Finance 537 | Financial Risk Management
Spring 2024 | Neil D. Pearson

Homework 4

Due Sunday, February 18, 2024 at 10:00 p.m. US central time

Total 10 points

Instructions. This is a group assignment. Groups may include up to 4 people. Please submit a Word or .pdf document with your solutions via the Canvas site prior to 10:00 p.m. on Sunday, February 18. You should also upload the computer codes and data files you use.

1. (2 points) Find Goldman Sachs' Annual Report for the year ended December 31, 2022. Page 104 of the Annual Report indicates that, "Net losses incurred on a single day for such [trading] positions exceeded our 95% one-day VaR (i.e., a VaR exception) on two occasions during 2022 and on one occasion during 2021."

(a) (1/2 point) If a VaR model is accurate, what is the expected number of times that the daily trading loss exceeds the VaR during a year? (Assume 252 trading days per year.)

(b) (1 point) Consider 252 independent Bernoulli trials (i.e., 252 trials of a binomial random variable) where on each trial the probability of a "success" is 0.95 and the probability of a "failure" is 0.05. What is the probability that the 252 trials result in two or fewer failures? What is the probability that 252 Bernoulli trials result in one or fewer failures? (You might want to look at your statistics book to refresh your memory about the binomial distribution and the distribution of Bernoulli trials.)

Hint. You might find the R functions `dbinom()` and/or `pbinom()` to be useful. Alternatively, you might find the Excel function `binom.dist()` to be helpful.

(c) (1/2 point) What does this analysis of Bernoulli trials tell you about whether Goldman's VaR model is or is not biased?

2. (1 point) You own a portfolio of CUB stock and options on CUB stock that has a current value of $10 million. The current price of CUB stock is $100 per share. Simple returns on CUB stock are described by the normal distribution with a mean of 0.5% per month and a standard deviation of 3.6538% per month. The figure below shows the value of your portfolio after one month. If you use the Monte Carlo method and a probability of 1%, what is the VaR of your portfolio?

Remark. The horizon of the VaR estimate is one month, so do not assume the expected return is zero. Do not do the simulation—rather, based on the figure, figure out what the outcome of the simulation would be.

1.a The expected number of days to exceed VaR @ 95% = 0.05 × 252

≈ 13 days

1.b $P(X \le 2)$; X is the number of failures in 252 days.

$$= \sum_{i=0}^{2} \underbrace{\binom{252}{i} \times (0.95)^{252-i} \times (0.05)^{i}}_{F(i)}$$

$\therefore F(0) = (0.95)^{252}$

$F(1) = 252 \times (0.95)^{251} (0.05)$

$$F(2) = \left(\frac{252 \times 251}{2}\right) \times (0.95)^{250} (0.05)^2$$

$$\therefore P(X \le 2) = F(0) + F(1) + F(2)$$

$$= 0.95^{252} + 252 \times 0.95^{251} (0.05) + \left(\frac{252 \times 251}{2}\right)(0.95)^{250} \times 0.05^2$$

$$P(X \le 1) = F(1) + F(0)$$

$$= 252 \times (0.95)^{251} \times 0.05 + 0.95^{252}$$

F(0)	2.43416E-06
F(1)	3.22846E-05
F(2)	0.000213248
P(X<=2)	0.000247967
P(X<=1)	3.47188E-05

1.c Yes, it is very biased as the analysis in 1.a tells us that it should breach the 5% VaR by 13 days whereas the model says it is 2 days.

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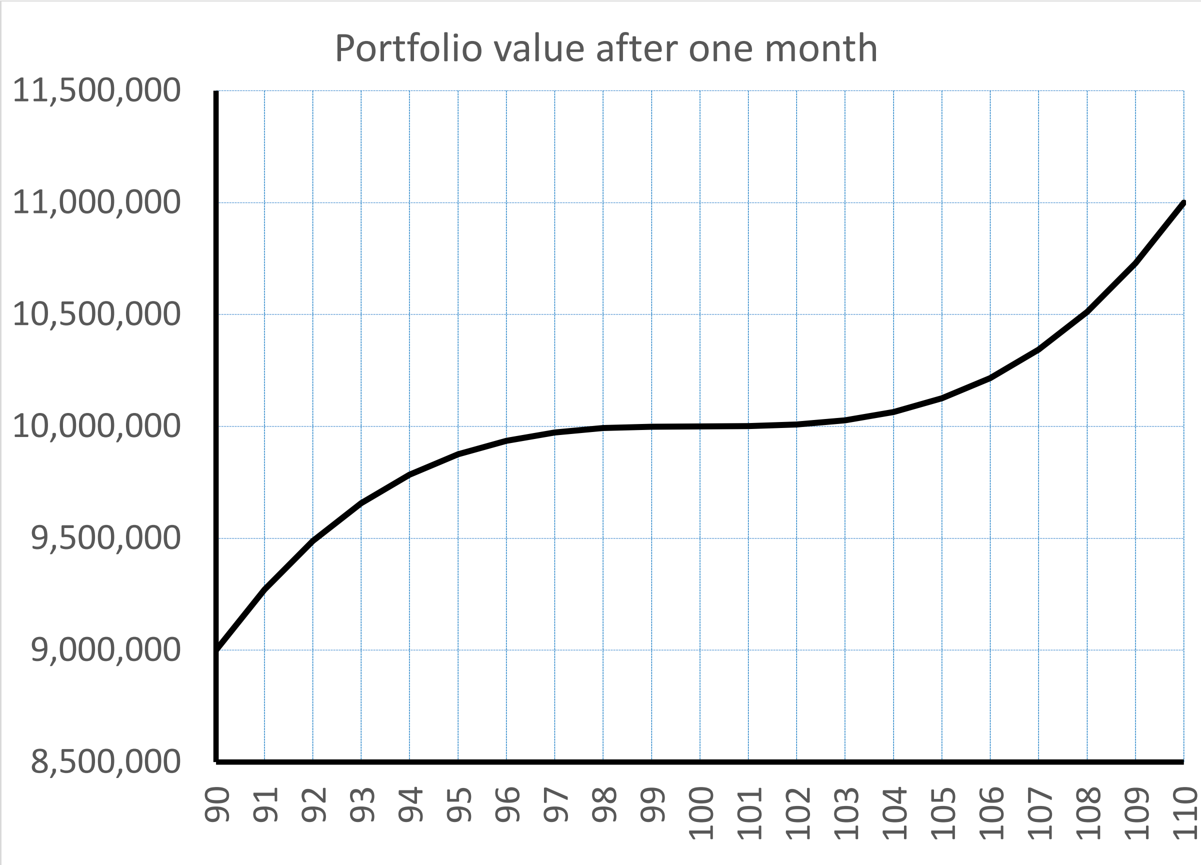
2. According to the graph, as the delta is always non negative for any possible S in the next 1 month, this means the VaR @ 1% happens at $S = 100(0.005 - 2.326 \times 0.036598 + 1) = 92.0013$.

So, from the graph, the VaR @ 1% should be 10M − 9.5M = 500,000 dollars.



3. (1 point) At the close of trading on Friday, December 16, 2022, the prices of XLE, XLF, and XLY are \$84.36, \$33.70, and \$133.90 per share. The implied volatilities of ATM options with 91 days to expiration are 0.317964, 0.227176, and 0.290478, and the continuously compounded 3-month interest rate is 0.0485 per year. XLE, XLF, and XLY will pay dividends of \$0.85963, \$0.20962, and \$0.36581, respectively, on December 22, 2022. The ex-dividend dates are Monday, December 19, 2022, the same for all three ETFs. Using these inputs, compute the values of a 91-day call option on XLE with a strike price of 84, a 91-day call option on XLF with a strike price of 34, and 91-day call option on XLY with a strike price of 134.

Remark. You might find the R function `am_call_bin_propdiv` in the R library `AmericanCallOpt` to be useful in computing the option prices. For this function, the dividend is entered as a fraction of the stock price. For the three ETFs, the fractions are \$0.85963/\$84.36, \$0.20962/\$33.70, and \$0.36581/\$133.90, respectively. The 91-day call options expire on March 17, 2023. In using the R function, compute the fraction of a year using trading days. December 16, 2022 to March 17, 2023 is 91 calendar days but only $13 \times 5 - 4 = 61$ trading days, due to the four holidays (Christmas, New Year, MLK, and President's Day). Also, the ex-dividend date of December 19 is one trading day after December 16. Use 1,000 timesteps for the binomial model.

4. (2 points) At the close of trading on December 16, 2022, you hold 30,000 shares of XLE, 60,000 shares of XLF, and 10,000 shares of XLY. In addition, you have written call options on 30,000 shares of XLE, 60,000 shares of XLF, and 10,000 shares of XLY. The call options all expire on March 17, 2022, that is, in 91 days. The strike prices of the options on XLE, XLF, and

3. See the code in "American_option_pricing_with_discrete_dividend.r"

The option price for XLE = $5.4360

" " XLF = $1.4393

" " XLV = $8.1891

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4. The 5% VaR is $98312.69

See "Historical_portfolio_VaR.R"

XLY are 84, 34, and 134, respectively. The continuously compounded interest rate is $r = 0.0485$. XLE, XLF, and XLY will pay dividends of \$0.85963, \$0.20962, and \$0.36581, respectively, on December 22, 2022. The ex-dividend dates are Monday, December 19, 2022, the same for all three ETFs.

Please use the simple historical simulation method to compute the VaR of this portfolio as of the close of trading on December 16, 2022. Use a probability of 5% and a holding period of one day. Use six market factors, these being the log returns on XLE, XLF, and XLY, and the log changes in the three ATM implied volatilities. Relevant data are in the three .csv files ETFprices.csv, ETFreturns.csv, and ETFATMvols.csv. For this question, use only 100 timesteps in the binomial model you use to compute the simulated options prices. Use the most recent 500 returns, including the return on December 16, in the historical simulation method.

5. (1 point) Use the most recent 500 daily log returns (including the return on December 16, 2022) for the three ETFs and most recent 500 daily log changes in the implied volatilities to compute the exponentially-weighted covariance matrix of the six market factors. Use $\lambda = 0.94$, and the expected log returns or log changes are zero when you compute the exponentially-weighted covariance matrix.

Remark. Compute the covariance matrix using the log returns and log changes, not the simple return and percentage changes. Note that while the question tells you to use 500 past returns, you do not actually need to use this many past returns. In thinking about how many past days should you use to compute the exponentially-weighted covariance matrix, you should think about how large is λ^{τ} for various values of τ.

6. (2 points) Consider the portfolio from Question 4. Please use the Monte Carlo method with holding period of one day, a probability of 5%, and the exponentially-weighted covariance matrix with $\lambda = 0.94$ from Question 5 to compute the Monte Carlo VaR of your portfolio as of the close of trading on December 16, 2022. Use 1,000 Monte Carlo trials to estimate the VaR.

Remark. You will make it easier for our TA to grade your homeworks if you set the seed equal to 137. Specifically, letting `mu` and `cov` be the mean vector and covariance matrix, generate the simulated log returns using the code

use $\mu = \vec{0}$

```
n <- 1000
set.seed(137)
simreturns <- mvrnorm(n, mu = mu, Sigma = cov)
```

5.

(See the way I created "returns.csv" at "save_returns.R")

```python
import pandas as pd
import numpy as np
```

```python
lambd = 0.94

assets = ['log_ret_xle', 'log_ret_xlf', 'log_ret_xly', 'log_vol_xle', 'log_vol_xlf', 'log_vol_xly']

df = pd.read_csv("returns.csv")[["Date"] + assets]
df['lambda weight'] = lambd ** np.arange(499, -1,-1)
df
```

	Date	log_ret_xle	log_ret_xlf	log_ret_xly	log_vol_xle	log_vol_xlf	log_vol_xly	lambda weight
0	20201223	0.021897	0.016391	0.002161	0.002757	-0.010368	0.000890	3.897615e-14
1	20201224	-0.005233	0.001383	0.002093	-0.018987	-0.051631	-0.027031	4.146399e-14
2	20201228	-0.006316	0.004824	0.011336	-0.011786	0.005380	-0.027269	4.411063e-14
3	20201229	-0.006622	-0.003444	0.000313	0.014387	0.004262	-0.000004	4.692620e-14
4	20201230	0.015557	0.004474	0.005992	0.004795	0.002240	-0.002879	4.992149e-14
...	...	...	...	...	...	...	...	...
495	20221212	0.025907	0.013234	0.002507	-0.020828	-0.013596	0.005684	7.807490e-01
496	20221213	0.018797	0.002569	0.000000	-0.048670	-0.037121	-0.026788	8.305840e-01
497	20221214	-0.006149	-0.012622	-0.007468	-0.002657	0.010016	-0.001623	8.836000e-01
498	20221215	-0.006069	-0.019825	-0.016571	0.005423	0.030087	0.034668	9.400000e-01
499	20221216	-0.012370	-0.007685	-0.018864	-0.075247	-0.047113	-0.028799	1.000000e+00

500 rows × 8 columns

```python
cov = []

for i in assets:
    for j in assets:
        cov.append((df[i] * df[j] * df['lambda weight']).sum() * (1-lambd))
cov = pd.DataFrame(np.reshape(cov, (len(assets), len(assets))), index=assets, columns=assets)
cov
```

	log_ret_xle	log_ret_xlf	log_ret_xly	log_vol_xle	log_vol_xlf	log_vol_xly
log_ret_xle	0.000332	0.000156	0.000176	-0.000314	-0.000261	-0.000209
log_ret_xlf	0.000156	0.000178	0.000187	-0.000170	-0.000258	-0.000221
log_ret_xly	0.000176	0.000187	0.000289	-0.000155	-0.000263	-0.000294
log_vol_xle	-0.000314	-0.000170	-0.000155	0.000996	0.000750	0.000597
log_vol_xlf	-0.000261	-0.000258	-0.000263	0.000750	0.000863	0.000687
log_vol_xly	-0.000209	-0.000221	-0.000294	0.000597	0.000687	0.000715

6. See "Monte_Carlo_Portfolio_VaR.R" for more details.

The 5% VaR = $95,390.55
