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1  ┌────────────────────────── MODULE Implementation ───────────────────────────┐
2  │
3  │ ALGORITHM:
4  │
5  │ 0: CHOOSE  $i, j \in [1 \dots n]$ :  $Find\_p(i) \vee Unite\_p(i, j)$ 
6  │
7  │    $Find\_p(c)$ :
8  │   F1:  $u = c$ 
9  │   F2: if  $F[u].bit = 1$  goto FR ELSE goto F3
10 │   F3:  $a = READ(u)$ ; goto F4 or F7
11 │   F4:  $b = READ(a.parent)$ 
12 │   F5: if  $b.bit = 1$ :  $u = a.parent$ ; goto FR
13 │   F6:  $CAS(F[u], [a.parent, a.rank, 0], [b.parent, a.rank, 0])$ ; goto F2
14 │   F7:  $u = v$ ; goto F2
15 │   FR: return u
16 │
17 │    $Unite\_p(c, d)$ :
18 │   U1:  $u = c$ ;  $v = d$ 
19 │        $u = Find\_p(u)$ 
20 │   U2:  $v = Find\_p(v)$ 
21 │   U3: if  $u = v$  goto UR
22 │   U4:  $a = [u\_p, u\_r, u\_b] = READ(u)$ 
23 │   U5:  $b = [v\_p, v\_r, v\_b] = READ(v)$ 
24 │   U6: if  $a.rank < b.rank$  then  $CAS(F[u], [a.parent, a.rank, 1], [v, a.rank, 0])$ 
25 │   U6: elif  $u\_r > v\_r$  then  $CAS(F[v], [b.parent, b.rank, 1], [u, b.rank, 0])$ 
26 │   U6: else:
27 │       if  $u < v$  then  $CAS(F[u], [a.parent, a.rank, 1], [v, a.rank, \$])$ 
28 │       else:  $CAS(F[v], [b.parent, b.rank, 1], [u, b.rank, \$])$ 
29 │   U7:  $u = Find\_p(u)$ 
30 │   U8:  $v = Find\_p(v)$ ; goto U3
31 │   UR: return ACK
32 │
33 │
34 EXTENDS FiniteSets, Integers
35 CONSTANT BOT, ACK, PROCESSES, N
36 VARIABLES  $pc, F, u\_F, a\_F, b\_F, u\_U, v\_U, a\_U, b\_U, c, d, M$ 
37  $NodeSet \triangleq 1 \dots N$ 
38 ASSUME  $NisNat \triangleq (N \in Nat) \wedge (N > 0)$ 
39 ASSUME  $AckBotDef \triangleq BOT \notin NodeSet \wedge ACK \notin NodeSet \wedge BOT \neq ACK$ 
40 ASSUME  $ExistProc \triangleq PROCESSES \neq \{\}$ 
41
42 Line Definitions
43  $varlist \triangleq \langle pc, F, u\_F, a\_F, b\_F, u\_U, v\_U, a\_U, b\_U, c, d, M \rangle$ 
44
45 Type Sets
46  $PCSet \triangleq \{ "0", "F1", "F2", "F3", "F4", "F5", "F6", "F7", "FR",$ 
47  $"U1", "U2", "U3", "U4", "U5", "U6", "U7", "U8", "UR",$ 

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48 "F1U1", "F2U1", "F3U1", "F4U1", "F5U1", "F6U1", "F7U1", "F8U1", "FRU1",  
 49 "F1U2", "F2U2", "F3U2", "F4U2", "F5U2", "F6U2", "F7U2", "F8U2", "FRU2",  
 50 "F1U7", "F2U7", "F3U7", "F4U7", "F5U7", "F6U7", "F7U7", "F8U7", "FRU7",  
 51 "F1U8", "F2U8", "F3U8", "F4U8", "F5U8", "F6U8", "F7U8", "F8U8", "FRU8"}  
 52  $FieldSet \triangleq$   $[parent : NodeSet, rank : Nat, bit : \{0, 1\}]$   
 53  $StateSet \triangleq$   $\{A \in [NodeSet \rightarrow NodeSet] : \forall i \in NodeSet : A[A[i]] = A[i]\}$   
 54  $ReturnSet \triangleq$   $[PROCESSES \rightarrow NodeSet \cup \{BOT\} \cup \{ACK\}]$   
 55  $OpSet \triangleq$   $[PROCESSES \rightarrow \{"F", "U", BOT\}]$   
 56  $ArgSet \triangleq$   $[PROCESSES \rightarrow \{BOT\} \cup NodeSet \cup NodeSet \times NodeSet]$   
 57  $Configs \triangleq$   $[\sigma : StateSet, ret : ReturnSet, op : OpSet, arg : ArgSet]$   
  
 59 **InitStates**  
 60  $InitState \triangleq$   $[i \in NodeSet \mapsto i]$   
 61  $InitF \triangleq$   $[i \in NodeSet \mapsto [parent \mapsto i, rank \mapsto 0, bit \mapsto 1]]$   
 62  $InitRet \triangleq$   $[p \in PROCESSES \mapsto BOT]$   
 63  $InitOp \triangleq$   $[p \in PROCESSES \mapsto BOT]$   
 64  $InitArg \triangleq$   $[p \in PROCESSES \mapsto BOT]$   
  
 67 **Initial state of algorithm**  
 68  $Init \triangleq$   $\wedge pc = [p \in PROCESSES \mapsto "0"]$   
 69  $\wedge F = InitF$   
 70  $\wedge a\_F \in [PROCESSES \rightarrow FieldSet]$   
 71  $\wedge b\_F \in [PROCESSES \rightarrow FieldSet]$   
 72  $\wedge u\_F \in [PROCESSES \rightarrow NodeSet]$   
 73  $\wedge a\_U \in [PROCESSES \rightarrow FieldSet]$   
 74  $\wedge b\_U \in [PROCESSES \rightarrow FieldSet]$   
 75  $\wedge u\_U \in [PROCESSES \rightarrow NodeSet]$   
 76  $\wedge v\_U \in [PROCESSES \rightarrow NodeSet]$   
 77  $\wedge c \in [PROCESSES \rightarrow NodeSet]$   
 78  $\wedge d \in [PROCESSES \rightarrow NodeSet]$   
 79  $\wedge M = \{[\sigma \mapsto InitState, ret \mapsto InitRet, op \mapsto InitOp, arg \mapsto InitArg]\}$   
  
 81 **Find operation**  
 82  $F1(p) \triangleq$   $\wedge u\_F' = [u\_F \text{ EXCEPT } !p] = c[p]$   
 83  $\wedge \vee pc[p] = "F1" \wedge pc' = [pc \text{ EXCEPT } !p] = "F2"]$   
 84  $\vee pc[p] = "F1U1" \wedge pc' = [pc \text{ EXCEPT } !p] = "F2U1"]$   
 85  $\vee pc[p] = "F1U2" \wedge pc' = [pc \text{ EXCEPT } !p] = "F2U2"]$   
 86  $\vee pc[p] = "F1U7" \wedge pc' = [pc \text{ EXCEPT } !p] = "F2U7"]$   
 87  $\vee pc[p] = "F1U8" \wedge pc' = [pc \text{ EXCEPT } !p] = "F2U8"]$   
 88  $\wedge \text{UNCHANGED } \langle F, a\_F, b\_F, u\_U, v\_U, a\_U, b\_U, c, d, M \rangle$   
  
 90  $F2(p) \triangleq$   $\wedge \text{IF } F[u\_F[p]].bit = 1$   
 91  $\text{THEN } \vee pc[p] = "F2" \wedge pc' = [pc \text{ EXCEPT } !p] = "FR"]$   
 92  $\vee pc[p] = "F2U1" \wedge pc' = [pc \text{ EXCEPT } !p] = "FRU1"]$   
 93  $\vee pc[p] = "F2U2" \wedge pc' = [pc \text{ EXCEPT } !p] = "FRU2"]$

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94          $\vee pc[p] = \text{"F2U7"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"FRU7"}]$ 
95          $\vee pc[p] = \text{"F2U8"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"FRU8"}]$ 
96     ELSE     $\vee pc[p] = \text{"F2"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F3"}]$ 
97              $\vee pc[p] = \text{"F2U1"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F3U1"}]$ 
98              $\vee pc[p] = \text{"F2U2"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F3U2"}]$ 
99              $\vee pc[p] = \text{"F2U7"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F3U7"}]$ 
100             $\vee pc[p] = \text{"F2U8"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F3U8"}]$ 
101     $\wedge \text{IF } F[u\_F[p]].bit = 1 \wedge pc[p] = \text{"F2"}$ 
102        THEN  $M' = \{t \in \text{Configs} : \exists told \in M : \wedge told.ret[p] = BOT$ 
103                 $\wedge t.sigma = told.sigma$ 
104                 $\wedge t.ret = [told.ret \text{ EXCEPT } ![p] = u\_F[p]$ 
105                 $\wedge t.op = told.op$ 
106                 $\wedge t.arg = told.arg\}$ 
107    ELSE  $M' = M$ 
108     $\wedge \text{UNCHANGED } \langle F, a\_F, b\_F, u\_F, u\_U, v\_U, a\_U, b\_U, c, d \rangle$ 

110  $F3(p) \triangleq$      $\wedge a\_F' = [a\_F \text{ EXCEPT } ![p] = F[u\_F[p]]]$ 
111     $\wedge \vee pc[p] = \text{"F3"} \quad \wedge (pc' = [pc \text{ EXCEPT } ![p] = \text{"F4"}] \quad \vee pc' = [pc \text{ EXCEPT } ![p] = \text{"F7"}]$ 
112         $\vee pc[p] = \text{"F3U1"} \quad \wedge (pc' = [pc \text{ EXCEPT } ![p] = \text{"F4U1"}] \quad \vee pc' = [pc \text{ EXCEPT } ![p] = \text{"F7U1"}]$ 
113         $\vee pc[p] = \text{"F3U2"} \quad \wedge (pc' = [pc \text{ EXCEPT } ![p] = \text{"F4U2"}] \quad \vee pc' = [pc \text{ EXCEPT } ![p] = \text{"F7U2"}]$ 
114         $\vee pc[p] = \text{"F3U7"} \quad \wedge (pc' = [pc \text{ EXCEPT } ![p] = \text{"F4U7"}] \quad \vee pc' = [pc \text{ EXCEPT } ![p] = \text{"F7U7"}]$ 
115         $\vee pc[p] = \text{"F3U8"} \quad \wedge (pc' = [pc \text{ EXCEPT } ![p] = \text{"F4U8"}] \quad \vee pc' = [pc \text{ EXCEPT } ![p] = \text{"F7U8"}]$ 
116     $\wedge \text{UNCHANGED } \langle F, u\_F, b\_F, u\_U, v\_U, a\_U, b\_U, c, d, M \rangle$ 

118  $F4(p) \triangleq$      $\wedge b\_F' = [b\_F \text{ EXCEPT } ![p] = F[a\_F[p].parent]]$ 
119     $\wedge \vee pc[p] = \text{"F4"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F5"}]$ 
120         $\vee pc[p] = \text{"F4U1"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F5U1"}]$ 
121         $\vee pc[p] = \text{"F4U2"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F5U2"}]$ 
122         $\vee pc[p] = \text{"F4U7"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F5U7"}]$ 
123         $\vee pc[p] = \text{"F4U8"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F5U8"}]$ 
124     $\wedge \text{UNCHANGED } \langle F, u\_F, a\_F, u\_U, v\_U, a\_U, b\_U, c, d, M \rangle$ 

126  $F5(p) \triangleq$      $\wedge \text{IF } b\_F[p].bit = 1$ 
127        THEN     $\wedge u\_F' = [u\_F \text{ EXCEPT } ![p] = a\_F[p].parent]$ 
128                 $\wedge \vee pc[p] = \text{"F5"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"FR"}]$ 
129                 $\vee pc[p] = \text{"F5U1"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"FRU1"}]$ 
130                 $\vee pc[p] = \text{"F5U2"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"FRU2"}]$ 
131                 $\vee pc[p] = \text{"F5U7"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"FRU7"}]$ 
132                 $\vee pc[p] = \text{"F5U8"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"FRU8"}]$ 
133        ELSE     $\wedge u\_F' = u\_F$ 
134                 $\wedge \vee pc[p] = \text{"F5"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F6"}]$ 
135                 $\vee pc[p] = \text{"F5U1"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F6U1"}]$ 
136                 $\vee pc[p] = \text{"F5U2"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F6U2"}]$ 
137                 $\vee pc[p] = \text{"F5U7"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F6U7"}]$ 
138                 $\vee pc[p] = \text{"F5U8"} \quad \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F6U8"}]$ 
139     $\wedge \text{IF } b\_F[p].bit = 1 \wedge pc[p] = \text{"F5"}$ 

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140 THEN  $M' = \{t \in Configs : \exists told \in M : \wedge told.ret[p] = BOT$ 
141  $\wedge t.sigma = told.sigma$ 
142  $\wedge t.ret = [told.ret \text{ EXCEPT } ![p] = a\_F[p].parent]$ 
143  $\wedge t.op = told.op$ 
144  $\wedge t.arg = told.arg\}$ 
145 ELSE  $M' = M$ 
146  $\wedge \text{UNCHANGED } \langle F, a\_F, b\_F, u\_F, u\_U, v\_U, a\_U, b\_U, c, d \rangle$ 
148  $F6(p) \triangleq$   $\wedge \text{IF } (F[u\_F[p]] = [parent \mapsto a\_F[p].parent, rank \mapsto a\_F[p].rank, bit \mapsto 0])$ 
149 THEN  $\wedge F' = [F \text{ EXCEPT } ![u\_F[p]] = [parent \mapsto b\_F[p].parent, rank \mapsto a\_F[p].rank]]$ 
150 ELSE  $\wedge F' = F$ 
151  $\wedge \vee pc[p] = \text{"F6"} \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F2"}]$ 
152  $\vee pc[p] = \text{"F6U1"} \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F2U1"}]$ 
153  $\vee pc[p] = \text{"F6U2"} \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F2U2"}]$ 
154  $\vee pc[p] = \text{"F6U7"} \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F2U7"}]$ 
155  $\vee pc[p] = \text{"F6U8"} \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F2U8"}]$ 
156  $\wedge \text{UNCHANGED } \langle a\_F, b\_F, u\_F, u\_U, v\_U, a\_U, b\_U, c, d, M \rangle$ 
159  $F7(p) \triangleq$   $\wedge u\_F' = [u\_F \text{ EXCEPT } ![p] = a\_F[p].parent]$ 
160  $\wedge \vee pc[p] = \text{"F7"} \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F2"}]$ 
161  $\vee pc[p] = \text{"F7U1"} \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F2U1"}]$ 
162  $\vee pc[p] = \text{"F7U2"} \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F2U2"}]$ 
163  $\vee pc[p] = \text{"F7U7"} \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F2U7"}]$ 
164  $\vee pc[p] = \text{"F7U8"} \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F2U8"}]$ 
165  $\wedge \text{UNCHANGED } \langle F, a\_F, b\_F, u\_U, v\_U, a\_U, b\_U, c, d, M \rangle$ 
167  $FR(p) \triangleq$   $\wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"0"}]$ 
168  $\wedge \vee pc[p] = \text{"FR"} \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"0"}]$ 
169  $\wedge u\_U' = u\_U$ 
170  $\wedge M' = \{t \in Configs : \exists told \in M : \wedge told.ret[p] = u\_F[p]$ 
171  $\wedge t.sigma = told.sigma$ 
172  $\wedge t.ret = [told.ret \text{ EXCEPT } ![p] = u\_F[p].parent]$ 
173  $\wedge t.op = [told.op \text{ EXCEPT } ![p] = u\_F[p].op]$ 
174  $\wedge t.arg = [told.arg \text{ EXCEPT } ![p] = u\_F[p].arg]\}$ 
175  $\vee pc[p] = \text{"FRU1"} \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"U2"}]$ 
176  $\wedge u\_U' = [u\_U \text{ EXCEPT } ![p] = u\_F[p]]$ 
177  $\wedge M' = M$ 
178  $\vee pc[p] = \text{"FRU2"} \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"U3"}]$ 
179  $\wedge v\_U' = [v\_U \text{ EXCEPT } ![p] = u\_F[p]]$ 
180  $\wedge M' = M$ 
181  $\vee pc[p] = \text{"FRU7"} \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"U8"}]$ 
182  $\wedge u\_U' = [u\_U \text{ EXCEPT } ![p] = u\_F[p]]$ 
183  $\wedge M' = M$ 
184  $\vee pc[p] = \text{"FRU8"} \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"U3"}]$ 
185  $\wedge v\_U' = [v\_U \text{ EXCEPT } ![p] = u\_F[p]]$ 

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186                                      $\wedge M' = M$ 
187  $\wedge \text{UNCHANGED } \langle F, a\_F, b\_F, u\_F, u\_U, v\_U, a\_U, b\_U, c, d \rangle$ 

189  $U1(p) \triangleq$   $\wedge pc[p] = \text{"U1"}$ 
190  $\wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F1U1"}]$ 
191  $\wedge u\_U' = [u\_U \text{ EXCEPT } ![p] = c[p]]$ 
192  $\wedge v\_U' = [v\_U \text{ EXCEPT } ![p] = d[p]]$ 
193  $\wedge \text{UNCHANGED } \langle F, u\_F, a\_F, b\_F, a\_U, b\_U, c, d, M \rangle$ 

195  $U2(p) \triangleq$   $\wedge pc[p] = \text{"U2"}$ 
196  $\wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F1U2"}]$ 
197  $\wedge c' = [c \text{ EXCEPT } ![p] = v\_U[p]]$ 
198  $\wedge \text{UNCHANGED } \langle F, u\_F, a\_F, b\_F, a\_U, b\_U, u\_U, v\_U, d, M \rangle$ 

200  $U3(p) \triangleq$   $\wedge pc[p] = \text{"U3"}$ 
201  $\wedge \text{IF } u\_U[p] = v\_U[p]$ 
202  $\quad \text{THEN } \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"UR"}]$ 
203  $\quad \wedge M' = \{t \in \text{Configs} : \exists told \in M :$ 
204  $\quad \quad \vee \wedge told.ret[p] = BOT$ 
205  $\quad \quad \wedge t.sigma = told.sigma$ 
206  $\quad \quad \wedge t.ret = [told.ret \text{ EXCEPT } ![p] = ACK]$ 
207  $\quad \quad \wedge t.op = told.op$ 
208  $\quad \quad \wedge t.arg = told.arg$ 
209  $\quad \quad \vee \wedge told.ret[p] = ACK$ 
210  $\quad \quad \wedge t.sigma = told.sigma$ 
211  $\quad \quad \wedge t.ret = told.ret$ 
212  $\quad \quad \wedge t.op = told.op$ 
213  $\quad \quad \wedge t.arg = told.arg\}$ 
214  $\quad \text{ELSE } \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"U4"}]$ 
215  $\quad \wedge M' = M$ 
216  $\wedge \text{UNCHANGED } \langle F, u\_F, a\_F, b\_F, u\_U, v\_U, a\_U, b\_U, u\_U, v\_U, c, d \rangle$ 

218  $U4(p) \triangleq$   $\wedge pc[p] = \text{"U4"}$ 
219  $\wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"U5"}]$ 
220  $\wedge a\_U' = [a\_U \text{ EXCEPT } ![p] = F[u\_U[p]]]$ 
221  $\wedge \text{UNCHANGED } \langle F, u\_F, a\_F, b\_F, u\_U, v\_U, b\_U, c, d, M \rangle$ 

223  $U5(p) \triangleq$   $\wedge pc[p] = \text{"U5"}$ 
224  $\wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"U6"}]$ 
225  $\wedge b\_U' = [b\_U \text{ EXCEPT } ![p] = F[v\_U[p]]]$ 
226  $\wedge \text{UNCHANGED } \langle F, u\_F, a\_F, b\_F, u\_U, v\_U, a\_U, c, d, M \rangle$ 

228  $U6(p) \triangleq$   $\wedge pc[p] = \text{"U6"}$ 
229  $\wedge \text{IF } a\_U[p].rank < b\_U[p].rank$ 
230  $\quad \text{THEN IF } F[u\_U[p]] = [parent \mapsto a\_U[p].parent, rank \mapsto a\_U[p].rank, bit \mapsto 1]$ 
231  $\quad \quad \text{THEN } \wedge F' = [F \text{ EXCEPT } ![u\_U[p]] = [parent \mapsto v\_U[p], rank \mapsto$ 
232  $\quad \quad \wedge M' = \{t \in \text{Configs} : \exists told \in M : \wedge t.ret = [told.ret \text{ EXCEPT } !$ 

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233                                      $\wedge t.\sigma = [i \in Node$ 
234
235
236                                      $\wedge t.op = told.op$ 
237                                      $\wedge t.arg = told.arg\}$ 
238                                     ELSE       $\wedge F' = F$ 
239                                                $\wedge M' = M$ 
240     ELSE IF  $a\_U[p].rank < b\_U[p].rank$ 
241       THEN IF  $F[v\_U[p]] = [parent \mapsto b\_U[p].parent, rank \mapsto b\_U[p].rank, bit \mapsto 1]$ 
242         THEN       $\wedge F' = [F \text{ EXCEPT } ![v\_U[p]] = [parent \mapsto u\_U[p], rank \mapsto$ 
243                    $\wedge M' = \{t \in Configs : \exists told \in M : \wedge t.ret = [told.ret \text{ EXC}$ 
244                                      $\wedge t.\sigma = [i \in Nod$ 
245
246
247                                      $\wedge t.op = told.op$ 
248                                      $\wedge t.arg = told.arg\}$ 
249                                     ELSE       $\wedge F' = F$ 
250                                                $\wedge M' = M$ 
251     ELSE
252       IF  $u\_U[p] < v\_U[p]$  ranks are equal
253         THEN IF  $F[u\_U[p]] = [parent \mapsto a\_U[p].parent, rank \mapsto a\_U[p].rank, b$ 
254           THEN       $\vee \wedge F' = [F \text{ EXCEPT } ![u\_U[p]] = [parent \mapsto v\_U[p]$ 
255                      $\wedge M' = \{t \in Configs : \exists told \in M : \wedge t.ret = [to$ 
256                                $\wedge t.\sigma =$ 
257
258
259                                      $\wedge t.op = to$ 
260                                      $\wedge t.arg = to$ 
261                                      $\vee \wedge F' = [F \text{ EXCEPT } ![u\_U[p]] = [parent \mapsto v\_U[p]$ 
262                                                $\wedge M' = M$ 
263                                     ELSE       $\wedge F' = F$ 
264                                                $\wedge M' = M$ 
265     ELSE IF  $F[v\_U[p]] = [parent \mapsto b\_U[p].parent, rank \mapsto b\_U[p].rank, b$ 
266       THEN       $\vee \wedge F' = [F \text{ EXCEPT } ![v\_U[p]] = [parent \mapsto u\_U[p]$ 
267                $\wedge M' = \{t \in Configs : \exists told \in M : \wedge t.ret = [to$ 
268                                $\wedge t.\sigma =$ 
269
270
271                                      $\wedge t.op = to$ 
272                                      $\wedge t.arg = to$ 
273                                      $\vee \wedge F' = F$ 
274                                                $\wedge M' = M$ 
275                                     ELSE       $\wedge F' = F$ 
276                                                $\wedge M' = M$ 
277      $\wedge pc' = [pc \text{ EXCEPT } ![p] = "U7"]$ 

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278  $\wedge \text{UNCHANGED } \langle u\_F, a\_F, b\_F, u\_U, v\_U, a\_U, b\_U, c, d \rangle$   
 280  $U7(p) \triangleq \wedge pc[p] = \text{"U7"}$   
 281  $\wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F1U7"}]$   
 282  $\wedge c' = [c \text{ EXCEPT } ![p] = u\_U[p]]$   
 283  $\wedge \text{UNCHANGED } \langle F, u\_F, a\_F, b\_F, u\_U, v\_U, a\_U, b\_U, d, M \rangle$   
 285  $U8(p) \triangleq \wedge pc[p] = \text{"U8"}$   
 286  $\wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F1U8"}]$   
 287  $\wedge c' = [c \text{ EXCEPT } ![p] = v\_U[p]]$   
 288  $\wedge \text{UNCHANGED } \langle F, u\_F, a\_F, b\_F, u\_U, v\_U, a\_U, b\_U, d, M \rangle$   
 290  $UR(p) \triangleq \wedge pc[p] = \text{"UR"}$   
 291  $\wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"0"}]$   
 292  $\wedge M' = \{t \in \text{Configs} : \exists told \in M : \wedge told.ret[p] = \text{ACK}$   
 293  $\wedge t.sigma = told.sigma$   
 294  $\wedge t.ret = [told.ret \text{ EXCEPT } ![p] = \text{BOT}]$   
 295  $\wedge t.op = [told.op \text{ EXCEPT } ![p] = \text{BOT}]$   
 296  $\wedge t.arg = [told.arg \text{ EXCEPT } ![p] = \text{BOT}]\}$   
 297  $\wedge \text{UNCHANGED } \langle F, u\_F, a\_F, b\_F, u\_U, v\_U, a\_U, b\_U, c, d \rangle$   
 299  $Decide(p) \triangleq \wedge pc[p] = \text{"0"}$   
 300  $\wedge \vee \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"F1"}]$   
 301  $\wedge \exists i \in \text{NodeSet} : \wedge c' = [c \text{ EXCEPT } ![p] = i]$   
 302  $\wedge M' = \{t \in \text{Configs} : \exists told \in M : \wedge told.ret[p] = \text{BOT}$   
 303  $\wedge told.op[p] = \text{BOT}$   
 304  $\wedge told.arg[p] = \text{BOT}$   
 305  $\wedge t.sigma = told.sigma$   
 306  $\wedge t.op = [told.op \text{ EXCEPT } ![p] = \text{BOT}]$   
 307  $\wedge t.arg = [told.arg \text{ EXCEPT } ![p] = \text{BOT}]$   
 308  $\wedge t.ret = told.ret\}$   
 309  $\wedge \text{UNCHANGED } \langle F, u\_F, a\_F, b\_F, u\_U, v\_U, a\_U, b\_U, d \rangle$   
 310  $\vee \wedge pc' = [pc \text{ EXCEPT } ![p] = \text{"U1"}]$   
 311  $\wedge \exists i \in \text{NodeSet} : \exists j \in \text{NodeSet} :$   
 312  $\wedge c' = [c \text{ EXCEPT } ![p] = i]$   
 313  $\wedge d' = [d \text{ EXCEPT } ![p] = j]$   
 314  $\wedge M' = \{t \in \text{Configs} : \exists told \in M : \wedge told.ret[p] = \text{BOT}$   
 315  $\wedge told.op[p] = \text{BOT}$   
 316  $\wedge told.arg[p] = \text{BOT}$   
 317  $\wedge t.sigma = told.sigma$   
 318  $\wedge t.op = [told.op \text{ EXCEPT } ![p] = \text{"U"}]$   
 319  $\wedge t.arg = [told.arg \text{ EXCEPT } ![p] = \langle i, j \rangle]$   
 320  $\wedge t.ret = told.ret\}$   
 321  $\wedge \text{UNCHANGED } \langle F, u\_F, a\_F, b\_F, u\_U, v\_U, a\_U, b\_U \rangle$   
 323  $Step(p) \triangleq \vee F1(p)$

```

324          ∨ F2(p)
325          ∨ F3(p)
326          ∨ F4(p)
327          ∨ F5(p)
328          ∨ F6(p)
329          ∨ F7(p)
330          ∨ FR(p)
331          ∨ U1(p)
332          ∨ U2(p)
333          ∨ U3(p)
334          ∨ U4(p)
335          ∨ U5(p)
336          ∨ U6(p)
337          ∨ U7(p)
338          ∨ U8(p)
339          ∨ UR(p)
340          ∨ Decide(p)

```

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342  Next  $\triangleq$        $\exists p \in PROCESSES : Step(p)$ 

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344  Spec  $\triangleq$       Init  $\wedge \Box[Next]_{varlist}$ 

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346  ┌───────────────────────────────────────────────────────────────────────────────────┐
    \| * Modification History
    \| * Last modified Tue Apr 22 17:47:15 EDT 2025 by karunram
    \| * Created Thu Apr 03 12:26:37 EDT 2025 by karunram

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