```
^{1} _{\sqcap}
                                                Module Inv
    EXTENDS Implementation, TypeSafety
    SameRoot(t, i, j) \triangleq
                                         t.sigma[i] = t.sigma[j]
     SigmaRespectsShared \stackrel{\Delta}{=}
                                          \forall t \in M : \forall i \in NodeSet :
                                                                               \wedge F[i].bit = 0
                                                                                                     \Rightarrow t.sigma[i] = t.sigma[F[i].parer]
                                                                               \wedge F[i].bit = 1
                                                                                                     \Rightarrow t.sigma[i] = i
 7
    InvF2All(p, t) \triangleq
                                \wedge SameRoot(t, c[p], u\_F[p])
    InvF3All(p, t) \triangleq
                                \wedge F[u_{-}F[p]].bit = 0
12
                                \wedge SameRoot(t, c[p], u\_F[p])
13
    InvF4All(p, t) \triangleq
                                \wedge F[u\_F[p]].bit = 0
15
                                \wedge a F[p].bit = 0
16
                                \land SameRoot(t, c[p], a\_F[p].parent)
17
                                \wedge SameRoot(t, c[p], u\_F[p])
18
     InvF5All(p, t) \triangleq
                                \wedge F[u_{-}F[p]].bit = 0
20
                                \wedge a F[p].bit = 0
21
                                \land SameRoot(t, c[p], a\_F[p].parent)
22
23
                                \wedge SameRoot(t, c[p], u\_F[p])
                                \land b\_F[p].bit = 0 \Rightarrow \land SameRoot(t, a\_F[p].parent, b\_F[p].parent)
24
                                                          \wedge F[a\_F[p].parent].bit = 0
25
     InvF6All(p, t) \triangleq
                                \wedge F[u_{-}F[p]].bit = 0
27
                                \wedge a F[p].bit = 0
28
                                \wedge F[a\_F[p].parent].bit = 0
29
                                \wedge b F[p].bit = 0
30
                                \land SameRoot(t, c[p], a\_F[p].parent)
31
                                \land \ SameRoot(t, \ c[p], \ u\_F[p])
32
33
                                \land SameRoot(t, a\_F[p].parent, b\_F[p].parent)
    InvF7All(p, t) \triangleq
                                \wedge F[u_{-}F[p]].bit = 0
35
                                \wedge a F[p].bit = 0
36
                                \land SameRoot(t, c[p], a\_F[p].parent)
37
                                \wedge SameRoot(t, c[p], u\_F[p])
38
    InvU2All(p, t) \triangleq
                                \land SameRoot(t, t.arg[p][1], u\_U[p])
40
                                \land SameRoot(t, t.arg[p][2], v\_U[p])
41
    InvU5All(p, t) \triangleq
                                \land SameRoot(t, t.arg[p][1], u\_U[p])
44
                                \land SameRoot(t, t.arg[p][2], v_{-}U[p])
45
                                \wedge u_{-}U[p] \neq v_{-}U[p]
46
                                \land a\_U[p].bit = 0 \Rightarrow SameRoot(t, a\_U[p].parent, u\_U[p])
47
                                \land t.ret[p] = ACK \Rightarrow SameRoot(t, u\_U[p], v\_U[p])
48
```

```
InvU6All(p, t) \triangleq
                                   \land SameRoot(t, t.arg[p][1], u\_U[p])
50
                                   \land SameRoot(t, t.arg[p][2], v_U[p])
51
                                   \wedge u_{-}U[p] \neq v_{-}U[p]
52
                                   \land a\_U[p].bit = 0 \Rightarrow SameRoot(t, a\_U[p].parent, u\_U[p])
53
                                   \land b\_U[p].bit = 0 \Rightarrow SameRoot(t, b\_U[p].parent, v\_U[p])
54
                                   \land t.ret[p] = ACK \Rightarrow SameRoot(t, u\_U[p], v\_U[p])
55
    InvU7All(p, t) \triangleq
                                   \land SameRoot(t, t.arg[p][1], u\_U[p])
57
58
                                   \land SameRoot(t, t.arg[p][2], v_{-}U[p])
                                   \land t.ret[p] = ACK \Rightarrow SameRoot(t, u\_U[p], v\_U[p])
59
     InvU8All(p, t) \triangleq
                                   \wedge SameRoot(t, t.arg[p][1], u_U[p])
61
                                   \land SameRoot(t, t.arg[p][2], v_U[p])
62
                                   \land t.ret[p] = ACK \Rightarrow SameRoot(t, u\_U[p], v\_U[p])
63
                                 \forall p \in PROCESSES : \forall t \in M :
    InvDecide \triangleq
65
                                                                    \wedge pc[p] = "0"
66
                                                                                              \Rightarrow \land t.ret[p] = BOT
                                                                                                    \wedge t.op[p] = BOT
67
                                                                                                    \wedge t.arg[p] = BOT
68
    InvF1 \triangleq
                                  \forall p \in PROCESSES : \forall t \in M :
69
                                                                    \wedge \ pc[p] = \text{``F1''}
                                                                                                \Rightarrow \land t.ret[p] = BOT
70
                                                                                                      \wedge t.op[p] = \text{``F''}
71
                                                                                                      \land \ t.arg[p] \in \mathit{NodeSet}
72
                                                                                                      \wedge SameRoot(t, c[p], t.arg[p])
73
                                                                    \land pc[p] = \text{``F1U1''} \Rightarrow \land t.ret[p] = BOT
74
                                                                                                      \wedge t.op[p] = \text{``U"}
75
                                                                                                      \land t.arg[p] \in NodeSet \times NodeSet
76
                                                                                                      \wedge SameRoot(t, c[p], u\_U[p])
77
                                                                    \land pc[p] = \text{``F1U2''} \Rightarrow \land t.ret[p] = BOT
78
                                                                                                      \wedge \ t.op[p] = \text{``U"}
79
                                                                                                      \land \ t.arg[p] \in NodeSet \times NodeSet
80
                                                                                                      \wedge InvU2All(p, t)
81
                                                                                                      \wedge SameRoot(t, c[p], v_{-}U[p])
82
                                                                    \land pc[p] = \text{``F1U7''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
83
                                                                                                      \wedge t.op[p] = \text{``U"}
84
                                                                                                      \land t.arg[p] \in NodeSet \times NodeSet
85
                                                                                                      \wedge InvU7All(p, t)
86
                                                                                                      \wedge SameRoot(t, c[p], u_U[p])
87
                                                                    \land \ \ pc[p] = \text{``F1U8''} \ \ \Rightarrow \ \ \land \ t.ret[p] \in \{BOT, \ ACK\}
88
                                                                                                      \wedge \; t.op[p] = \text{``U"}
89
                                                                                                      \land \ t.arg[p] \in \mathit{NodeSet} \times \mathit{NodeSet}
90
91
                                                                                                      \wedge InvU8All(p, t)
                                                                                                      \wedge SameRoot(t, c[p], v_U[p])
92
    InvF2 \triangleq
                                  \forall p \in PROCESSES : \forall t \in M :
93
                                                                    \land pc[p] = \text{``F2''}
                                                                                                \Rightarrow \land t.ret[p] = BOT
94
                                                                                                      \wedge t.op[p] = \text{``F''}
95
```

```
\land t.arg[p] \in NodeSet
 96
                                                                                                      \wedge SameRoot(t, c[p], t.arg[p])
 97
                                                                                                      \wedge InvF2All(p, t)
98
                                                                     \land pc[p] = \text{``F2U1''} \Rightarrow \land t.ret[p] = BOT
 99
                                                                                                      \wedge t.op[p] = "U"
100
                                                                                                      \land t.arg[p] \in NodeSet \times NodeSet
101
                                                                                                      \wedge SameRoot(t, c[p], u_U[p])
102
                                                                                                      \wedge InvF2All(p, t)
103
                                                                     \land pc[p] = \text{``F2U2''} \Rightarrow \land t.ret[p] = BOT
104
                                                                                                      \wedge t.op[p] = \text{``U"}
105
                                                                                                      \land t.arg[p] \in NodeSet \times NodeSet
106
                                                                                                      \wedge InvU2All(p, t)
107
                                                                                                      \wedge SameRoot(t, c[p], v_U[p])
108
                                                                                                      \wedge InvF2All(p, t)
109
                                                                     \land pc[p] = \text{``F2U7''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
110
                                                                                                      \land \ t.op[p] = \text{``U"}
111
                                                                                                      \land \ t.arg[p] \in \mathit{NodeSet} \times \mathit{NodeSet}
112
                                                                                                      \wedge InvU7All(p, t)
113
                                                                                                      \wedge SameRoot(t, c[p], u_U[p])
114
115
                                                                                                      \wedge InvF2All(p, t)
                                                                     \land pc[p] = \text{``F2U8''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
116
                                                                                                      \wedge t.op[p] = \text{``U"}
117
                                                                                                      \land t.arg[p] \in NodeSet \times NodeSet
118
                                                                                                      \wedge InvU8All(p, t)
119
120
                                                                                                      \wedge SameRoot(t, c[p], v_{-}U[p])
121
                                                                                                      \wedge InvF2All(p, t)
     InvF3 \stackrel{\triangle}{=}
                                   \forall p \in PROCESSES : \forall t \in M :
124
                                                                     \land pc[p] = \text{``F3''}
                                                                                                \Rightarrow \land t.ret[p] = BOT
125
                                                                                                      \wedge t.op[p] = \text{``F''}
126
                                                                                                      \land t.arg[p] \in NodeSet
127
                                                                                                      \land SameRoot(t, c[p], t.arg[p])
128
                                                                                                      \wedge InvF3All(p, t)
129
                                                                     \land pc[p] = \text{``F3U1''} \Rightarrow \land t.ret[p] = BOT
130
                                                                                                      \wedge t.op[p] = "U"
131
132
                                                                                                      \land t.arg[p] \in NodeSet \times NodeSet
                                                                                                      \wedge SameRoot(t, c[p], u_{-}U[p])
133
                                                                                                      \wedge InvF3All(p, t)
134
                                                                     \land pc[p] = \text{``F3U2''} \Rightarrow \land t.ret[p] = BOT
135
                                                                                                      \wedge t.op[p] = \text{``U"}
136
                                                                                                      \land t.arg[p] \in NodeSet \times NodeSet
137
                                                                                                      \wedge InvU2All(p, t)
138
                                                                                                      \wedge SameRoot(t, c[p], v_{-}U[p])
139
                                                                                                      \wedge InvF3All(p, t)
140
```

```
\land pc[p] = \text{``F3U7''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
141
                                                                                                        \wedge t.op[p] = "U"
142
                                                                                                        \land t.arg[p] \in NodeSet \times NodeSet
143
                                                                                                        \wedge InvU7All(p, t)
144
                                                                                                        \wedge SameRoot(t, c[p], u\_U[p])
145
                                                                                                        \wedge InvF3All(p, t)
146
                                                                      \land pc[p] = \text{``F3U8''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
147
                                                                                                        \wedge t.op[p] = "U"
148
                                                                                                        \land t.arg[p] \in NodeSet \times NodeSet
149
                                                                                                        \wedge InvU8All(p, t)
150
151
                                                                                                        \wedge SameRoot(t, c[p], v\_U[p])
                                                                                                        \wedge InvF3All(p, t)
152
      InvF4 \stackrel{\triangle}{=}
                                   \forall p \in PROCESSES : \forall t \in M :
154
                                                                       \wedge \ pc[p] = \text{``F4''}
155
                                                                                                  \Rightarrow \land t.ret[p] = BOT
                                                                                                        \wedge t.op[p] = \text{``F''}
156
                                                                                                        \land t.arg[p] \in NodeSet
157
                                                                                                        \wedge SameRoot(t, c[p], t.arg[p])
158
                                                                                                        \wedge InvF4All(p, t)
159
                                                                       \land \ pc[p] = \text{``F4U1''} \ \Rightarrow \ \land t.ret[p] = BOT
160
161
                                                                                                        \wedge t.op[p] = "U"
                                                                                                        \land t.arg[p] \in NodeSet \times NodeSet
162
                                                                                                        \wedge SameRoot(t, c[p], u_{-}U[p])
163
                                                                                                        \wedge InvF4All(p, t)
164
                                                                       \land pc[p] = \text{``F4U2''} \Rightarrow \land t.ret[p] = BOT
165
                                                                                                        \wedge t.op[p] = "U"
166
                                                                                                        \land t.arg[p] \in NodeSet \times NodeSet
167
                                                                                                        \wedge InvU2All(p, t)
168
                                                                                                        \land SameRoot(t, c[p], v\_U[p])
169
                                                                                                        \wedge InvF4All(p, t)
170
                                                                       \land pc[p] = \text{``F4U7''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
171
                                                                                                        \wedge t.op[p] = \text{``U"}
172
                                                                                                         \land t.arg[p] \in NodeSet \times NodeSet
173
                                                                                                        \wedge InvU7All(p, t)
174
                                                                                                        \wedge SameRoot(t, c[p], u\_U[p])
175
176
                                                                                                        \wedge InvF4All(p, t)
                                                                      \land \ \ pc[p] = \text{``F4U8''} \ \Rightarrow \ \land t.ret[p] \in \{BOT, \, ACK\}
177
                                                                                                        \wedge t.op[p] = \text{``U"}
178
                                                                                                        \land \ t.arg[p] \in NodeSet \times NodeSet
179
                                                                                                        \wedge InvU8All(p, t)
180
                                                                                                        \wedge SameRoot(t, c[p], v_{-}U[p])
181
182
                                                                                                        \wedge InvF4All(p, t)
185 \quad InvF5 \quad \stackrel{\triangle}{=} \quad
                                   \forall p \in PROCESSES : \forall t \in M :
                                                                      \wedge pc[p] = \text{``F5''}
                                                                                                  \Rightarrow \land t.ret[p] \in \{BOT\} \cup NodeSet
186
```

```
\land t.op[p] = \text{``F''}
187
                                                                                                     \land t.arg[p] \in NodeSet
188
                                                                                                     \land SameRoot(t, c[p], t.arg[p])
189
                                                                                                     \wedge InvF5All(p, t)
190
                                                                                                     \land \ b\_F[p].bit = 0 \Rightarrow t.ret[p] = BOT
191
                                                                                                     \wedge b_{-}F[p].bit = 1 \Rightarrow t.ret[p] = a_{-}F[p].pa
192
                                                                    \land pc[p] = \text{``F5U1''} \Rightarrow \land t.ret[p] = BOT
193
                                                                                                     \wedge t.op[p] = "U"
194
                                                                                                     \land t.arg[p] \in NodeSet \times NodeSet
195
                                                                                                     \wedge SameRoot(t, c[p], u_{-}U[p])
196
                                                                                                     \wedge InvF5All(p, t)
197
                                                                    \land pc[p] = \text{``F5U2''} \Rightarrow \land t.ret[p] = BOT
198
                                                                                                     \wedge t.op[p] = "U"
199
                                                                                                     \land t.arg[p] \in NodeSet \times NodeSet
200
                                                                                                     \wedge InvU2All(p, t)
201
                                                                                                     \wedge SameRoot(t, c[p], v_{-}U[p])
202
                                                                                                     \wedge InvF5All(p, t)
203
                                                                     \land pc[p] = \text{``F5U7''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
204
                                                                                                     \land \ t.op[p] = \text{``U"}
205
                                                                                                     \land t.arg[p] \in NodeSet \times NodeSet
206
                                                                                                     \wedge InvU7All(p, t)
207
                                                                                                     \wedge SameRoot(t, c[p], u_{-}U[p])
208
                                                                                                     \wedge InvF5All(p, t)
209
                                                                     \land pc[p] = \text{``F5U8''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
210
                                                                                                     \wedge t.op[p] = "U"
211
                                                                                                     \land t.arg[p] \in NodeSet \times NodeSet
212
                                                                                                     \wedge InvU8All(p, t)
213
                                                                                                     \land SameRoot(t, c[p], v\_U[p])
214
                                                                                                     \wedge InvF5All(p, t)
215
      InvF6 \triangleq
                                  \forall p \in PROCESSES : \forall t \in M :
217
                                                                    \land pc[p] = \text{``F6''}
                                                                                               \Rightarrow \land t.ret[p] = BOT
218
                                                                                                     \wedge t.op[p] = \text{``F''}
219
                                                                                                     \land t.arg[p] \in NodeSet
220
                                                                                                     \wedge SameRoot(t, c[p], t.arg[p])
221
222
                                                                                                     \wedge InvF6All(p, t)
                                                                    \land pc[p] = \text{``F6U1''} \Rightarrow \land t.ret[p] = BOT
223
                                                                                                     \land \ t.op[p] = \text{``U"}
224
                                                                                                     \land t.arg[p] \in NodeSet \times NodeSet
225
                                                                                                     \wedge SameRoot(t, c[p], u\_U[p])
226
                                                                                                     \wedge InvF6All(p, t)
227
                                                                     \land pc[p] = \text{``F6U2''} \Rightarrow \land t.ret[p] = BOT
228
                                                                                                     \wedge t.op[p] = "U"
229
                                                                                                     \land t.arg[p] \in NodeSet \times NodeSet
230
                                                                                                     \wedge InvU2All(p, t)
231
```

```
\wedge SameRoot(t, c[p], v_{-}U[p])
232
                                                                                                      \wedge InvF6All(p, t)
233
                                                                     \land pc[p] = \text{``F6U7''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
234
                                                                                                      \wedge t.op[p] = "U"
235
                                                                                                      \land t.arg[p] \in NodeSet \times NodeSet
236
                                                                                                      \wedge InvU7All(p, t)
237
                                                                                                      \wedge SameRoot(t, c[p], u\_U[p])
238
                                                                                                      \wedge InvF6All(p, t)
239
                                                                     \land \ pc[p] = \text{``F6U8''} \ \Rightarrow \ \land t.ret[p] \in \{BOT, ACK\}
240
                                                                                                      \wedge t.op[p] = "U"
241
                                                                                                      \land t.arg[p] \in NodeSet \times NodeSet
242
                                                                                                      \wedge InvU8All(p, t)
243
                                                                                                      \wedge SameRoot(t, c[p], v_{-}U[p])
244
245
                                                                                                      \wedge InvF6All(p, t)
      InvF7 \triangleq
                                   \forall p \in PROCESSES : \forall t \in M :
248
                                                                     \land \ \ pc[p] = \text{``F7''}
                                                                                                \Rightarrow \land t.ret[p] = BOT
249
                                                                                                      \wedge t.op[p] = \text{``F''}
250
                                                                                                      \land t.arg[p] \in NodeSet
251
                                                                                                      \land SameRoot(t, c[p], t.arg[p])
252
                                                                                                      \wedge InvF7All(p, t)
253
                                                                     \land pc[p] = \text{``F7U1''} \Rightarrow \land t.ret[p] = BOT
254
                                                                                                      \wedge t.op[p] = "U"
255
                                                                                                      \land t.arg[p] \in NodeSet \times NodeSet
256
                                                                                                      \wedge SameRoot(t, c[p], u_U[p])
257
                                                                                                      \wedge InvF7All(p, t)
258
                                                                     \land pc[p] = \text{``F7U2''} \Rightarrow \land t.ret[p] = BOT
259
                                                                                                      \wedge t.op[p] = "U"
260
                                                                                                      \land \ t.arg[p] \in \mathit{NodeSet} \times \mathit{NodeSet}
261
262
                                                                                                      \wedge InvU2All(p, t)
                                                                                                      \wedge SameRoot(t, c[p], v_U[p])
263
                                                                                                      \wedge InvF7All(p, t)
264
                                                                     \land pc[p] = \text{``F7U7''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
265
                                                                                                      \wedge t.op[p] = "U"
266
                                                                                                      \land t.arg[p] \in NodeSet \times NodeSet
267
268
                                                                                                      \wedge InvU7All(p, t)
                                                                                                      \wedge SameRoot(t, c[p], u_{-}U[p])
^{269}
                                                                                                      \wedge InvF7All(p, t)
270
                                                                     \land pc[p] = \text{``F7U8''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
271
                                                                                                      \land t.op[p] = \text{``U"}
272
                                                                                                      \land \ t.arg[p] \in \mathit{NodeSet} \times \mathit{NodeSet}
273
274
                                                                                                      \wedge InvU8All(p, t)
                                                                                                      \wedge SameRoot(t, c[p], v_{-}U[p])
275
                                                                                                      \wedge InvF7All(p, t)
276
```

```
InvFR \triangleq
                                   \forall p \in PROCESSES : \forall t \in M :
279
                                                                     \land pc[p] = \text{"FR"}
                                                                                                \Rightarrow \land t.ret[p] = u_F[p]
280
                                                                                                      \wedge t.op[p] = \text{``F''}
281
                                                                                                      \land t.arg[p] \in NodeSet
282
                                                                                                      \land SameRoot(t, t.arg[p], u\_F[p])
283
                                                                                                      \wedge SameRoot(t, c[p], u\_F[p])
284
                                                                     \land pc[p] = \text{``FRU1''} \Rightarrow \land t.ret[p] = BOT
285
                                                                                                      \wedge t.op[p] = "U"
286
                                                                                                      \land t.arg[p] \in NodeSet \times NodeSet
287
                                                                                                      \wedge SameRoot(t, c[p], u_U[p])
288
                                                                                                      \land SameRoot(t, c[p], u\_F[p])
289
                                                                     \land pc[p] = \text{"FRU2"} \Rightarrow
                                                                                                      \wedge t.ret[p] = BOT
290
                                                                                                      \land \ t.op[p] = \text{``U"}
291
                                                                                                      \land t.arg[p] \in NodeSet \times NodeSet
292
                                                                                                      \wedge InvU2All(p, t)
293
294
                                                                                                      \wedge SameRoot(t, c[p], v_{-}U[p])
                                                                     \land pc[p] = \text{``FRU7''} \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
296
                                                                                                      \wedge t.op[p] = "U"
297
                                                                                                      \land \ t.arg[p] \in \mathit{NodeSet} \times \mathit{NodeSet}
298
299
                                                                                                      \wedge InvU7All(p, t)
                                                                                                      \wedge SameRoot(t, c[p], u\_U[p])
300
                                                                     \land pc[p] = \text{"FRU8"} \Rightarrow
                                                                                                      \land t.ret[p] \in \{BOT, ACK\}
301
                                                                                                      \wedge t.op[p] = "U"
302
                                                                                                      \land t.arg[p] \in NodeSet \times NodeSet
303
304
                                                                                                      \wedge InvU8All(p, t)
                                                                                                      \wedge SameRoot(t, c[p], v_{-}U[p])
305
     InvU1 \triangleq
                                   \forall p \in PROCESSES : \forall t \in M :
307
                                                                    pc[p] = "U1"
                                                                                          \Rightarrow \land t.ret[p] = BOT
308
309
                                                                                                 \wedge t.op[p] = \text{``U"}
                                                                                                 \land \ t.arg[p] \in \mathit{NodeSet} \times \mathit{NodeSet}
310
      InvU2 \triangleq
                                   \forall p \in PROCESSES : \forall t \in M :
313
                                                                    pc[p] = "U2"
                                                                                                \wedge t.ret[p] = BOT
314
                                                                                                 \wedge t.op[p] = \text{``U"}
315
                                                                                                 \land t.arg[p] \in NodeSet \times NodeSet
316
317
                                                                                                 \wedge InvU2All(p, t)
      InvU3 \triangleq
                                   \forall p \in PROCESSES : \forall t \in M :
319
                                                                    pc[p] = "U3"
                                                                                          \Rightarrow \land t.ret[p] \in \{BOT, ACK\}
320
                                                                                                 \wedge t.op[p] = \text{``U"}
321
                                                                                                 \land t.arg[p] \in NodeSet \times NodeSet
322
                                                                                                 \land SameRoot(t, t.arg[p][1], u\_U[p])
323
                                                                                                 \land SameRoot(t, t.arg[p][2], v_U[p])
324
```

```
\land t.ret[p] = ACK \Rightarrow SameRoot(t, u\_U[p], t
325
     InvU4 \stackrel{\triangle}{=}
                                  \forall p \in PROCESSES : \forall t \in M :
326
                                                                   pc[p] = "U4"
                                                                                             \land t.ret[p] \in \{BOT, ACK\}
327
                                                                                              \wedge t.op[p] = \text{``U"}
328
329
                                                                                              \land t.arg[p] \in NodeSet \times NodeSet
                                                                                              \land SameRoot(t, t.arg[p][1], u\_U[p])
330
                                                                                              \land SameRoot(t, t.arg[p][2], v_U[p])
331
                                                                                              \land t.ret[p] = ACK \Rightarrow SameRoot(t, u\_U[p], t
332
                                                                                              \wedge u_{-}U[p] \neq v_{-}U[p]
333
      InvU5 \stackrel{\triangle}{=}
                                  \forall p \in PROCESSES : \forall t \in M :
335
                                                                   pc[p] = \text{"U5"}
                                                                                              \land t.ret[p] \in \{BOT, ACK\}
336
                                                                                              \wedge t.op[p] = "U"
337
                                                                                              \land t.arg[p] \in NodeSet \times NodeSet
338
                                                                                              \wedge InvU5All(p, t)
339
      InvU6 \triangleq
                                  \forall p \in PROCESSES : \forall t \in M :
340
                                                                  pc[p] = "U6"
341
                                                                                             \land t.ret[p] \in \{BOT, ACK\}
342
                                                                                             \wedge t.op[p] = "U"
343
                                                                                             \land t.arg[p] \in NodeSet \times NodeSet
344
345
                                                                                             \wedge InvU6All(p, t)
      InvU7 \triangleq
                                  \forall p \in PROCESSES : \forall t \in M :
347
                                                                   pc[p] = "U7"
                                                                                             \land t.ret[p] \in \{BOT, ACK\}
348
                                                                                              \wedge t.op[p] = \text{``U"}
349
                                                                                              \land t.arg[p] \in NodeSet \times NodeSet
350
                                                                                              \wedge InvU7All(p, t)
351
      InvU8 \triangleq
                                  \forall p \in PROCESSES : \forall t \in M :
353
                                                                   pc[p] = "U8"
                                                                                              \land t.ret[p] \in \{BOT, ACK\}
354
                                                                                              \land \ t.op[p] = \text{``U"}
355
                                                                                              \land \ t.arg[p] \in NodeSet \times NodeSet
356
                                                                                              \wedge InvU8All(p, t)
357
      InvUR \triangleq
                                  \forall p \in PROCESSES : \forall t \in M :
359
                                                                   pc[p] = "UR"
                                                                                              \wedge t.ret[p] = ACK
360
                                                                                               \wedge \ t.op[p] = \text{``U"}
361
                                                                                               \land t.arg[p] \in NodeSet \times NodeSet
362
                                                                                               \land SameRoot(t, t.arg[p][1], u\_U[p])
363
                                                                                               \wedge SameRoot(t, t.arg[p][2], v_U[p])
364
                                                                                               \wedge SameRoot(t, u\_U[p], v\_U[p])
365
      Linearizable \triangleq
367
                                 M \neq \{\}
      Inv \triangleq
                  \land TypeOK
369
                  \land InvDecide
370
                  \wedge InvF1
371
```

```
\land \mathit{InvF} 2
372
                          \land \mathit{InvF3}
373
                          \wedge InvF4
374
                          \land \mathit{InvF}5
375
376
                          \land \mathit{InvF} 6
                          \land \mathit{InvF7}
377
                          \land \mathit{InvFR}
378
                          \wedge \operatorname{Inv} U1
379
                          \wedge \mathit{InvU}2
380
                          \wedge \, \mathit{InvU} 3
381
                          \wedge \, \mathit{InvU} \, 4
382
                          \wedge \, \mathit{InvU} 5
383
                          \wedge \, \mathit{InvU} 6
384
                          \wedge \mathit{InvU7}
385
                          \wedge \, \mathit{InvU} \, 8
386
                          \wedge InvUR
387
                          \land \ SigmaRespectsShared
388
                          \land \ Linearizable
389
391 L
```

^{*} Modification History

^{*} Last modified Fri May 02 00:44:09 EDT 2025 by karunram

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