Control Theory homework 2

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BS18-02

1 Introduction

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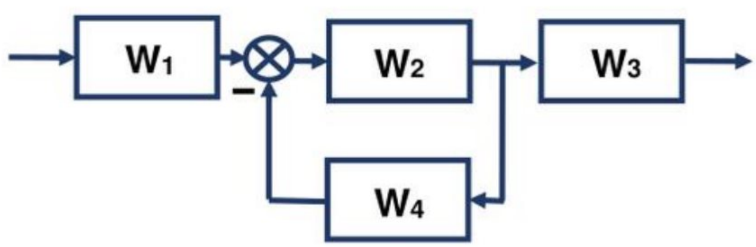
Link to my GitHub repository is here

2 Transfer function calculations

$$W_1 = \frac{2}{s+1}, W_2 = \frac{1}{s}, W_3 = \frac{2}{s+1.5}, W_4 = \frac{1}{s+3}$$



(A) Calculate the total Transfer Function of the system. To calculate this we need to know two properties:

1. Series connection

$$W(s) = W_1(s) * W_2(s) * ... * W_n(s)$$

2. Feedback loop

$$W(s) = \frac{W_1}{1 \pm W_1 W_2}$$

In our case total transfer function is

$$W(s) = W_1 * \frac{W_2}{1 + W_2 W_4} * W_3$$

$$W(s) = \frac{2}{s+1} * \frac{1}{s(1 + \frac{1}{s}\frac{1}{s+3})} * \frac{2}{s+1.5} = \frac{4s+12}{s^4 + 5.5s^3 + 10s^2 + 7s + 1.5}$$

(B) Build initial system shown in the block diagram and simplified in one Simulink schema and analyze its Step, Impulse and Frequency responses.

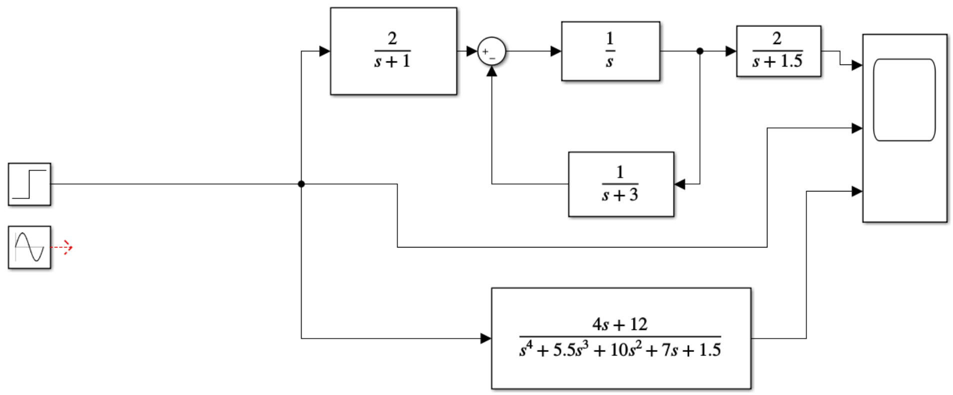


Figure 1: Simulink schema with both initial systems: system of transfer functions and total transfer function of this system + input signal itself.

On all three next pictures blue plot is input, yellow and red plots (same) are outputs of system of transfer functions and total transfer function

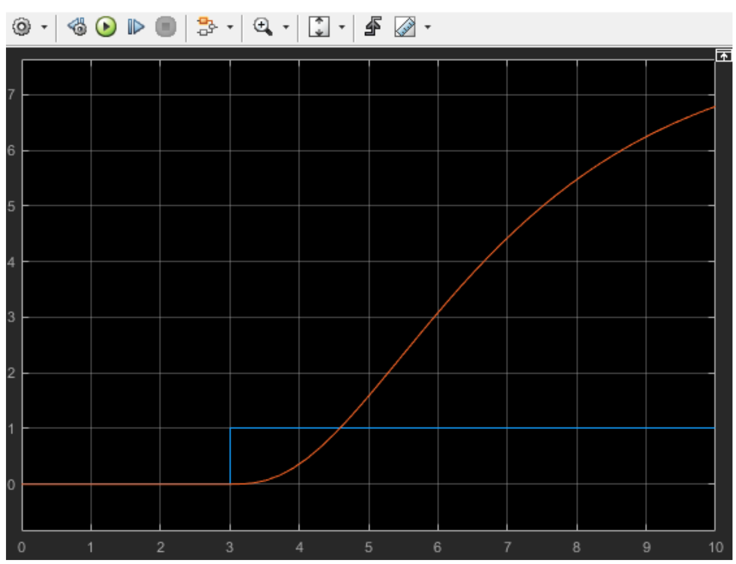


Figure 2: Plot's input is STEP block with Step time = 3. As we can see, all three plots starts from x = 3. If we have Step time = 1 plots would start from x = 1 and so on.

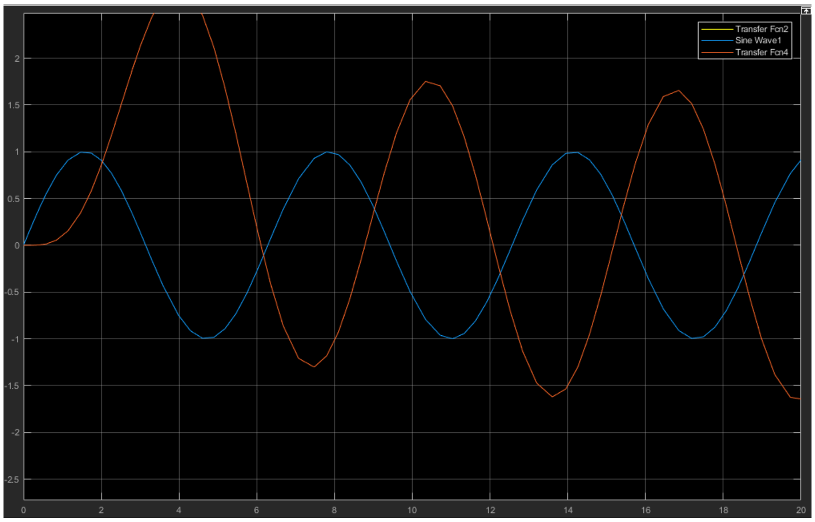


Figure 3: Plot's input is SINE WAVE1 block with frequency = 1. When the frequency of the input plot increases, the frequency of the output plot increases in direct proportion. At the same time, the amplitude of plot of the output function decreases.

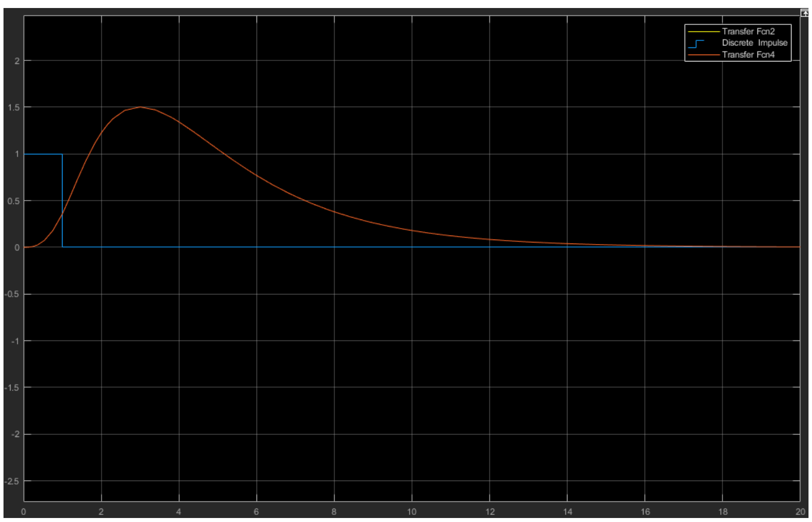


Figure 4: Plot's input are two STEP block, which are making impulse. Step time of first = 1, step time of second = 2. Amplitude of output plots increases as difference between two STEP block's step time increase

(C) generate a Bode and Pole-Zero map plots. Put plots and result - stable or unstable is system and why.

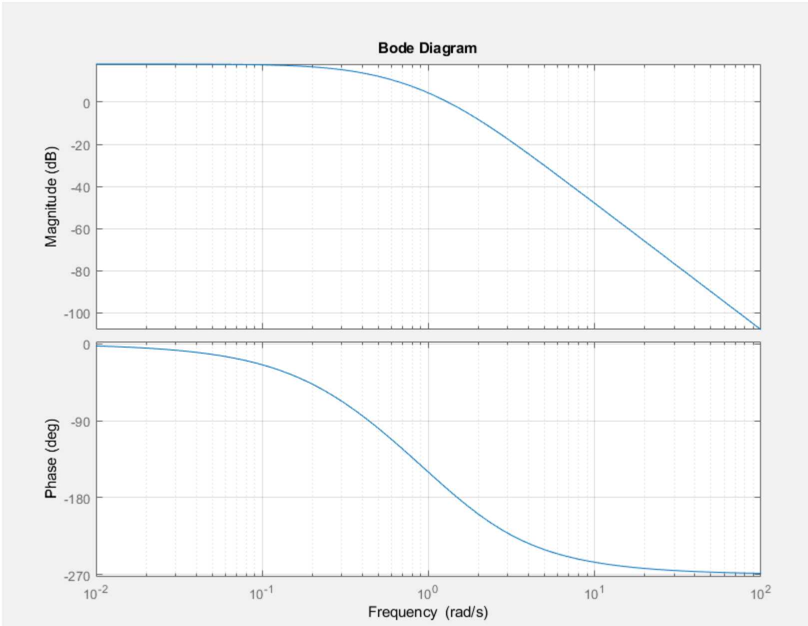


Figure 5: Bode plot for STEP block input

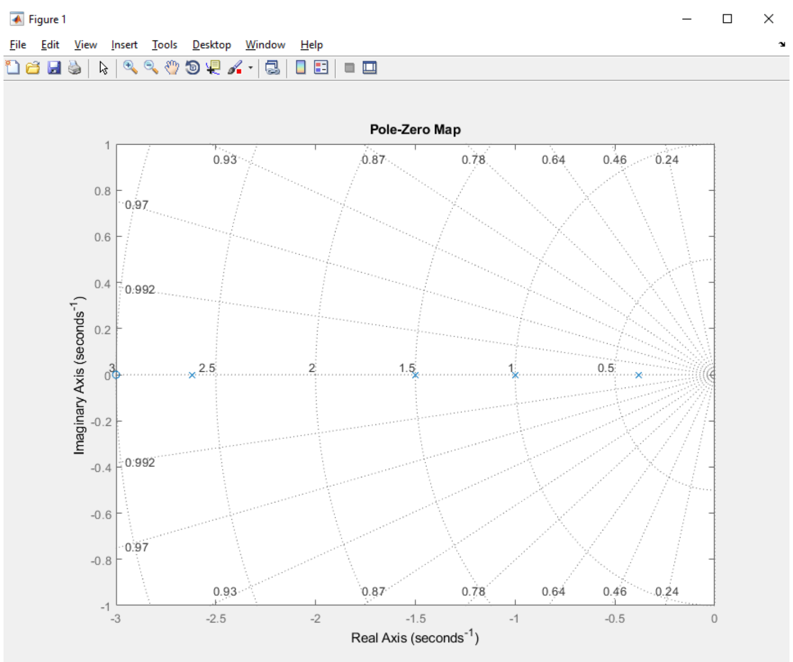


Figure 6: Pole-Zero plot for STEP block input

Let's find is a system stable or not.
For Stable System: Both the margins should be positive or phase margin should be greater than the gain margin.
For Marginal Stable System: Both the margins should be zero or phase margin should be equal to the gain margin.
Unstable System: If any of them is negative or phase margin should be less than the gain margin.
In our system
Gain Margin is 2.41dB (Y-axis value of magnitude plot at the X-axis point, there phase is equal 180)
Phase Margin is 9.54 deg (Y-axis value of phase plot at the X-axis point, there magnitude is equal 0)
System is **stable**

(D) Analyze Bode plot - calculate asymptotes and frequence breaks and put calculations in report

$$T(s) = \frac{4s+12}{s^4+5.5s^3+10s^2+7s+1.5} =$$
$$= \frac{2}{s+1} * \frac{s+3}{s^2+3s+1} * \frac{2}{s+1.5} =$$
$$= \frac{2}{s+1} * \frac{s+3}{s^2+3s+1} * \frac{2}{1.5(\frac{2}{3}s+1)} =$$
$$= \frac{12(\frac{s}{3}+1)}{1.5(s+1)(s^2+3s+1)(\frac{2}{3}s+1)}$$

Constant is 12/1.5 = 8

$$Zero : -3$$

$$Poles : -1, -1.5, \frac{3}{2} - \frac{sqrt(5)}{2}, \frac{sqrt(5)}{2} - \frac{3}{2}$$

3 Find total transfer function for a closed-loop system

$$W(s) = \frac{s-1}{s^2-s+1}, M(s) = \frac{s+1}{s+3}$$

We will use these formulas (X is total TF):

$$\phi(s) = \frac{X}{G} = \frac{W(s)}{1+W(s)}$$

$$\phi_f(s) = \frac{X}{F} = \frac{M(s)}{1+W(s)}$$

$$X = \phi(s)G + \phi_f(s)F = \frac{W(s)}{1+W(s)}G + \frac{M(s)}{1+W(s)}F$$

$$X = \frac{\frac{s-1}{s^2-s+1}}{1+\frac{s-1}{s^2-s+1}}G + \frac{\frac{s+1}{s+3}}{1+\frac{s-1}{s^2-s+1}}F$$

$$X = \frac{s-1}{s^2}G + \frac{1+s^3}{3s^2+s^3}F$$

4 Find transfer function of the system

A = $\begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix}$ B = $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ C = $\begin{pmatrix} 0 & 1 \end{pmatrix}$ D = $\begin{pmatrix} 3 \end{pmatrix}$
We will use next formula to find transfer function using state space model:

$$T(s) = \frac{Y(s)}{U(s)} = C(sI-A)^{-1}B + D$$

$$(sI-A)^{-1} = \begin{pmatrix} s+1 & -2 \\ 0 & s-1 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{s-1}{s^2-1} & \frac{2}{s^2-1} \\ 0 & \frac{s+1}{s^2-1} \end{pmatrix}$$

$$C(sI-A)^{-1}B = \begin{pmatrix} 0 & 1 \end{pmatrix} \frac{\begin{pmatrix} s-1 & 2 \\ 0 & s+1 \end{pmatrix}}{s^2-1} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{s+1}{s^2-1}$$

$$T(s) = \frac{s+1}{s^2-1} + 3 = \frac{3s^2+s-2}{s^2-1}$$

5 Find transfer function of the system

A = $\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$ B = $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ C = $\begin{pmatrix} 3 & 0 \end{pmatrix}$ D = $\begin{pmatrix} 0 & 3 \end{pmatrix}$
We will use next formula to find transfer function using state space model:

$$T(s) = \frac{Y(s)}{U(s)} = C(sI-A)^{-1}B + D$$

$$(sI-A)^{-1} = \begin{pmatrix} s-1 & 2 \\ -1 & s-1 \end{pmatrix}^{-1} = \frac{1}{s^2-2s+3} \begin{pmatrix} s-1 & 2 \\ -1 & s-1 \end{pmatrix}$$

$$C(sI-A)^{-1}B = \begin{pmatrix} 3 & 0 \end{pmatrix} \frac{1}{s^2-2s+3} \begin{pmatrix} s-1 & 2 \\ -1 & s-1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} = \frac{3}{s^2-2s+3} * \begin{pmatrix} s-3 & 2s \end{pmatrix}$$

$$T(s) = \frac{3}{s^2 - 2s + 3} * \begin{pmatrix} s-3 & 2s+3 \end{pmatrix} = \begin{pmatrix} \frac{3(s-3)}{s^2-2s+3} & \frac{3(2s+3)}{s^2-2s+3} \end{pmatrix}$$

6 Simplify the system step by step and calculate total transfer function

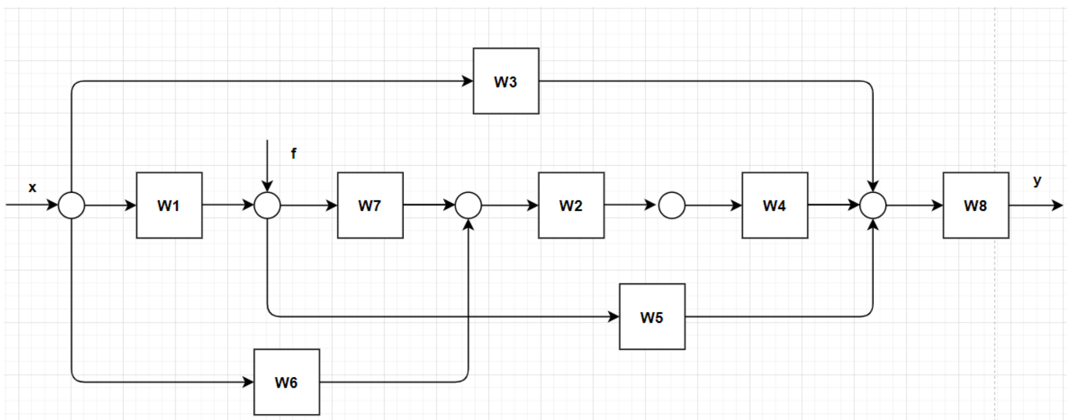


Figure 7: Initial system

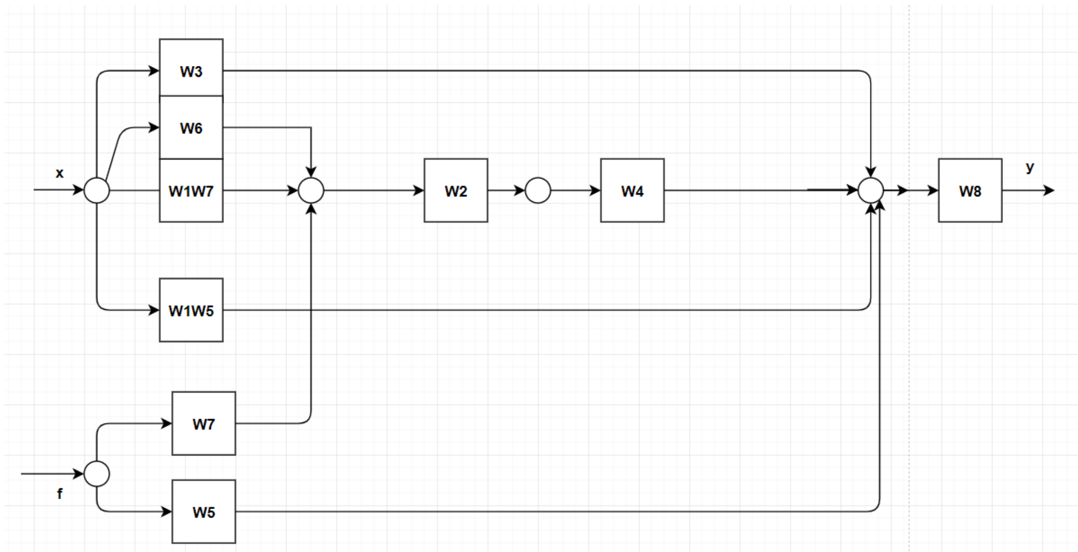


Figure 8: We move f and simplify W7 block

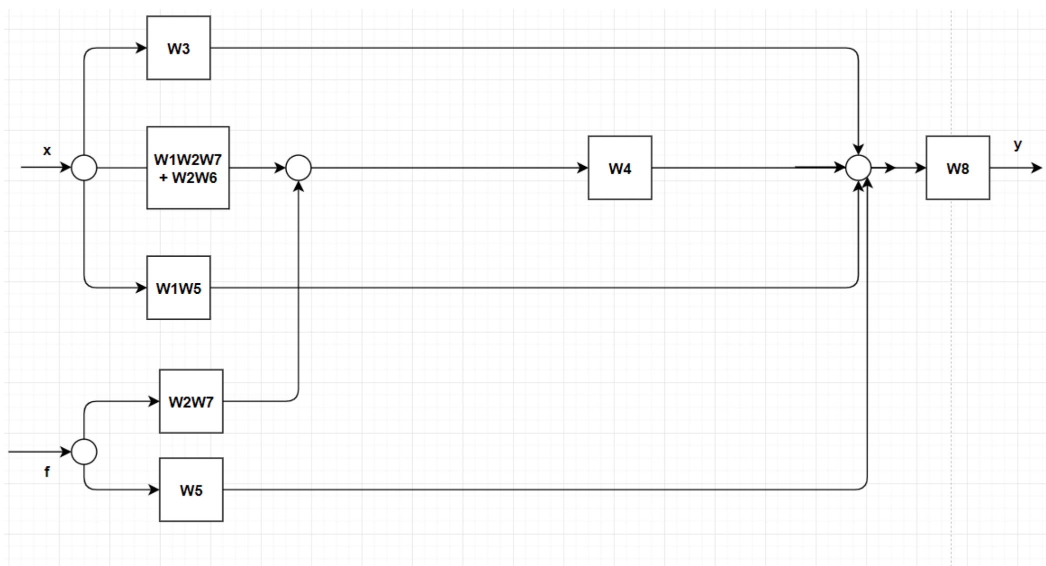


Figure 9: We add W6 to W1W7, because they are parallel. Then we simplify W2 block

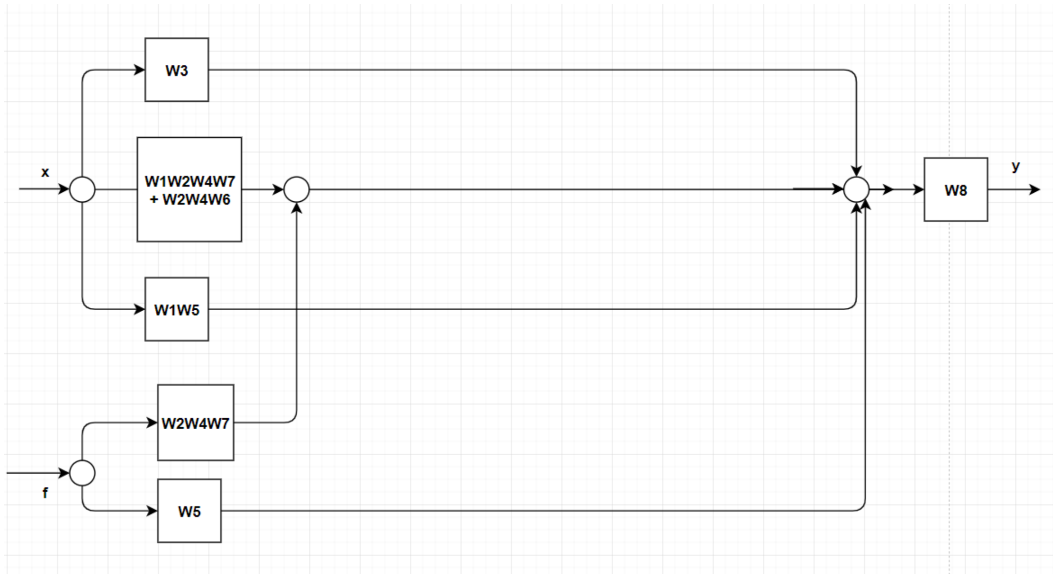


Figure 10: We simplify W4

Figure 11: We merge 3 parallel connections from x and 2 parallel connections from f

Figure 12: We multiply everything by W8. System simplifyed