The Art of Deception in Finance: Machine Learning and Downside Risk Control through Clever Index Replication

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SUMMARY: This paper introduces a novel approach to downside risk portfolio construction using Machine Learning techniques, termed "Deceptive Replication." Our method leverages the power of Machine Learning algorithms to replicate market indices while focusing exclusively on positive returns, thereby creating portfolios with enhanced upside potential and mitigated downside risk. Our "Deceptive Replication" technique employs a unique strategy that "tricks" Machine Learning models into replicating only the positive aspects of an index's performance. This approach addresses a significant gap in current research by integrating advanced Machine Learning techniques with traditional portfolio theory in a novel way. We demonstrate the efficacy of our method through empirical analysis, comparing its performance to traditional index replication strategies and other downside risk mitigation techniques. Our results show that "Deceptive Replication" portfolios exhibit improved risk-adjusted returns and better downside protection than conventional approaches. This paper contributes to the growing body of literature on the application of Machine Learning in finance, offering a new perspective on downside risk management and index replication. Our findings have important implications for both academic researchers and practitioners in the field of portfolio management, particularly in the context of passive investment strategies and the construction of investment products with enhanced downside protection.

Key words. Downside Risk, Portfolio Construction, Machine Learning, Index Replication, Deceptive Replication

1. INTRODUCTION

We will address the problem of replicating an index by primarily capturing its positive returns and avoiding its negative ones using Machine Learning (ML). The purpose of this paper is to study the behavior of this technique under various market conditions, examining its adaptability and effectiveness across different market regimes. This is an interesting topic with numerous applications both in the "smart" replication of indices, even when they are opaque in their structure (or when they are so large that their replica is impractical), and in the construction of portfolios that want to be exposed to certain markets, but aim at a better upside risk/downside risk ratio. Let us briefly and (without claiming to be exhaustive) go over the cornerstones of both the management of portfolios with protection from downside risk and the replication of indices whose composition is not known.

1.1. Downside Risk Portfolio Construction

The field of downside risk portfolio construction has evolved significantly since the seminal work of Markowitz (1952), which introduced the mean-variance framework. While this approach remains foundational, researchers have increasingly recognized the limitations of using variance as a risk measure, particularly its inability to capture the asymmetric risk preferences of investors (Sortino and Price (1994)). Over the years, several approaches have emerged to address the challenge of creating portfolios with positive asymmetric return distributions, thus limiting the downside risk. Let us take a look at some of the most significant contributions (without claiming to be exhaustive, given the vastness of the topic), which we can group into four types of models:

1.1.1. Risk Minimization through Optimization

This approach focuses on minimizing specific risk measures through optimization techniques. Key developments include:

- Lower Partial Moments (LPM) introduced by Bawa (1975) and Fishburn (1977), focusing on returns below a target threshold.
- The Sortino ratio, proposed by Sortino and Van Der Meer (1991), which uses downside deviation instead of standard deviation.
- Value at Risk (VaR) and Conditional Value at Risk (CVaR) measures, with Artzner et al. (1999) introducing the concept of coherent risk measures.
- Rockafellar and Uryasev (2000) developed a linear programming approach for portfolio optimization by minimizing CVaR.
- Alexander et al. (2006) analyzed the problem of computing optimal VaR and CVaR portfolios for derivatives.

Significant models in this category include:

- Risk Parity: Introduced by Qian (2005), this approach equalizes risk contribution from each asset, leading to more balanced portfolios. Maillard et al. (2010) provided a formal mathematical treatment of risk parity portfolios.
- Maximum Diversification: Proposed by Choueifaty and Coignard (2008), Choueifaty et al. (2013), this strategy maximizes the ratio of weighted average asset volatilities to portfolio volatility.
- Minimum Variance: While rooted in Markowitz (1952)'s work, Clarke et al. (2006) popularized this approach, which focuses solely on minimizing portfolio variance without considering expected returns.
- Portfolio Resampling: This approach improves portfolio rebalancing by constructing resampled efficient frontiers using techniques such as bootstrapping. It mitigates the impact of estimation errors commonly found in asset allocation decisions, leading to more robust portfolio optimization. For further details, refer to Michaud and Michaud (2007).
- Inverse Volatility Weighting: A simple but effective approach where asset weights are inversely proportional to their volatilities, as discussed by Chow et al. (2011).

These models represent significant advancements in portfolio optimization, offering unique approaches to risk minimization and diversification.

A popular portfolio baseline model, which is often associated with the idea of diversification and therefore risk control is the equally weighted one. However, equally weighted portfolios are not always truly diversified. As Pola (2020) points out, equally weighting assets does not guarantee diversification, especially when the assets have correlated risks. A more diversified portfolio would account for differences in correlations between assets, which is not captured by naive equal weighting. Fusai et al. (2020) further explore this issue, comparing equally diversified portfolios to equally weighted ones, and demonstrating that equally diversified portfolios can achieve superior risk-adjusted returns by better accounting for the correlation structure of assets. This has led to the development of new diversification measures, such as the Diversification Ratio (DR) by Choueifaty and Coignard (2008) and the Effective Number of Bets (ENB) proposed by Meucci (2009), aimed at better quantifying how well-diversified a portfolio truly is. These measures allow for portfolios to be built that are not only theoretically diversified but also practically so, even under real-world constraints.

1.1.2. Low-Risk Portfolio Construction

This method involves ranking securities based on risk measures and selecting the least risky ones. A seminal work in this area is the "Betting Against Beta" strategy by Frazzini and Pedersen (2014), which constructs portfolios using the bottom 25% of securities in terms of risk. Another influential study is by Blitz and Van Vliet (2007), who introduced a ranking-based approach to constructing low-volatility portfolios, which has become a foundation for many subsequent studies. Building on this, Asness et al. (2014) presented a method for constructing low-risk portfolios that avoids industry concentration, also using a ranking-based approach.

1.1.3. Option Replication Strategies

These strategies aim to explicitly create convexity in returns, mimicking the behavior of options. Key examples include:

- Constant Proportion Portfolio Insurance (CPPI), was introduced by Black and Jones (1987).

- Dynamic Proportion Portfolio Insurance (DPPI), is an extension of CPPI (Hamidi et al. (2014)).

1.1.4. *Risk Regime Detection*

This approach involves detecting risk-on and risk-off market situations (i.e., systemic risk), often using an index of financial distress and adjusting portfolio allocations accordingly. Recent advancements include:

- The development of financial distress indices, such as the CFNAI (Chicago Fed National Activity Index) for macroeconomic risk assessment (Brave and Butters (2012)), can be used to identify different market regimes.
- ML-based anomaly detection for market regime identification (Kou et al. (2019)).

1.2. Machine Learning in Downside Risk Portfolio Construction

Recent years have seen increased application of ML techniques in the field of Downside Risk Portfolio Construction, including:

- Jiang et al. (2019) proposed a deep Reinforcement Learning approach incorporating downside risk constraints for portfolio management.
- Coqueret (2020) utilized ML algorithms to predict drawdowns and integrate this information into portfolio construction.
- Zhang and Maringer (2022) introduced a hybrid model combining evolutionary algorithms with neural networks for multi-objective portfolio optimization.

1.3. Index and Hedge Fund Replication

Index replication aims to construct portfolios that closely mimic the performance of a benchmark index. This approach is fundamental to passive investment strategies such as ETFs and has gained popularity due to its cost-effectiveness and ability to provide broad market exposure.

Replicating indices without known composition presents a unique challenge, as investors lack transparency regarding the specific assets and their respective weights within the index. The idea was born from the desire to replicate the performance of Hedge Funds with more liquid assets. In recent years, Hedge Fund replication has encountered growing interest both from an academic and a practitioner perspective. However, this topic is broader and relevant when dealing with large or proprietary indices or those based on complex algorithms, where full disclosure is unavailable. In such cases, investors aim to replicate the index's behavior using observable market data, such as returns, which can be achieved through advanced modeling techniques including ML. Successful replication in these scenarios provides investors access to desired risk-return profiles without requiring detailed knowledge of index composition. Thus, the importance of replicating indices without known composition has grown significantly, particularly in markets where index providers do not disclose full constituent details or weights. The range of business applications is wide, including:

- Creation of liquid clones where full index replication is challenging or suboptimal - e.g., alternative investments such as Hedge Funds and other Alternative investments - using liquid assets such as Futures contracts.
- Identification of the financial drivers (for example the main asset classes), which determine the performance of an opaque investment product, for risk management purposes.
- Using liquid replication to dynamically hedge less liquid investment products.

Linear cloning techniques have been extensively studied for replicating Hedge Fund returns and indexes with unknown compositions. Hasanhodzic and Lo (2007) introduced a linear factor model approach to replicate hedge fund returns using a set of liquid, tradable instruments. Building on this work, Roncalli and Teiletche (2008) proposed an enhanced method for Hedge Fund replication using Kalman filtering techniques. This approach allows for time-varying exposures to risk factors, providing a more dynamic and adaptive replication strategy. Further advancing the field, Roncalli and Weisang (2012) explored the use of more sophisticated Bayesian filtering techniques, such as particle filters, to capture non-linear exposures and higher moments of Hedge Fund returns. In recent years, ML techniques have been increasingly applied to the problem of index replication with unknown composition. Ridge regression, Lasso regression, and Elastic Net have shown promise in this domain due to their ability to handle high-dimensional data and multicollinearity among predictors. Ridge regression, introduced by Hoerl and Kennard (1970), adds a penalty term to the ordinary least squares objective function, which helps to stabilize coefficient estimates in the presence of multicollinearity. Cule and De Iorio (2013) demonstrated the effectiveness of ridge regression in high-dimensional prediction problems and proposed a method for automatic selection of the ridge parameter. Lasso regression, developed by Tibshirani (1996), introduces an L1 penalty term that can perform both regularization and variable selection. This makes it particularly useful for index replication when dealing with a large number of potential factors. Elastic Net, proposed by Zou and Hastie (2005), combines the penalties of ridge and lasso regression, offering a compromise between the two methods. This approach can be especially effective when dealing with groups of correlated predictors. Giamouridis and Paterlini (2010) applied these regularization techniques to the problem of hedge fund replication, demonstrating their effectiveness in capturing the risk exposures of complex investment strategies. These ML approaches offer powerful tools for index replication with unknown composition, allowing for more flexible and robust models that can adapt to the complexities of financial markets.

1.4. Research Gaps and Opportunities

While significant progress has been made in downside risk portfolio construction, index replication, and the application of ML, several challenges remain in the current research landscape, such as:

- Limited integration of ML and traditional methods: Despite the growing use of ML in finance, there's a lack of approaches that seamlessly integrate advanced ML techniques with traditional portfolio theory, particularly in the context of downside risk management.
- Behavioral Finance considerations: Many existing models don't fully account for investor psychology and behavioral biases, particularly regarding asymmetric risk preferences.
- Asymmetric learning in index replication: Current index replication methods typically aim to mimic both positive and negative returns. There's a gap in exploring methods that selectively replicate only the desirable characteristics of an index.
- Adaptability to Market Regimes: There's a lack of portfolio construction methods that can dynamically adapt to changing market conditions without relying on explicit regime-switching models.

This paper aims to address several of these gaps, particularly focusing on the novel integration of ML with asymmetric index replication, thereby contributing to the evolving landscape both of downside risk portfolio construction and index replication.

1.5. Introducing "Deceptive Index Replication" Models

Building upon these advancements, we introduce a novel class of methods for obtaining investment portfolios with protection from downside risk, using index replication techniques based on ML: the "Deceptive Index Replication" models. This brand new approach involves "tricking" ML replication models of an index by providing only positive returns as input. The goal is to create portfolios that capture upside potential while mitigating downside risk. The core idea behind Deceptive Index Replication is simple, and can be summarized in a few steps:

1. Identify a financial index of interest, which can be as large as desired, or can be a composite index (a linear combination), and can have an unknown composition.
2. Get the time series of values.
3. Compute the time series of the returns and set to zero all negative ones.
4. Apply an ML-based linear cloning model (e.g., Elastic Net), using the computed returns as the response variable and time series data from liquid assets, such as Futures, as explanatory variables. This will estimate the portfolio weights.
5. Use the model output (i.e., the liquid asset weights) to build the replication portfolio.
6. Reiterate 2-4.

This innovative method addresses the growing demand for sophisticated risk management in both active and passive investment strategies and offers a new perspective on index replication without full constituent information. The Deceptive Index Replication approach offers improved risk-adjusted performance compared to conventional techniques, providing investors with a more robust tool for portfolio construction. This opens the way to numerous applications in the financial services industry, both in the creation of investment products with a positive asymmetric distribution of returns, therefore with a better relationship between upside potential and downside risk, and in the smart replication of indices, especially if they have an opaque composition. We will show a significant example of using this approach to index replication using a target built specifically for its difficult replicability.

2. METHODOLOGY

2.1. Dataset Description

The example of the proposed replication methodology uses as its target a broad and very opaque index, built by combining two very large indexes of relatively liquid assets, a global bond and a global stock, and a large index representative of the world of Hedge Funds. Thus, we created an ad hoc target to demonstrate the validity of the method: it is a very opaque target, with a huge number of underlying securities and factors, almost impossible to replicate with direct investment in the underlying assets. The dataset we are using consists of weekly data spanning from October 2007 to April 2021. This period is significant because of its length and because it includes the Great Financial Crisis and the Eurozone Crisis. The dataset includes a target time series that represents a weighted combination of major indices we aim to replicate. Specifically, the target index is constructed as 50% HFRX Global Hedge Fund Index (HFRXGL), 25% MSCI World Index (MXWO), and 25% Bloomberg Global Aggregate Bond Index (LEGATRUU). To replicate this target index, we utilize a portfolio composed of the following Futures contracts: RX1 (Euro-Bund), TY1 (10-Year Treasury Note), GC1 (Gold), CO1 (Crude Oil), ES1 (S&P 500), VG1 (EuroStoxx 50), NQ1 (Nasdaq 100), LLL1 (Long Gilt), TP1 (Topix), DU1 (DAX), and TU2 (2-Year Treasury Note). These Futures contracts, representing a variety of asset classes, serve as the building blocks for replicating the performance of the target indices. Futures contracts are very liquid, have low transaction costs, and cover a large part of the financial factors, therefore they are an ideal ingredient in a portfolio replication process. The data is from Bloomberg.

2.2. Introduction to the cloning model

The heart of this work lies in how we handle the data. We take the original dataset and create a new version that keeps all the positive returns and sets negative ones to zero. Our goal is to figure out the effectiveness of this method on a baseline model that consists of a simple linear regression on the entire dataset.

After taking the time series of the prices of the target index and the futures, we differentiate at first order to make them stationary, then let $Y(t)$ be the time series of the returns of our target index and $F_i(t)$ for $i \in I$ the set of futures' returns that we use to approximate it. Since we are working with real-world data and we do not have continuous functions, we represent the discretized time series as $Y = \{Y_j\}_{j=0}^{N}$, $F_i = \{F_{ij}\}_{i \in I, j=0}^{N}$ where N represents the total number of discretized time steps. Then we build our dataset as:

$$\tilde{Y} = \{\tilde{y}_j\}_{j=0}^{N} = \{\max(0, y_j)\}_{j=0}^{N}$$

See figure 1.

This innovative dual-dataset approach is designed to manage and limit downside risk by separating positive returns and transforming negative ones. By isolating these two types of returns, we establish a framework to analyze their distinct effects on the model's performance. The positive returns dataset highlights growth potential, while the transformed negative returns provide a unique lens for assessing risk. This dual perspective enables a more comprehensive evaluation of our regression model's effectiveness across different market conditions, offering valuable insights into how varying return transformations can help mitigate potential losses.

2.3. Mathematical Formulation

2.3.1. Mathematical Assumptions

After a preliminary check, the time series is assumed to be stationary after differentiation, meaning the key statistical properties do not change over time. The relationships between the target index returns and the futures returns are assumed to be linear since, by definition, a portfolio is a linear combination of assets. The error terms ϵ_t are assumed to follow a normal distribution with mean zero and constant variance and are assumed to be uncorrelated with each other (i.e., "iid"). The model's validity is constrained by the accuracy of these underlying assumptions. The more the data deviates from this ideal hypothesis (which notoriously does not often occur in "fat-tailed" financial markets), the more the results can worsen, i.e., the model's performance may degrade. But, as will be seen from the results, the model is quite robust to the hypotheses.

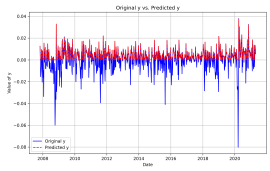

Fig. 1: Dataset of returns with Cropped Negative values (red) vs Original Dataset of returns (blue)

2.3.2. Domain Assumptions

There is sufficient historical data available to accurately estimate the weights and other model parameters.

2.3.3. Constraints

The quality and completeness of the data can constrain the model's accuracy. Missing or inaccurate data can lead to incorrect estimations. The model's performance may be constrained to the specific period for which the data is available. Results might not generalize to different periods.

2.3.4. Regressions

Let I denote the set of Futures used to approximate the target index returns $\tilde{Y}$, with F_i representing the returns of these futures in our discrete time interval and $F = [F_1|...|F_N]$. Our objective is to determine optimal weights $\underline{w}$ such that:

$$\tilde{Y} = \underline{w}F + \underline{\varepsilon}$$

where ϵ is a vector of error terms with zero mean and unitary standard deviation. To achieve this, we minimize the following linear regression problem:

$$\underline{w}^* = \arg\min_{\underline{w}} \|\tilde{Y} - \underline{w}F\|_2^2$$

where w^* are the optimal weights in our portfolio. To enhance our model, we explore penalized regression methods such as Ridge, Lasso, and Elastic Net in which we add a norm penalty to reduce the norm of the weights.

$$w^* = \arg\min_w \|\tilde{Y} - \underline{w}F\|_2^2 + \lambda\|\underline{w}\| + \alpha\|\underline{w}\|_2^2 \quad (1)$$

The λ and α coefficients are optimized through cross-validation on the training set.

To ensure our model remains accurate and responsive to changing market conditions, we employ a dynamic approach to parameter tuning. Specifically, the hyperparameters of our penalized regression methods (Ridge, Lasso, and Elastic Net) are retrained using cross-validation at every new window of data. This approach allows the model to adapt to new information and maintain its predictive performance over time.

This dynamic retraining process is crucial for maintaining the robustness of our model. By continuously updating the hyperparameters and weights, we ensure that the model adapts to new patterns and trends in the data, thereby enhancing its predictive power and reliability.

2.3.5. State Space Models

Following the analysis of regressions, we want to check if we can improve our replication by considering eventual time-dependent patterns while minimizing:

$$w^* = \arg\min_w \|\tilde{Y} - \underline{w}F\|_2^2 \quad (2)$$

To do this, we introduce in our models an autoregressive part and a moving average one, and we consider our futures as exogenous variables. This model can be formulated as:

$$\tilde{Y} = \Theta(L)^p Y + \Phi(L)^q + \underline{w}F + \underline{\epsilon} \quad (3)$$

where L is the lag operator of order p, q, Θ, Φ are the parameters of the standard ARIMA model and ϵ is a vector of error terms with zero mean and unitary standard deviation.

Following the previous reasoning, we also add seasonality features to our models, thus getting a SARIMAX:

$$\tilde{Y} = \Theta(L)^p Y + \Phi(L)^q + \Psi(L^s)^p Y + \Upsilon(L^s)^q + \underline{w}F + \underline{\epsilon} \quad (4)$$

where L^s represents the lag operator on seasonal time lags and Ψ, Υ are the parameters that control regression on seasonal time lags.

In this context, we can apply Kalman filtering to further enhance the model's adaptability to time-varying dynamics in both the index and the exogenous variables (futures). Kalman filters offer a recursive estimation technique that allows us to update the portfolio weights in real time as new data becomes available. The system can be modeled in state-space form, where the state vector $\underline{w_j}$ evolves in $j = 1, ..., N$, and observations are updated as follows:

$$\underline{w_j} = A\underline{w_{j-1}} + \eta_j \quad (5)$$

$$\tilde{Y}_j = H\underline{w_j} + \underline{\epsilon_j} \quad (6)$$

where, A is the state transition matrix, H is the observation matrix, η_t represents the process noise, and $\underline{\epsilon_t}$ is the observation noise. Kalman filtering continuously refines the estimates of the state vector $\underline{w_t}$ based on the latest data, capturing latent states and adapting the replication process accordingly.

3. EMPIRICAL ANALYSIS

To verify whether setting the negative returns to zero effectively reduces downside risk in replicating the indexes, we test all the models explained in the previous section out-of-sample on the altered (”cropped”) dataset, using the original dataset as a baseline for comparison. Several key performance metrics are monitored to evaluate both return and risk.

This evaluation will prioritize widely recognized and interpretable metrics that assess both return and risk. For return, we analyze effective return along with its simulated probability distribution function for future returns. The risk evaluation includes a range of advanced metrics such as standard deviation, downside deviation, lower and upper partial moments (LPM/UPM), Value at Risk (VaR), Conditional Value at Risk (CVaR), Conditional Drawdown at Risk (CDaR), and the Sortino ratio.

- **Lower Partial Moment (LPM)**: This metric assesses downside risk by estimating the expected deviation of returns that fall below a specified target level, which is set at zero with a degree of 2. It highlights the magnitude of negative deviations from the target.
- **Upper Partial Moment (UPM)**: In contrast to LPM, UPM focuses on potential gains by measuring the returns that exceed a defined threshold (set at zero). This metric captures the positive deviations from the target, providing insights into the upside potential of an investment, calculated with a degree of 2.
- **Value at Risk (VaR)**: VaR estimates the maximum potential loss that a portfolio might face over a specified time frame at a given confidence level (in our case, 95%). While it identifies a threshold for expected losses, it does not account for the magnitude of losses that may occur beyond this threshold.
- **Conditional Value at Risk (CVaR)**: CVaR builds upon VaR by quantifying the expected losses that exceed the VaR threshold, offering a deeper understanding of the risks associated

with extreme loss events. This metric is valuable for understanding the potential severity of tail risks.

- **Lower Partial Standard Deviation (LPSD)**: LPSD focuses specifically on downside risk and is related to the second-order LPM (with degree 2 and pole at zero). Unlike standard deviation, which treats positive and negative volatility equally, LPSD emphasizes only the downside deviations, providing a more precise measure of risk associated with unfavorable outcomes.
- **Sortino Ratio**: The Sortino ratio refines the traditional Sharpe ratio by considering only downside risk. It evaluates the excess return relative to the risk of negative deviations (with the pole at zero), providing a clearer assessment of risk-adjusted returns in the context of downside volatility.
- **Conditional Drawdown at Risk (CDaR)**: CDaR assesses the potential maximum drawdown during a specified period, taking into account both the severity and duration of drawdowns. This metric is useful for evaluating the risks associated with prolonged periods of loss.
- **Turnover**: The turnover represents the total volume of trading activity within a given period. It is calculated using the standard turnover measure, which involves dividing the total by 2. Note that this turnover is annualized.

Additionally, we seek to determine whether optimal results are achieved using the entire historical dataset up to the present or by applying a rolling window of data.

Each price observation is recorded weekly, and for model retraining, we incorporate the last 156 weeks of data to reflect a long-term investment scenario. This technique can be adapted to shorter periods, provided a thorough analysis of trading costs is performed.

4. RESULTS

For the sake of brevity, we present the best and worst results obtained, which respectively correspond to a Lasso regression model trained from 30-08-2011 to 19-08-2014 and tested from 19-08-2014 to 20-04-2021, and to a Kalman Filter trained and tested in the same way. This time window works well for this paper - whose main objective is to show the validity of the approach based on cropped returns -, but of course, the choice of optimal time windows can be subject to specific calibration.

We can see in Figure 2 how the model effectively replicates the index out-of-sample, and this makes sense since the regression is made on both positive and negative returns, and thus it's aimed to replicate the "full" behavior of the time series.

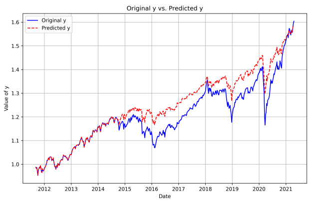

Fig. 2: Out-of-sample replication of the last 400 time-steps, with training conducted from 30-08-2011 to 19-08-2014 and testing from 19-08-2014 to 20-04-2021 on the original dataset using an optimized Lasso regression model.

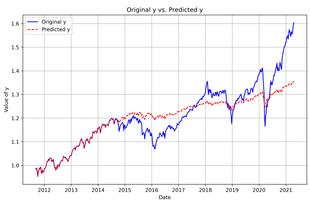

Fig. 3: Out-of-sample replication of the last 400 time-steps, with training conducted from 30-08-2011 to 19-08-2014 and testing from 19-08-2014 to 20-04-2021 on the cropped dataset using an optimized Lasso regression model.

Then we see how the dataset with cropped values $\tilde{y}(t)$ in Figure 3 mimics the uptrend and effectively limits the downside parts, thus minimizing the downside risk.

Finally, we can analyze the worst-case scenario (Kalman Filter) in Figure 4 and observe that, even in this situation, the model still yields a relatively low-risk replica; however, it results in an underestimation of the index. This indicates that the technique is robust, and the choice of the model acts merely as a hyperparameter.

For each of these tests, we represent what we believe are the most important metrics, the return, the lower partial moments, and the Sortino ratio. As

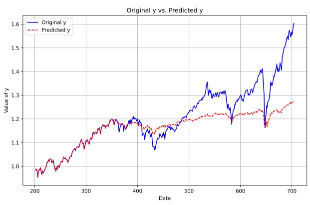

Fig. 4: Out-of-sample replication of the last 400 time-steps, with training conducted from 30-08-2011 to 19-08-2014 and testing from 19-08-2014 to 20-04-2021 on the cropped dataset using a Kalman Filter state space model.

shown in Figure 5, in the optimal window of training, the replicated index outperforms the original one by lowering significantly the downside risk at the same return as the original one.

Moreover, when examining the estimated probability distribution functions of the returns, as shown in Figure 6, we observe that our replica effectively reduces negative returns. This adjustment results in a slightly reduced risk-adjusted return while significantly trimming the lower tail of the distribution, thereby limiting downside exposure. Furthermore, the distribution is positively skewed.

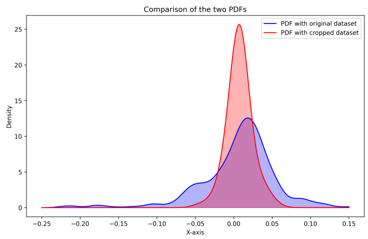

Fig. 6: Comparison of estimated probability density functions: replication with original dataset vs replication with cropped dataset in case of Lasso regression, with training conducted from 30-08-2011 to 19-08-2014 and testing from 19-08-2014 to 20-04-2021.

We now present Table 1, which summarizes the key results from the simulation in the optimal training window.

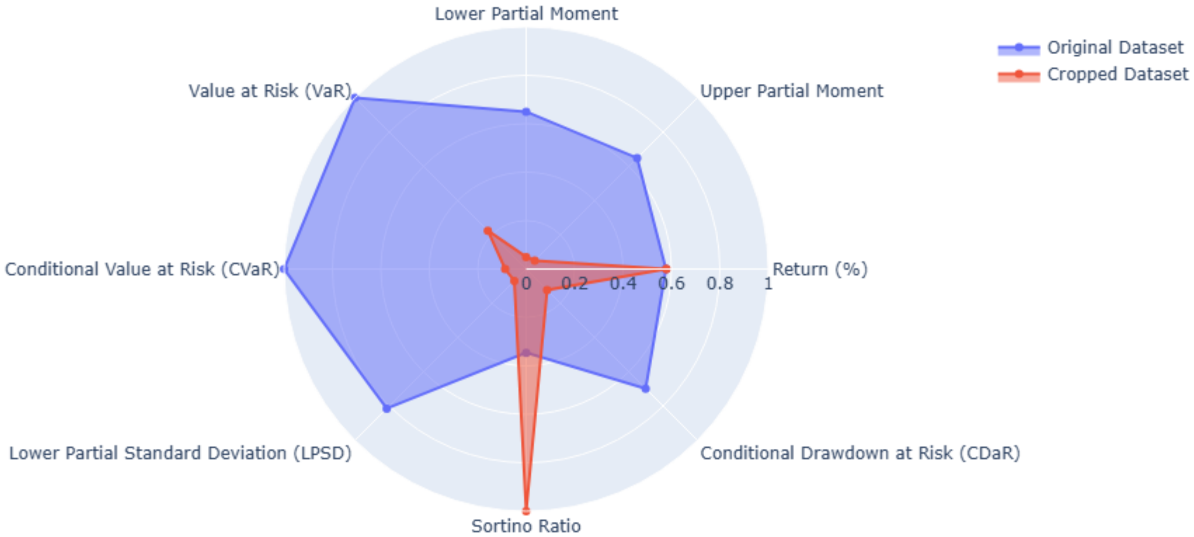

Fig. 5: Out-of-sample comparison of metrics between replication with original dataset vs cropped dataset in case of Lasso regression, with training conducted from 30-08-2011 to 19-08-2014 and testing from 19-08-2014 to 20-04-2021. All the metrics are normalized in the interval [0, 1] for visibility purposes.

Model	Return (%)	LPSD (%)	Sortino Ratio	Turnover (%)
OLS original	35.22	1.27	0.0521	69.73
OLS cropped	43.10	0.64	0.1586	59.79
Lasso original	35.30	1.27	0.0521	28.66
Lasso cropped	35.30	0.57	0.1426	40.78
Ridge original	35.22	1.27	0.0521	34.39
Ridge cropped	35.74	0.57	0.1444	14.68
ElasticNet original	35.22	1.27	0.0521	26.31
ElasticNet cropped	34.24	0.57	0.1387	9.21
ARIMA original	35.22	1.27	0.0521	70.08
ARIMA cropped	37.33	0.59	0.1482	37.39
SARIMAX original	35.22	1.27	0.0521	85.20
SARIMAX cropped	37.20	0.58	0.1489	32.75
Kalman Filter original	35.22	1.27	0.0521	25.35
Kalman Filter cropped	27.63	0.59	0.1145	36.30

Table 1: Return, Lower Partial Standard Deviation (LPSD), and Sortino Ratio for different models. Return is the total return over the entire period, LPSD is calculated on weekly returns, Sortino ratio is calculated as the ratio between the average weekly return and the square root of the LPM of the weekly returns of the replication portfolio.

5. DISCUSSION

5.0.1. Interpretation of Results

As demonstrated in the previous section, using the original dataset closely replicates the overall performance of the index. However, when applying the dataset with cropped returns, we see a distinct advantage in managing downside risk. This new methodology of modifying the dataset effectively captures the uptrend while significantly reducing exposure to negative returns, providing a more balanced approach that preserves growth potential while minimizing potential losses. This combination offers a more resilient replication of the index under varying market conditions.

5.0.2. Strengths and limitations

A major strength of this approach is that it doesn't add unnecessary layers of complexity to the replication strategy and has almost no computational overhead since only a little pre-processing is needed. Additionally, the method effectively limits downside risk without significantly reducing returns. However, a limitation is that the model's effectiveness can vary based on the type of dataset used, and the approach may not capture all the nuances of market behavior.

5.0.3. Implications for practitioners

For practitioners, this approach offers a simplified yet effective method for index replication that minimizes downside risk. The minimal pre-processing required means that it can be easily integrated into existing workflows without significant changes to infrastructure. Practitioners should consider the type of dataset used, as it impacts the model's performance, and continuously monitor and adjust the model based on real-time data to maintain its effectiveness.

5.1. Future Research Directions

Several avenues for further research could enhance the model's performance, particularly in the area of downside risk management. Key directions include:

- To examine the impact of trading costs on the model. An important aspect to investigate is determining the frequency at which the model can be retrained and the portfolio position updated without incurring excessive trading costs. By addressing this aspect, researchers can identify the optimal balance between the frequency of model retraining and the associated trading costs. Achieving this balance will facilitate the creation of a more adaptable portfolio that can hedge risk more effectively.

- To explore integrating more sophisticated techniques, such as neural networks, to further enhance the model's predictive accuracy and robustness. Specifically, integrating neural networks could capture potential non-linearity in the data. Although portfolio construction is intrinsically linear, the relationships between futures returns and the index may exhibit nonlinear dynamics that could improve replication accuracy.

- To explore how the model behaves across various market regimes. While the method adapts to changing data, a more detailed analysis of its performance during extreme market events (e.g., financial crises) could offer valuable insights for both academic researchers and practitioners.

6. CONCLUSION

The main conclusions that can be drawn from the results obtained can be summarized as follows:

- **Effective risk mitigation:** The "Deceptive Replication" approach introduced in this paper demonstrates promise in constructing portfolios that successfully capture upside potential while significantly mitigating downside risk. Empirical analysis reveals that this method outperforms traditional index replication strategies, offering superior downside protection without excessively sacrificing returns, particularly when using cropped returns.

- **Innovative use of Machine Learning:** By focusing selectively on positive returns, this approach leverages ML to address limitations inherent in conventional methods that replicate both positive and negative returns. The use of penalized linear regression models, such as Ridge, Lasso, and Elastic Net, proves effective in replicating indices with unknown composition, offering a robust solution for index replication.

- **Improved risk-adjusted performance:** The results indicate that the model not only mirrors the upward trajectory of the target index but also minimizes downside risk, as evidenced by an improved Sortino ratio. This underscores the effectiveness of the approach in balancing return generation with risk management.

- **Adaptability:** The dynamic tuning of model parameters through cross-validation in each new data window maintains model robustness in response to evolving market conditions. This feature is particularly valuable for practitioners seeking adaptive risk management strategies in volatile environments.

- **Behavioral Finance considerations:** While the paper primarily addresses the technical aspects of index replication, it also incorporates Behavioral Finance principles by accounting for the asymmetric risk preferences of investors. This enhances the practical relevance of the model, particularly in its sensitivity to downside risk aversion.

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