Control Theory homework 3

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BS18-02

1 Introduction

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My variant is ”e”
Link to my GitHub repository is here

2 Implement given tasks using python

(A) Design PD-controller that tracks time varying reference states i.e. [x*(t), x'*(t)] as closely as possible.

Lets consider second order linear ODE

$$x'' + \mu x' + kx = u, \mu = 14, k = 85 \tag{1}$$

To create Proportional-derivative controller we should have P control:

$$u = k_p(x^* - x) \tag{2}$$

And add derivative term to it

$$u = k_p(x^* - x) + k_d(x'^* - x') \tag{3}$$

Where control error equals

$$e = x^* - x \tag{4}$$

Finally, PD controller:

$$u = k_p e + k_d e' \tag{5}$$

Link to python code in Google Colaboratory is here

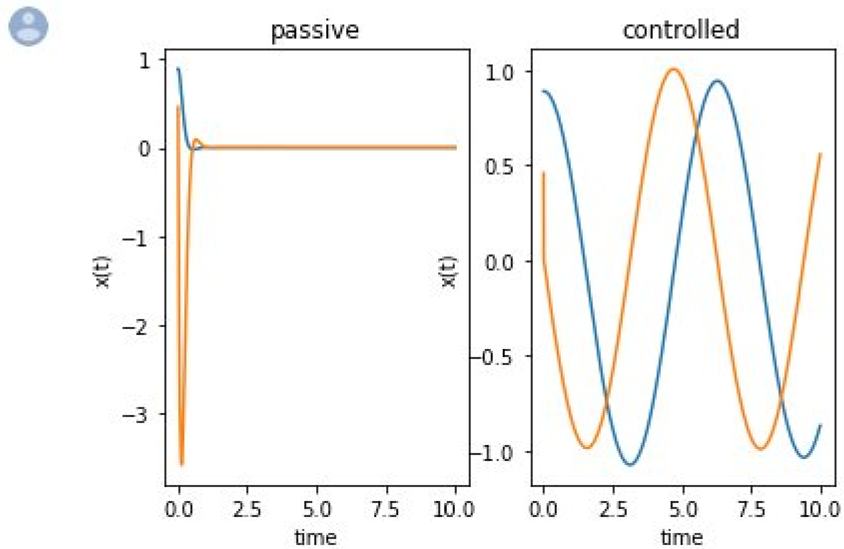


Figure 1: for COS function result is following, kp = 1000, kd = 10000

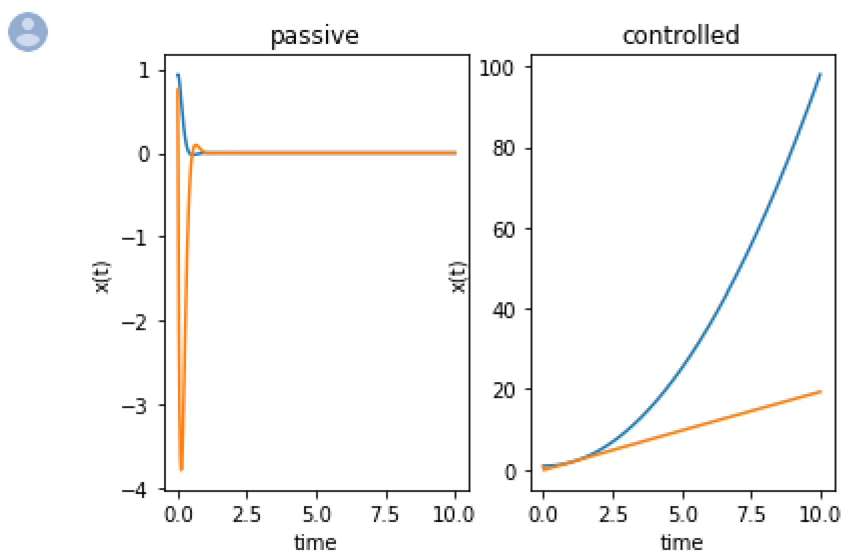


Figure 2: for t*t function result is following, kp = 1000, kd = 10000

(B) Tune controller gains kp and kd . Find gains that provide no oscillations and no overshoot. Prove it with step input.

My choice of gains is:

$$k_p = 1000, k_d = 10000 \tag{6}$$

Link to python code in Google Colaboratory is here

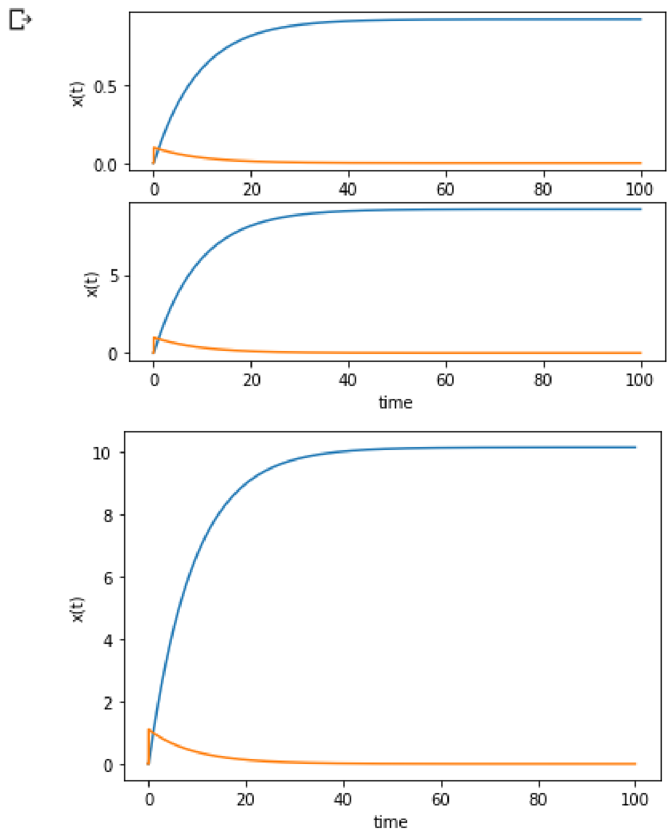


Figure 3: As we can see, no oscillations and no overshoot

(C) Prove that controlled oscillator dynamics is stable for your choice of kp, kd

Lets consider second order linear ODE

$$x'' + 14x' + 85x = 1000 * (x^* - x) + 10000 * (x'^* - x') \tag{7}$$

$$x'' = -14x' - 85x + 1000 * x^* - 1000 * x + 10000 * x'^* - 10000 * x' \tag{8}$$

$$\begin{pmatrix} x'' \\ x' \end{pmatrix} = \begin{pmatrix} -14 - 10000 & -85 - 1000 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x' \\ x \end{pmatrix} + \begin{pmatrix} 10000 & 1000 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x^{*\prime} \\ x^* \end{pmatrix} \tag{9}$$

Now let's find eigenvalues of matrix A

$$det(A - \lambda I) = (-10014 - \lambda)(-\lambda) + 1085 = \lambda^2 + 10014\lambda + 1085 = 0 \quad (10)$$

$$\lambda_1, \lambda_2 = -0.10834, -10013.89 \quad (11)$$

Both eigenvalues less than zero, system is **stable**

(D) Think of how you would implement PD control for a linear system:

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 10 & 3 \\ 5 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad (12)$$

,

To implement PD controller I would add control law

$$u = k_p e + k_d e' \quad (13)$$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 10 & 3 \\ 5 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} (k_p e + k_d e') \quad (14)$$

I would translate this state space form into Linear Time Varying system and apply 1st task's code to this LTV.

(E) Implement a PI/PID controller for the system: x + 14x' +85x + 9.8 = u. Test your controller on different trajectories, at least two.
Link to python code in Google Colaboratory is here

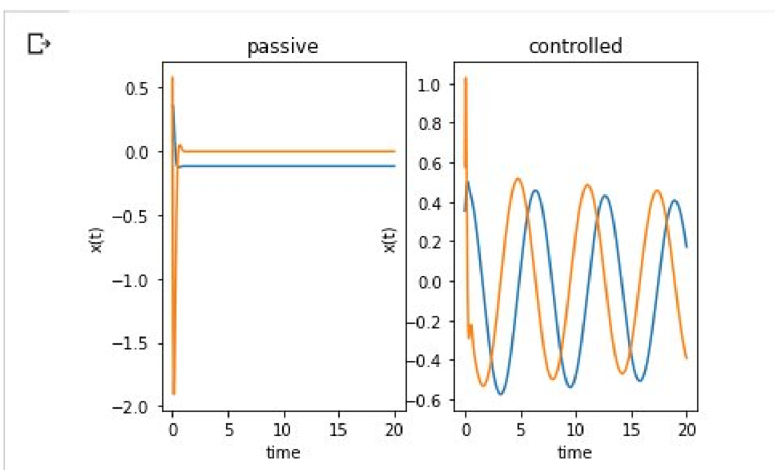


Figure 4: for COS function result is following, kp = 2, kd = 100

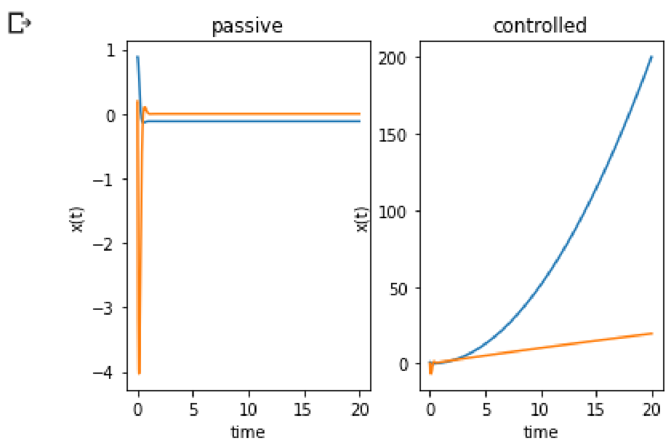


Figure 5: for t*t function result is following, kp = 2, kd = 100

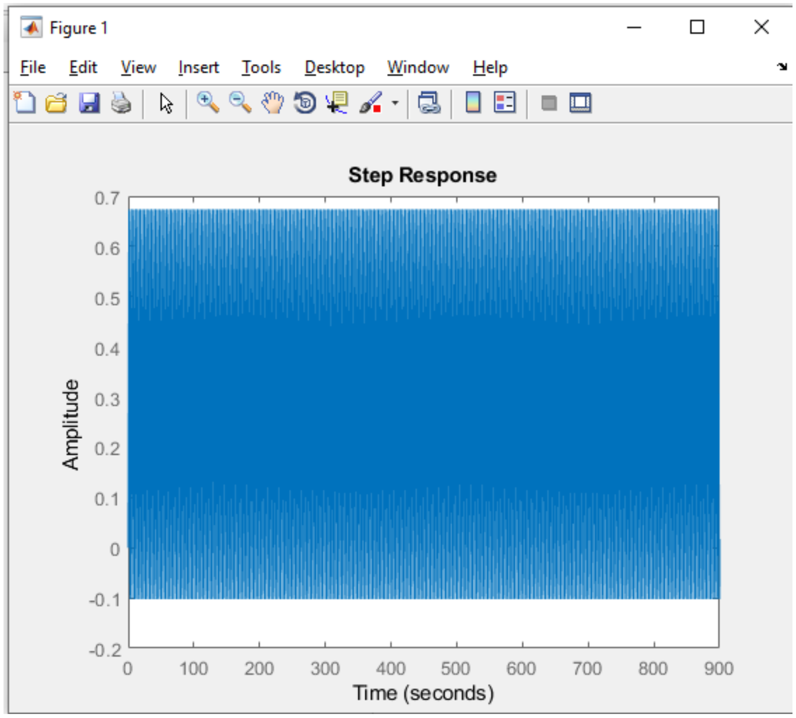
3 Design a PID controller. Use step input function and try to improve rise time, overshoot and steady-state error, comparing with no controller system

$$W(s) = \frac{s+2}{2s^2+7} \tag{15}$$

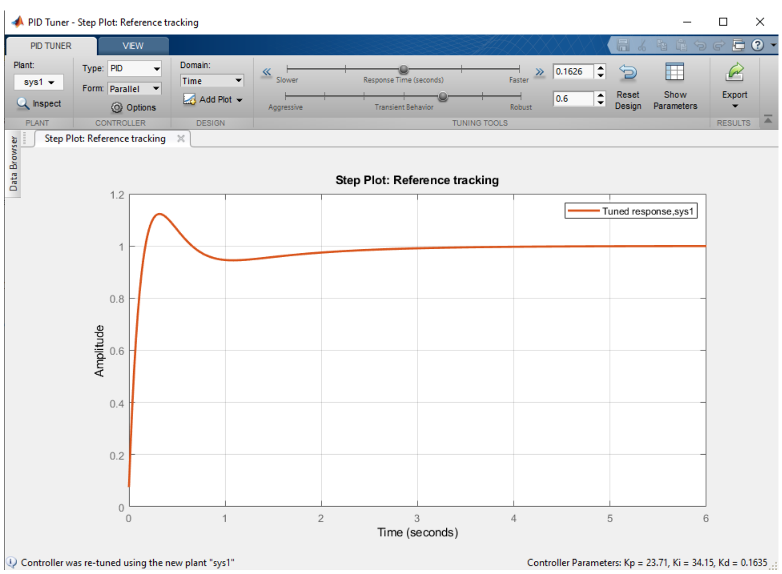
Let's wtire such code in matlab

```
numerator = [1, 2];
denominator = [2,0,7];
sys1 = tf(numerator,denominator)
step(sys1)
```

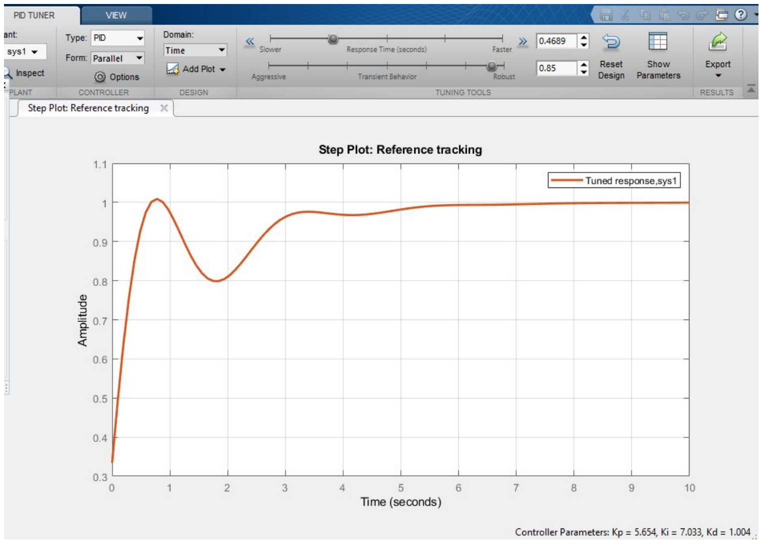
non controlled step function



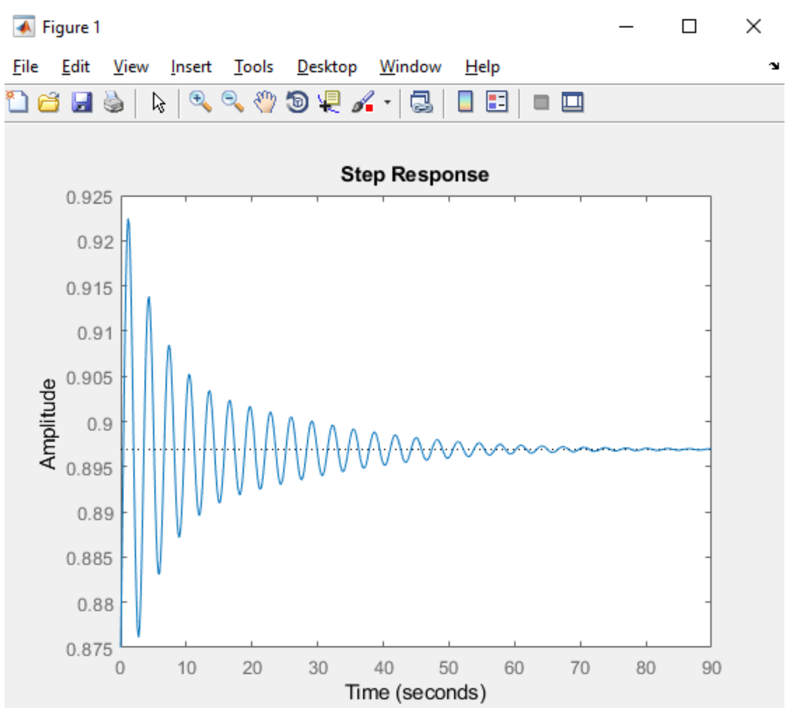
In PID Tuner it looks like that



Now we tune it a bit



and get this step function



Was:

Controller Parameters

	Tuned
Kp	4.2362
Ki	5.9182
Kd	0.59223
Tf	n/a

Performance and Robustness

	Tuned
Rise time	0.486 seconds
Settling time	5.15 seconds
Overshoot	2.71 %
Peak	1.03
Gain margin	Inf dB @ NaN rad/s
Phase margin	64.6 deg @ 3.46 rad/s
Closed-loop stability	Stable

Now:

Controller Parameters

	Tuned
Kp	5.6536
Ki	7.0333
Kd	1.004
Tf	n/a

Performance and Robustness

	Tuned
Rise time	0.458 seconds
Settling time	5.23 seconds
Overshoot	0.894 %
Peak	1.01
Gain margin	Inf dB @ NaN rad/s
Phase margin	89.9 deg @ 4.27 rad/s
Closed-loop stability	Stable

Results became better.

4 Design a lag or lead compensator (if applicable), play with zero and pole to find optimal values (of overshoot, peak time, transient process time, stationary error, etc.) for transient process.

$$W(s) = \frac{s+4}{s^2+3s+15} \tag{16}$$

Lag or lead compensator looks like that:

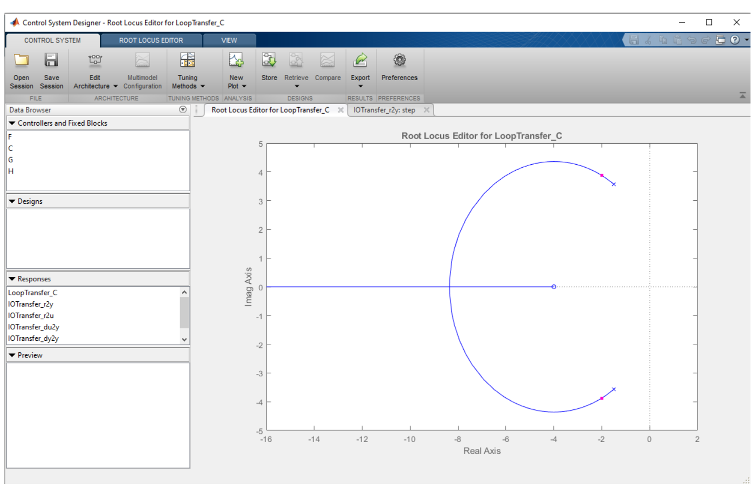
$$C(s) = K_c \frac{(s - z_0)}{(s - p_0)}$$

Figure 6: first-order lead compensator C(s)

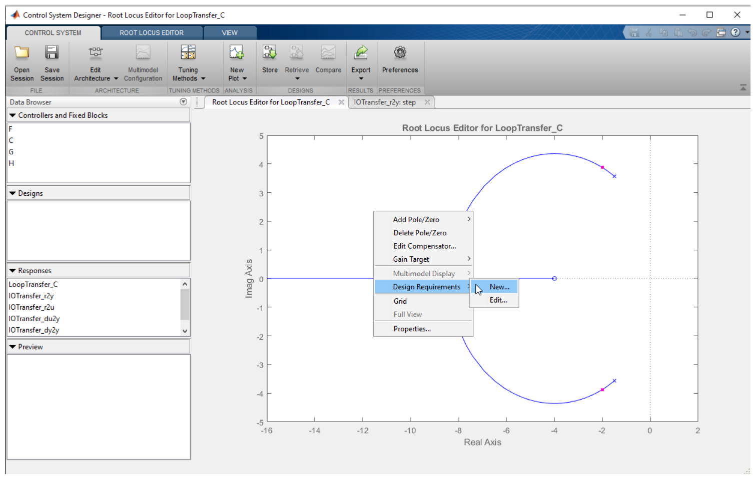
Let's run such program in matlab:

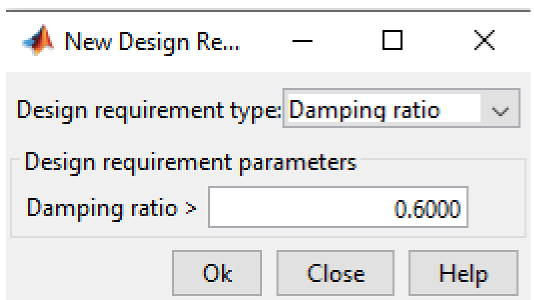
```
numerator = [1, 4];
denominator = [1,3,15];
sys = tf(numerator,denominator)
rltool(sys)
```

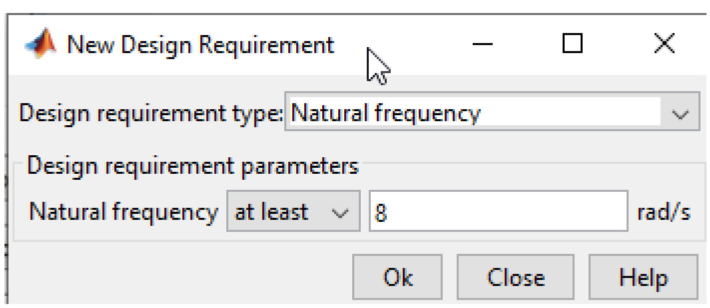
We can see, that Root Locus of our transfer function in Control System Designer.



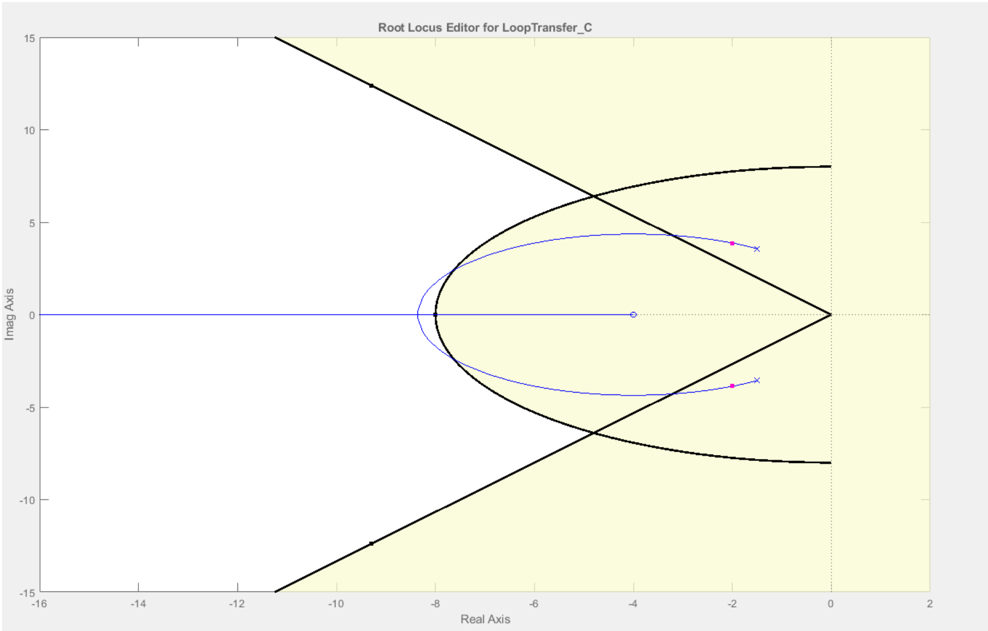
Let's add some design requirements



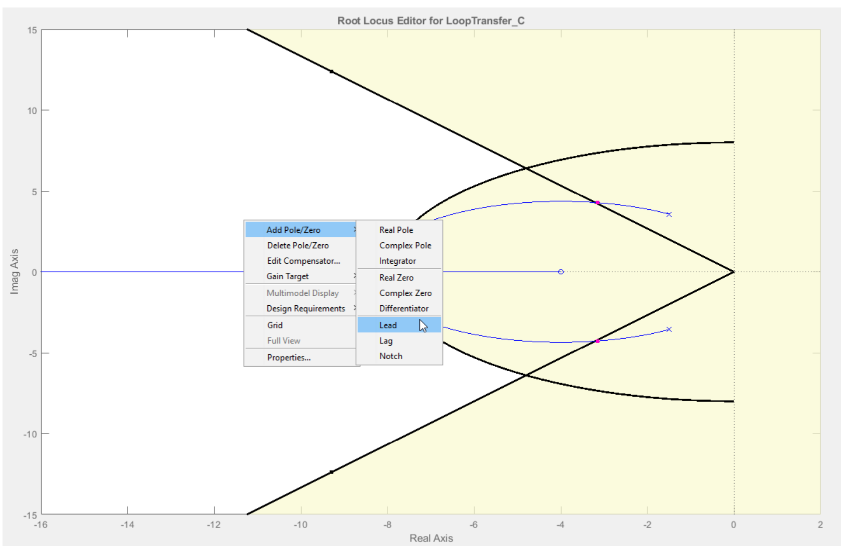




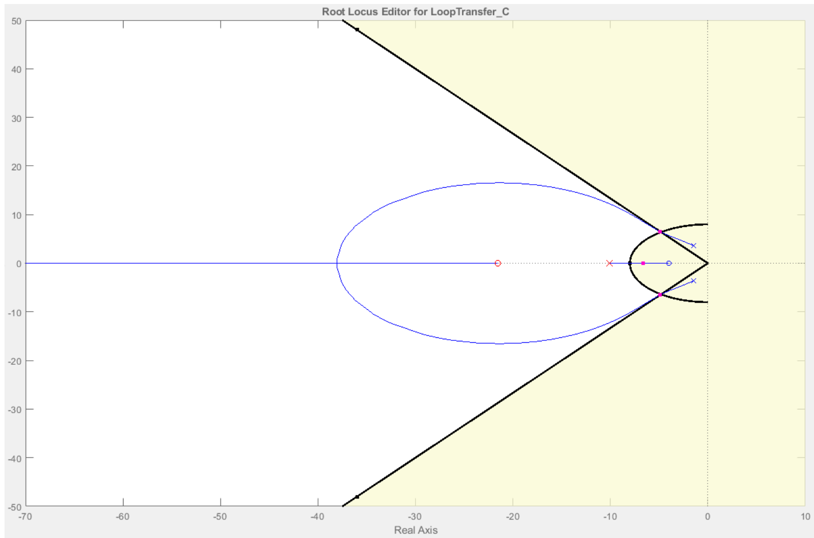
We have:



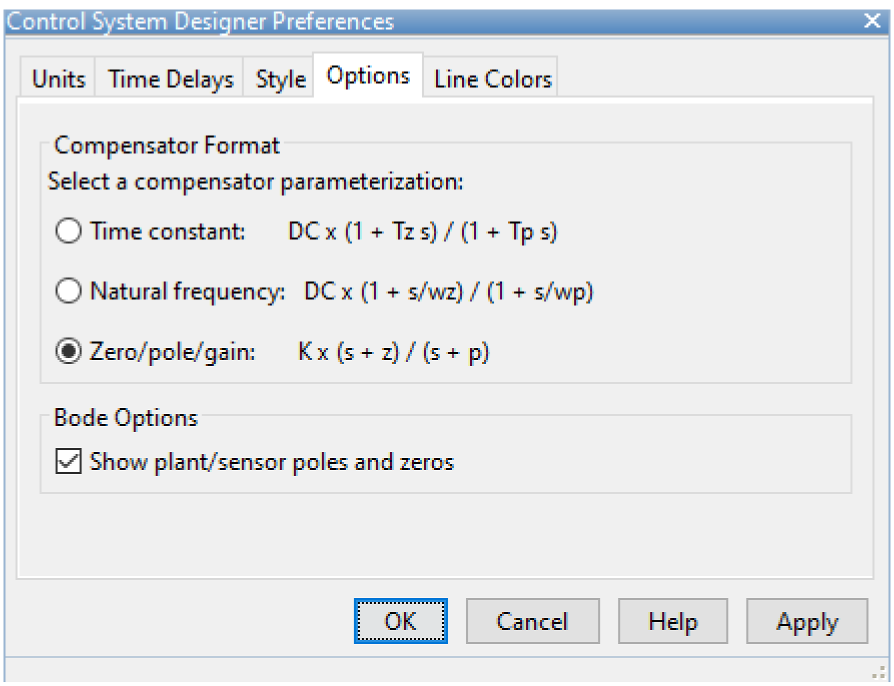
Our goal is to place red squares in intersection of damping ratio and natural frequency graph.
To do this let's add Lead



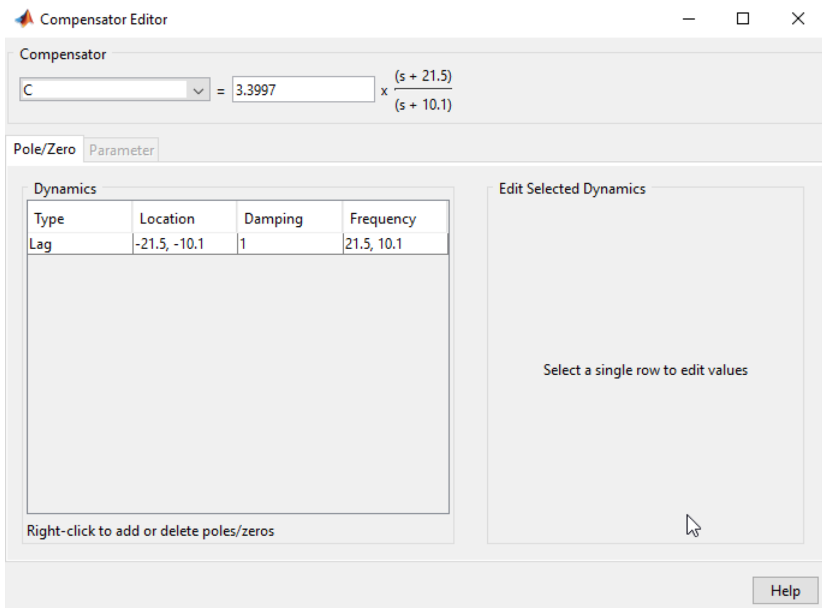
Now we move new zero point



In preferences let's change compensator format to Zero/Pole/Gain



Now in C controller we have our solution.


Compensator Editor
Compensator
C
=
3.3997
x
(s + 21.5)
(s + 10.1)
Pole/Zero
Parameter
Dynamics
Type
Location
Damping
Frequency
Lag
-21.5, -10.1
1
21.5, 10.1
Right-click to add or delete poles/zeros
Edit Selected Dynamics
Select a single row to edit values
Help