Control Theory homework 5

Kamil Hayrullin
BS18-02

1 Introduction

My Innopolis mail is k.hayrullin@innopolis.ru
My variant is ”b”
Link to my GitHub repository is here

2 Consider classical benchmark system in control theory - inverted pendulum on a cart

$$(M+m)\ddot{x} - mlcos(\theta)\ddot{\theta} + mlsin(\theta)\dot{\theta}^2 = F$$

$$-cos(\theta)\ddot{x} + l\ddot{\theta} - gsin(\theta) = 0$$

$$g = 9.81, M = 15.1, m = 1.2, l = 0.35$$

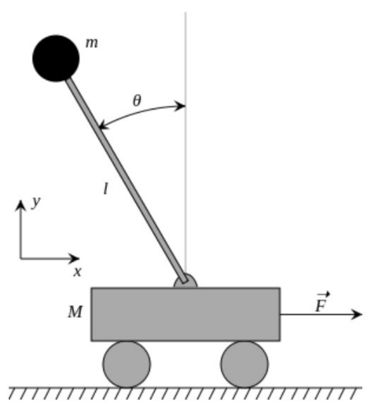


Figure 1: A schematic drawing of the inverted pendulum on a cart. The rod is considered massless. The mass of the cart and the point mass at the end of the rod are denoted by M and m. The rod has a length l.

A. Prove that it is possible to design state observer of the linearized system

Lets consider linearalized system from previous homework.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.7796 & 0 & 0 \\ 0 & 30.256 & 0 & 0 \end{bmatrix} C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

We know what system will be observable if and only if the (nr n) matrix is of rank n.
Rank of A = 4, so system is observable.

$$\begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0.7796 & 0 & 0 \\ 0 & 0 & 0 & 0.7796 \end{bmatrix}$$

B. For open loop state observer, is the error dynamics stable?

Error between model state and observer state based on matrix

$$A - LC$$

The open loop state observer is absolutely stable if all the poles of the open loop transfer function present in left half of ‘s’ plane. Let’s find eigenvalues of matrix A

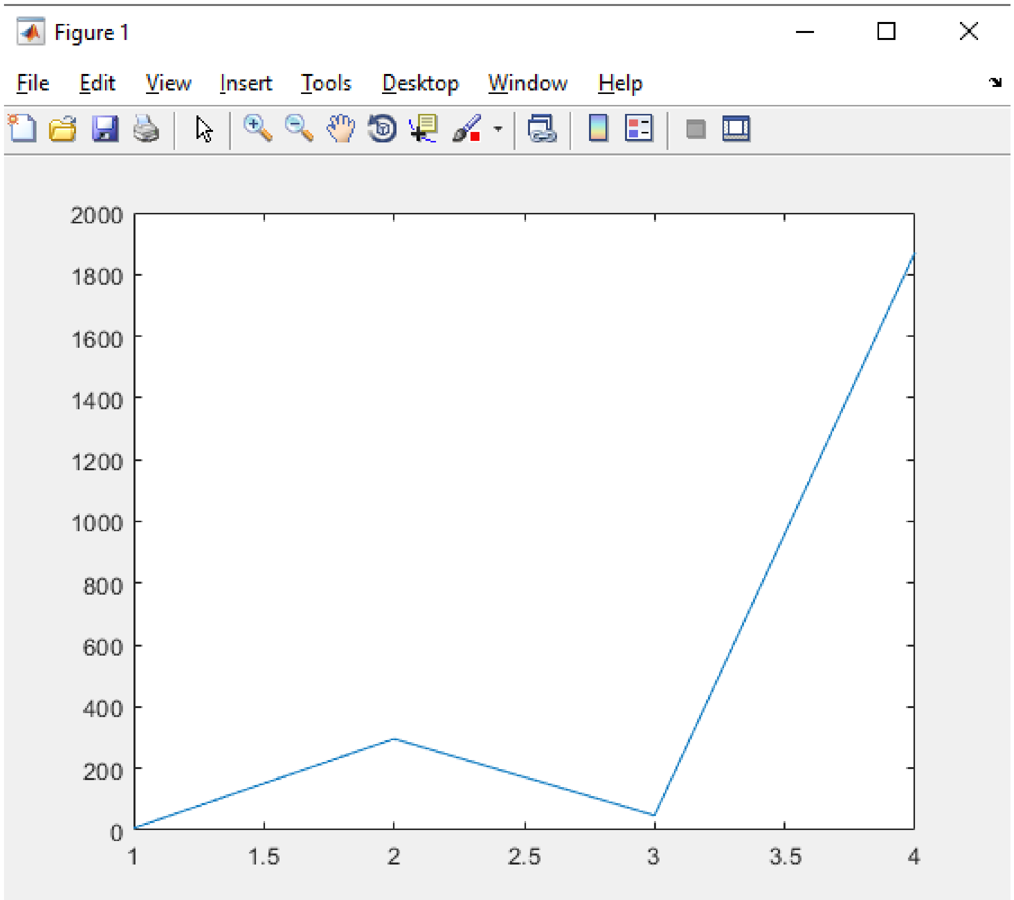
```
>>
A = [0 0 1 0; 0 0 0 1; 0 0.7796 0 0 ; 0 30.256 0 0]
eig(A)

A =

         0         0    1.0000         0
         0         0         0    1.0000
         0    0.7796         0         0
         0   30.2560         0         0


ans =

         0
         0
    5.5005
   -5.5005
```

As we can see, system is unstable

C. Design Luenberger observer for linearized system using both pole placement and LQR methods

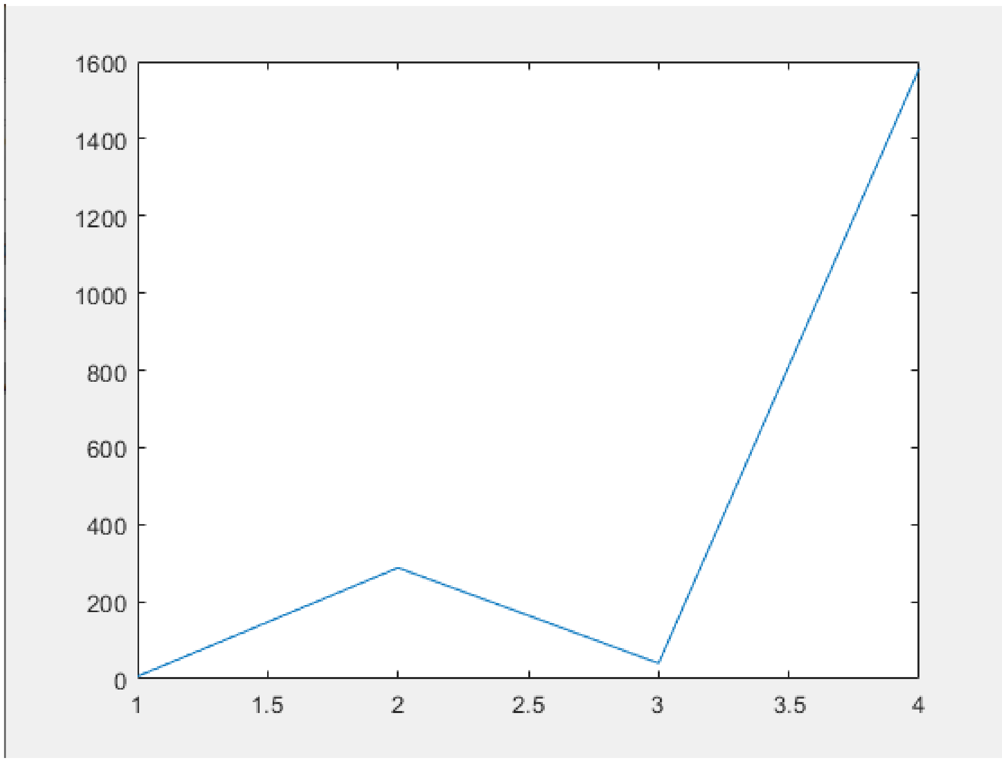
Using pole placement method:

```
clear
syms L
A = [0 0 1 0; 0 0 0 1; 0 0.7796 0 0 ; 0 30.256 0 0]
C = [1 0 0 0]
P=[-1, -1.5, -2, -2.5]
Mo = ctrb(A', C')
rank(Mo)
L = place(A', C', P)
L = L'
plot(L)
```



LQR:

```
clear
A = [0 0 1 0; 0 0 0 1; 0 0.7796 0 0 ; 0 30.256 0 0]
C = [1 0 0 0]
Q = [1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 0 1]
R = [[1]]
L = 0.5 * lqr(A', C', Q, R)
L = L'
plot(L)
```



D. Design Luenberger observer for linearized system using both pole placement and LQR methods

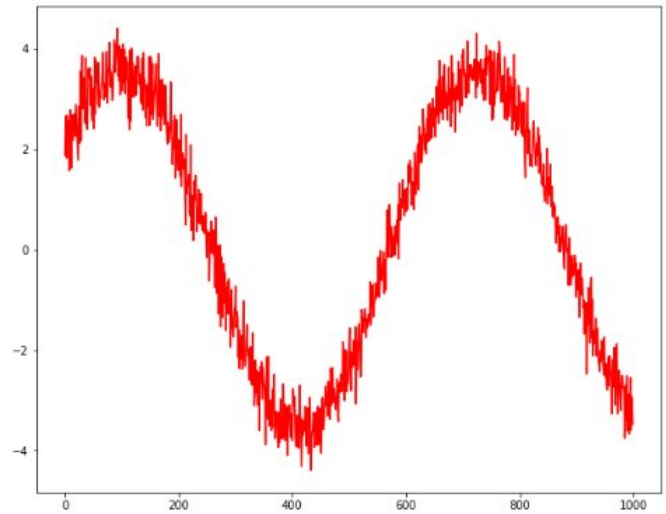
```python
from scipy import signal
import numpy as np
A = np.array([[0, 0, 1, 0],
              [0, 0, 0, 1],
              [0, 0.7796, 0, 0],
              [0, 30.256, 0, 0]])
B = np.array([[0],[0],
              [1/0.06622],
              [1/0.1892]])
p = np.array([-1, -2, -3, -4])
K = signal.place_poles(A, B, p)
K.gain_matrix
```

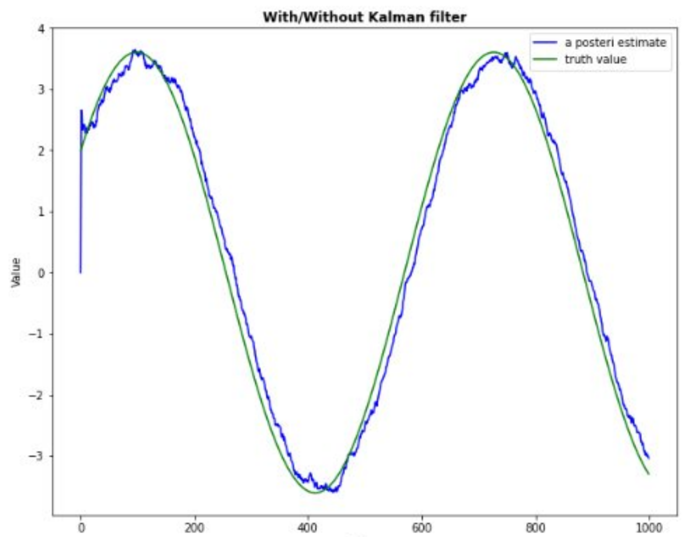
```
array([[-0.05300579, 12.49788031, -0.11042873,  2.20751065]])
```

```matlab
A = [0, 0, 1, 0; 0, 0, 0, 1; 0, 0.7796, 0, 0; 0, 30.256, 0, 0]
B = [0; 0; 1/0.06622; 1/0.1892]
p = [-1, -2, -3, -4]
K = place(A,B,p)
```

Command Window

```
K =

   -0.0530   12.4979   -0.1104    2.2075
```





H.Implement Kalman Filter (you can use libraries with KF if this task is not for you to get some points in next ones)