                                       \wedge SameRoot(t, c[p], v_{-}U[p])
296
                                                                      \land pc[p] = \text{``FRU7''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
298
                                                                                                       \wedge t.op[p] = \text{``U"}
299
                                                                                                       \land t.arg[p] \in NodeSet \times NodeSet
300
                                                                                                       \wedge InvU7All(p, t)
301
                                                                                                       \wedge SameRoot(t, c[p], u_{-}U[p])
302
                                                                      \land pc[p] = \text{"FRU8"} \Rightarrow
303
                                                                                                      \land t.ret[p] \in \{BOT, ACK\}
                                                                                                       \wedge t.op[p] = \text{``U"}
304
                                                                                                       \land \ t.arg[p] \in \mathit{NodeSet} \times \mathit{NodeSet}
305
                                                                                                       \wedge InvU8All(p, t)
306
                                                                                                       \wedge SameRoot(t, c[p], v_{-}U[p])
307
      InvU1 \triangleq
                                   \forall p \in PROCESSES : \forall t \in M :
309
                                                                     pc[p] = \text{``U1''}
                                                                                           \Rightarrow \land t.ret[p] = BOT
310
                                                                                                 \wedge t.op[p] = \text{``U"}
311
                                                                                                 \land t.arg[p] \in NodeSet \times NodeSet
312
      InvU2 \triangleq
                                   \forall p \in PROCESSES : \forall t \in M :
315
                                                                     pc[p] = "U2"
                                                                                                \wedge t.ret[p] = BOT
316
                                                                                                 \wedge t.op[p] = \text{``U"}
317
                                                                                                 \land t.arg[p] \in NodeSet \times NodeSet
318
                                                                                                 \wedge InvU2All(p, t)
319
     InvU3 \triangleq
                                   \forall p \in PROCESSES : \forall t \in M :
321
                                                                     pc[p] = "U3"
                                                                                           \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
322
                                                                                                 \wedge t.op[p] = \text{``U"}
323
```

```
\land \ t.arg[p] \in \mathit{NodeSet} \times \mathit{NodeSet}
324
                                                                                              \land SameRoot(t, t.arg[p][1], u\_U[p])
325
                                                                                              \land SameRoot(t, t.arg[p][2], v\_U[p])
326
                                                                                              \land t.ret[p] = ACK \Rightarrow SameRoot(t, u\_U[p], \bullet)
327
     InvU4 \stackrel{\triangle}{=}
                                  \forall p \in PROCESSES : \forall t \in M :
328
                                                                   pc[p] = "U4"
                                                                                             \land t.ret[p] \in \{BOT, ACK\}
329
                                                                                              \wedge t.op[p] = \text{``U"}
330
                                                                                              \land t.arg[p] \in NodeSet \times NodeSet
331
                                                                                              \land SameRoot(t, t.arg[p][1], u\_U[p])
332
                                                                                              \land SameRoot(t, t.arg[p][2], v_U[p])
333
                                                                                              \land t.ret[p] = ACK \Rightarrow SameRoot(t, u\_U[p], \bullet)
334
                                                                                              \wedge u_{-}U[p] \neq v_{-}U[p]
335
      InvU5 \triangleq
                                  \forall p \in PROCESSES : \forall t \in M :
337
                                                                   pc[p] = "U5"
338
                                                                                             \land t.ret[p] \in \{BOT, ACK\}
                                                                                              \wedge t.op[p] = \text{``U"}
339
                                                                                              \land t.arg[p] \in NodeSet \times NodeSet
340
                                                                                              \wedge InvU5All(p, t)
341
     InvU6 \triangleq
                                  \forall p \in PROCESSES : \forall t \in M :
342
                                                                 pc[p] = "U6"
343
344
                                                                                             \land t.ret[p] \in \{BOT, ACK\}
                                                                                             \land \ t.op[p] = \text{``U"}
345
                                                                                             \land t.arg[p] \in NodeSet \times NodeSet
346
                                                                                             \wedge InvU6All(p, t)
347
      InvU7 \triangleq
                                  \forall p \in PROCESSES : \forall t \in M :
349
                                                                   pc[p] = "U7"
                                                                                             \land t.ret[p] \in \{BOT, ACK\}
350
                                                                                              \wedge t.op[p] = \text{``U"}
351
                                                                                              \land t.arg[p] \in NodeSet \times NodeSet
352
                                                                                              \wedge InvU7All(p, t)
353
     InvU8 \triangleq
                                  \forall p \in PROCESSES : \forall t \in M :
355
                                                                   pc[p] = "U8"
                                                                                              \land t.ret[p] \in \{BOT, ACK\}
356
                                                                                              \wedge t.op[p] = \text{``U"}
357
                                                                                              \land t.arg[p] \in NodeSet \times NodeSet
358
                                                                                              \wedge InvU8All(p, t)
359
      InvUR \triangleq
                                  \forall p \in PROCESSES : \forall t \in M :
361
                                                                   pc[p] = \text{"UR"}
                                                                                              \wedge t.ret[p] = ACK
362
                                                                                               \wedge t.op[p] = "U"
363
                                                                                               \land t.arg[p] \in NodeSet \times NodeSet
364
                                                                                               \land SameRoot(t, t.arg[p][1], u\_U[p])
365
                                                                                               \land SameRoot(t, t.arg[p][2], v_U[p])
366
                                                                                               \wedge SameRoot(t, u_{-}U[p], v_{-}U[p])
367
```

 $Linearizable \triangleq$ 

369

 $M \neq \{\}$ 

```
\land TypeOK
      Inv \stackrel{\triangle}{=}
371
                       \land \ InvDecide
372
                       \land \mathit{InvF} 1
373
                       \wedge \mathit{InvF} 2
374
375
                       \land InvF3
                       \land \mathit{InvF4}
376
                       \land \mathit{InvF5}
377
                       \land \mathit{InvF} 6
378
                       \land \mathit{InvF7}
379
                       \land \mathit{InvFR}
380
                       \wedge InvU1
381
                       \wedge \operatorname{Inv} U2
382
                       \wedge \mathit{InvU}3
383
                       \wedge InvU4
384
                       \wedge \, \mathit{InvU} 5
385
                       \wedge \mathit{InvU}6
386
                       \land \mathit{InvU7}
387
                       \wedge \mathit{InvU}8
388
                       \wedge \; InvUR
389
                       \land SigmaRespectsShared
390
                       \land \ SharedRespectsSigma
391
                       \land \ Linearizable
392
394 L
```

**<sup>\\*</sup>** Modification History

<sup>\\*</sup> Last modified Thu Apr 17 20:52:02 EDT 2025 by karunram

<sup>\ \*</sup> Created Thu Apr 03 22:44:42 EDT 2025 by karunram